1. From the Example 1.1.2(One Sample Problem) (i) of the Lecture note 1, page 4, briefly explain why it may not be identifiable without the condition

$$\int x dG(x) dx = 0.$$

What happens if there is another condition below?

$$\inf\{x : G(x) \geq 1/2\} = 0$$

$G_1(x-\mu_1) = G_2(x-\mu_2)$, let $t = x-\mu_1$

$G_1(t) = G_2(t+\mu_1-\mu_2)$

$\Rightarrow \inf\{x ; G_1(x) \geq \frac{1}{2}\} = \inf\{x ; G_2(x+\mu_1-\mu_2) \geq \frac{1}{2}\}$

by condition

$\Rightarrow 0 = -\mu_1 + \mu_2$.

$\Rightarrow \mu_1 = \mu_2$

2. Let $X_1, \ldots, X_n$ be a random sample from Cauchy$(\theta; 1)$ with pdf

$$f(x;\theta) = \frac{1}{\pi(1+(x-\theta)^2)}.$$

Show that the order statistics $(X_{(1)}, \cdots, X_{(n)})$ is a minimal sufficient statistics.

$$\prod_{i=1}^{n} f(x_i;\theta) = \left(\frac{1}{\pi}\right)^n \cdot \prod_{i=1}^{n} \frac{1}{1+(x_i-\theta)^2} = \frac{1}{\pi^n} \cdot \prod_{i=1}^{n} \frac{1}{1+(x_{(i)}-\theta)^2}$$

$$= h(x_{(1)}, \cdots, x_{(n)}) \cdot g_\theta(x_{(1)}, \cdots, x_{(n)})$$

$\therefore$ $(X_{(1)}, \cdots, X_{(n)})$ is sufficient statistics.

$\exists \theta_0 \in \mathbb{R}$; s.t. $\operatorname{supp}(P_\theta) \subset \operatorname{supp}(P_{\theta_0}) = \mathbb{R}$

T; $\dfrac{P_\theta(x)}{P_{\theta_0}(x)}$ is m.s.s. for θ.

$$\therefore T = \frac{\prod_{i=1}^{n}(1+(x_i-\theta_0)^2)}{\prod_{i=1}^{n}(1+(x_i-\theta)^2)} \quad \text{is m.s.s. for } \theta.$$

$$\prod_{i=1}^{n}\left(\frac{1+(x_i-\theta_0)^2}{1+(x_i-\theta)^2}\right) = \prod_{i=1}^{n}\left(\frac{1+(x_{(i)}-\theta_0)^2}{1+(x_{(i)}-\theta)^2}\right)$$

$\Rightarrow T$ is func of $(X_{(1)}, \cdots, X_{(n)})$

$\therefore$ $(X_{(1)}, \cdots, X_{(n)})$ is m.s.s.