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Faculty of Mathematics

UV Relax: Reducing the Distortion of UVs in Houdini

Side Effects Software, Inc.

Remote

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Dear Mr. Moser,

This report, entitled "UV Relax: Reducing the Distortion of UVs in Houdini" was prepared as my 3A Work Report for Side Effects Software, Inc. This is my third work term report. The purpose of this report is to compare two potential approaches for the new UV Relax tool, which reduces the distortion of UVs. The report determines what use cases each approach is most suited for.

The Research and Development team, in which I was employed, develops new tools and features for Houdini. Houdini is a 3D software for modeling, animation and special effects and is most commonly used in films and video games. As a 3D Software Developer, I researched and implemented various prototypes for the UV Relax tool.

This report was written entirely by me and has not received any previous academic credit at this or any other institution. I received no assistance.

Sincerely,

Sophia Pietsch
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Executive Summary

In Houdini, users use two-dimensional representations of a surface in three dimensions, called UVs, to map textures onto three-dimensional models. The creation of textures becomes easier as the amounts of distortion of a UV decreases. Existing tools in Houdini create UVs with minimal amounts of angular distortion. The UV Relax tool proposed here reduces the amount of area distortion and gives users control over the relative size of areas in the UVs.

Using Least Squares Conformal Mapping (LSCM) and discrete conformal equivalence for UV Relax ensures there is no significant increase in angular distortion. This prototype successfully reduced the amount of area distortion, but did not always correctly change the size of areas in response to user input. Adding a normalization step improves the response to user input but significantly weakens the default behaviour of the prototype.

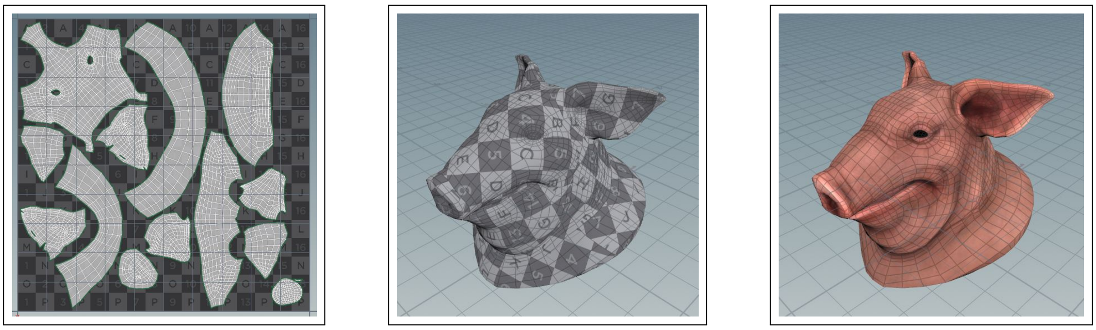
Using a harmonic map for the UV Relax tool allows the interior of the UV to fluctuate more while fixing the boundary points. The fixed boundary prevents the prototype from significantly reducing the amount of area distortion. However, this approach allowed areas to change size in response to user input. The addition of a frame surrounding the UV would allow some movement of the boundary and potentially improve the default behaviour.

The report suggests using LSCM to reduce the amount of area distortion and a harmonic map to change the size of areas in response to user input. Alternatively, UV Relax could let the user decide which approach is more suitable for their use case.

1.0 Introduction

Houdini is a 3D software for modeling, animation and special effects and is most commonly used in films and video games. Users create and texture three-dimensional models, which are commonly represented by polygon meshes approximating a smooth surface. Often, the textures have a much finer resolution than the mesh, so the texture needs to be mapped onto the surface of the model.

For texture mapping, Houdini creates a two-dimensional representation of the surface of the model, called a UV. Textures are applied to the UV, each part of which corresponds to a certain part of the model. Using this correspondence, textures are transferred to the surface of the three-dimensional model (see Figure 1).



(a) The UVs of a model (b) UVs map to a model (c) Textures map to a model

Figure 1: The UVs and texture mappings of the Pig Head test geometry in Houdini.

Often, some surface areas have to be distorted to create a two-dimensional representation of a surface in three dimensions. Area distortions make texturing a model more difficult, since areas of the same size in the UV correspond to areas of different size on the 3D surface. Similarly, angular distortions cause a straight line on the three-dimensional surface to bend in the UV. Thus, UVs are most useful when they have minimal distortion.

In Houdini, users can use the UV Flatten tool to create a UV for a model. The tool minimizes angular distortion, ensuring that angles between the edges around a vertex in the UV are proportional to corresponding angles on the model's surface. However, the resulting UVs could still have significant amounts of area distortion. A UV Relax tool that allows users to modify the relative size of areas of a UV to reduce the amount of area distortion could mitigate this issue.

The goal of this report is to compare several implementations of a UV Relax tool. First, the report briefly analyzes the requirements for such a tool. Then, the report compares the use of discrete conformal equivalence and Least Squares Conformal Mapping with the use of harmonic maps.

2.0 Analysis

2.1 Requirements for UV Relax

The aim of the UV Relax tool is to make UVs easier to work with by modifying the relative size of areas in the UVs. By default, the tool should reduce the amount of area distortion over the whole UV. However, users may care more about distortion in areas with many details than in other areas in the model. For this case, the tool should provide an attribute for specifying what areas users want contracted or expanded. The tool should automatically move other areas of the UV to accommodate the change.

The UV Relax tool should focus on improving UVs created by other tools in Houdini. Thus, the implementation may assume that the input UVs have small amounts of angular distortion and focus on minimizing the amount of area distortion. The modified UVs should continue to have small amounts of angular distortion, as a high amount of angular distortion would make texture mapping equally difficult for users.

2.2 Approaches and Evaluation

For one model, there can be multiple different UVs, each with minimal angular distortion but different amounts of area distortion. Thus, the report will first consider changing the UV such that the angular distortion remains close to minimal and the amount of area distortion is reduced. The approach uses the concept of discrete conformal equivalence and a flattening algorithm called Least Squares Conformal Mapping.

Next, the report considers using a harmonic map to manipulate the input UV. Harmonic maps do not minimize angular distortion, but still ensure that the angular distortion remains small. Thus, using a harmonic map allows for more flexible results than the first approach.

To evaluate these approaches, the report compares their performance on two test models. The input UVs are created from the models using the UV Flatten tool available in Houdini. The default performance as well as the responsiveness to user-provided attributes is considered.

The first model is a sphere with one cap removed. Since the UV has a large amount of area distortion, this model evaluates how well the UV Relax tool can reduce it. The UV and model, both displaying the area distortion, are shown in Figure 2.

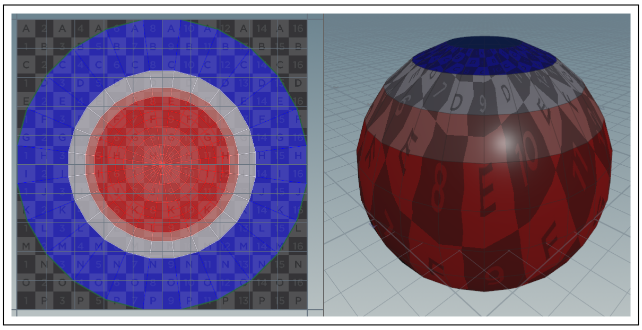
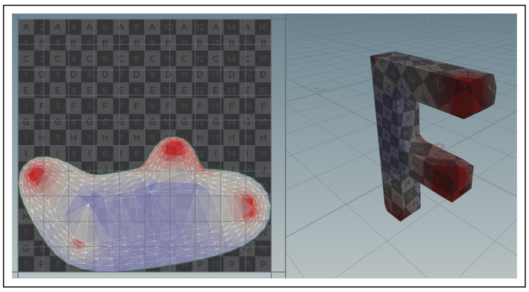
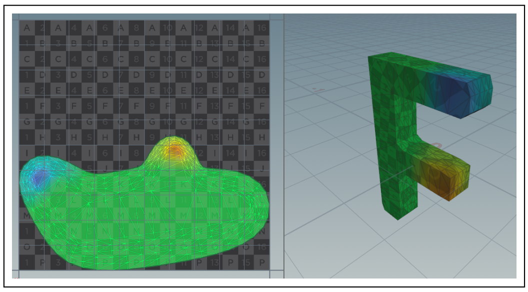


Figure 2: The first test model (right) and its UV (left), as created by the UV Flatten tool. Red areas are proportionally smaller and blue areas proportionally larger in the UV than in the model.

The second model is a representation of the letter 'F'. This model is more complex and evaluates how well the UV Relax tool responds to the user-specified attribute. The UV and model are displayed in Figure 3, once showing the area distortion and once showing the attribute values used.



(a) The distortion of the test model. Red areas are proportionally smaller and blue areas proportionally larger in the UV than in the model.



(b) The attribute values used for the test model. Orange areas should be expanded, while blue areas should be scaled down.

Figure 3: The second test model and its UV, as created by the UV Flatten tool.

2.3 Conformal equivalence and Least Squares Conformal Mapping

The first prototype of UV Relax determines what areas of the triangle mesh have high area distortion in the input UV and uses a flattening algorithm called Least Squares Conformal Mapping (LSCM) to create a similar UV. While LSCM usually takes the three-dimensional surface as an input, UV Relax modifies some aspects of the input UV to reduce the area distortion and uses LSCM to ensure the result is a two-dimensional surface. The modification of the input UV uses discrete conformal equivalence to ensure that there is no significant increase in angular distortion with respect to the original model.

2.3.1 Discrete Conformal Equivalence

The notion of discrete conformal equivalence is formally defined by Springborn et al. in "Conformal Equivalence of Triangle Meshes". Informally, if two meshes are discretely conformally equivalent then they have the same geometry and each has low angular distortion with respect to the other. Given the edge lengths l_{ij} of a mesh, the paper provides a way to construct a discretely conformally equivalent mesh. The edge length of edge e_{ij} in the conformally equivalent mesh is $e^{(u_i+u_j)/2}l_{ij}$, where the u_i's are coefficients assigned to the vertices (Springborn et al., 2008).

The UV Relax tool constructs the coefficients u_i based on the area distortion and the user-specified attribute and uses the new edge lengths to construct the input to LSCM. Vertices whose surrounding area is stretched in the input UV are assigned a coefficient less than one, resulting in a reduction of the surrounding edge lengths. Similarly, vertices whose surrounding area is contracted are assigned a coefficient greater than one. If an attribute is specified by the user, then the values for each vertex are added to the coefficients calculated above. Thus, a positive attribute value will increase the size of an area and a negative value will decrease the size of an area.

2.3.2 Least Squares Conformal Mapping

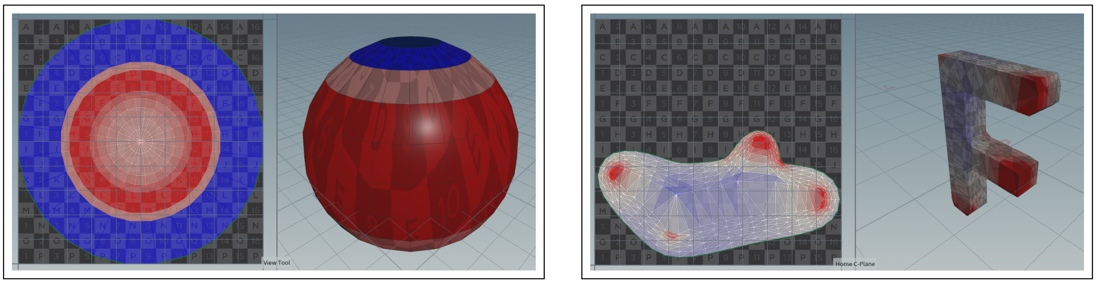
LSCM is a linear flattening algorithm that uses information about a triangle mesh to create a UV with almost no angular distortion. Unlike other methods of creating UVs, the borders of the resulting UVs can have an arbitrary shape and no triangles can flip upside down. The two UV coordinates are directly linked, such that the resulting UVs are fairly rigid (Lévy et al., 2002).

The UV Relax tool splits the input mesh into triangles so that the LSCM algorithm can be used. Then, the tool uses the edge lengths calculated using discrete conformal equivalence to calculate the angles and areas of the modified triangles. This information is passed to LSCM, which creates the final UV.

2.3.3 Benefits and Drawbacks

The first prototype succeeded in distributing the area distortion across the first test model (see Figure 4(a)). The model was chosen to have a default UV with very large amount of distortion, so some remaining distortion is expected.

The default behaviour also modified the UV of the second test model, with the resulting UV having slightly straighter edges and a slightly smaller amount of distortion in some areas (see Figure 4(b)). The model contains many small triangles oriented in different ways, which limits the amount that LSCM can move the vertices since LSCM does not allow any angular distortion.



(a) Results for the first test model (b) Results for the second test model

Figure 4: Default behaviour of the first UV Relax prototype on the test models.

Unfortunately, the UV Relax prototype did not respond well when supplied with an attribute, with the area that was marked for expansion shrinking instead (see Figure 5). The high attribute values may cause the computed angles to increase, so that the sum of angles incident to boundary vertices exceeds 180 degrees. Since the sum was previously less than 180 degrees, the boundary would collapse inward to get closer to the computed sum. This may be what leads to the observed contraction of the area.

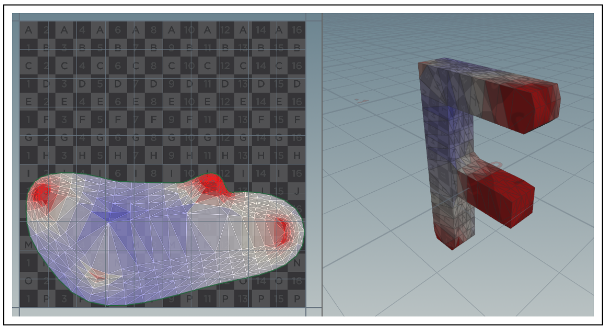
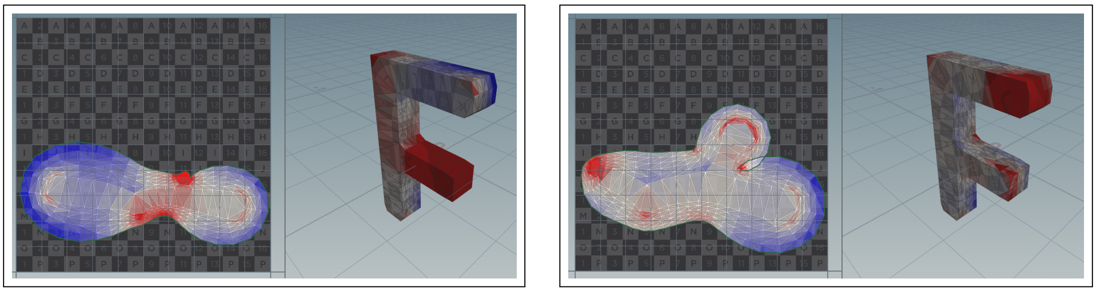


Figure 5: The behaviour of the first UV Relax prototype when passed the attribute from Figure 3(b)

To test this hypothesis, the prototype is modified to normalize the angles around each vertex. While the default behaviour is significantly worse using this modification, the prototype is a lot more responsive to the attribute. Figure 6(b) shows that the expansion and contractions works very well in this case. Thus, further development of this modification may lead to very promising results.



(a) The default behaviour

(b) The behaviour when passed the attribute from Figure 3(b)

Figure 6: The behaviour of the first UV Relax prototype when modified to normalize angles.

2.4 Harmonic Maps

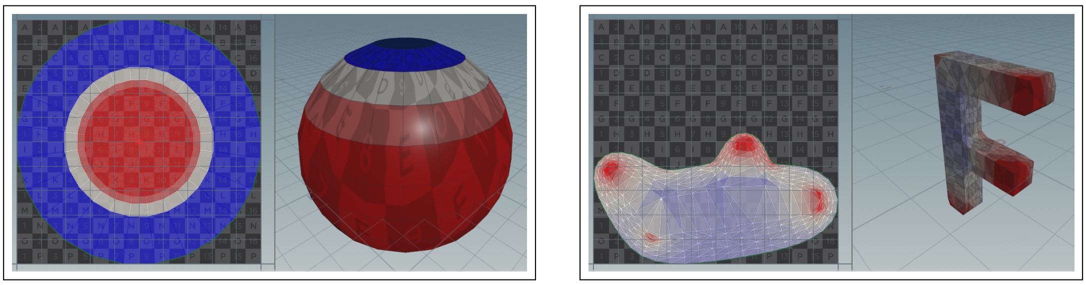
The second prototype of UV Relax uses a harmonic map with adjusted weights to modify the input UVs and minimize the area distortion. The tool adjusts the weights of each vertex based on the size of the area around it on the three-dimensional surface. The weights used are further modified when the user specifies what areas they want contracted or expanded.

Harmonic maps are a linear method of creating UVs that requires the boundary points of the UV to be fixed. They are more flexible than LSCM since the u and v coordinates of vertices can vary more independently (Springborn et al., 2008). While the resulting UV can have some angular distortion, harmonic maps are guaranteed to result in only small amounts of angular distortion (Floater & Hormann, 2005).

The UV Relax prototype initializes the weights of the harmonic map using the input UV. To adjust the sizes of UV areas to match more closely those of the model, the UV Relax tool multiplies the matrix used for the linear system of equations by a diagonal matrix. This matrix is called a lump mass matrix and the entries correspond to the areas around vertices (Patané & Spagnuolo, 2013). Here, the areas of the three-dimensional surface are used. If a user supplies an attribute, the attribute value for each vertex is added to the corresponding weight in the lump mass matrix.

2.4.1 Benefits and Drawbacks

The default behaviour of the second UV Relax prototype resulted in only small improvements of the area distortion for both models (see Figure 7). For the second model, this can be in part attributed to the fixed boundary, which prevents expansion of the contracted areas. Improvements, in particular in the case of the first model, may be achievable with fine-tuning of the weights in the lump mass matrix.



(a) Results for the first test model

(b) Results for the second test model

Figure 7: Default behaviour of the second UV Relax prototype on the test models.

When passed the attribute, the expansion and contraction of the corresponding area is noticable in the resulting UV (see Figure 8). However, the changes only affect the vertices in the immediate vicinity of the areas where a nonzero attribute was passed in. This behaviour combined with the fixed boundary makes this prototype less viable for UVs where significant expansion or contraction of areas close to a boundary is desired.

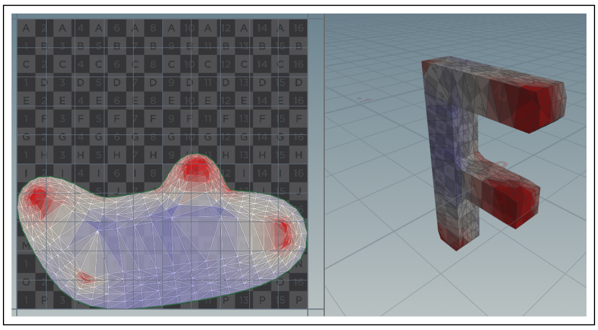


Figure 8: The behaviour of the second UV Relax prototype when passed the attribute from Figure 3(b)

The limitations of this prototype could be mitigated by creating and fixing a frame outside of the input UV's boundary, allowing limited movement of the actual boundary points. However, movement would still be constrained by the frame, making the prototype unsuitable should users desire to expand the boundary without limitations.

3.0 Conclusion

The UV Relax tool makes UVs easier to work with by reducing the distortion of the UV with respect to the three-dimensional model. For additional flexibility, users should be able to influence the results by specifying what areas they want expanded or contracted.

Using LSCM for the UV Relax tool ensures that the angular distortion of the UV does not increase. As the boundary of the UV is not fixed, this approach is well-suited to expanding or contracting areas close to the boundary. However, the rigidity of LSCM prevents the shape of interior areas from changing drastically, limiting the amount distortion can be reduced and the responsiveness to user attributes. Further adjustments, such as the normalization of angles, may improve the response to user-specified attributes.

In contrast, the use of harmonic maps for the UV Relax tool allows UVs to change flexibly in small areas as specified by an attribute. However, the prototype does reduce the overall distortion well since the boundary of the UV remains fixed. The addition of a fixed frame allows limited movement for the boundary and may improve the default behaviour of the prototype.

The two approaches could be combined by using LSCM to distribute the area distortion and the harmonic map only to influence the size of areas using the user-specified attribute. Alternatively, both algorithms could be available, so the user could choose which approach is better suited for their use case.

References

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