Your coursework 1

- There are three procedures to be investigated.
- For each of the procedures, you have to prove or disprove that the prcedure satisfies a certain specification.
- You have to implement the procedures, and check them on the graph given by a matrix $\boldsymbol{W}$.
- Your matrix $\boldsymbol{W}$ is determined below.

1. Dijkstra's procedure 1

```
int n; // nodes are numbered with 1,..., n
int j0;// j0 is a fixed node
m = 1;
for each edge (1,v)
  l[v]=w(1,v) // fill up details ..
while (m < n) {
  m = m+1;
  for k = 1 to n
  l[j] = min(l[j], l[k]+ w(k,j));};
return (l[j0]);
```

(a) With loop invariants, give a full proof that, given a graph with n nodes, the procedure computes correctly **the minimal weight** of a path from 1 to $j0$.

(b) In accordance with your matrix W, draw your graph G with 6 nodes.

(c) Print out your code (in your beloved PL) to implement the above procedure.

(d) For a path from 1 to 5, print out the result returned by your code. Is it really minimal?

2. The maximum: Procedure 2

```
int n; // nodes are numbered with 1,..., n
int j0;// j0 is a fixed node
m = 1;
for each edge (1,v)
  l[v]=w(1,v) // fill up details ..
while (m < n) {
  m = m+1;
  for k = 1 to n
  l[j] = max(l[j], l[k]+ w(k,j));};
return (l[j0]);
```

(a) In accordance with your matrix W, draw your graph G with 6 nodes.

(b) Print out your code (in your beloved PL) to implement the above procedure.

(c) For a path from 1 to 5, print out the result returned by your code. Is it really maximal?

(d) Give a full proof that, given a graph with n nodes, the procedure computes correctly **the maximal weight** of a path from 1 to $j0$, or find a **counter-example**.

3. Procedure 3

```
int n; // nodes are numbered with 1,..., n
int j0;// j0 is a fixed node
k = 0;
for each edge (u,v)
  d[u][v] = w(u,v) // the weight of (u,v)
while (k < n) {
  k = k+1;
  for i and j from 1 to n
    d[i][j] =
      min(d[i][j], d[i][k] + d[k][j]);};
return(d[1][j0]);
```

(a) With loop invariants, give a full proof that, given a graph with n nodes, the procedure computes correctly **the minimal weight** of a path from 1 to $j0$.

(b) In accordance with your matrix W, draw your graph G with 6 nodes.

(c) Print out your code (in your beloved PL) to implement the above procedure.

(d) For a path from 1 to 5, print out the result returned by your code. Is it really minimal?

How to choose your own graph

Let

(a) $a =$ the number of vowels in your family name $\pmod{2}$,

(b) $b =$ the number of consonants in your family nar $\pmod{2}$,

(c) $c =$ the number of vowels in your given name $\pmod{2}$,

(1) For $\boldsymbol{abc} = 000$,

$$\boldsymbol{W} = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & - & \mathbf{1} & \mathbf{1} & - & - & - \\ \mathbf{2} & \mathbf{1} & - & - & \mathbf{2} & \mathbf{1} & - \\ \mathbf{3} & \mathbf{1} & - & - & \mathbf{4} & \mathbf{1} & - \\ \mathbf{4} & - & \mathbf{2} & \mathbf{4} & - & - & \mathbf{1} \\ \mathbf{5} & - & \mathbf{1} & \mathbf{1} & - & - & \mathbf{1} \\ \mathbf{6} & - & - & - & \mathbf{1} & \mathbf{1} & - \end{array}$$

(2) For $\boldsymbol{abc} = 001$,

$$\boldsymbol{W} = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & - & \mathbf{2} & \mathbf{1} & - & - & - \\ \mathbf{2} & \mathbf{2} & - & - & \mathbf{2} & \mathbf{1} & - \\ \mathbf{3} & \mathbf{1} & - & - & \mathbf{4} & \mathbf{1} & - \\ \mathbf{4} & - & \mathbf{2} & \mathbf{4} & - & - & \mathbf{1} \\ \mathbf{5} & - & \mathbf{1} & \mathbf{1} & - & - & \mathbf{1} \\ \mathbf{6} & - & - & - & \mathbf{1} & \mathbf{1} & - \end{array}$$

(3) For $abc = 010$,

$$W = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & - & \mathbf{1} & \mathbf{2} & - & - & - \\ \mathbf{2} & \mathbf{1} & - & - & \mathbf{2} & \mathbf{1} & - \\ \mathbf{3} & \mathbf{2} & - & - & \mathbf{4} & \mathbf{1} & - \\ \mathbf{4} & - & \mathbf{2} & \mathbf{4} & - & - & \mathbf{1} \\ \mathbf{5} & - & \mathbf{1} & \mathbf{1} & - & - & \mathbf{1} \\ \mathbf{6} & - & - & - & \mathbf{1} & \mathbf{1} & - \end{array}$$

(4) For $abc = 011$,

$$W = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & - & \mathbf{2} & \mathbf{2} & - & - & - \\ \mathbf{2} & \mathbf{2} & - & - & \mathbf{2} & \mathbf{1} & - \\ \mathbf{3} & \mathbf{2} & - & - & \mathbf{4} & \mathbf{1} & - \\ \mathbf{4} & - & \mathbf{2} & \mathbf{4} & - & - & \mathbf{1} \\ \mathbf{5} & - & \mathbf{1} & \mathbf{1} & - & - & \mathbf{1} \\ \mathbf{6} & - & - & - & \mathbf{1} & \mathbf{1} & - \end{array}$$

(5) For $abc = 100$,

$$W = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & - & \mathbf{1} & \mathbf{1} & - & - & - \\ \mathbf{2} & \mathbf{1} & - & - & \mathbf{3} & \mathbf{1} & - \\ \mathbf{3} & \mathbf{1} & - & - & \mathbf{4} & \mathbf{1} & - \\ \mathbf{4} & - & \mathbf{3} & \mathbf{4} & - & - & \mathbf{1} \\ \mathbf{5} & - & \mathbf{1} & \mathbf{1} & - & - & \mathbf{1} \\ \mathbf{6} & - & - & - & \mathbf{1} & \mathbf{1} & - \end{array}$$

(6) For $abc = 101$,

$$W = \begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \hline \mathbf{1} & - & \mathbf{2} & \mathbf{1} & - & - & - \\ \mathbf{2} & \mathbf{2} & - & - & \mathbf{3} & \mathbf{1} & - \\ \mathbf{3} & \mathbf{1} & - & - & \mathbf{4} & \mathbf{1} & - \\ \mathbf{4} & - & \mathbf{3} & \mathbf{4} & - & - & \mathbf{1} \\ \mathbf{5} & - & \mathbf{1} & \mathbf{1} & - & - & \mathbf{1} \\ \mathbf{6} & - & - & - & \mathbf{1} & \mathbf{1} & - \end{array}$$

(7) For $abc = 110$,

$$W = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & - & 1 & 2 & - & - & - \\ 2 & 1 & - & - & 3 & 1 & - \\ 3 & 2 & - & - & 4 & 1 & - \\ 4 & - & 3 & 4 & - & - & 1 \\ 5 & - & 1 & 1 & - & - & 1 \\ 6 & - & - & - & 1 & 1 & - \end{array}$$

(8) For $abc = 111$,

$$W = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & - & 2 & 2 & - & - & - \\ 2 & 2 & - & - & 3 & 1 & - \\ 3 & 2 & - & - & 4 & 1 & - \\ 4 & - & 3 & 4 & - & - & 1 \\ 5 & - & 1 & 1 & - & - & 1 \\ 6 & - & - & - & 1 & 1 & - \end{array}$$