CBowersox_Assign14

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Taylor approximation for functions

function: $f(x) = \frac{1}{1-x} x \in (-1, 1)$

$$f(x) = (1-x)^{-1}$$
$$f'(x) = (1-x)^{-2}$$
$$f''(x) = 2(1-x)^{-3}$$
$$f'''(x) = 6(1-x)^{-4}$$
$$\ldots$$
$$f^{(n)}(x) = n!(1-x)^{-(n+1)}$$

$$a = 0$$
$$f(a) = 1$$
$$f'(a) = 1$$
$$f''(a) = 2$$
$$f'''(a) = 6$$
$$\ldots$$
$$f^{(n)}(a) = n!$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$a = 0:$$
$$f(x) = \sum_{n=0}^{\infty} \frac{n!}{n!}(x-0)^n$$
$$f(x) = \sum_{n=0}^{\infty} x^n$$

in general $a \in (-1, 1)$

$$f(x) = \sum_{n=0}^{\infty} \frac{n!(1-a)^{-(n+1)}}{n!}(x-a)^n$$
$$f(x) = \sum_{n=0}^{\infty} \frac{(x-a)^n}{(1-a)^{n+1}}$$

function: $f(x) = e^x$

$$f(x) = e^x$$
$$f'(x) = e^x$$
$$f''(x) = e^x$$
$$f'''(x) = e^x$$
$$\ldots$$
$$f^{(n)}(x) = e^x$$

$$a = 0$$
$$f(a) = 1$$
$$f'(a) = 1$$
$$f''(a) = 1$$
$$f'''(a) = 1$$
$$\ldots$$
$$f^{(n)}(a) = 1$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!}(x-0)^n$$
$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

in general $a \in R$

$$f(x) = \sum_{n=0}^{\infty} \frac{e^a}{n!}(x-a)^n$$

function: $f(x) = ln(1+x) x \in (-1,1)$

$$f(x) = ln(1+x)$$
$$f'(x) = (1+x)^{-1}$$
$$f''(x) = -(1+x)^{-2}$$
$$f'''(x) = 2(1+x)^{-3}$$
$$f^{(4)}(x) = -6(1+x)x^{-4}$$
$$\dots$$
$$f^{(n)}(x) = -1^{n+1}(n-1)!(1+x)^{-n}, n > 0$$

$$a = 0$$
$$f(a) = 0$$
$$f'(a) = 1$$
$$f''(a) = -1$$
$$f'''(a) = 2$$
$$f'''(a) = -6$$
$$\dots$$
$$f^{(n)}(a) = -1^{n+1}(n-1)!, n > 0$$

$$f(x) = \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$f(x) = \sum_{n=1}^{\infty} \frac{-1^{n+1}(n-1)!}{n!}(x-0)^n$$
$$f(x) = \sum_{n=1}^{\infty} \frac{-1^{n-1}x^n}{n}$$

in general $a \in (-1,1)$

$$f(x) = ln(1+a) + \sum_{n-1}^{\infty} \frac{-1^{n-1}(1+a)^{-n}}{n}(x-a)^n$$