Econometrics 2 – M1 PPD, Paris School of Economics – 2024-2025

Problem set no 4

Exercice 1: A simple matching method

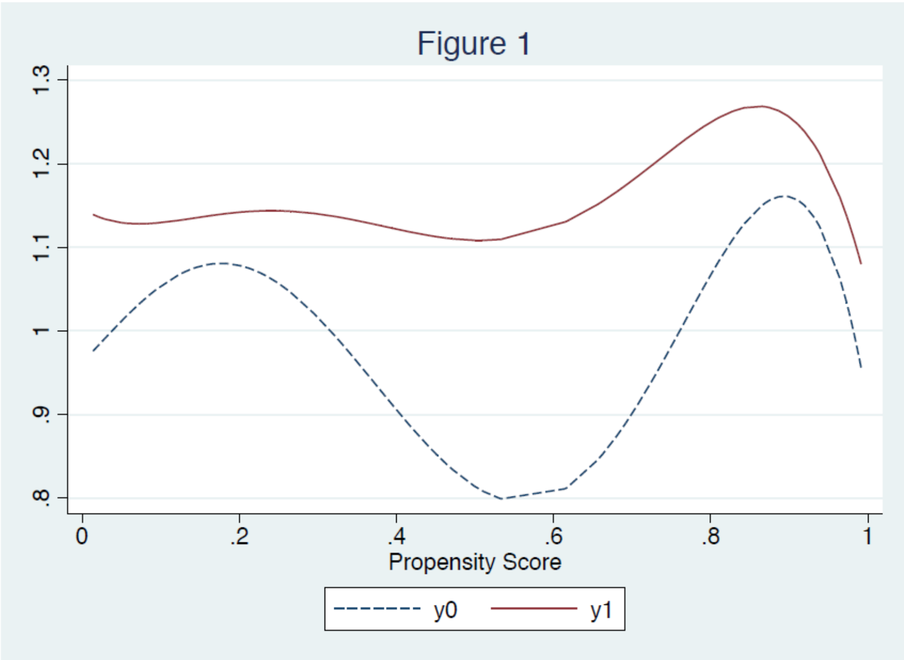
Consider an outcome y and a treatment $T = 0/1$. X is a vector of control variables determined before treatment happens (such as education, age, etc.). All are measured in a dataset for a sample of observations.

1. Note y_0 and y_1 the counterfactuals associated to treatment status. We assume ignorability:
$$y_0, y_1 \perp T|X$$
Show that Average Treatment Effect (ATE) is equal to:
$$E_X(E(y|T = 1, X) - E(y|T = 0, X))$$
(where E_X indicates expectancy over all values of X).

2. Counterfactuals are functions of X, noted $y_0(X)$ and $y_1(X)$. Assume the shape of those functions are:
$$y_0(X) = \alpha_0 + X\beta_0 + \varepsilon_0$$
$$y_1(X) = \alpha_1 + X\beta_1 + \varepsilon_1$$
How can you estimate ATE using this hypothesis? Give one method and describe it precisely.

3. Define the propensity score as $p(X) = P(T = 1|X)$. Counterfactuals are also functions of the propensity score, noted $y_0(p(X))$ and $y_1(p(X))$. Figure 1 shows the two *true* functions (unknown to the econometrician). Is treatment effect positive? Is it heterogeneous? Explain.

4. Show that, under ignorability above, $ATE = E_{p(X)}(E(y|T = 1, p(X)) - E(y|T = 0, p(X)))$.



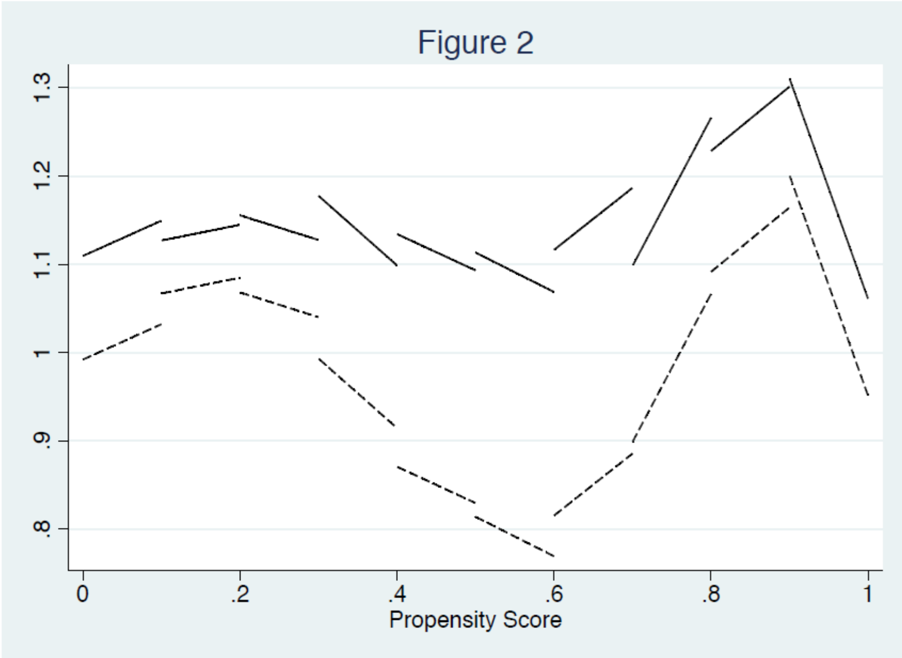
5. Assume that we have estimated $p(X)$, so that we have one value $p(X_i)$ for each individual i in the sample. We build J blocks of values for $p(X)$, each labelled B_j, $j = 1...J$ such that:

$$\begin{array}{ll} \text{Observation } i \text{ belongs to } B_1 \text{ if} & 0 \leq p(X_i) < c_1 \\ \text{Observation } i \text{ belongs to } B_2 \text{ if} & c_1 \leq p(X_i) < c_2 \\ \text{etc...} & \\ \text{Observation } i \text{ belongs to } B_J \text{ if} & c_{J-1} \leq p(X_i) \end{array}$$

for a series of thresholds c_1 to c_{J-1}, all between 0 and 1.
For each block B_j, we take the observations belonging to that block and estimate by OLS:

$$y_i = \alpha_j + \tau_j T_i + \beta_j p(X_i) + \varepsilon_{ij}$$

Figure 2 plots the predicted values from those regression when $J = 10$ and the c_j's are spaced equally, every 0.1 units of propensity score. Explain this figure and how it was built. How can you use those regressions to compute ATE? How accurate would this estimation be? How could you improve accuracy (and with what constraint)?



6. We now use the same blocks, but do not run regressions. Rather, within each block j we compute the empirical means of y among the treated ($\bar{y}_{j1}$) and the untreated ($\bar{y}_{j0}$); we then compute $\mu_j = \bar{y}_{j1} - (\bar{y}_{j0})$ for each block. Finally, we estimate ATE as $\sum_j \pi_j \mu_j$, where π_j is the share of the sample that belongs to block j.
   Show that this is the same matching method as in question 5, but with some restrictions on the specification.

Exercice 2: Common support

Consider a training program offered to a large population of workers for which you will obtain the necessary data. For any individual, the potential outcome (say wage) is y_0 if not treated and y_1 if treated. Call x a vector of observed variables (including a constant) and β a corresponding vector of parameters. We assume that:

$$y_1 = y_0 + \beta x$$

We also assume that $y_0 = f(x) + u$ where $f(.)$ is some function of the observed variables and u is an unobserved random variable independent from x. Individuals can freely enter the training program: T is a dummy variable equal to 1 if the individual enters the training program (treated), and to 0 otherwise (not treated).

1. What is the treatment impact in this model? Is it homogenous, heterogeneous?

We assume that there is perfect information, no uncertainty and that agents are rational in the sense that they enter the training program offered to them whenever $y_1 > y_0$.

2. Show that $E(y_0|T = 1) \neq E(y_0)$. What do you think of comparing the average outcomes of treated and not treated workers?

We now wonder if we could use matching on x to estimate the treatment impact.

3. Assume that x only contains 2 variables: a constant and age (a), so that $\beta x = \beta_0 + \beta_1 a$. Define the propensity score as $P(T = 1|x)$. Is there a common support (i.e. values of x for which $0 < P(T = 1|x) < 1$)? What do you think of using a matching estimator to estimate the program impact, such as nearest neighbor, or forming cells?

4. We observe the outcome $y = Ty_1 + (1 - T)y_0$. Write the regression of y on T and x that derives from this model in the case where x is a constant and age. Assuming $f(.)$ is linear in a, can we estimate the parameters of this model using ordinary least squares? Explain intuitively why: to that aim draw a figure in the (y, a) space where you distinguish the relation between y and a for the treated and for the untreated. Is the answer to this question compatible with the answer to the previous question?

We now assume that

$$y_1 = y_0 + [\beta x + \varepsilon]$$

with ε an unobserved random variable independent from x and u.

5. Show that $E(y_0|x, T = 1) = E(y_0|x)$.

6. Write the propensity score again. Is there a common support now? (for instance, you can assume $\varepsilon \sim N(0, 1)$).

7. Assume from now that ε is correlated with u. Can we apply a matching method?