Econometrics 3: Linear panel data models

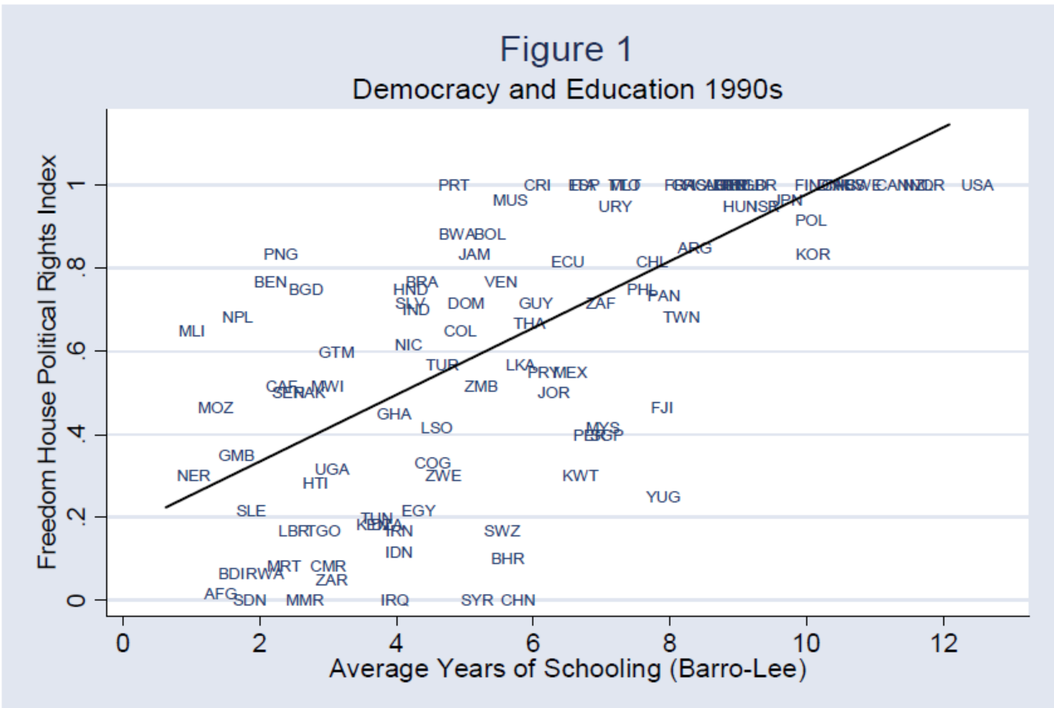
Problem Set 4

Exercises labeled with (*) are optional

1. **The modernization theory**

   The modernization theory, popularized by Lipset (1959), states that economic growth and education are key drivers of democratization. Following Acemoglu, Johnson, Robinson, and Yared [AJRY] (2005), using a panel of countries, you are asked to test the second part of this hypothesis: is there a positive impact of the level of education in a country on the degree of democracy in this country?

   AJRY use a standard index of political freedom (the Freedom House Political Rights Index), which takes values between 0 and 1, and data from Barro and Lee (2000) on the average level of schooling in the population. They plot the following graph, Figure 1.



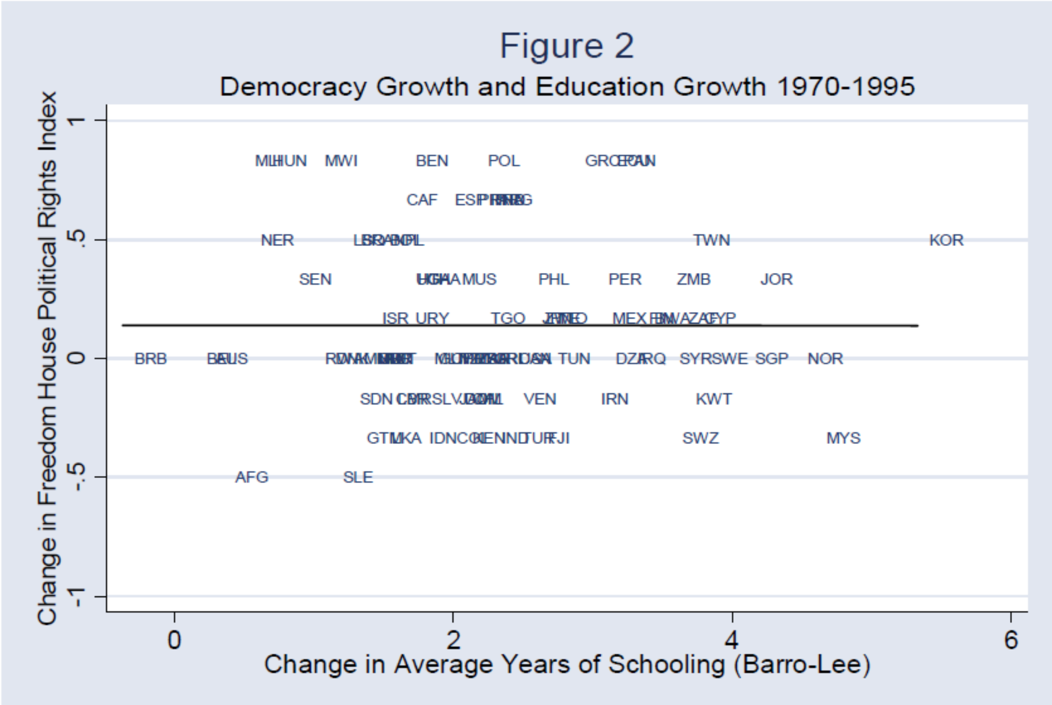
Freedom House Political Rights Index is normalized from 0 to 1 and is from Freedom House (2004). Average years of schooling in the adult population (25 years of age and older) is from Barro and Lee (2000). Values are averaged by country from 1990 to 1999.

   (a) What is the statistical model underlying the regression line seen in Figure 1?

   (b) Is the observed relationship likely to be causal? Explain.

   Then, AJRY plot the following graph, Figure 2.

   (c) What is the statistical model underling the regression line seen in Figure 2?



Freedom House Political Rights Index is normalized from 0 to 1 and is from Freedom House (2004). Average years of schooling in the adult population (25 years of age and older) is from Barro and Lee (2000). Changes are total differences between 1970 and 1995.

(d) Is the observed relationship more likely to be causal than that observed in Figure 1? Explain.

AJRY then turn to regression analysis. They use an unbalanced panel where they observe the level of education and the index of political freedom at 5-year intervals. They estimate the following model

$$d_{it} = \alpha d_{it-1} + \gamma s_{it-1} + \mu_t + v_{it}, \tag{1}$$

where s is education, d is the political index, μ are time dummies, and v are error terms. i indexes the countries and t the periods.

(e) What is an unbalanced panel?

(f) The model in equation (1) is estimated by (pooled) OLS. What are the assumptions on the error terms v_{it}s required for consistency? (Hint: Note the presence of d_{it-1}.)

(g) What is the rationale of including d_{it-1} as a regressor?

Table 1 contains the regression output obtained by AJRY.

TABLE 1—FIXED EFFECTS RESULTS

	Base sample, 1965–2000 (5-year data)								
Independent variable	Pooled OLS (i)	Fixed-effects OLS (ii)	Arellano-Bond GMM (iii)	Fixed-effects OLS (iv)	Arellano-Bond GMM (v)	Fixed-effects OLS (vi)	Arellano-Bond GMM (vii)	Fixed-effects OLS (viii)	Arellano-Bond GMM (ix)
Democracy_{t-1}	0.709	0.385	0.507	0.362	0.493	0.369	0.510	0.351	0.499
	(0.035)	(0.053)	(0.096)	(0.053)	(0.101)	(0.054)	(0.094)	(0.055)	(0.097)
Education_{t-1}	0.027	−0.005	−0.017	0.005	−0.013	−0.012	−0.013	−0.007	−0.020
	(0.004)	(0.019)	(0.022)	(0.020)	(0.024)	(0.019)	(0.026)	(0.020)	(0.026)
Age-structure F test:				[0.08]	[0.31]			[0.19]	[0.27]
Log population_{t-1}				−0.124	−0.023			−0.042	0.049
				(0.101)	(0.115)			(0.108)	(0.143)
Log GDP per capita_{t-1}						−0.012	−0.187	−0.001	−0.121
						(0.042)	(0.110)	(0.049)	(0.182)
Time-effects F test:	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Hansen J test:			[0.31]		[0.21]		[0.44]		[0.15]
AR(2) test:			[0.81]		[0.89]		[0.96]		[0.88]
Number of observations:	765	765	667	746	652	684	595	676	589
Number of countries:	108	108	104	104	101	97	93	95	92
R^2:	0.71	0.78		0.78		0.76		0.77	

Notes: Fixed-effects OLS regressions are reported in columns (ii), (iv), (vi), and (viii) with country dummies and robust standard errors clustered by country in parentheses. Columns (iii), (v), (vii), and (ix) use GMM of Manuel Arellano and Stephen R. Bond (1991), with robust standard errors; columns (vii) and (ix) treat log GDP per capita_{t-1} as predetermined and instrument its first difference with log GDP per Capita_{t-2}. Year dummies are included in all regressions, and the time effects F test gives the p value for their joint significance. The dependent variable is the augmented Freedom House Political Rights Index. The base sample is an unbalanced panel, 1965–2000, with data at five-year intervals in levels where the start date of the panel refers to the dependent variable (i.e., $t = 1965$, so $t - 1 = 1960$). Columns (iv), (v), (viii), and (ix) include but do not display the median age of the population at $t - 1$ and four covariates corresponding to the percentage of the population at $t - 1$ in the following age groups: 0–15, 15–30, 30–45, and 45–60. The age structure F test gives the p value for the joint significance of these variables. Countries enter the panel if they are independent at $t - 1$. See text for data definitions and sources.

(h) The estimation results obtained for equation (1) are given in column (i) of Table 1. Is there evidence in favor of the modernization theory or not?

Then, AJRY also estimate the following model

$$d_{it} = \alpha d_{it-1} + \gamma s_{it-1} + \delta_i + \mu_t + v_{it}, \tag{2}$$

where δ are individual effects.

(i) AJRY do not want to assume that the δ_is are random effects and consider a first difference transformation. Write down the above model (in equation (2)) in first differences.

(j) Why is (pooled) OLS inconsistent (for the model in first differences)?

(k) Under the assumption that the original error terms, the v_{it}s in equation (2), are uncorrelated over time, suggest an instrumental variable strategy (based on the model in first differences).

(l) The estimation results for the instrumental variable strategy you suggested in (k) are given in column (iii) of Table 1. Is there evidence in favor of the modernization theory or not?

AJRY then compare their results to Glaser et al. (2004), who find a positive impact of education on democracy using the following model:

$$d_{it} = \alpha d_{it-1} + \gamma s_{it-1} + \xi \log \mathrm{GDP}_{it-1} + \delta_i + v_{it}, \tag{3}$$

(m) What is the rationale for introducing $\log \mathrm{GDP}_{it-1}$ as a regressor?

(n) Explain why the omission of time dummies, μ_t, could lead to biased estimates of the coefficients of interest?

(o) AJRY reproduce the results of Glaser et al. (2004) with and without time dummies. The regression results are reported in Table 2 below. What do you conclude?

2. **Differences-in-differences with heterogeneous treatment effects**

Stevenson and Wolfers (2006) study the impact of no-fault divorce reforms on female suicide. These reforms allowed women to file for divorce unilaterally (i.e., without the need for the husband to agree) and the study tries to see if this lowered suicide. Stevenson and Wolfers use state-level data in the US between 1969 and 1985, and the "quasi-experimental" variation resulting from the adoption of unilateral divorce laws in 37 states, in various years. They analyze the following "two-way fixed effects" (TWFE) model:

$$y_{it} = \alpha_i + \lambda_t + \beta D_{it} + e_{it}, \tag{4}$$

where y_{it} is female suicide rate in year t and state i, D_{it} is an indicator variable equal to 1 if the state has passed a unilateral divorce law in year t or before, α_is and λ_ts are individual state and time effects, and e_{it} is an error term.

Figure 1 (Figure 2 from Goodman-Bacon, 2021) illustrates the identifying variation used to estimate β in such an empirical design: by comparing a state that adopted a unilateral divorce law (henceforth: a law) to a state that did not adopt over the period (panels A and B); by comparing a state that adopted early to a state that adopted later, *before* the latter adopted (panel C); by comparing a state that adopted late to a state that adopted earlier, *afte*r the latter adopted (panel D) .

Following Rubin's potential outcome framework, define $y_{it}(0)$ as the potential outcome if the state has not passed a law by year t. Define $y_{it}(t_i)$ as the potential outcome at time t if the state has passed a law at time $t_i \leq t$. Note that potential outcome varies with the time elapsed since the law was passed, $t - t_i$. The time-varying effect of the reform in year t, for a state i that passed a law in t_i, is then defined as:

$$\Delta_i(t) = y_{it}(t_i) - y_{it}(0) \tag{5}$$

for $t \geq t_i$.

(a) Explain why $\Delta_i(t)$ is not identified.

(b) Define $ATT_k(W)$ as

$$ATT_k(W) \equiv \frac{1}{T_W} \sum_{t \in W} E(y_{it}(k) - y_{it}(0) | t_i = k) \tag{6}$$

where W is a date range (e.g. between 1970 and 1975) and T_W is the number of years in that range (e.g. $T_W = 6$). Interpret $ATT_k(W)$.
(Hint: $ATT_k(W)$ concerns states that passed a law at the beginning or before period W.)

(c) Consider the quantity identified by the difference-in-differences (DD) represented in panel A. To simplify, consider that there are only two years, denoted by $t = post$ and $t = pre$. Then denote the DD associated with panel A as:

$$\begin{aligned} DD_A \equiv & \; E(y|t = post, state = K) - E(y|t = pre, state = K) \\ & -(E(y|t = post, state = U) - E(y|t = pre, state = U)). \end{aligned}$$

(Indexes are dropped for convenience: $y \equiv y_{it}$.) Show that

$$DD_A = E(y(1) - y(0)|t = post, state = K),$$

where $y(1)$ denotes the potential outcome in a state one year after adoption. Specify the parallel trend assumption that you need to make.
(Hint on notations: (i) for some reason, Stevenson and Wolfers denote with K the states that adopted the reforms early in the observation window, and by U the states that did not adopt at all. L corresponds to states that adopted late; (ii) note that we simplified the notation for potential outcomes: in keeping with the above definition of $y_{it}(t_i)$, one would write $y(1)$ as $y_{i,post}(post)$).)

(d) Now consider the quantity identified by the difference-in-differences (DD) represented in panel D. To simplify, consider that there are only two years, denoted by $t = post$ and $t = pre$. Then denote the DD associated with panel D as:

$$\begin{aligned} DD_D \equiv & \; E(y|t = post, state = L) - E(y|t = pre, state = L) \\ & -(E(y|t = post, state = K) - E(y|t = pre, state = K)). \end{aligned}$$

i. Show that under the standard parallel trend assumption

$$\begin{aligned} DD_D = & \; E(y(1) - y(0)|t = post, state = L) \\ & +E(y(1) - y(0)|t = pre, state = K) - E(y(1) - y(0)|t = post, state = K). \end{aligned}$$

ii. What happens if treatment effects are stable over time (i.e. do not depend on t)? If treatment effects are homogeneous (i.e. do not vary across states)?

iii. Conversely, what is a potential problem with the causal interpretation of DD_D if treatment effects vary over time and across states?

(e) Now consider a simplified staggered treatment model with 2 states and 3 periods. State 1 is untreated in period 1 but treated in periods 2 and 3. State 2 is untreated in periods 1 and 2, but treated in period 3. i in the subscript denotes state and t denotes period.

i. As per the slides, the β in model 4 can be estimated as a weighted average of state and time specific treatment effects, with the weights being $\frac{D_{it}e_{it}}{e_{it}^2}$. Compute those weights for the aforementioned staggered treatment model. Hint: You can compute the residuals by running the relevant regression on stata.

ii. Show that the β in model 4 identifies the same quantity as the simple mean of $DID_1 = y_{12} - y_{11} - (y_{22} - y_{21})$ and $DID_2 = y_{23} - y_{22} - (y_{13} - y_{12})$. Hint: DID_1 and DID_2 are similar to the DD_A and DD_D that you estimated in parts (c) and (d) respectively.

iii. Why does β not have a causal interpretation? Try providing two distinct explanations based on what you have learnt from the previous two subquestions.

iv. In a more general staggered treatment model with multiple state and multiple periods, which states and periods in your opinion are the most likely to be assigned negative weights?

v. Can you think of alternative estimators that get rid of problems with the TWFE β?

(f) Goodman-Bacon (2021) shows that β in model 4 can be decomposed as a weighted average of the four types of differences-in-differences shown in Figure 1. The corresponding differences-in-differences (for various periods W and pairs of states) and their weights can be estimated from the data. The results are displayed in Figure 2 (Figure 6 from Goodman-Bacon). Comment on the figure. Is there evidence of treatment effect heterogeneity?

TABLE 2—FIXED-EFFECTS RESULTS: EDUCATION, DEMOCRACY, AND POLITICAL INSTITUTIONS: GLAESER ET AL. (2004) SAMPLE, 1965–2000 (FIVE-YEAR DATA)

Independent variable	Executive (i)	Autocracy (ii)	Democracy (iii)	Autocracy (iv)
A. *No Time Effects*				
Institutions$_{t-1}$	−0.572	−0.547	−0.515	−0.864
	(0.072)	(0.068)	(0.065)	(0.103)
Education$_{t-1}$	0.498	0.909	0.700	0.096
	(0.119)	(0.179)	(0.180)	(0.071)
Log GDP per capita$_{t-1}$	0.038	−0.508	0.292	0.267
	(0.403)	(0.630)	(0.606)	(0.202)
R^2:	0.33	0.32	0.30	0.47
B. *Including Time Effects*				
Institutions$_{t-1}$	−0.618	−0.616	−0.580	−0.897
	(0.073)	(0.071)	(0.067)	(0.106)
Education$_{t-1}$	−0.163	−0.318	−0.432	−0.137
	(0.192)	(0.267)	(0.298)	(0.126)
Log GDP per capita$_{t-1}$	0.168	−0.317	0.477	0.292
	(0.360)	(0.550)	(0.561)	(0.192)
Time-effects F test:	[0.01]	[0.00]	[0.00]	[0.08]
R^2:	0.39	0.40	0.37	0.50
C. *Including Time Effects*				
Institutions$_{t-1}$	−0.617	−0.615	−0.579	−0.891
	(0.073)	(0.071)	(0.068)	(0.107)
Education$_{t-1}$	−0.125	−0.389	−0.324	−0.088
	(0.182)	(0.229)	(0.289)	(0.125)
Time-effects F test	[0.01]	[0.00]	[0.00]	[0.07]
R^2:	0.39	0.40	0.37	0.49
Number of observations:	499	499	499	349

Notes: The table reports fixed effects OLS regressions in all columns, with country dummies and robust standard errors clustered by country in parentheses. Year dummies are included in panels B and C, and the time-effects F test gives the p value for their joint significance. The dependent variable in column (i) is change in Constraint on the Executive from Polity. The dependent variable in column (ii) is change in negative Autocracy Index from Polity. The dependent variable in column (iii) is change in Democracy Index from Polity. The dependent variable in column (iv) is change in negative Autocracy Index from Przeworski et al. (2000). The base sample in all columns is an unbalanced panel, 1965–2000, with data at five-year intervals, where the start date of the panel refers to the dependent variable (i.e., t = 1965, so $t - 1 = 1960$). See Glaeser et al. (2004) for data definitions and sources.

Figure 1: Figure 2 from Goodman-Bacon (2021)

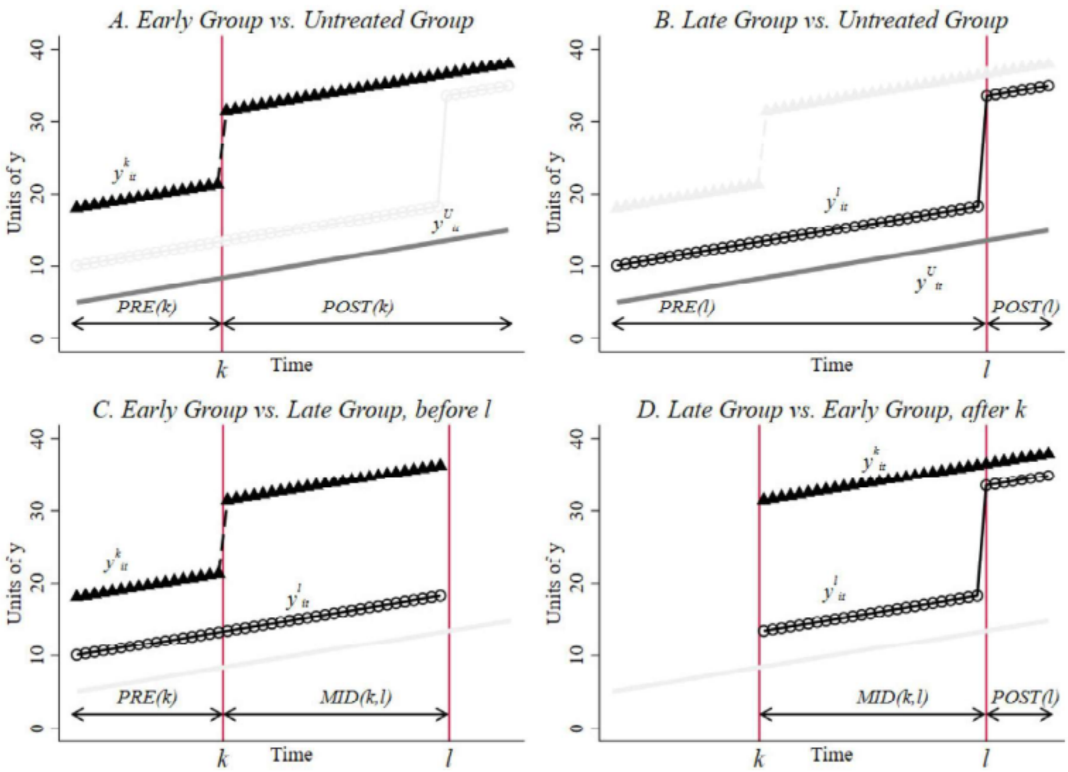


Fig. 2. The four simple (2x2) difference-in-differences estimates in the three group case. Notes: The figure plots outcomes for the subsamples that generate the four simple 2x2 difference-in-difference estimates in the three timing group case from Fig. 1. Each panel plots the data structure for one 2x2 DD. Panel A compares early treated units to untreated units ($\hat{\beta}^{DD}_{kU}$); panel B compares late treated units to untreated units ($\hat{\beta}^{DD}_{\ell U}$); panel C compares early treated units to late treated units during the late timing group's pre-period ($\hat{\beta}^{DD,k}_{k\ell}$); panel D compares late treated units to early treated units during the early timing group's post-period ($\hat{\beta}^{DD,\ell}_{k\ell}$). The treatment times mean that $\overline{D}_k = 0.67$ and $\overline{D}_\ell = 0.16$, so with equal group sizes, the decomposition weights on the 2x2 estimate from each panel are 0.365 for panel A, 0.222 for panel B, 0.278 for panel C, and 0.135 for panel D.

Figure 2: Figure 6 from Goodman-Bacon (2021)

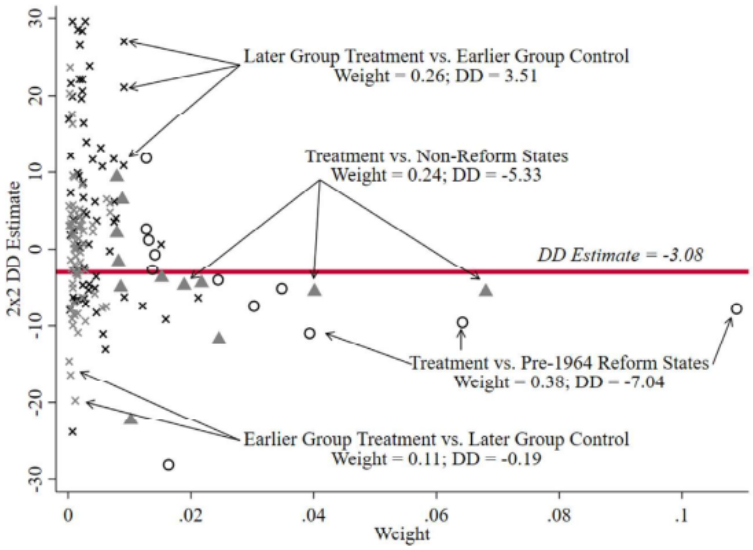


Fig. 6. Difference-in-differences decomposition for unilateral divorce and female suicide. Notes: The figure plots each 2x2 DD components from the decomposition theorem against their weight for the unilateral divorce analysis. The open circles are terms in which one timing group acts as the treatment group and the pre-1964 reform states act as the control group. The closed triangles are terms in which one timing group acts as the treatment group and the non-reform states act as the control group. The *x*'s are the timing-only terms. The figure notes the average DD estimate and total weight on each type of comparison. The two-way fixed effects estimate, −3.08, equals the average of the *y*-axis values weighted by their *x*-axis value.

4.1. Describing the design

Fig. 6 uses the DD decomposition theorem to illustrate the sources of variation. I plot each 2x2 DD against its weight and calculate the average effect and total weight for the three types of 2x2 comparisons: treated/untreated, early/late, late/early.[24] The two-way fixed effects estimate, −3.08, is an average of the *y*-axis values weighted by their *x*-axis values. Summing the weights on timing terms ($s^k_{k\ell}$ and $s^\ell_{k\ell}$) shows how much of $\hat{\beta}^{DD}$ comes from timing variation (37 percent). The large untreated group puts a lot of weight on $\hat{\beta}^{2x2}_{kU}$ terms, but more on those involving pre-1964 reform states (38.4 percent) than non-reform states (24 percent). Fig. 6 also highlights the role of a few influential 2x2 DDs. Comparisons between the 1973 states and non-reform/pre-1964 reform states account for 18 percent of the estimate, and the ten highest-weight 2x2 DDs account for over half.

The bias resulting from time-varying effects is also apparent in Fig. 6. The average of the post-treatment event-study estimates in Fig. 5 is −4.92, but the DD estimate is 60 percent as large. The difference stems from the comparisons of later- to earlier-treated groups. The average treated/untreated estimates are negative (−5.33 and −7.04) as are the comparisons of earlier- to later-treated states (although less so: −0.19).[25] The comparisons of later- to earlier-treated states, however, are *positive* on average (3.51) and account for the bias in the overall DD estimate. Using the decomposition theorem to take these terms out of the weighted average yields an effect of −5.44—close to the average of the event-study coefficients.