Paris School of Economics - Master in Public Policy and Development (M1)
Microeconomics, Markets and market failures - Coordination With and Without Markets

Problem Set 2

Due at 23.59, 20th March 2025

1 Coasian bargaining

Suppose that individual's utility can be represented with the following function that depends on health (either physical health P or mental health M) and money m

$$U = m + P \times M$$

One of the key health measures to prevent the spread of Covid-19 in France was curfew. It is safe to assume that a later (sooner) curfew has negative (positive) effects on physical health but positive (negative) effects on mental health. We thus model physical and mental health in the following way.

$$P = \alpha P_0 - x$$
$$M = (1-\alpha)M_0 + x$$

With x the time of the curfew (normalized so that $x \in [0,1]$ - think of 0 as a 3 p.m. curfew, and 1 as a 3 a.m. curfew for instance), P_0 physical health stock, M_0 mental health stock and α the relative quantity of physical health stock compared to mental health stock. The value of α depends heavily on age. In particular, the young have relatively more health stock than the elderly. If we consider those two groups - the elderly (E) and the young (Y) - α_E is thus probably smaller than α_Y. In the following, we set $\alpha_E = .5$ and $\alpha_Y = 1$. For simplicity, also assume that $P_0 = M_0 = 2$.

In this exercise, we will study how the young and the elderly can bargain over the curfew hour. We will consider that one group can transfer money to the other group and we denote by y the amount that the young transfer to the elderly (if $y < 0$, $-y$ becomes an amount that the elderly transfer to the young). Thus we can write, each group's utility function as:

$$u_E = \quad y + (\alpha_E P_0 - x) \times \Big((1-\alpha_E)M_0 + x\Big) = y + 1 - x^2 \tag{1}$$

$$u_Y = \quad -y + (\alpha_Y P_0 - x) \times \Big((1-\alpha_Y)M_0 + x\Big) = -y + 2x - x^2 \tag{2}$$

1. What other major difference between the elderly and the young with respect to COVID-19 could have been incorporated into the model? How?

2. What is the preferred curfew time of each group?

3. What is the socially optimal curfew time, if the social optimum is defined as the sum of the two group utilities ? What can you say about the relationship between the social optimum and each group's preferred curfew time? Do you think this is a general result that does not depend on the specific functional form of utility?

Suppose that the curfew is set at midnight ($x = .75$) and Y can design a take it or leave it offer to E promising (we assume credibly) to voluntarily submit to an earlier curfew in return for a side payment from E to Y (equal to $-y$).

4. What offer will Y make? Explain why the voluntary curfew is identical to the social optimum.
5. Explain why had the initial curfew been set at 6 p.m., the selection of x as a result of Coasian bargaining would have been the same as that resulting from a curfew set at midnight and the social planner's optimum. This is what Coase meant when he wrote that 'all that matters (questions of equity aside) is that the rights of the various parties should be well defined and the results of legal actions easy to forecast.'

Assume that E has limited resources and cannot make a payment to Y in excess of y^{max} (and the curfew is set at midnight).

6. What is the smallest value of y^{max} that will induce Y to implement the socially optimal outcome (assuming, as above that he can make a take it or leave it offer to E)?
7. Now assume that E rather than Y is in a position to make the take-it-or-leave-it offer. (The official curfew is still midnight) What is the smallest value of y^{max} that will induce E to implement the social optimum? Why are your answers to this and the previous question different?

Suppose that the amount E has available to make a side payment to Y is equal to 0.1 ($-y$ cannot be greater than .1) that is, positive, but too small to support a bargain between the two resulting in the social optimum curfew (whether Y or E is in the position to make a take it or leave it offer).

8. What's the latest time (earlier than midnight but later than the social optimum) that, if imposed by the social planner, would allow the social optimum bargained curfew to be implemented under each bargaining rules (either Y or E makes the take it or leave it offer)
9. Why can the social planner plus Coasean bargaining together accomplish what Coasean bargaining alone could not in this case?

2 The tragedy of the fishers

Let's assume that 2 fishermen, A and B, can fish on a lake.

Fisherman A has a production function $y_A = \alpha(1-\beta e_B)e_A$ where y_i stands for the value of total catches, and e_i the fishing effort of fishermen $i \in \{A, B\}$. The parameter α captures the productivity and the parameter β depicts the extent of the inter-fisherman externality. The profit of fisherman A is given by the expression $\pi_A = y_A - e_A^2$.

To simplify, let's assume that fishermen are symmetric and that fisherman B faces the production function $y_B = \alpha(1 - \beta e_A)e_B$. The profit function is $\pi_B = y_B - e_B^2$

1. Say how you would determine the maximum A would be willing to pay B to purchase private ownership of the lake, assuming that ownership would allow A to regulate B's access to the lake and that without this assignment of rights the two would fish at the Nash equilibrium. To further simplify the problem, let's assume that $\alpha = 12$ and $\beta = \frac{1}{12}$.

2. Consider the following allocations

   (a) The Nash equilibrium

   (b) The most egalitarian social welfare optimum

   (c) A case in which A is the first mover and B decides on its effort level

   (d) A can make a take-it-or-leave-it offer to B and the underlying contract is credible by assumption.

   Let's assume that $u_i = \pi_i$. First compute the utility of each fisherman in the 4 cases. Based on these conclusions, which pairs can you Pareto rank? Continue to assume that $\alpha = 12$ and $\beta = \frac{1}{12}$.

3. How can being a second mover be advantageous, relative to being a first mover? (Hint: consider that fishing can also be a group activity and that one's catch varies positively with the effort level of the other. Modify the production function accordingly by changing the value of one parameter)

4. Suppose that the access to the lake is now open to anyone. How many (symmetric) fishermen can the lake sustain if the profit function of fisherman i is given by the expression $\Pi_i = 12 \times (1 - \frac{1}{12}\sum_{j \neq i} e_j)e_i - e_i^2$.