Paris School of Economics - Master in Public Policy and Development (M1)
Microeconomics, Markets and market failures - Coordination With and Without Markets

Problem Set 3

Due at 23.59, 3rd April 2025

1 Name the game

North and South are selecting environmental policies. The well-being of each is interdependent, in part due to global environmental effects. Each has a choice of two strategies: Emit or Restrict emission. Suppose this is just a two-person game. It may clarify things to let the representative citizen in each region have a reduced form utility function $u_i(e_i, e_j)$, where e is the level of emissions (0 or 1) and the subscripts i and j refer to North and South. (It is a reduced form because the citizens' well-being is proximately affected not by emissions per se but by the things with which emissions are associated positively (consumption) or negatively (health status).) Some have modeled this problem as a prisoners' dilemma, while others have proposed the Assurance Game or even the Chicken (Hawk Dove) Game (Taylor 1987).

- Illustrate each of these possibilities with a payoff matrix and explain why it might be a reasonable depiction of the interaction.
- Suppose North's utility function has the form

$$u_i = \alpha e_i + \beta e_i + \gamma e_i e_j$$

and South's is identical (with appropriate substitution of superscripts). What values of the parameters of these utility functions would make each of these three games the appropriate model of the North-South Emissions Game?

2 Bad chemistry

Consider the generic coordination problem given by

$$u = \alpha + \beta a + \gamma A + \delta a A + \lambda a^2$$

$$U = \alpha + \beta A + \gamma a + \delta a A + \lambda A^2$$

1. For the Nash equilibrium (i.e., a^{star} and A^{star}), give conditions under which the external effect is positive or negative and the two strategies a and A are substitutes or complements.

2. What is the first order condition for a symmetric Pareto-efficient allocation? Using this first order condition (assuming the second order condition holds) and your expression for the Nash equilibrium above to show that a^{star} and A^{star} exceed the Pareto-efficient levels if and only if the external effect is negative. Explain why this is so.

3. Assuming that the Nash equilibrium is in pure strategies, show that there will always be a first mover advantage, and that the second mover will do worse (than in the Nash equilibrium) if strategies are substitutes and better if strategies are complements. Explain why this is so.

4. Two adjacent farmers (Lower and Upper) choose whether to use a chemical intensive anti-pest strategy or a less chemical-intensive approach that uses natural predators to control the pests which threaten their crops (integrated pest management or IPM). The use of chemicals generates negative external effects (the chemicals kill the natural predators as well), while IPM generates positive external effects (the natural predators do not respect the farmer's property boundaries and prey on the pests throughout the area). Specifically, increased use of chemicals by one raises output of the user and lowers the output and raises the marginal productivity of chemical use in the other farm for any given level of other inputs. Letting a and A be the level of chemical use by the two, give the values of the parameters of the above utility functions that describe this interaction.

3 Contracts

Each agent has an identical utility function $u(y, e)$, where y is hourly income measured in units of goods (all payments are made in units of goods) and e is work effort per hour and the function is increasing and concave in the first and decreasing and convex in the second argument. Goods (Q) may be produced on an hourly basis according to the production function $Q(E)$, where E is the sum of effort devoted to production of goods (either by a single worker or by the combined members of a team) and $Q' > 0$, $Q'' < 0$. The level of effort not verifiable. Property rights consist of permission to use the production function (there are no inputs other than effort, but use of the production function requires permission from the "owner"). Where property rights are held by someone other than the agent (say an owner), you may assume that the property owner maximizes profits. Suppose that for each agent, the alternative to working is to receive zero utility. Consider the following situations:

a. The agent owns the right to use the production function and works for himself and owns the resulting output.

b. The agent works under a contract where a fraction s of the output is claimed by some other agent (called the "owner") who also determines s.

c. The agent pays a fixed sum, k, per period to the "owner" for permission to use the production function above, and owns the residual income. The owner determines k.

d. The owner offers the agent (who is a member of a team of identical agents) a contingent renewal contract, with wage w.

e. The agent is one of a team of n identical agents who share equally in the product resulting from their efforts.

f. The owner employs a team of workers, offering to pay each worker $Q - x$ per period, where x is some positive constant.

g. The owner offers the agent (one of a team of identical workers) a contingent renewal contract, charging the agent a one time fee equal to B for permission to begin work.

1. Describe how the level of effort of each agent will be determined under the above situations. Give the relevant maximizing problem or problems and derive the relevant first order conditions, adding whatever additional information you need to do this.

2. Describe how the values of w, s, k, x, and B will be determined in the above situations.

3. In each of the above situations, determine if the agent's level of effort and income determined by the relevant first order conditions is or is not a Pareto optimum. Explain why the results differ.