

Microeconomics – PPD M1

Problem set for 14 February 2025 tutorial

Problem 1.

An individual faces the risk of having an accident leading to loss of 36 euros with probability $3/4$. If he undertakes a precautionary action at cost $c = 4/10$ he may reduce this probability to $1/2$. In addition, the individual may subscribe an insurance contract specifying a premium of x euros and a compensation of y euros in case of accident. The von Neumann-Morgenstern utility function of the individual is $u(w) = \sqrt{w}$ and his initial wealth is $W = 100$ euros. Whether or not he undertakes the precautionary action is not observed, but the occurrence of the accident is a contractible event. Determine the optimal insurance contract assuming that the insurance company is risk neutral.

Problem 2.

Consider the contract design problem of a principal who maximizes expected profit. The principal's revenue is a random variable whose distribution depends on the effort exerted by an agent, $e \in \{0, 1\}$, as described in the following table:

	$x_1 = 0$	$x_2 = 10$	$x_3 = 25$
$p(0)$	$\frac{4}{10}$	$\frac{4}{10}$	$\frac{2}{10}$
$p(1)$	$\frac{2}{10}$	$\frac{4}{10}$	$\frac{4}{10}$

The reservation utility of this agent is $\underline{u} = 1$, his cost of effort is $v(e) = e^2$ and his von Neumann-Morgenstern utility function is $u(w) = \sqrt{w}$. Determine the optimal contract when effort level is observed and when it is not.

Problem 3.

Consider a principal who wants to hire a risk neutral agent for a job. In exchange for the effort e made by the agent, which is observed by the principal, the agent would receive a wage w . Effort level e generates a revenue $x(e) = \sqrt{e}$ to the principal. This agent can be of two types: either A with probability q , or B with probability $1 - q$. Agents' utilities are given:

$$U^A(w, e) = w - e \quad \text{and} \quad U^B(w, e) = w - 2e$$

- (a) Find the contract(s) offered by P if she can distinguish the type of the agent.
- (b) Find the contract(s) offered by P in the case of adverse selection and provide an explanation. How do the contracts vary as a function of q ?

Problem 4.

Consider a market for used cars whose qualities, indexed by the sellers' cost, are uniformly distributed in the interval $[2, 6]$. Buyers are risk-neutral and value each quality 20% more than sellers. Naturally, each seller knows the quality of the good he sells, but quality is not observable to buyers prior to purchase. Assume that there are more buyers than sellers.

- (a) Determine the market supply and the average quality of the cars offered at each price.
- (b) Calculate the market equilibrium.