Microeconomics – PPD M1

Problem set for 20 December 2024 tutorial

Answers should be precise, complete and as short as possible.

Problem 1. Collusion and multi-market contact (12 points)

The following assumptions are made throughout the problem set.

- We consider two markets labelled Market A and Market B.
- In each market, there is a continuum of consumers with total mass 1. In each market, consumers have unit demand, with a uniform willingness to pay $V > 0$ for one unit of the good. Consumers in Market A care only about product A, consumers in Market B care only about product B - they are separate sets of consumers.
- In Market A, two firms are active: Firm 1 and Firm 2.
- In Market B, three firms are active: Firm 1, Firm 2 and Firm 3. This means that Firms 1 and 2 can produce both Product A and Product B, whereas Firm 3 can produce Product B, but not Product A.
- Firms' production costs are zero for both products.
- Each firm maximises the sum of its future discounted profits, on the basis of a rate of time preference δ, with $0 < \delta < 1$. In other words, an amount X earned k periods from now is valued $\delta^k X$ today.

Question 1. If there were a single firm active in Market A, or in Market B (a monopolist), which price would it set? What would the sum of the monopolist's future discounted profits be?

Question 2. Going back to the above assumptions (with 2 firms in Market A and 3 in Market B), what is the Nash equilibrium of the one-period simultaneous price-setting game in each of these two markets?

Question 3. Consider Market A alone and consider an infinitely repeated game, such that in each period both firms simultaneously set prices.

(a) Describe the hypothetical collusive equilibrium based on the "tit for tat" strategy seen in class. Your description of the "candidate" collusive equilibrium should include a precise explanation on strategies after off-equilibrium deviations (i.e.,

what happens after a deviation, and why that would be an equilibrium of a post-deviation subgame).

(b) For which values of δ is this candidate equilibrium indeed an equilibrium?

Question 4. Answer the same questions for Market B (considered in isolation).

Question 5. We consider now a possible "collusive linkage" between both markets. This expression means that we will consider a candidate equilibrium involving strategies that establish a link between both markets. The goal of this question is to check for which values of δ this candidate equilibrium is indeed an equilibrium, and what it implies in terms of the overall feasibility of collusion when some firms compete in several markets.

We consider a candidate equilibrium characterised by the following strategies.

- In any period, all firms set the monopoly prices in both markets (namely, V) unless in some earlier period at least one firm did not set the monopoly price in one market or the other or both.
- In Market A, Firms 1 and 2 have equal market shares, implying that each sells exactly a quantity ½.
- In Market B, Firm 3 has market share $s < 1$ (it sells a quantity s), and Firms 1 and 2 divide equally the remaining market share, meaning that each has market share $\frac{1-s}{2}$ (each sells a quantity $\frac{1-s}{2}$).
- If in any period, in either market, a firm sets a price different from V, then in all subsequent periods all firms set a price equal to zero, *in both markets*. This creates a link between Markets A and B, because a deviation in one market would lead to zero profits in both markets forever.

(a) Write the condition ensuring that Firm 3 has no incentive to undercut its competitors and prefers to play the equilibrium collusive strategy (and get market share s). Express the answer as a lower bound on δ, as a function of s.

(b) Among all possible deviations away from the strategy prescribed by the candidate equilibrium, which would be most profitable for Firm 1?

(c) Write the condition ensuring that Firm 1 has no incentive to undertake such a deviation, given the above characterisation of what would happen after a

deviation (last bullet point before Question a). Express it as a lower bound on δ, as a function of s. Obviously the same condition holds for Firm 2.

(d) Collusion is possible only if no firm has an interest in deviating, that is, if the conditions found in your answers to (a) and to (c) are both satisfied. Under which conditions on δ is collusion possible? Express the answer as a lower bound on δ, as a function of s.

(e) Collusion is most likely if the lower bound found in (d) is lowest. For which value of s is collusion most likely? Your reply should be based on an exact calculation, but you are expected to also explain the economic intuition behind the result.

Problem 2. Price competition and investment in cost reduction (8 points)

We consider the standard model of price competition seen in class, with two firms. As a reminder, its main results are as follows, with obvious notations (*t* being the differentiation parameter, exogenously given).

- Firm i's demand function is $D_i(p_1, p_2) = \frac{1}{2} + \frac{p_j - p_i}{2t}$, with $i \neq j$.
- Firm i's price in equilibrium is $p_i^* = t + \frac{2c_i + c_j}{3}$

We assume that each firm can invest in the reduction of its marginal cost according to the following technology: iIf a firm (Firm 1 or Firm 2) does not invest in cost reduction, then its marginal cost is $c > 0$ (the same for both firms). A firm can reduce it marginal cost from c to $c - \delta$ by investing $\gamma(\delta) = \alpha\delta^2$ ($\alpha > 0$ is "large enough" so that you do not have to care about costs being negative). γ is the "investment cost function".

Question 1. We consider the following game. In Period 1, both firms choose their level of cost reduction. The cost reduction level chosen by one firm is *not observable* by the other firm. In Period 2, both firms simultaneously set prices.

(a) Since the game is symmetric, we consider only a hypothetical equilibrium such that, in the first period, both firms choose the same cost reduction level δ^*. What is each firm's price and sales volume in that hypothetical equilibrium (possibly as a function of δ^*)?

(b) Assume that Firm 1's contemplates a very small increase ε in its cost reduction, from δ^* to $\delta^* + \varepsilon$. What is the corresponding extra investment cost (based on the derivative of the investment cost function)? What is the corresponding

increase in profit thanks to the reduction in production costs? *Hint:* to answer this last question, you may apply the marginal cost reduction to the sales volume found in your answer to to question (a).

(c) In equilibrium, the extra investment cost and the extra profit thanks to the reduction in production costs should cancel out. Using your answer to (b), derive the equilibrium value of the cost reduction δ^*.

Question 2. We consider the same game, with one crucial difference: the cost reduction level chosen by a firm in Period 1 is *observed* by the other firm before the beginning of Period 2, that is, before firms set prices.

(a) Since the game is symmetric, we consider only a hypothetical equilibrium such that, in the first period, both firms choose the same cost reduction level δ^{**}. What is each firm's price, margin, and sales volume in that hypothetical equilibrium?

(b) Assume that Firm 1's contemplates a very small increase ε in its cost reduction, from δ^* to $\delta^* + \varepsilon$. What is the impact on the price set by Firm 2 in Period 2?

(c) What is the impact of Firm 2's price increase on the demand for Firm 1's product?

(d) What is the impact of Firm 2's price increase on Firm 1's profit?

(e) The hypothetical very small increase ε in Firm 1's cost reduction now generates three effects: the two effects quantified in Question 1, which remain identical (namely, a small increase in the investment cost paid by Firm 1, and a small decrease in production costs). But now, there is also a third effect, the one covered in questions (b), (c) and (d) above, related to the impact on Firm 2's prices and, indirectly, on Firm 1's profits. In equilibrium, the net effect of these three mechanisms should be zero. Using your answers to Questions 1a), 1b), and 2d), derive the equilibrium value of the cost reduction δ^{**}.

Question 3. Compare δ^* and δ^{**}. Could this result be expected without any calculation? Explain the economic intuition.