transformation

$$P' = P + t = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

uniform scaling

$$P' = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= \mathrm{scale}(\alpha) \cdot P$$

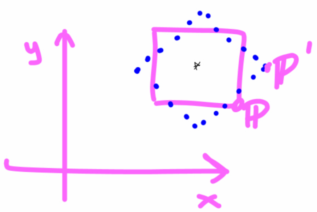
Rotation

$$P' = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} P$$

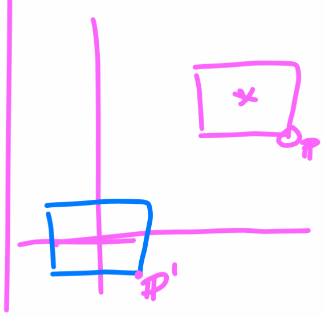
$$= \begin{pmatrix} c\alpha & -s\alpha \\ s\alpha & c\alpha \end{pmatrix} P = \mathrm{Rot}(\alpha) \cdot P$$

"Defining more general transforms"

Ex: Rotation w.r.t arbitrary center $C = \begin{pmatrix} c_x \\ c_y \end{pmatrix}$

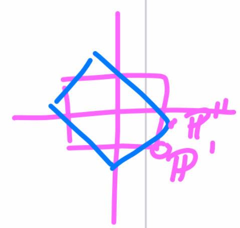


1. Translate $(-C)$



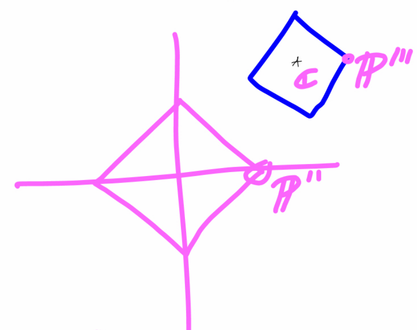
$P' = P - C$

2. Rotate (α)



$P'' = \mathrm{Rot}(\alpha) \cdot P'$

3. Translate $(+C)$

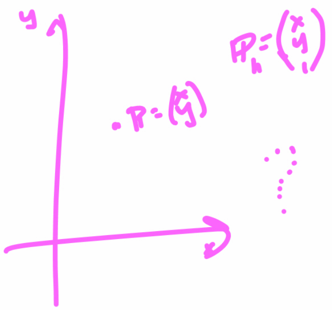


$P''' = P'' + C$

<u>Goal</u>: Eliminate need to write translation vector 'explicitly'

$\Rightarrow$ Want just $\underline{1}$ matrix to define all linear transforms

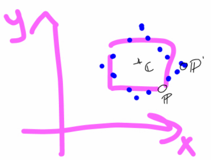
<u>Homogeneous</u> coordinates permit exactly this!



Write all transform with $\underline{1}$ matrix:

$$\mathbb{P}' = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & t_x \\ m_{21} & m_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Ex: Rotation w.r.t. center $\mathbb{C}$ in HOMOGENOUS form



1) Translate$(-\mathbb{C})$
2) Rotate(α)
3) Translation$(+\mathbb{C})$

$$\mathbb{P}' = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \text{Translation}(\mathbb{C}) \cdot \text{Rotation}(\alpha) \cdot \text{Translate}(-\mathbb{C}) \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -C_x \\ 0 & 1 & -C_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C\alpha & -S\alpha & -C_x C\alpha + C_y S\alpha \\ S\alpha & C\alpha & -C_x S\alpha - C_y C\alpha \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

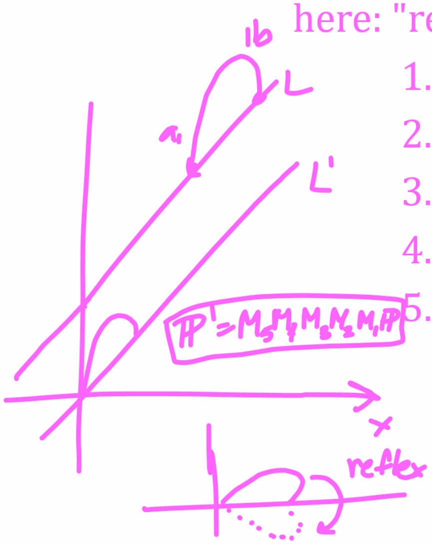
$$= \begin{pmatrix} C\alpha & -S\alpha & -C_x C\alpha + C_y S\alpha \\ S\alpha & C\alpha & -C_x S\alpha - C_y C\alpha \\ 0 & 0 & \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = M \cdot \mathbb{P}$$

Ex3: Refl. w.r.t any line L

(-> reduce to refl. w.r.t. coordinate system axis)

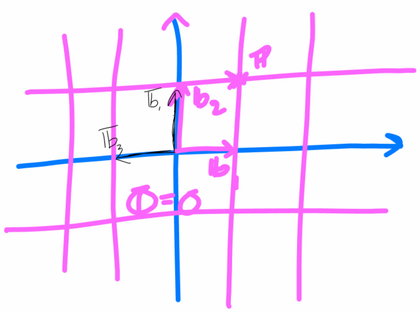
here: "reduce" to refl. w.r.t. x-axis:

1. Translate $(-\mathbb{C})$ $= \begin{pmatrix} 1 & 0 & -a_x \\ 0 & 1 & -a_y \\ 0 & 0 & 1 \end{pmatrix} = M_1$
2. Rotate $(-\alpha)$ $= \begin{pmatrix} c(-\alpha) & -s(-\alpha) & 0 \\ s(-\alpha) & c(-\alpha) & 0 \\ 0 & & 1 \end{pmatrix} = M_2$
3. Reflx $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = M_3$
4. Rotate$(+\alpha)$ $= \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = M_4$
5. Translate$(+\mathbb{C})$ $= \begin{pmatrix} 1 & 0 & a_x \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{pmatrix} = M_5$



$\boxed{\mathbb{P}' = M_5 M_4 M_3 M_2 M_1 \mathbb{P}}$

<u>Coordinates</u>



"Rotate" Sys1 by $\alpha(=90°)$

$Sys1 = \{\mathbb{O}, \mathbb{b}_1, \mathbb{b}_2\}$

$Sys2 = \{\bar{\mathbb{O}}, \bar{\mathbb{b}}_1, \bar{\mathbb{b}}_2\}$

$\mathbb{P}_{sys1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\mathbb{P}_{sys2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

To obtain $\mathbb{P}$'s coordinates rel. to sys2, apply Rotate$(-90°)$ to $\mathbb{P}$ sys1

$$\mathbb{P}_{sys2} = \text{Rotate}\left(-\tfrac{\pi}{2}\right) - \mathbb{P}_{sys1}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{c} 1 \\ 1 \\ \hline 1 \end{array}\right) = \left(\begin{array}{c} -1 \\ 1 \\ \hline 1 \end{array}\right)$$