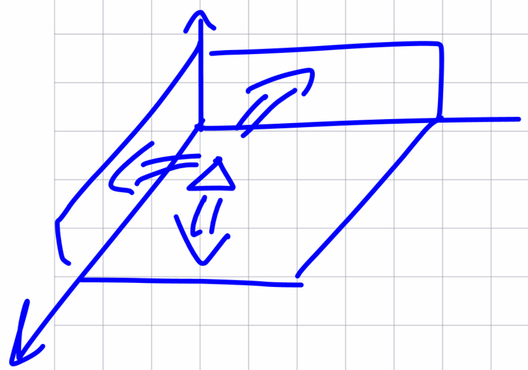
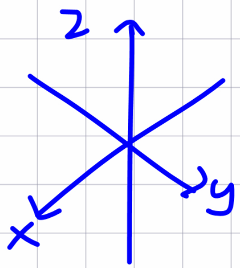
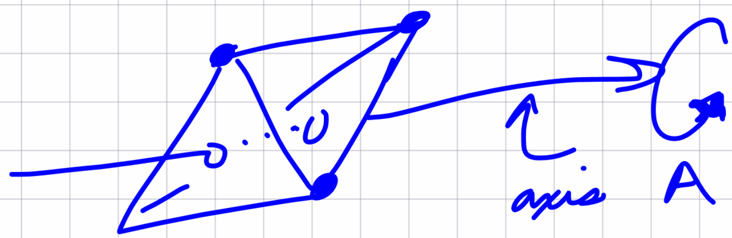
Axonometric Projections

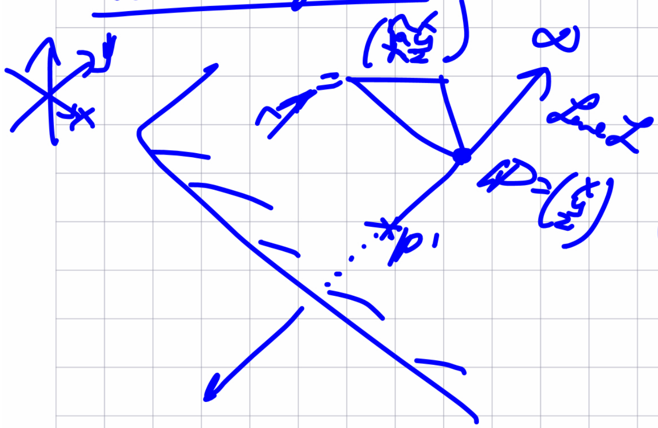


= Create "animations" by rotating an object by $0°, 1°, 2°, 3°, \ldots, 360°$ around an axis in 3D space and after each incremental rotation, perform an axonometric projection!





"Generalization": Ortho Projection onto arbitrary plane:



① Plane P given implicitly:

$$Ax + By + Cz + D = 0$$

② Line L defined parametrically

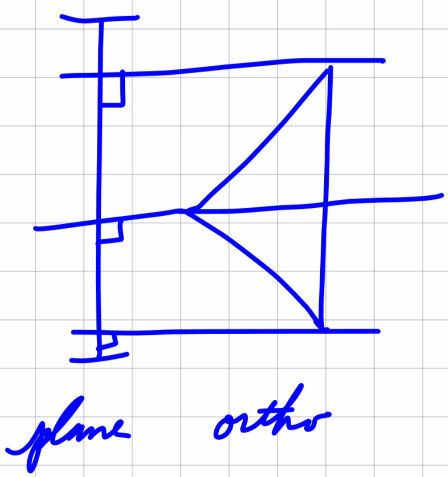
$$\mathbb{P}(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

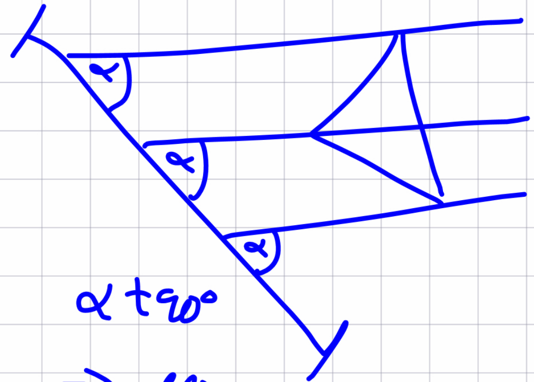
=> Intersection point $\mathbb{P}'$: insert ② into ①

$$A(x + tn_x) + B(y + tn_y) + C(z + tn_z) + D = 0$$

$x = \ldots$ ⟹ insert this t-value into ② => p'

Oblique Projection

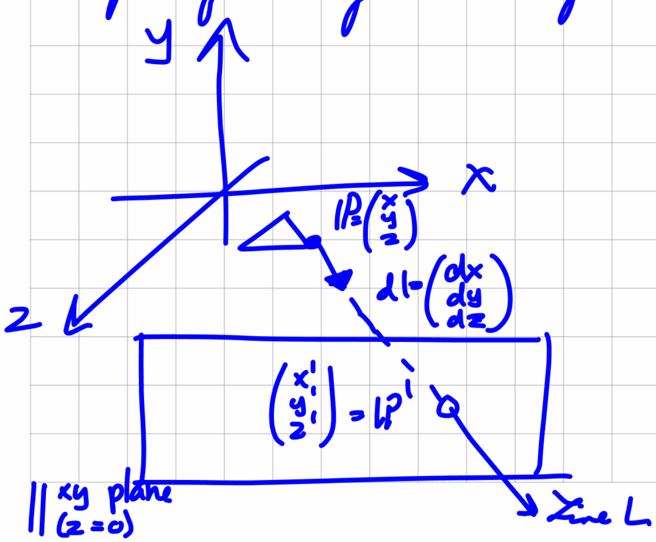




$\alpha \neq 90°$

$\Rightarrow$ oblique

Specifically: Oblique projection onto xy plane $(z=0)$:



given: xy-plane $(z=0)$ as projection plane
point $\mathbb{P}$, direction vector d
line L in parametric form:

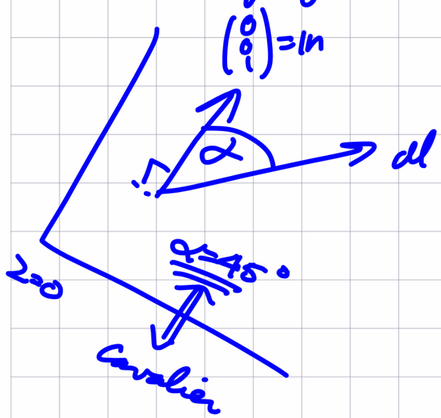
$$\mathbb{P}(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t \cdot \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \overset{!}{=} \begin{pmatrix} x' \\ y' \\ 0 \end{pmatrix}$$

$$t = -z/dz$$

$$\Rightarrow OBL: \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 0 & -dx/dz & 0 \\ 0 & 1 & -dy/dz & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Important : oblique projection Cavalier, Cabinet

① Cavalier projection



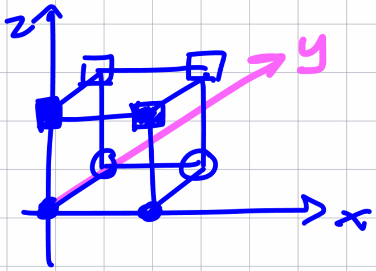
given: $\mathbb{n} = \begin{pmatrix}0\\0\\1\end{pmatrix}$, $dl \begin{pmatrix}1\\1\\\sqrt{2}\end{pmatrix}$

$\measuredangle(\mathbb{n}, dl) \stackrel{?}{=} 45°$

$\left\{ \text{check } \cos(\alpha) = \mathbb{n} \cdot dl / \| = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 45° \right\}$

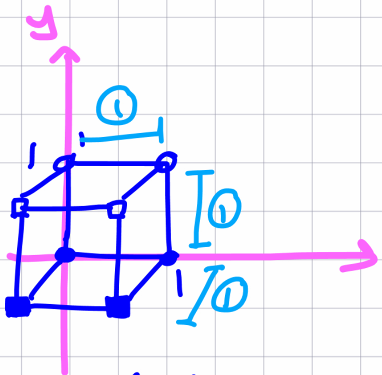
$$\Rightarrow CAV = \left(\begin{array}{ccc|c} 1 & 0 & -\sqrt{2}/2 & 0 \\ 0 & 1 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

Example: Project a unit cube



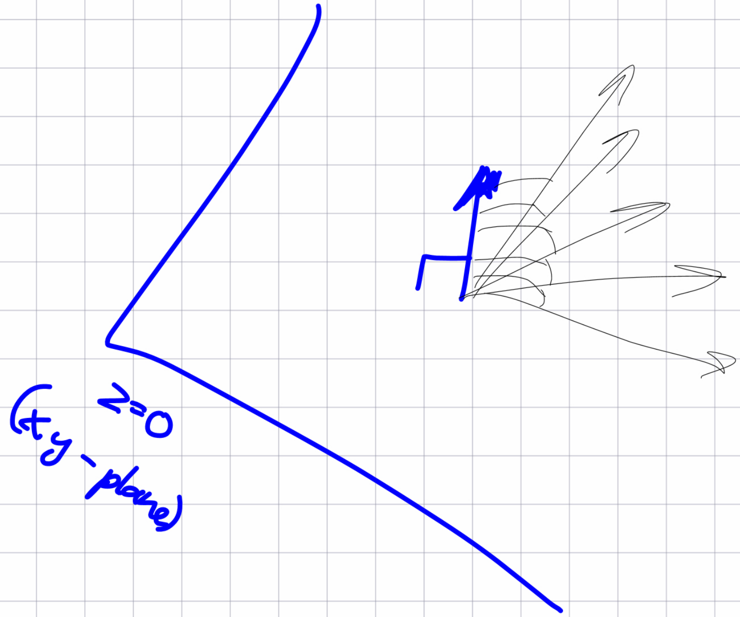
table

x	y	z	x'	y'
0	0	0	0	0
1	0	0	1	0
0	1	0	0	1
1	1	0	1	1
0	0	1	$-\sqrt{2}/2$	$-\sqrt{2}/2$
1	0	1	$1-\sqrt{2}/2$	$-\sqrt{2}/2$
0	1	1	$-\sqrt{2}/2$	$1-\sqrt{2}/2$
1	1	1	$1-\sqrt{2}/2$	$1-\sqrt{2}/2$



length preserved in projection

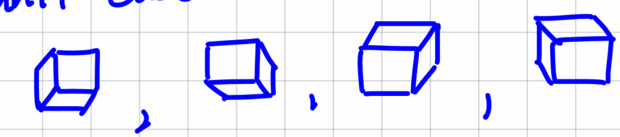
But: there is an infinite # of all-vectors producing valid CAV projections onto xy-plane



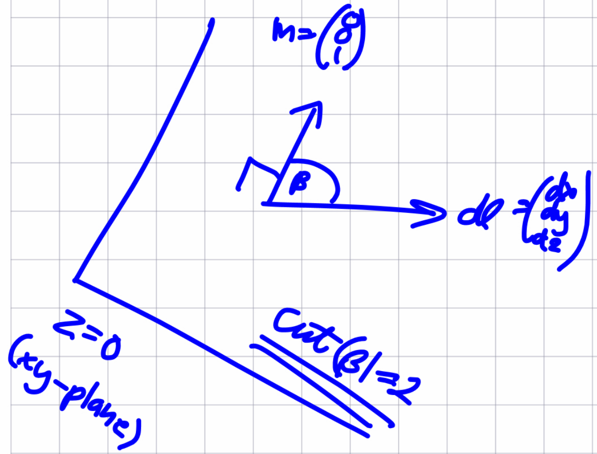
$\Rightarrow$ can rotate $\begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix}$ around z-axis using 1° increments

$\Rightarrow$ set of Cavalier projections

unit cube



Cabinet Projection



example: $n \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $dl \begin{pmatrix} 1 \\ 1 \\ 2\sqrt{2} \end{pmatrix}$

$\{$ check: $\measuredangle(n, dl) = \dots = \dots \beta \Rightarrow \cot(\beta) = 2 \}$

$$CAB = \left(\begin{array}{ccc|c} 1 & 0 & -\sqrt{2}/4 & 0 \\ 0 & 1 & -\sqrt{2}/4 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

Ex: CAB .. projection of unit cube

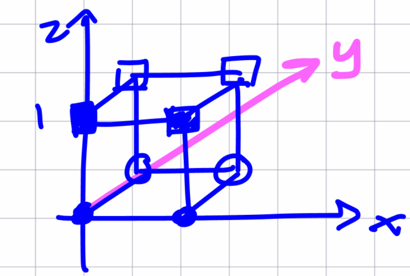
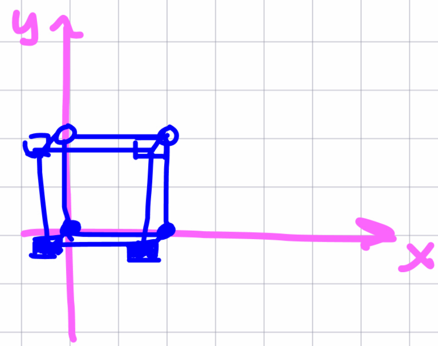
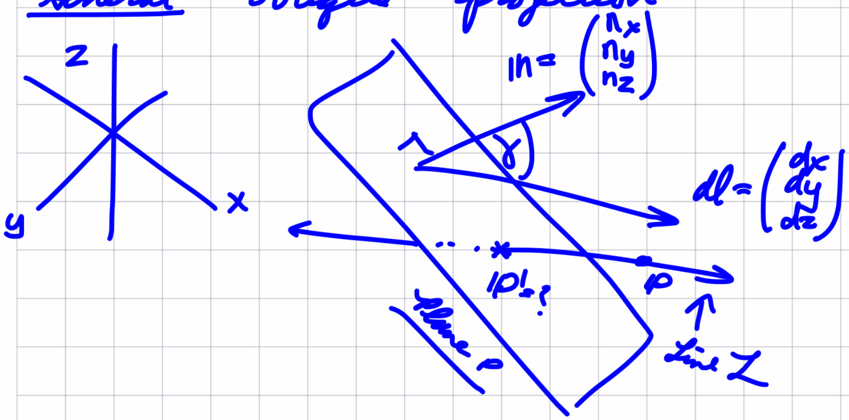


table:

x	y	z	x'	y'
0	0	0	0	0
1	0	0	1	0
0	1	0	0	1
1	1	0	1	1
0	0	1	$-\sqrt{2}/4$	$-\sqrt{2}/4$
1	0	1	$1-\sqrt{2}/4$	$-\sqrt{2}/4$
0	1	1	$-\sqrt{2}/4$	$1-\sqrt{2}/4$
1	1	1	$1-\sqrt{2}/4$	$1-\sqrt{2}/4$



General oblique projection:



① Plane $\mathcal{P}$

$$Ax + By + Cz + D = 0$$

② $$p(x) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \mathbb{p} + t \cdot d\ell$$

$$= \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t \cdot \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

insert ② into ①:

$$A(x + t\,dx) + B(y + t\,dy) + C \cdot (z + t\,dy) + D = 0$$

$\underline{\underline{t = \ldots}}$ $\Rightarrow$ insert t into ②

$\Rightarrow$ obtain $\underline{\underline{\mathbb{P}'}}$

renderman camera viewing model