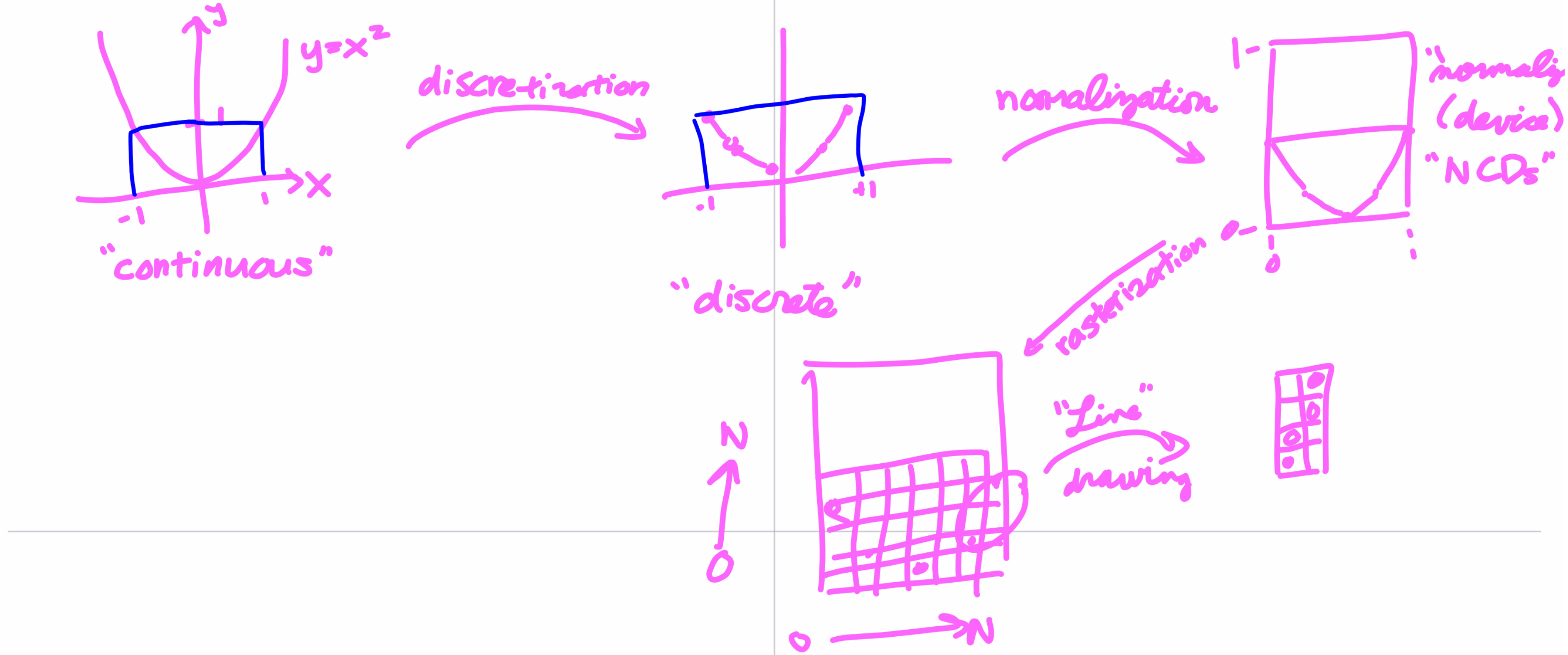
Lines, Curves, ...



Line Drawing

Two "famous" methods:

i) DDA – digital differential analysis ← floats, slow

ii) Bresenham's algorithm ← integer only, Fast

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i) DDA Just low-slope lines, i.e.,

$0 < m < 1$, with m coming from $y = mx + b$

move from first pixel "1 by 1" in x-direction (i-direction)

for (i=6) to (i=9)

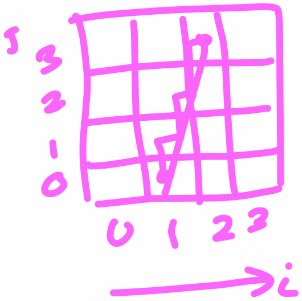
1) "determine correct y-value @ i";

2) "turn on the correct pixel";

Numerically

i	y-value (float)	pix
6	$11 + 0 \cdot m = 0$	(6, 11)
7	$11 + 1 \cdot m = 11\frac{1}{3}$	(7, 11)
8	$11 + 2 \cdot m = 11\frac{2}{3}$	(8, 12)
9	$11 + 3 \cdot m = 12$	(9, 12)

But: steep lines — $|1 < m < \infty|$



$(x_1, y_1) = (1,0)$

$(x_2, y_2) = (2,3)$

$$\left.\begin{array}{l}\Delta x = 1\\ \Delta y = 3\end{array}\right\} \Rightarrow m = \frac{\Delta y}{\Delta x} = \left(\frac{3}{1}\right) = 3$$

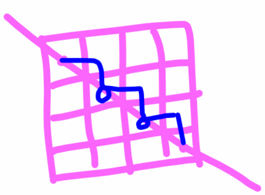
$\Rightarrow$ define $(\hat{m}) = \frac{1}{m} = 1/3$

for (j=0) to (j=3 do

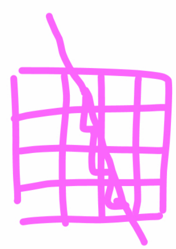
1) determine exact x-value;

2) turn on current pixel

But: Negative slopes



$-1 < m < 0$



Summary

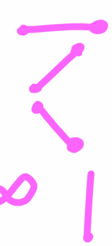
"special cases":

$m = 0$

$m = 1$

$m = -1$

$m = \pm\infty$



1) "Difficult Cases"

$0 < m < 1$

$1 < m < \infty$

$-1 < m < 0$

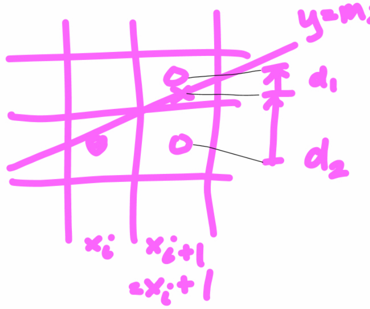
$-\infty < m < -1$

Bresenham's Algo.

Case: $0 < m < 1$

= Means: "Move from left to right 1 by 1 and decide whether to go up by 1 for a specific i-location"

$y = mx + b = \frac{\Delta Y}{\Delta x} x + b$



"Test" If $(d_1 - d_2) < 0$

Then $y_{i+1} = y_i$

else $y_{i+1} = y_i + 1$

Compute exactly:

$$(d_1 - d_2) = m(x_i + 1) + b - y_i - (y_i + 1 - \langle m(x_i + 1) + b \rangle)$$

$$= 2mx_i - 2y_i + 2m + 2b - 1 \quad \Big| \cdot \Delta x \text{ on both sides}$$

$$\Delta x \cdot (d_1 - d_2) = 2\Delta Y x_i - 2\Delta x y_i + \underbrace{\Delta x \cdot (2m + 2b - 1)}_{C\text{onstant}}$$

$$\Leftarrow = 2\Delta Y x_i - 2\Delta x y_i + C =: P_i$$

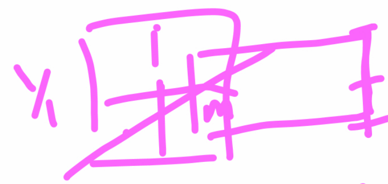
Get rid of "C" (a fact!)

$P_i = 2\Delta Y x_i - 2\Delta X y_i + C$

$P_{i+1} = 2\Delta Y x_i - 2\Delta X y_{i+1} + C$

$P_{i+1} = P_i + 2\Delta Y - 2\Delta X (y_{i+1} - y_i)$

$P_1 = 2\Delta Y - \Delta X$

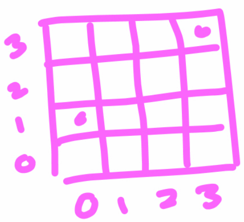


$\left.\begin{array}{l} 1-m = d_2 \\ m = d_1 \end{array}\right\} \Rightarrow$

$P_1 = \Delta X(m-(1-m)) = \Delta X(2m-1)$

$= 2\Delta Y - \Delta X$

Examples



$\left.\begin{array}{l} (x_1, y_1) = (0,1) \\ (x_2, y_2) = (3,3) \end{array}\right\} \Rightarrow \Delta X = \underline{3}, \Delta Y = \underline{2}$

Step i	PIX	Predicates P_i
1	(0,1)	$P_1 = 4 - 3 = 1 \geq 0 \Rightarrow +1$
2	(1,2	$P_2 = 1 + 4 - 6\cdot(1) = -1 < 0 \Rightarrow \checkmark$
3	(2,2	$P_3 = -1 + 4 - 6(0) = 3 \geq 0 \Rightarrow +1$
4	(3,3	P_4 Done