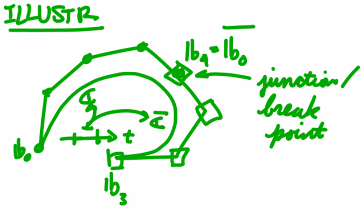
Composite Bezier Curves

→ "Building complicated shapes using simple 'building block'"

POLYNOMIAL CURVES = BEZIER CURVES

① Connecting 2 Bézier Curves of <u>different degrees</u>

ILLUSTR



$\mathbb{C}(t) = \sum_{i=0}^{m} b_i B_i^m(t)$

$\bar{\mathbb{C}}(t) = \sum_{i=0}^{n} \bar{b}_i B_i^n(t)$, $t \in [0,1]$

⇒ Conditions for "continuity" @ junction point:

i) C^0-continuity: $\mathbb{C}$ & $\bar{\mathbb{C}}$ are continuous @ junction point:

⇒ $\mathbb{C}(1) = b_m$

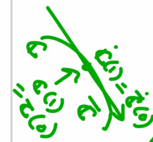
$\bar{\mathbb{C}}(0) = \bar{b}_0$ "position continuity"

ii) C^1-continuity: (in addition to being C^0 continuous)

⇕ continuity of velocity

$\mathbb{C}$ & $\bar{\mathbb{C}}$ have same 1^{st} derivative @ junction point

⇒ 1^{st} derivative of $\mathbb{C}$ & $\bar{\mathbb{C}}$:



$\dot{\mathbb{C}}(1) = m(b_m - b_{m-1})$

$\dot{\bar{\mathbb{C}}}(0) = n(\bar{b}_1 - \bar{b}_0) = n(\bar{b}_1 - b_m)$

$m(b_m - b_{m-1}) = n(\bar{b}_1 - b_m)$

$(m+n)\, b_m = m\, b_{m-1} + n\, \bar{b}_1$

$$\boxed{b_m = \frac{m}{m+n} b_{m-1} + \frac{n}{m+n} \bar{b}_1}$$

condition for C^1-continuity

⇒ geometrical meaning:

Ex

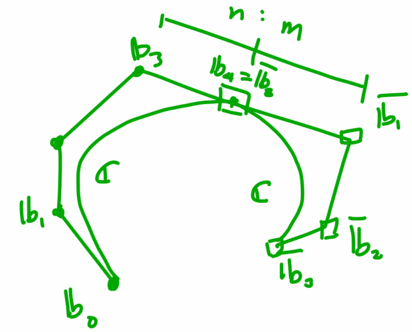
$m = 4$

$n = 3$

ratio

$n : m$

$= 3:4$



iii) C^2-continuity: c & $\bar{c}$ are C^0 and C^1-continuity and have same 2^{nd} derivative @ junction point

$\Updownarrow$

"continuity of acceleration"

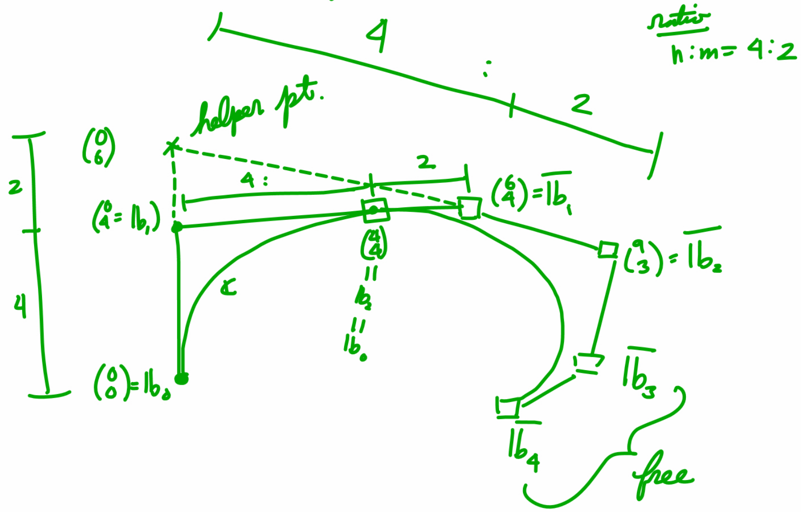
$$\Rightarrow \ddot{c}(1) = m(m-1)(b_m - 2b_{m-1} + b_{m-2})$$

$$\ddot{\bar{c}}(0) = n(n-1)(\bar{b}_2 - 2\bar{b}_1 + \bar{b}_0)$$

. . .

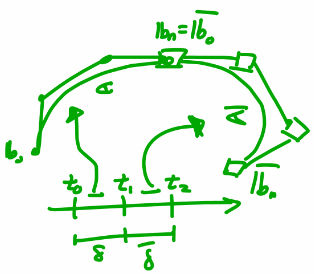
$\Rightarrow$ 'simple' geometric meaning:

Ex of C^2 continuity: $m=2,\ n=4$



② Merging 2 Bézier Curves of same degree, but defined over 2 different (time) parameter intervals: $[t_0, t_1] \sim c$, $[t_1, t_2] \sim \bar{c}$

ILLUSTR



i) C^0-continuity: $\boxed{\bar{b}_0 = b_n}$

ii) C^1-continuity:

$$\dot{c}(t_1) = \frac{n}{\delta}(b_n - b_{n-1})$$

$$\dot{\bar{c}}(t_1) = \frac{n}{\bar{\delta}}(\bar{b}_1 - \bar{b}_0) = (\bar{b}_1 - b_n)$$

$$\frac{n}{\delta}(b_n - b_{n-1}) = \frac{n}{\bar{\delta}}(\bar{b}_1 - b_n)$$

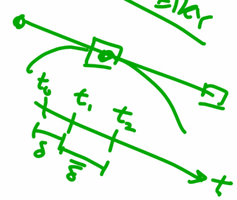
$$\bar{\delta}(b_n - b_{n-1}) = \delta(\bar{b}_1 - b_n)$$

$$(\delta + \bar{\delta}) b_n = \bar{\delta} b_{n-1} + \delta \bar{b}_1$$

$$\boxed{b_n = \frac{\bar{\delta}}{\delta + \bar{\delta}} b_{n-1} + \frac{\delta}{\delta + \bar{\delta}} \bar{b}_1}$$

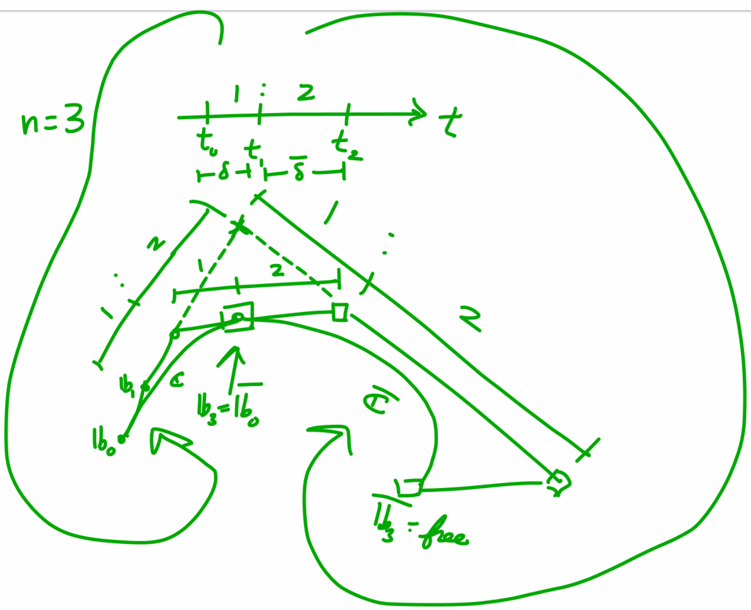
CONDITION FOR C^1-CONTINUITY

GEOMETRY



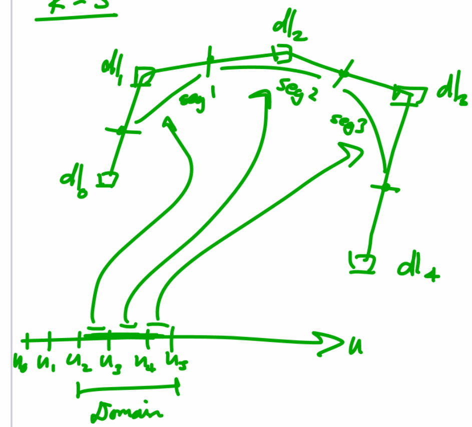
iii) C^2 continuity condition:

$$\ddot{c}(t_1) = \ldots \quad \ddot{\bar{c}}(t_1) = \ldots \Rightarrow \ldots \Rightarrow$$ geometrical meaning:



B-spline Curve

$k=3$



· control pnts: $d_0, \ldots, d_n$

(k) "order";

$k \in \{1, 2, 3 \ldots\}$

· knot sequence / knots:

$u_0, u_1, \ldots, u_{n+k}$