More B-splines

Def: A B-spline curve is defined as

$$\mathbb{C}(u) = \sum_{i=0}^{n} d_i \, N_i^k(u), \text{ where}$$

- k is the order of the curve;
- $d_0, \ldots, d_n$ are the coefficients / control points
- $u_0 < u_1 < \ldots < u_{n+k}$ is the knot-sequence; &
- $N_i^k(u)$ are normalized B-spl-basis fcts.

Further, the functions $N_i^k(u)$ are defined as

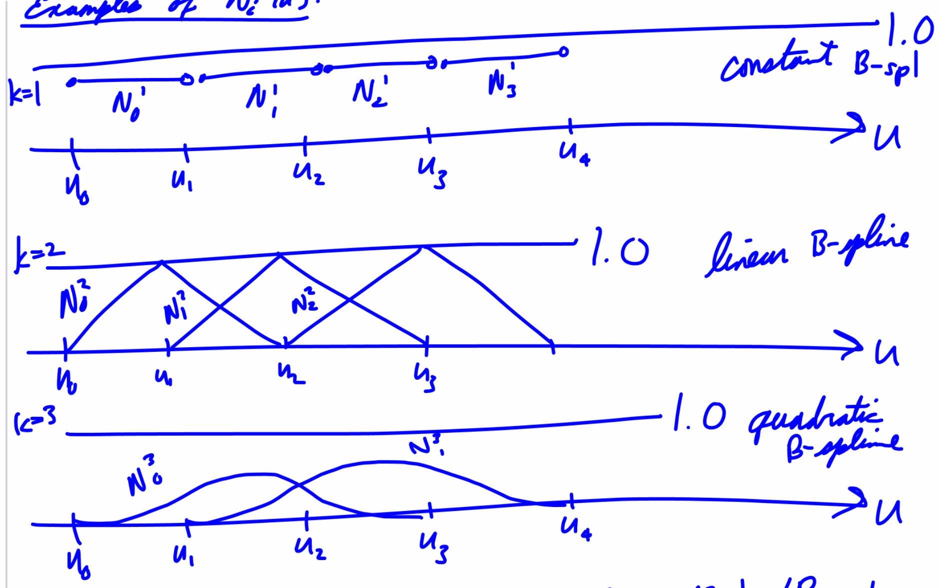
$$\boxed{N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u)}$$

for $k > 1$,

and

$$N_i^1(u) = \begin{cases} 1, & \text{if } u \in [u_i, u_{i+1}) \\ 0, & \text{otherwise} \end{cases}$$

Examples of $N_i^k(u)$:



Beatty / Barsky / Bartels

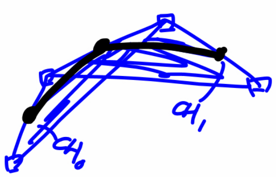
Carl de Boor

Important property of B-spline curves:

"Theorem": Each segment of a B-spline curve lies inside the convex hull CH of its defining $\leq k$ control points...

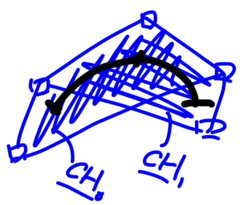
Ex:

$k=3$



$k=4$

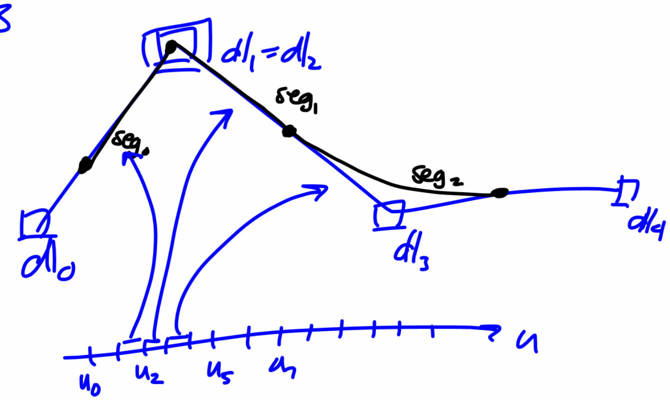
$$\boxed{\mathbb{C}(u) \subseteq \bigcup CH_i}$$



"Multiplicities"

① Control points with multiplicity greater than one

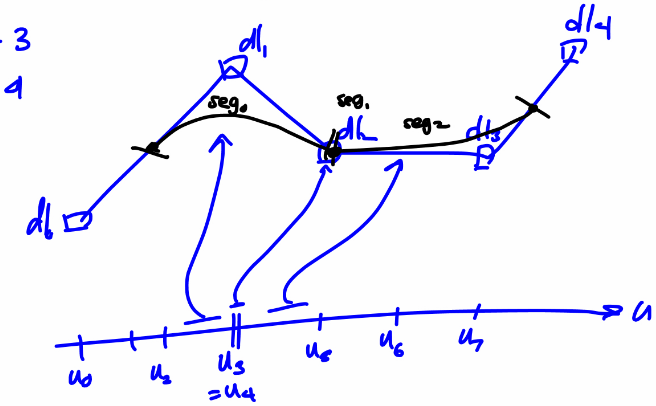
$k=3$



Knots with multiplicity > 1 :

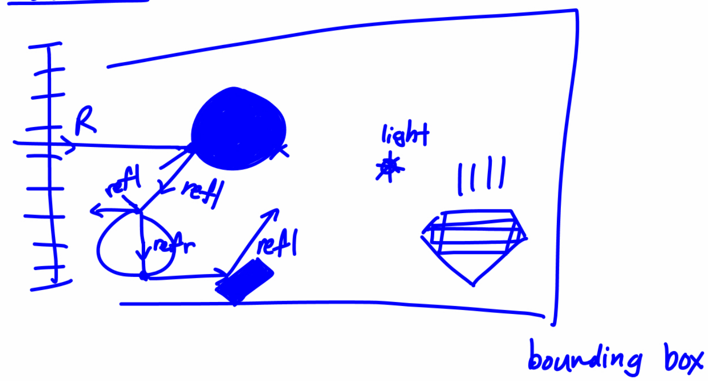
$k = 3$

$n = 4$



Illustration

expensive

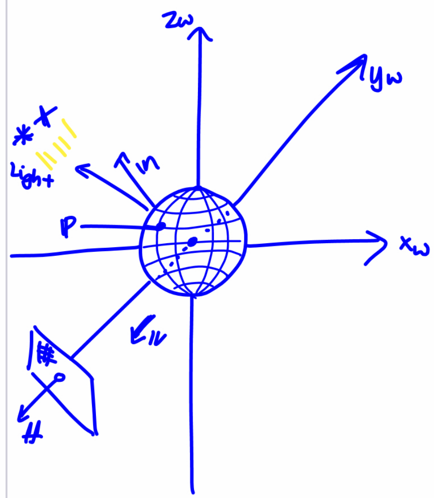


Ray Tracing

- Image based algorithm
- Traces rays 'backwards' into 3D scene, by tracing a ray through each pixel (I, J)
- Combines direct illumination (Phong model) and global illumination (reflection, transmission through transparent objects / refractions)

Example:

Ray-trace SIMPLE UNIT SPHERE:



Set-up: (camera viewing model)

camera + phong parameters

$\mathbb{E} = \begin{pmatrix}0\\-\infty\\0\end{pmatrix}$, $al = \begin{pmatrix}0\\0\\0\end{pmatrix}$

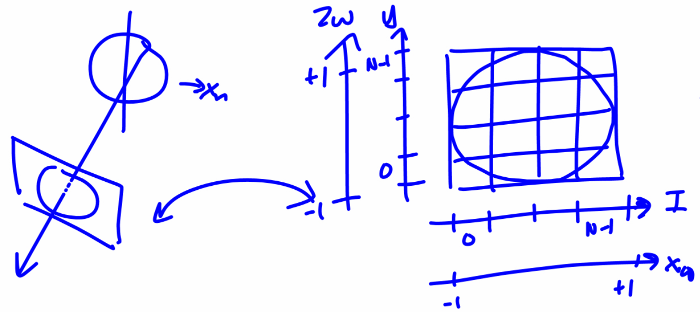
$\mathbb{V} = \begin{pmatrix}0\\-1\\0\end{pmatrix}$, $\mathbb{P}$ pnt on sphere, $\mathbb{n}(p) = \mathbb{P}$,

$\mathbb{L} = \mathbb{x} - p / ||\mathbb{x} - p||$

$\mathbb{r} = -\mathbb{L} + 2(\mathbb{n} \cdot \mathbb{L})\mathbb{n}$

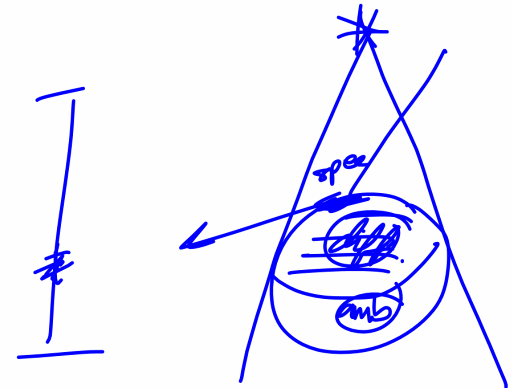
Sphere definition: $x_w^2 + y_w^2 + z_w^2 = 1$

→ ray-trace such that sphere occupies maximal area on a N by N screen:



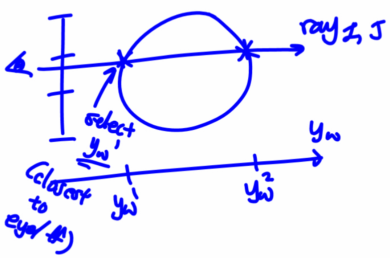
$$\underline{I} \mapsto x_w = \frac{2I}{N-1} - 1$$

$$\underline{J} \mapsto z_w = \frac{2J}{N-1} - 1$$



need y_w coordinate:

2D sketch



$$x_w^2 + y_w^2 + z_w^2 = 1$$

$$y_w = \pm\sqrt{1 - x_w^2 - z_w^2}$$

$$= \pm\underbrace{\sqrt{1 - \left(\frac{2I}{N-1} - 1\right)^2 - \left(\frac{2J}{N-1} - 1\right)^2}}_{\geq 0}$$