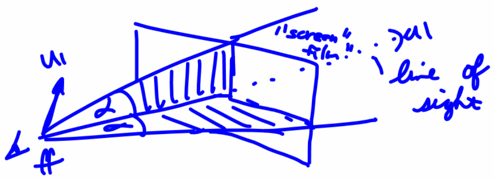
Camera Viewing Model



ff from point

al at point

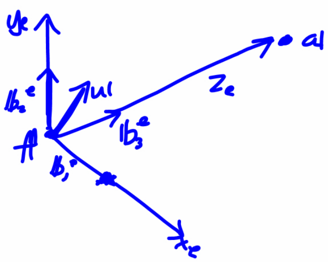
ul up point

α vertical angle

Processing Pipeline / Data

1) Scene / geometry - in WORLD coords
2) Def. of camera model parameters
3) Establish eye coordinate sys.
4) Map WORLD coords. to EYE coords.
5) CLIP geometry against VIEWING FRUSTUM
6) Map from eye coords to
   i) SCREEN coords, then to
   ii) NDCs
7) Raster geometry in Pixel space

· The Eye coord system



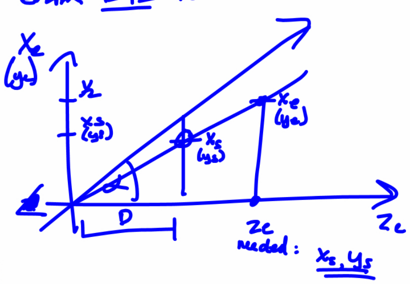
EYE

$$b_3^e = \frac{al - ff}{\|al - ff\|}$$

$$b_1^e = \frac{ul \times al}{\|ul \times al\|}$$

$$b_2^e = b_1^e \times b_3^e$$

· From EYE to SCREEN/NDC

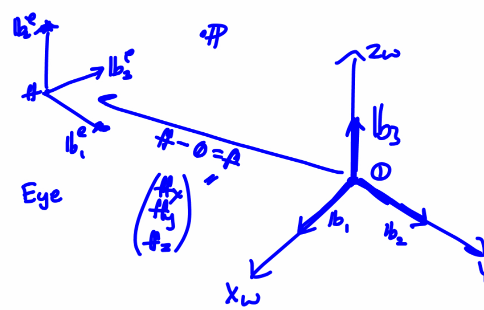


$$\tan\alpha = \frac{1}{2D} \Rightarrow D = \frac{1}{2\tan\alpha}$$

$$x_s = \frac{1}{2\tan\alpha}\ \frac{x_e}{z_e}$$

$$y_s = \frac{1}{2\tan\alpha}\ \frac{y_e}{z_e}$$

· From World to Eye coord



Map WORLD sys to EYE sys

① Change in orientation of b_i's

$b_i \mapsto b_i^e$

② Translate origin 0 to ff

matrices:

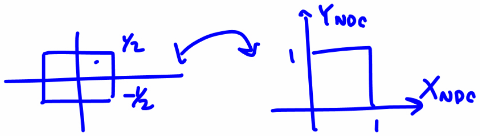
① $$\left(\begin{array}{ccc|c} [b_1^e] & [b_2^e] & [b_3^e] & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

② $$\left(\begin{array}{ccc|c} 1 & 0 & 0 & ff_x \\ 0 & 1 & 0 & ff_y \\ 0 & 0 & 1 & ff_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

Thus: Apply "inverses" of ① & ② in reverse order to P_w to obtain P_e:

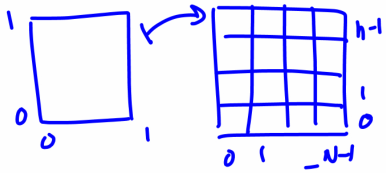
$$P_e = \left(\begin{array}{c|c} [\ b_1^e\ ] & 0 \\ [\ b_2^e\ ] & 0 \\ [\ b_3^e\ ] & 0 \\ \hline 0\ \ 0\ \ 0 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 0 & 0 & -ff_x \\ 0 & 1 & 0 & -ff_y \\ 0 & 0 & 1 & -ff_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right) P_w$$

· From Screen to NDC



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_s \\ y_s \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

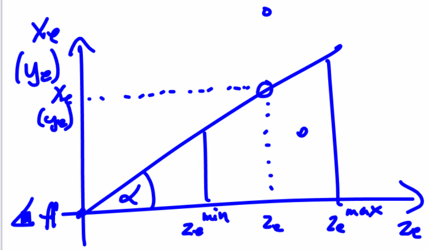
· From NDC to PIXELS



$$\begin{pmatrix} I \\ J \end{pmatrix} = \begin{pmatrix} \text{round}\left((N-1)\, X_{NDC}\right) \\ \text{round}\left((N-1)\, Y_{NDC}\right) \end{pmatrix}$$

CLIPPING (Done after mapping from WORLD to EYE)

→ need to understand which point is on boundary of viewing frustum



3 conditions must be met by met by point to lie inside frustum:

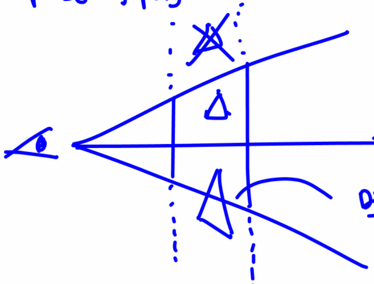
i) $0 < z_e^{min} < z_e < z_e^{max} < \infty$

ii) $-z_e \tan\alpha \leq x_e \leq z_e \tan\alpha$

iii) $-z_e \tan\alpha \leq y_e \leq z_e \tan\alpha$

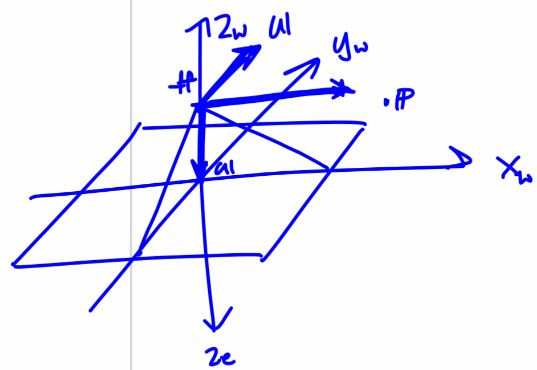
$$\tan\alpha = \frac{x_e}{z_e} \Rightarrow x_e = z_e \tan\alpha$$
$$\tan\alpha = \frac{y_e}{z_e} \Rightarrow y_e = z_e \tan\alpha$$

But: We must perform Clipping not only for isolated points but for polygons, polyhedra



Difficult Clip Polyhedron IN 3D

Example:



$$P_w = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$\alpha = 45° \Rightarrow \tan 45° = 1$

$f = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $a = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $u = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\Rightarrow b_3^e = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix},\ b_2^e = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},\ b_1^e = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$P_e = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = P_e$$

$$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = \frac{1}{2\tan\alpha} \begin{pmatrix} x_e/z_e \\ y_e/z_e \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \Bigg| \begin{pmatrix} X_{NDC} \\ Y_{NDC} \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Bigg\| \begin{pmatrix} I \\ J \end{pmatrix} = \begin{pmatrix} N-1 \\ N-1 \end{pmatrix}$$