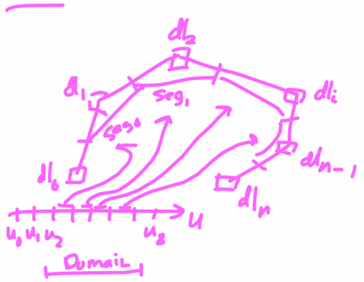
# B-splines – Schoenberg | Curvy | Carl R. de Boor

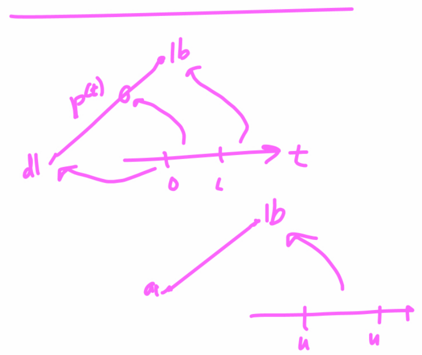
- are piecewise polynomial curves;
- support local curve/shape modeling;
- consist of multiple polynomial segments:

Illst:

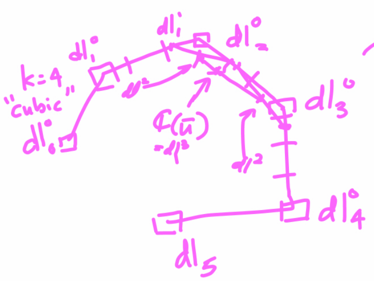


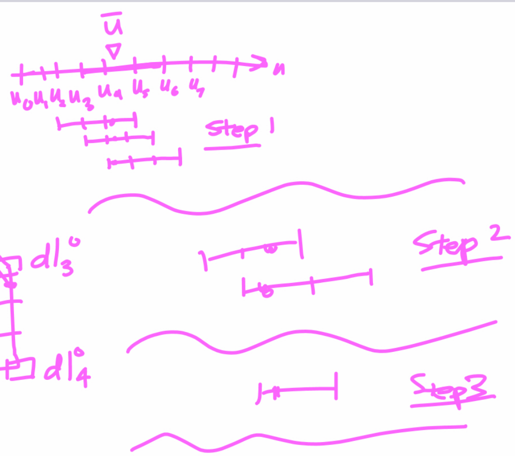

Defining parameters:

- $dl_0, dl_1, \dots, dl_n$ (n+1) de Boor control points
- $K \in \{1,2,3,\dots\}$ order
  /* $n \geq k-1$ */
- $u_0, u_1, \dots u_{n+k}$, $u_i \leq u_{i+1}$,
  knots / knots sequence

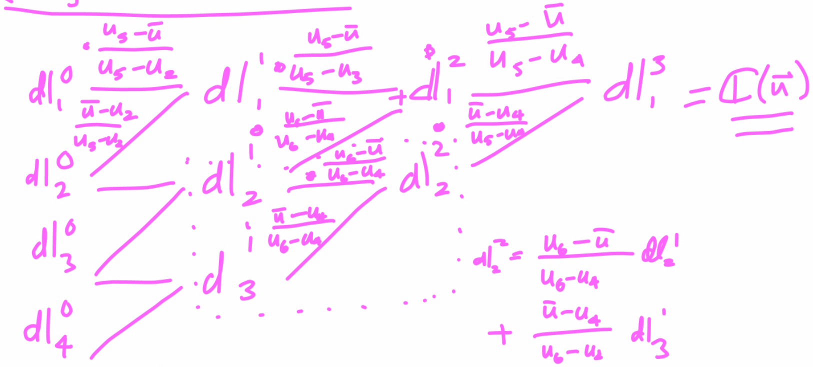



## de Boor Algorithm




## (triangular) de Boor scheme




$$dl_2^2 = \frac{u_6-\bar u}{u_6-u_4}\, dl_2^1 + \frac{\bar u-u_4}{u_6-u_4}\, dl_3^1$$

General:

$$dl_i^j = \frac{u_{i+k}-\bar u}{u_{i+k}-u_{i+j}}\, dl_i^{j-1} + \frac{\bar u-u_{i+j}}{u_{i+k}-u_{i+j}}\, dl_{i+1}^{j-1}$$

## de Boor Algorithm

**IN:** $d l_0, \dots d l_n$ ; $K$ ; $u_0 \le u_1 \le \dots \le u_{n+k}$ ; $\quad \bar{u} \in [u_{k-1}, u_{n+1}]$ "Domain"

**OUT:** $C(\bar{u})$

**Algo:** "determine index $I$ such that $\bar{u} \in [u_I, u_{I+1}]$ ;

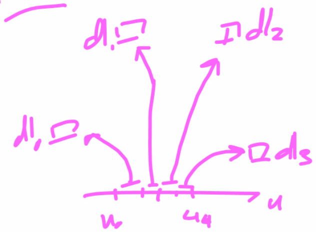
**for** $(j=1)$ to $(k-1)$ **do**

**for** $(i = (I-(k-1))$ to $(I-j)$

$$dl_i^j = \frac{u_{i+k} - \bar{u}}{u_{i+k} - u_{i+j}} \, dl_i^j + \frac{\bar{u} - u_{i+j}}{u_{i+k} - u_{i+j}} \, dl_{i+1}^{j+1}$$

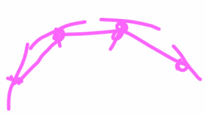
return $dl^{k-1}_{I-(k-1)}$ $= C(\bar{u})$

K=1




K=2

K=3



K=4