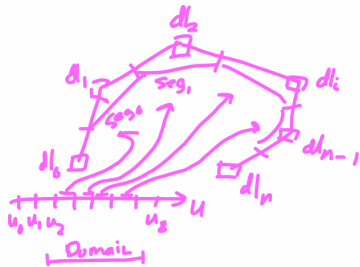


B-splines - Schoenberg Curve | Carl R. de Boor

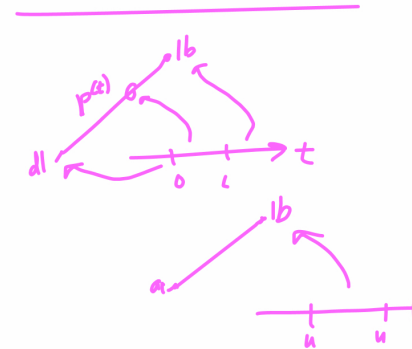
- are piecewise polynomial curves;
- support local curve/shape modeling;
- consist of multiple polynomial segments:

Illustration:

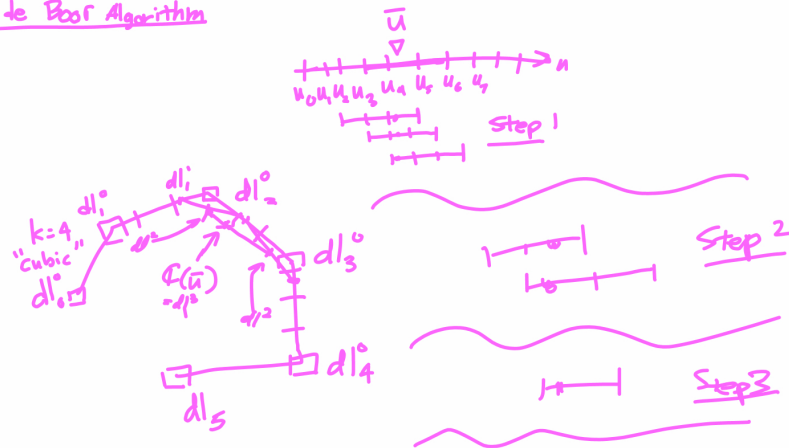


Defining parameters:

- d_0, d_1, \dots, d_n ($n+1$) de Boor control points
- $K \in \{1, 2, 3, \dots\}$ order
/* $n \geq k-1$ */
- $u_0, u_1, \dots, u_{n+k}, u_i \leq u_{i+1}$,
knots / knots sequence



de Boor Algorithm



(triangular) de Boor scheme

$$d_1^0 \xrightarrow{\frac{u_5 - \bar{u}}{u_5 - u_2}} d_1^1 \xrightarrow{\frac{u_5 - \bar{u}}{u_5 - u_3}} d_1^2 \xrightarrow{\frac{u_5 - \bar{u}}{u_5 - u_4}} d_1^3 = \underline{\underline{F(\bar{u})}}$$

$$d_2^0 \xrightarrow{\frac{\bar{u} - u_2}{u_5 - u_2}} d_2^1 \xrightarrow{\frac{u_5 - \bar{u}}{u_5 - u_3}} d_2^2 \xrightarrow{\frac{\bar{u} - u_3}{u_5 - u_3}} d_2^3$$

$$d_3^0 \xrightarrow{\frac{\bar{u} - u_2}{u_5 - u_2}} d_3^1 \xrightarrow{\frac{\bar{u} - u_3}{u_5 - u_3}} d_3^2 \xrightarrow{\frac{\bar{u} - u_4}{u_5 - u_4}} d_3^3$$

$$d_4^0 \xrightarrow{\frac{\bar{u} - u_2}{u_5 - u_2}} d_4^1 \xrightarrow{\frac{\bar{u} - u_3}{u_5 - u_3}} d_4^2 \xrightarrow{\frac{\bar{u} - u_4}{u_5 - u_4}} d_4^3$$

$$d_5^0 \xrightarrow{\frac{\bar{u} - u_2}{u_5 - u_2}} d_5^1 \xrightarrow{\frac{\bar{u} - u_3}{u_5 - u_3}} d_5^2 \xrightarrow{\frac{\bar{u} - u_4}{u_5 - u_4}} d_5^3$$

General:

$$d_i^j = \frac{u_{n+k} - \bar{u}}{u_{n+k} - u_{i+j}} d_i^{j-1} + \frac{\bar{u} - u_{i+j}}{u_{n+k} - u_{i+j}} d_{i+1}^{j-1}$$

de Boor Algorithm

IN: d_0, \dots, d_n ; K ; $u_0 \leq u_1 \leq \dots \leq u_{n+K}$; $\bar{u} \in [u_{n+1}, u_{n+K}]$
 "Domain"

OUT: $C(\bar{u})$

Algo: "determine index I such that $\bar{u} \in [u_I, u_{I+1}]$;

for $(j=1)$ to $(K-1)$ do

for $(i=(I(K-1))$ to $(I-j)$

$$d_i^j = \frac{u_{I+1} - \bar{u}}{u_{I+K} - u_{I+j}} d_i^{j+1} + \frac{\bar{u} - u_{I+j}}{u_{I+K} - u_{I+j}} d_{i+1}^{j+1}$$

return $d_{I(K-1)}^{K-1}$
 $= C(\bar{u})$

