Write up - Eve's Group

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Key Science questions:

1. What is the formation condition for atmosphere to occur assuming current Neon content?

1 Methodology

1.1 Henry's Law

During the early formation of solar system, dust and gas ~~are~~ accrete onto planets, developing ~~its~~ primordial atmosphere. This atmosphere, under the gravitational influence of its own, is compressed and dissolved into the molten magma ocean. By looking at the current gas reservoir on Earth's atmosphere and deep mantle, we are able to constrain the atmospheric characteristic during its early formation period. We are interested in non-reacting gas, the noble gases, reservoir on Earth. Specifically, we look at the Neon reservoir as it does not escape as easily as that of Helium atoms. The concentration of non-reacting gas obeys that of Henry's law, ~~as shown in Equation 1.~~

$$k_{Ne} = \frac{\chi_{Ne}}{p_{Ne}} \tag{1}$$

Here, k_{Ne} is the solubility coefficient of Neon with unit kg/kg/Pa. χ_{Ne} is the concentration of Ne reservoirs in deep mantle, with unit of kg/kg. And p_{Ne} is the ~~partial pressure of surface atmospheric pressure for Ne~~ with units of Pascal. The partial pressure of a gas is calculated using:

$$p = x/P \tag{2}$$

where x is the molar fraction of the gas and P is the total pressure.

In this paper, we take Neon solubility to be $k_{Ne} = 2.7 \times 10^{-12}$ kg/kg/Pa [Olson and Sharp, 2019] and the concentration of current deep mantle reservoir to be $\chi_{Ne} = 3.42 \times 10^{-16}$ mol/g [Marty, 2012]. Assuming a 99% degassing rate (insert citation), we assume that the total amount of Neon reservoir dissolved during early Earth formation to be $\chi_{Ne} = 3.42 \times 10^{-14}$ mol/g. This is comparable to the current atmospheric reservoir of Neon, which gives $\chi_{Ne-atm} = 2.27 \times 10^{-14}$ mol/g. The molar fraction of Neon in the atmosphere is $x_{Ne} = 2.1 \times 10^{-5}$.

Next, we made certain assumptions on the proto-Earth that accreted the primordial atmosphere. We are interested in proto-Earth with a mass range between $0.1 - 1\ M_{\oplus}$. In addition, we assume that 70% of the proto-Earth are silicate, with the rest as an iron core.

Using Equation (1), we can deduce the concentration of Neon in the mantle at a given surface pressure P_{Ne}. Hence, for a given mantle mass proto-earth and a surface pressure, the total amount of dissolvable gas into the magma ocean could be acquired. Figure 1 ~~provide~~ a schematic presentation of the dissolved Neon gas into the

~~melten~~ magma ocean. We overlay red contour lines on-top for easier visualization. The white line shows the required surface pressure to dissolve the current Neon concentration $\chi_{Ne} = 3.42 \times 10^{-14}$ mol/g. We see that it is independent of the mantle mass and set constant at $P = 0.121$ bar.

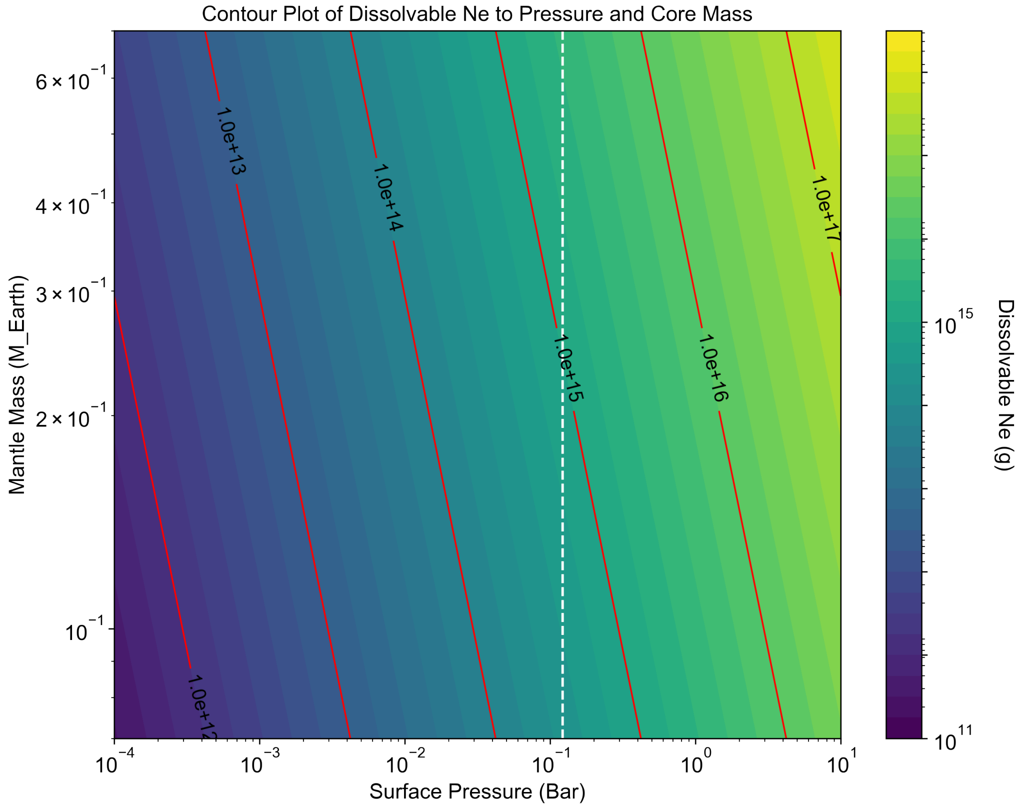

Figure 1: The amount of dissolvable neon in grams given a surface pressure and melten mantle mass. On top of the contour, red contour lines are overplotted. The white line shows the surface pressure and mantle mass needed to dissolve $\chi_{Ne} = 3.42 \times 10^{-14}$ mol/g of Neon.

Next, we are interested in what is the temperature profile of the proto-Earth in order to acquire such a surface pressure. We first begin by looking at ~~a~~ atmosphere under hydrostatic equilibrium.

$$\frac{dP}{dr} = -\rho \frac{GM}{r^2} \tag{3}$$

Where P is the pressure, ρ is the density of the atmosphere . G is the gravitational constant, and M is the mass of the atmosphere. All parameters are a function of the radius r, which goes from the surface of the planet, R_{bot}, to the outer boundary, R_{out}. Here, R_{bot} can be ~~deduce~~ directly from the given core mass M through the terrestrial planet mass-radius relation [Valencia et al., 2006]:

$$R_{bot} = R_E \times M_{\oplus}^{0.25} \tag{4}$$

R_{out} is given by the minimum of the Hill Radius, R_H, or Bondi Radius, R_B. Hill radius is the distance from a planet which the planet's gravitational influence overcomes that of the central star, as shown in Equation (5). Here, a is the semi-major axis of the planet, m_p is the mass of the planet and $M_\star$ is that of the central star:

$$R_H = a \times \left(\frac{m_p}{3M_\star}\right)^{1/3} \tag{5}$$

R_B is the radius which the escape velocity equals the sound speed. Beyond this limit, the gas particle would escape the planet's attraction.

$$R_B = \frac{GM}{Cs^2} \tag{6}$$

Here, the sound speed at a given temperature is calculated using Equation (7), where k_B is the boltzmann constant, μ is the mean molecular mass and m_p is the mass of proton.

$$Cs^2 = \frac{k_B T}{\mu m_p} \tag{7}$$

1.2 Isothermal Approximation

First, we consider an isothermal approximation of the proto-planet atmosphere. Under an isothermal condition, the density profile $\rho(r)$ can be ~~calculate~~ using Equation (8).

$$\rho(r) = \frac{P(r)}{Cs(T)^2} \tag{8}$$

We set the boundary condition at $r = R_{bot}$ and $r = R_{out}$. By using the surface pressure we calculated in the previous section, we can constrain $P_{bot} = 0.121$ bar. Since we are approximating an isothermal atmosphere, $T_{bot} = T_{out}$. Here, the temperature of the outer boundary is set by Equation (9) [D'Alessio et al., 1998], which is the temperature profile of a flared disk with a central star as a classical T Tauri stars. a is the semi-major axis of the proto-planet.

$$T = 1000K \left(\frac{a}{0.1AU}\right)^{-3/7} \tag{9}$$

Hence, for an isothermal atmosphere with a given surface pressure P_{bot} and temperature T_{bot}, we have the density profile of the atmosphere given by:

$$\rho_{bot} = P_{bot}/Cs_{bot} \tag{10}$$

By solving Equation (3) and (8), the mass profile of the atmosphere can be written as:

$$M_{atm}(r) = 4\pi\rho_{bot} \int r^2 \exp(A/R_{bot} - A/R_{out}) dr \tag{11}$$

where A is defined as,

$$A = \frac{GM_{core}}{Cs^2} \tag{12}$$

with the assumption that the core-mass of the proto-Earth is much greater than that of the atmosphere and the atmospheric mass does not influence the gravity field of the planet.

The Gas-to-Core Ratio (GCR) of a planet is calculated using M_{atm}/M_{core}, which M_{core} includes both the mantle and the iron core. Using Equation (11), we can compute the GCR of a proto-planet for a given mass and surface temperature. The isothermal limits allows us to estimate the maximum accreted atmospheric mass of a planet. Figure 2 allows us to interpret the relation bewteen the M_{core}, T_{bot} and GCR under the surface pressure of $P = 0.121$ bar.

Using Equation (9), we can convert the T_{bot} to the location of the proto-planet in the solar-system disk. Figure 3 gives out the relation between the semi-major axis of the proto-planet, its mass and the maximum GCR it can acquire.

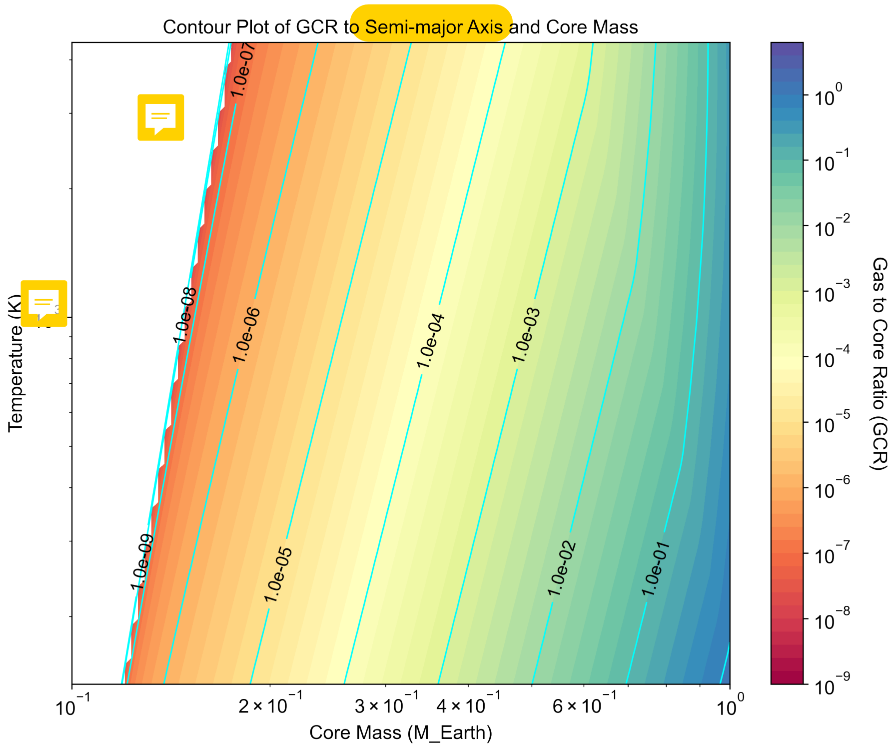

Figure 2: The maximum Gas to Core (GCR) of a proto-Earth object can accrete given a core mass and temperature assuming an isothermal atmosphere. The x-axis gives out the core mass from 0.1 to 1 Earth mass. The y-axis gives out the temperature in K. The color code is the GCR of the planet from 10^{-9} to 10.

In addition, the density of the nebula can also be calculated by combining Equation (3) and (8), where ρ_{neb} is the nebular density:

$$\rho_{neb} = \rho(R_{bot}) \times \exp\left(\frac{A}{R_{out}} - \frac{A}{R_{bot}}\right) \tag{13}$$

Here, we assume that the vertical temperature and vertical density across the solar-system disk is constant. Figure 4 shows required nebular density to form a desired isothermal atmosphere with core mass M_{core}.

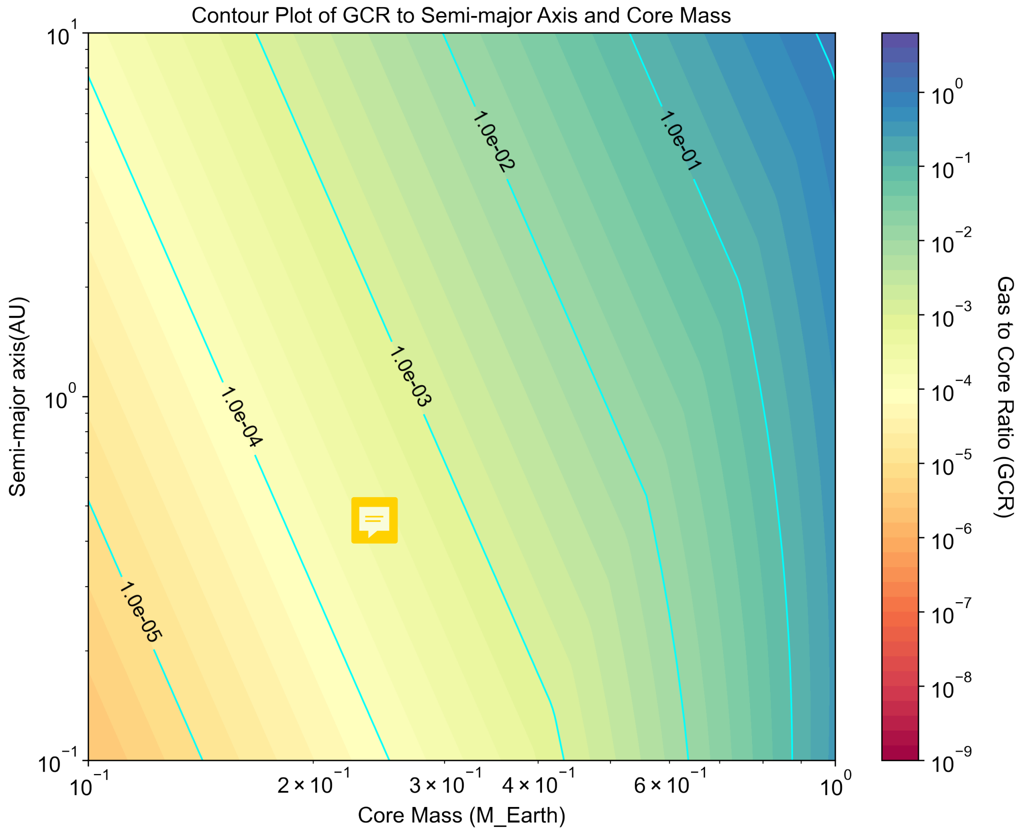

Figure 3: The maximum Gas to Core (GCR) of a proto-Earth object can accrete given a core mass and semi-major assuming an isothermal atmosphere. The x-axis gives out the core mass from 0.1 to 1 Earth mass. The y-axis gives out the semi-major axis from 0.1 to 10 AU. The color code is the GCR of the planet from 10^{-9} to 10.

References

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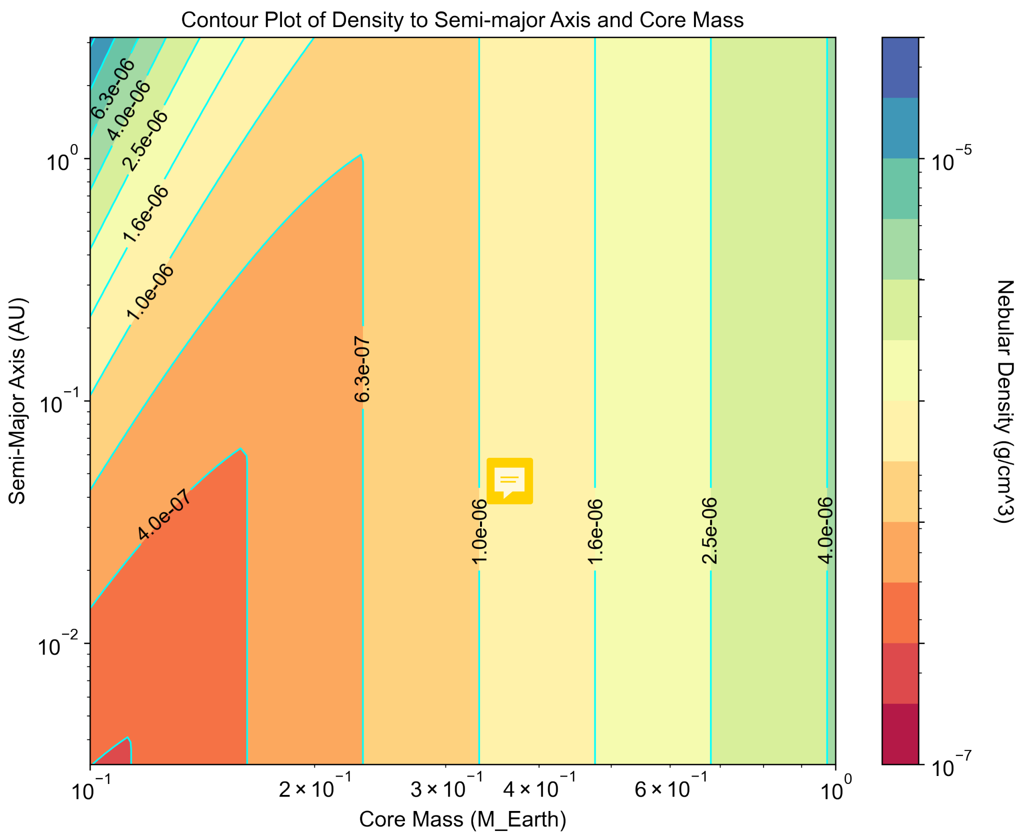

Figure 4: The contour plot of the required nebular density ρ_{neb} in order to form an isothermal atmophere at a given location with M_{core}. The x-axis labels the core mass of Earth from 0.1 to 1 Earth mass. The y-axis is the semi-major axis from 0.003 to 3 AU. The color code is the nebular density at the given location.