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Problem Set 6

I have modified Jon's simple_read_ligo to loop around the four events in two different detectors. I have called this py file in my own ps6_1 to use its strain and template data.

1 Noise Model

Our initial data would have the problem of leakage and bias, which we would deal with individually using a window function and pre-whitening.

Here, we uses a window function of Plank-Taper, as shown in Figure 1. The window function is perfectly flat in the middle and does off to zeros on the ends[1].

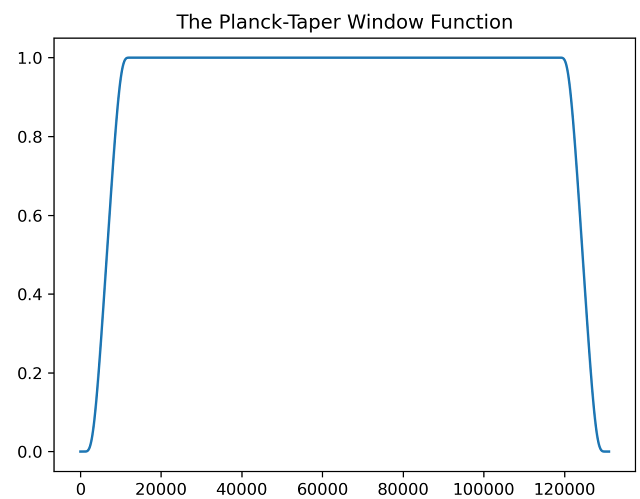

Figure 1: The Planck Taper window function.

The pre-whitening of the data could be achieve by dividing our signal $A(t)$ with the arbitrary spectrum $S_n(f)$, which is the square-root of the Noise function $N(f)$.

$$\tilde{h}(f) \propto \frac{\tilde{A}(f)}{S_n(f)} \tag{1}$$

In addition, I smooth out the Noise function using a scipy medfilt function with kernel $= 11$.

Here, we listed our noise model for each event in Figure 2.

[1] The detailed piece-wise function can be found here: https://en.wikipedia.org/wiki/Window_function

2 Match Filtering

In Figure 3, we show that the match filtering results of the four events at two different sites.

3 Signal-to-Noise Ratio

We calcuate the Noise of each event by taking the standard deviation of the first 2000 points. The Signal to Noise Ratio (SNR) is calculated by taking the ratio between the highest amplitude and the Noise.
1 GW event at Hanford has Noise of: 0.197 and SNR = 25.662
1 GW event at Livingston has Noise of: 0.172 and SNR = 20.502
2 GW event at Hanford has Noise of: 0.095 and SNR = 14.177
2 GW event at Livingston has Noise of: 0.066 and SNR = 10.711
3 GW event at Hanford has Noise of: 0.166 and SNR = 12.579
3 GW event at Livingston has Noise of: 0.204 and SNR = 13.522
4 GW event at Hanford has Noise of: 0.134 and SNR = 11.208
4 GW event at Livingston has Noise of: 0.119 and SNR = 9.934

4 Analytic Noise Model

All of our analytic model produce higher SNR than our numerical model. This is because of the computer round-off error that we experience. In addition, the SNR would fluctuates up and down according to the amount of points we used to measure the standard deviation of the noise. Usually, the more points we take, the higher the SNR we obtained.

The analytic Model is calculated by taking the square root of the absolute value of the match filtering of the whitened template with itself.
1 Noise Model at Hanford has Noise of: 1.520 and SNR = 36.648
1 Noise Model at Livingston has Noise of: 1.311 and SNR = 37.045
2 Noise Model at Hanford has Noise of: 0.804 and SNR = 32.231
2 Noise Model at Livingston has Noise of: 0.478 and SNR = 37.911
3 Noise Model at Hanford has Noise of: 1.308 and SNR = 35.254
3 Noise Model at Livingston has Noise of: 1.736 and SNR = 33.608
4 Noise Model at Hanford has Noise of: 1.161 and SNR = 32.259
4 Noise Model at Livingston has Noise of: 0.996 and SNR = 33.914

5 Frequency of Half Event

Here, we calculate the frequency of the half weight by doing an integration of the match filter frequencies. Then we find the frequency that summed up to the half of the total

integration. The results are listed below:
The half weight frequency for the 1 Event at Hanford is 3.990e+03 Hz
The half weight frequency for the 2 Event at Hanford is 4.134e+03 Hz
The half weight frequency for the 3 Event at Hanford is 4.021e+03 Hz
The half weight frequency for the 4 Event at Hanford is 3.628e+03 Hz
The half weight frequency for the 1 Event at Livingston is 4.234e+03 Hz
The half weight frequency for the 2 Event at Livingston is 5.420e+03 Hz
The half weight frequency for the 3 Event at Livingston is 3.648e+03 Hz
The half weight frequency for the 4 Event at Livingston is 4.135e+03 Hz

Since the frequency range of LIGO lies within 10Hz to 10kHz. The frequency of half-weight signal is within the reasonable range.

6 Time of Arrival

We can localized the event to the smallest unit of time available: $dt = 0.000244140625$s. Hence, after shifting the event to the middle, we have:

The time of arrival of the GW events at Hanford is:
[16.43164062 16.64770508 16.61279297 16.44335938]
The time of arrival of the GW events at Livingston is:
[16.44042969 16.64794922 16.60791016 16.44140625]
The difference between the time of arrivals are (in s):
[0.00878906 0.00024414 -0.00488281 -0.00195312]

The detectors are about 3000 km apart. It takes light 0.01s to travel across. Given that $dt = 0.000244140625$s, the positional uncertainty is around

$$\delta = \frac{0.00024414}{0.01} = 0.0244$$

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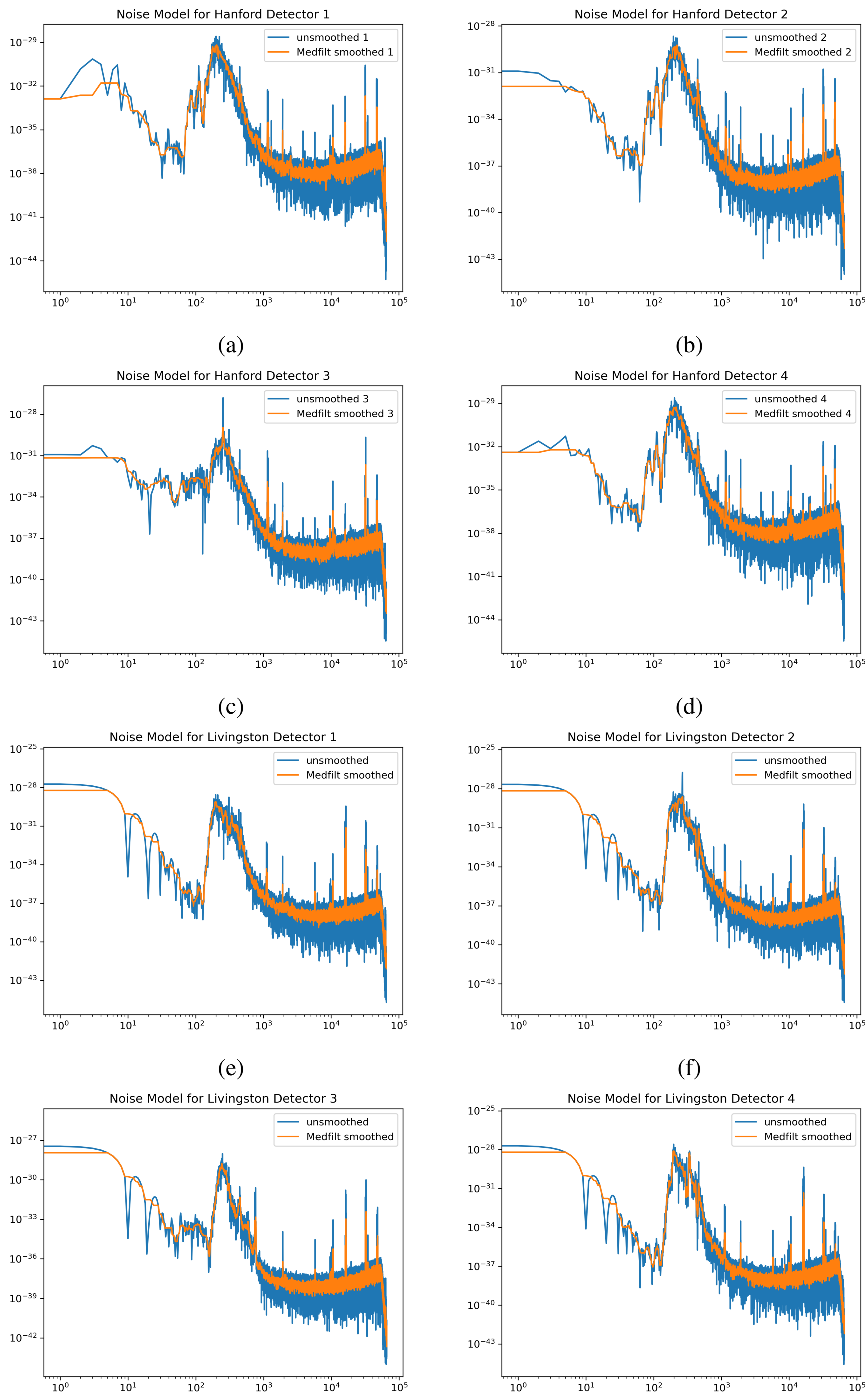
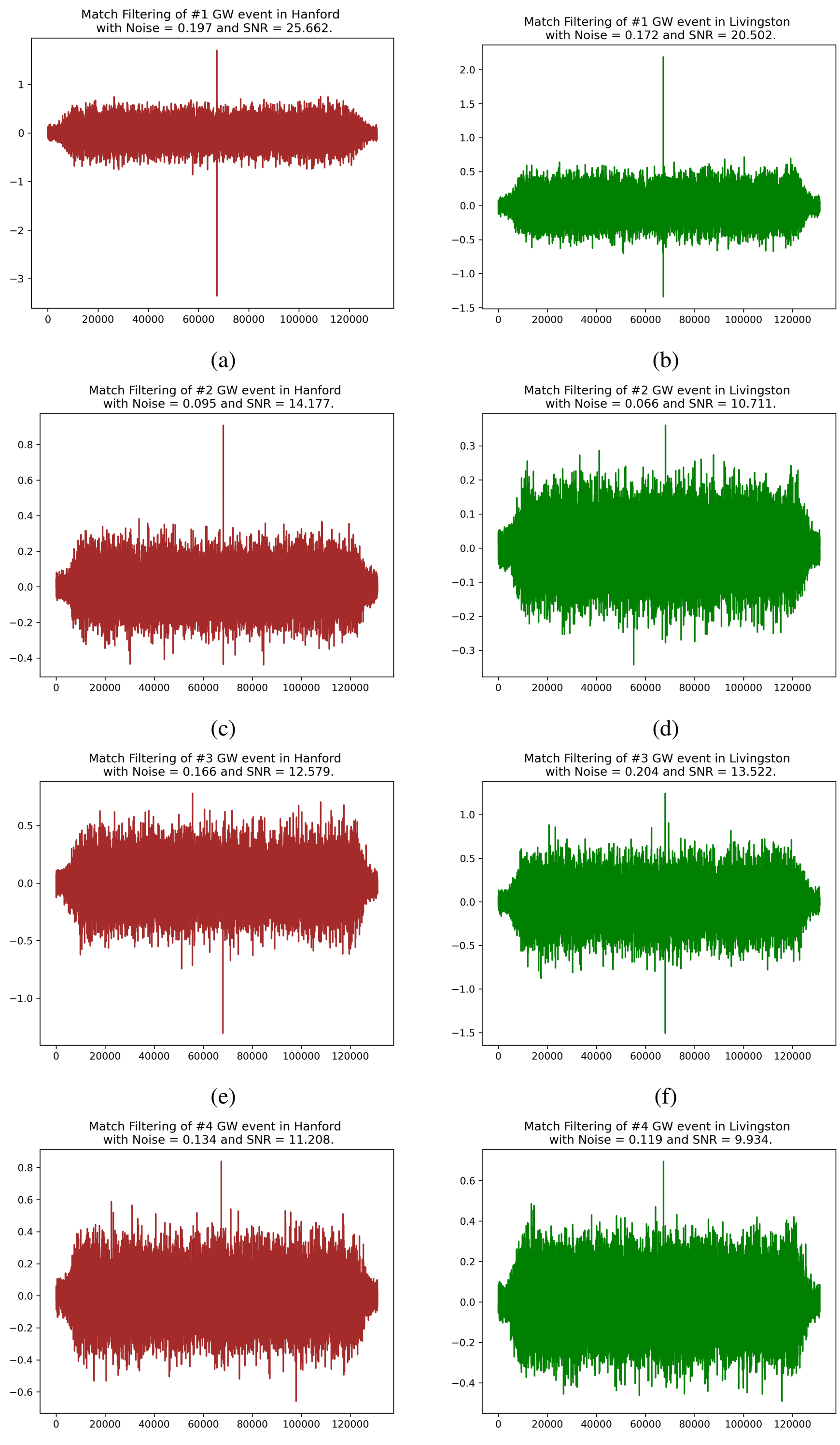


Figure 2: Noise Model for event Events. (a) - (d) listed out the Noise Model for Hanford, while (e) - (h) listed out the Noise models for Livingston.



(a) (b) (c) (d) (e) (f) (g) (h)

Figure 3: Match Filter of of the Four Signals in two sites