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PHYS 512
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Problem Set 4

1

The χ^2 using the current parameters is 15267.937150261656, with 2501 degrees of freedom. When using the new parameters: $[69, 0.022, 0.12, 0.06, 2.1e-9, 0.95]$, the new χ^2 is calculated as: 3272.2053559202204. Hence the current parameters in Jon's model is not an acceptable fit and could be reduced significantly.

2

Here, we use both the Newton's method and the Levenberg-Marquardt to find the best-fit parameters. We will be using the four-point method numerical derivative derived in PS1. Here, we take $dx = 0.01x$.

Using the Newton's method, we need to constrain our value of τ to be greater than 0. Using this method, the sequence quickly breaks and our final parameters are given by starting from $[60, 0.02, 0.1, 0.05, 2.00e-9, 1.0]$:

```
Best fit: 6.22792705e+01 2.12801872e-02 1.22393797e-01
4.76814149e-02 2.07195060e-09 9.37303882e-01
With error: 9.71662882e-03 1.88853245e-06 1.87115317e-05
 3.23021137e-04 1.24286102e-12 6.26441905e-05
```

Using LM technique, we could achieve a better estimation, even without a constrain on τ. Starting with $[60, 0.02, 0.1, 0.05, 2.00e-9, 1.0]$, we have the parameters and estimates as:

```
Best-fit:
6.449436695309610457e+01 2.161054600180116469e-02
1.237807830529353004e-01 4.359724846973285495e-02
2.069642215672208679e-09 9.459750229946621847e-01
Error:
9.719646272544468085e-03 1.886750492609992116e-06
1.871060505832912522e-05 3.225315494226174608e-04
1.241200546232076509e-12 6.256747539328629990e-05
```

Starting with $[69, 0.022, 0.12, 0.06, 2.10e-9, 0.95]$, we have:

```
Best-fit:
6.735929421941352757e+01 2.227255217879068119e-02
```

```
1.1907509011068068l4e-01  5.758979581397048741e-02
2.107725268395881251e-09  9.651673183157759572e-01
Error:
9.719646272544468085e-03  1.886750492609992116e-06
1.871060505832912522e-05  3.225315494226174608e-04
1.241200546232076509e-12  6.256747539328629990e-05
```

We save the curvature matrix for the next question.

Here, will be using [65,0.021,0.11,0.055,2.05e-9,0.97], for our next two sets of calculations. This gives a parameter and error estimation of:

```
Best-fit:
6.941980211744811413e+01  2.241431720437129377e-02
1.140806682767117469e-01  5.234993879342723921e-02
2.058117715642832844e-09  9.753528549662625613e-01
Error:
9.614071941703751512e-03  1.933031386636351386e-06
1.933578575453760102e-05  2.791590070884725108e-04
1.091717214117128327e-12  6.152411088036627892e-05
```

3

In the MCMC method, we take our step size from the curvature matrix from the previous section. The final data are included in the planck_chain_3.txt with starting values [65,0.021,0.11,0.055,2.05e-9,0.97]. We ran the MCMC chain for 10,000 steps and the final fitting parameters as:
[68.17 2.14e-02 1.12e-01 2.92e-02 1.92e-09 9.72e-01] with $\chi^2 = 2690.09$, this have error terms:
[9.45574426e+01 3.98298987e-01 1.50632898e-04 6.68668096e-04 1.04459166e-02 4.02055268e-11 1.36167533e-03] corresponding to H_0, $\Omega_b h^2$, $\Omega_c h^2$, τ, A_s, n_s respectively.
Here, we plot the Fast Fourier Transform of H_0 Here in Figure 1, The chain only looked flat for a small portion of the data points. However, comparing 10k step-run with the 3k step run, the data points definitely converged more.

Here, the estimation of the mean value of dark energy Ω_Γ is given as:

```
The estimation of Dark energy is 7.122e-01,
with uncertainty 3.208e-03.
```

4

In this section, we first constrain our previous parameters with $0.0466 < \tau < 0.0614$. We first calculate the χ^2 between the parameters we have from Section 3 and the con-

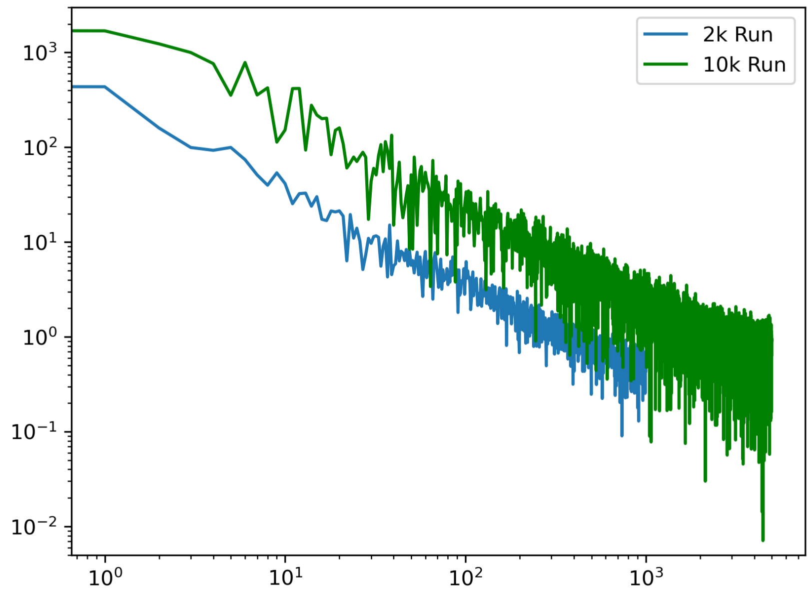

Figure 1: The FFT of the parameter H_0 after running MCMC method for 2k and 10k points with constrain on $\tau > 0.01$.

strain, and then use it to importance sample out a new best-fit parameter. This set of parameter is: [67.46, 0.021, 0.11, 0.045, 1.98e-09, 0.97].

Then, we use this new sets of parameter to make a new trial step of the new MCMC. In addition, during this new MCMC run, we included the constrain $0.0466 < \tau < 0.0614$. Without satisfying this condition, the code will not take the next step. We again run this new MCMC code for 10k steps, and output the results in planck_chain_tauprior_2.txt. The final fitting parameters are:

[68.56 2.16e-02 1.12e-01 5.35e-02 2.03e-09 9.84e-01] with χ^2 as 2665.98.

This has an error term of [3.322e-01 1.497e-04 9.452e-04 4.23e-03 1.70e-11 3.364e-03].

In figure 2, we have similar patterns as that of Figure 1. Both the pink and red run converges in the end. Although if run with more steps, the pattern of convergence might appear better.

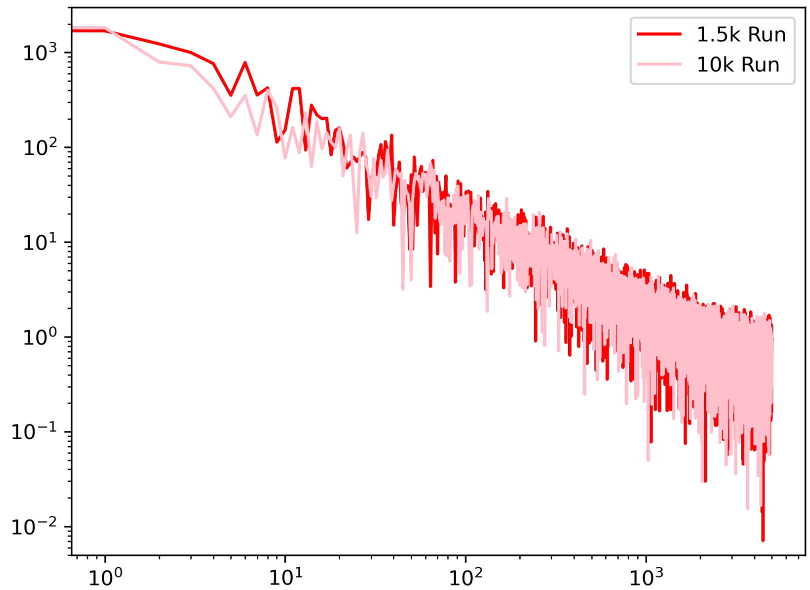

Figure 2: The FFT of the parameter H_0 after running MCMC method with additional τ constraints for 1.5k and 10k points.

5 Code label

Python file name	Usage
p41.py	Newton's method
p4_lm.py	Levenberg-Marquardt Method
p4_mcmc_tau.py	Importance Sampling of previous parameters
p4_mcmc.py	First MCMC
p4_mcmc_im.py	Second MCMC after Importance Sampling
p4_3_cal.py	Dark Matter Calculation and Plotting
planck_fit_params.txt	Newton's Method Parameters and Errors
planck_fit_params_lm_new.txt	Levenberg-Marquardt Method Parameter with Error
planck_chain_3.txt	Final data of MCMC with 10k points
planck_chain_tauprior_2.txt	Final data of MCMC with constrains with 10k points

Because of the huge number of runs I carried, the final name of the data file might appears different then what is asked for. I here labelled all of them of clear references.