Algorithm 1: Preconditioned Conjugate Gradient (PCG)

Input : Matrix A, preconditioner M, right-hand side b, tolerance ϵ, maximum iteration $k_{\max}$

Output: Solution x

$x_0 \leftarrow$ initial guess (e.g., zero)
$r_0 \leftarrow b - Ax_0$
$z_0 \leftarrow M^{-1} r_0$
$p_0 \leftarrow z_0$
for $k \leftarrow 0$ **to** $k_{\max}$ **do**
    if $\|r_k\| < \epsilon$ **then**
        return x_k
    end
    $w_k \leftarrow Ap_k$
    $\alpha_k \leftarrow \frac{(r_k, z_k)}{(p_k, w_k)}$
    $x_{k+1} \leftarrow x_k + \alpha_k p_k$
    $r_{k+1} \leftarrow r_k - \alpha_k w_k$
    $z_{k+1} \leftarrow M^{-1} r_{k+1}$
    $\beta_k \leftarrow \frac{(r_{k+1}, z_{k+1})}{(r_k, z_k)}$
    $p_{k+1} \leftarrow z_{k+1} + \beta_k p_k$
end
return $x_{k_{\max}}$