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Assignment

Name : Cheshta Bhala
Roll No. : 102103545
class : 3CO20

1.) mean = θ_1
variance = θ_2

Liklihood function →

$$L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{x-\theta_1}{2\theta_2}\right)^2}$$

Log on both sides.

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Diffrentiate wrt θ_1 & θ_2, then == 0.

for θ_1 :-

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So MLE for θ_1 is sample mean.

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for θ_2.

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 | x_1, x_2 \dots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

so θ_2 MLE is sample variance

2) Bernoulli distribution

Parameter $\rightarrow \theta \in \Theta = (0, 1)$ unknown

$\rightarrow m$ (known $+$ve Z)

Liklihood function $\rightarrow$

$$L(\theta | x_1, x_2 \dots x_n) = \prod_{i=1}^{n} P(x = x_i | \theta)$$

since x_i follows bernoulli distribution.

$$P(X_i = x_i | \theta) = \theta^{x_i} (1-\theta)^{m-x_i} \text{ for each } i$$

Taking log on both sides.

$$\ln L(\theta | x_1, x_2, \dots x_n) = \sum_{i=1}^{n} \ln(\theta^{x_i} (1-\theta)^{m-x_i})$$

$$= \sum_{i=1}^{n} (x_i \ln \theta \; (m - x_i) \ln(1-\theta)$$

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Differentiate wrt θ

$$\frac{d}{d\theta} \ln L\left(\theta \mid x_1, x_2 \ldots x_n\right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = mn - \sum_{i=1}^{n} \frac{x_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$

So max likelihood estimate of θ is

$$\theta_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m},$$