Slides, notes, and exercises at:
https://github.com/cheshyre/nptls-hf

Many-body theory

Nuclear Physics Turtle Lecture Series 2025:
Ab initio Hartree-Fock calculations of nuclei

Lecture 2

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Recap

- We have some V_{NN}, V_{3N}
- We want to construct our many-body Hamiltonian H
- And we want to solve the Schrödinger equation

$$H|\Psi\rangle = E|\Psi\rangle$$

- $H, |\Psi\rangle$ both complicated (imagine 200 particles → 200 $\vec{r}, \vec{p}$)
- **This lecture:** How do we do this efficiently?

Main messages

- Systems of many fermions are complicated to describe theoretically
- Second quantization → creation and annihilation operators $a^\dagger, a$
- Key benefits:
  - **Easy construction** of antisymmetric A-particle states
  - **General representation** of many-body operators
  - Able to easily describe processes where **particle number changes** (finite temperature, knockout reactions on nuclei)
- Main result: Matrix elements H_{pq} and H_{pqrs} and particle number A are **all we need** to simulate a nucleus → simple problem

Second quantization on whiteboard

Summary

- Creation and annihilation operators for fermions
  - $a_p^\dagger |0\rangle = |p\rangle, a_p |p\rangle = |0\rangle$
- Slater determinant = A-particle antisymmetric state
  - $|\Phi\rangle = a_{p_1}^\dagger a_{p_2}^\dagger \dots a_{p_A}^\dagger |0\rangle$
- Operators easily represented in terms of many-body matrix elements
  - $O_1 = \sum_{pq} O_{pq} a_p^\dagger a_q, \; O_2 = \frac{1}{4} \sum_{pqrs} O_{pqrs} a_p^\dagger a_q^\dagger a_s a_r$