

Hartree-Fock for nuclei

Key concepts:

- HO basis provides good starting point
→ especially spherical nuclei
- Standard shell model ordering to charge occupied states
- Many-body Hamiltonian includes kinetic energy T , two-body interactions V_{NN} , three-body interactions V_{3N}
- Hartree-Fock will give us an optimized basis, but limited by basis truncation parameters

Basis states, spherical harmonic oscillator

Choose 1-body basis $|p\rangle$ as eigenstates of spherical HO:

$$H = \frac{p^2}{2m} + \frac{m\omega}{2} r^2$$

HO frequency sets (inverse) "width" of oscillator potential

→ should not be too far away from size of nucleus

HO states

$$|n_x n_y n_z\rangle = |n l m\rangle = |p\rangle$$

$$n = 0, 1, 2, \dots$$

$$l = 0, 1, 2, \dots$$

$$m = -l, -l+1, \dots l$$

Useful: HO energy number $E = 2n + l$

\Rightarrow energy of $|nlm\rangle$ is $(E + \frac{1}{2})\hbar\omega$

\Rightarrow HO energy number E allows one to organize basis

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$E=3$

$\overline{n=1, l=1}$

$\overline{n=0, l=3}$

$E=2$

$\overline{l=0, n=1}$

$\overline{l=2, n=0}$

$E=1$

$\overline{n=0 \quad l=1}$

$E=0$

allowed states only $n=0, l=0$

Naming scheme:

s: $l=0$

d: $l=2$

p: $l=1$

f: $l=3$

Final step couple to total angular momentum $\vec{J} = \vec{L} + \vec{S}$

why?

- nucleons are spin $1/2$
- nature is rotationally invariant

$\Rightarrow J$ is a good quantum number of physical states

$$|nlsm_s m_e\rangle \rightarrow |n(ls)jm_j\rangle$$

$$= \sum_{m_s m_e} C_{l m_e s m_s}^{j m_j} |nlsm_s m_e\rangle$$

Clebsch-Gordan coefficient

\Rightarrow add iso spin $m_t = \frac{1}{2}$ (proton)
 $-\frac{1}{2}$ (neutron)

$|n(ls)jm_j m_t\rangle$ = single-particle basis

Notation: $0s\frac{1}{2} = n=0 \quad l=0 \quad j=\frac{1}{2}$

H₀ H₀ + spin-orbit

$e = 3$



of $5/2$	6
1P $1/2$	2
1P $3/2$	4
of $7/2$	8

$e = 2$

20	od $3/2$	4
	1s $1/2$	2
	od $5/2$	6

$e = 1$

	op $3/2$	2
	op $5/2$	4

$e = 0$

2	os $1/2$	2
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\Rightarrow (2), (8), (20) are "magic"

numbers \Rightarrow especially deeply bound

Use this ordering to determine occupied levels

Q: What about m_j ?

- Basis ✓

→ Next: Hamiltonian

$$H = T + V_{NN} + V_{3N}$$

Problem: \bar{T} is total kinetic energy
including center of mass ($C.M.$)

→ But binding energy does not depend
on C.M. kinetic energy

Solution:

subtract C.M. kinetic energy

$$T_{cm} = \frac{1}{2A^m} \left(\sum_i \vec{p}_i \right)^2$$

Our Hamiltonian is A -dependent due
to C.M. correction

$$H = T - T_{cm}(A) + V_{NN} + V_{3N}$$

What we have:

$$T_{pq}(\hbar\omega) \quad T_{cm, pqrs}(\hbar\omega) \quad U_{NN, pqrs}(\hbar\omega)$$



T_{cm} is a 1+2-body operator

$\Rightarrow V_{3N, pqrs t u}$ after lunch



Final considerations:

- Basis size will be finite

example: include all states with

$$e \leq e_{max} = 4$$

- Basis depends also on $\hbar\omega$ (HO width)

- Q: How do we choose $e_{max}, \hbar\omega$?

\Rightarrow Vary and check convergence

- $e_{max} \rightarrow e_{max+1}$ does not change?

- trw behavior can be optimized using variational principle

