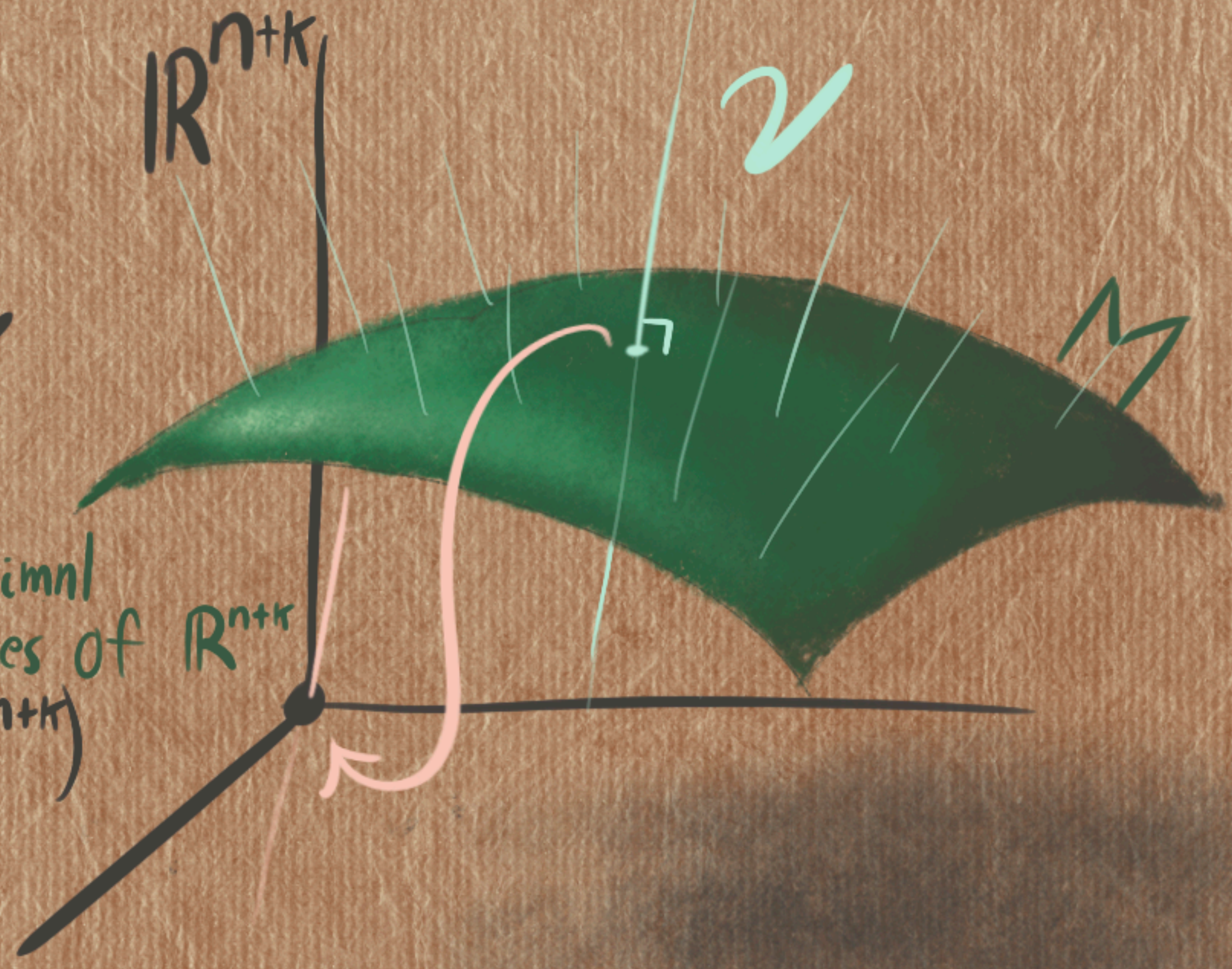
normal bundle w/ tubular nbhd
assigns pt to 'displacement' from M

$\Rightarrow$ Want homotopy classification for ν



for $M \hookrightarrow \mathbb{R}^{n+k}$, $\nu_p \subset T_p\mathbb{R}^{n+k} \cong \mathbb{R}^{n+k}$ k dimnl subspaces of $\mathbb{R}^{n+k}$

ν_p canonically associated to pt in $Gr(k, \mathbb{R}^{n+k})$

$\Rightarrow$ map $f: M \to Gr(k, \mathbb{R}^{n+k})$

Tautological k-bundle:

$$\varepsilon^k \longrightarrow Gr(k, \mathbb{R}^{n+k})$$

fiber @ p = subspace for p

$$\nu_p = f^*\varepsilon^k_{f(p)} \ \forall p \Rightarrow \nu = f^*\varepsilon^k$$

Universal bundle

every k-bundle is pullback of $\varepsilon^k \to Gr(k,\mathbb{R}^{n+k})$ for some n

Glue all $Gr(k,\mathbb{R}^{n+k})$ together:

$\mathbb{R}^{n+k} \hookrightarrow \mathbb{R}^{n+k+1}$ induces $Gr(k,\mathbb{R}^{n+k}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+1})$

$BO(k) :=$ telescoped mapping cylinder of inclusions

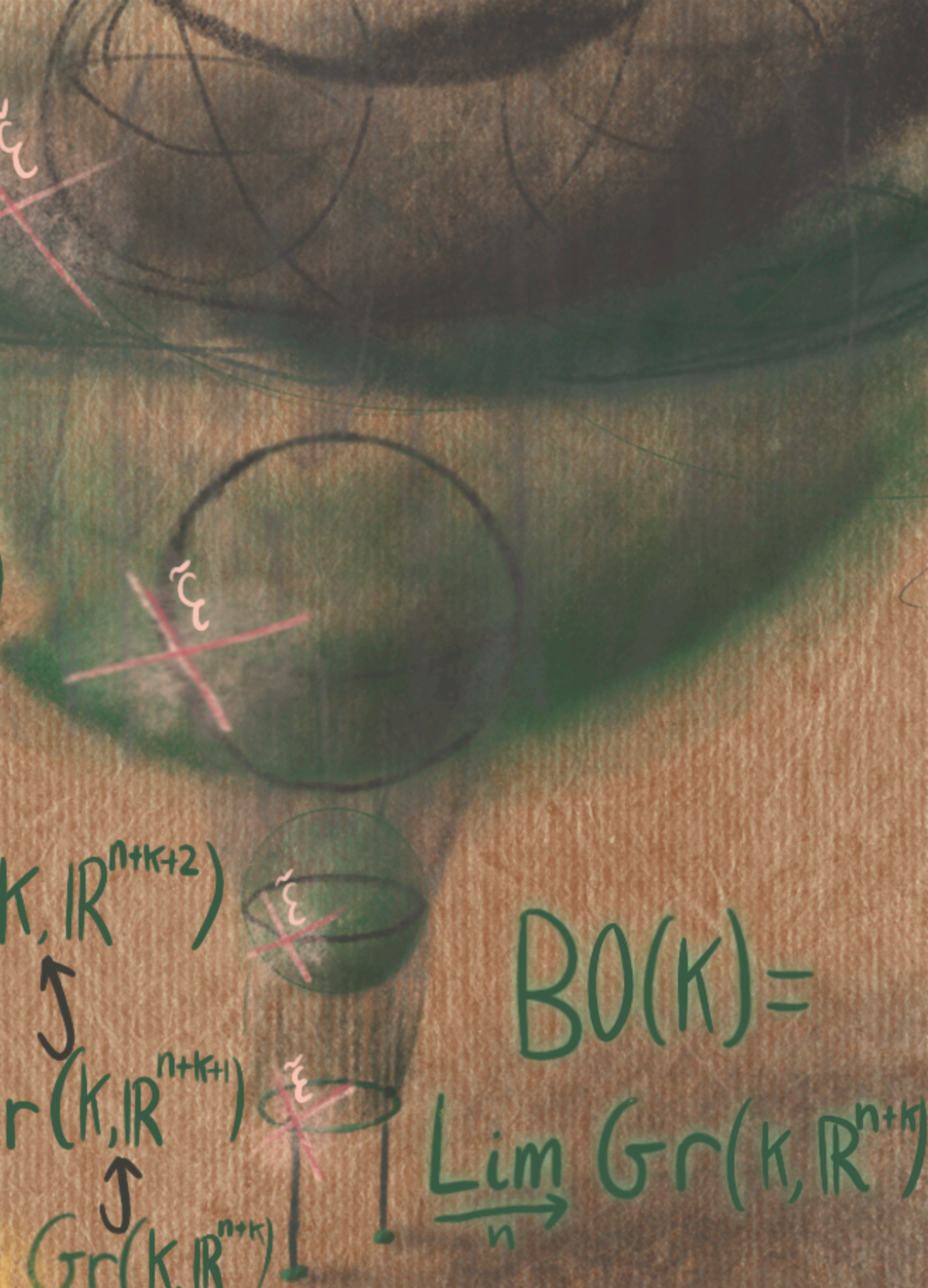
<u>every</u> k-bundle is a pullback of $\tilde{\varepsilon}^k$!!

induced by classifying map $f: X \to BO(k)$

invariants of $[X, BO(k)] \leftrightarrow$ invariants of V

cohomology of $BO(k)$ = charactistic Classes

Chern-Weil theory



$$Gr(k,\mathbb{R}^{n+k}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+1}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+2}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+3}) \hookrightarrow \cdots Gr(k,\mathbb{R}^{n+k}) \to$$

$$BO(k) = \varinjlim_n Gr(k,\mathbb{R}^{n+k})$$

$$\tilde{\varepsilon}^k \to BO(k)$$