

Differential topology

Whitney embedding Thm:

↪ all mflds embed in some \mathbb{R}^{n+k}

all cobordisms can embed like

Tubular nbhd Thm:

i extends to embedding

$$\mathcal{V} \hookrightarrow \mathbb{R}^{n+k}$$



$M \xrightarrow{i} \mathbb{R}^{n+k}$
 $\mathcal{V} \rightarrow M$ Normal bundle $\mathcal{V} = TM^\perp \subset T\mathbb{R}^{n+k}$



normal bundle w/ tubular nbhd
assigns pt to 'displacement' from M

\Rightarrow Want homotopy classification for \mathcal{V}

for $M \hookrightarrow \mathbb{R}^{n+k}$ $\mathcal{V}_p \subset T_p \mathbb{R} \cong \mathbb{R}^{n+k}$

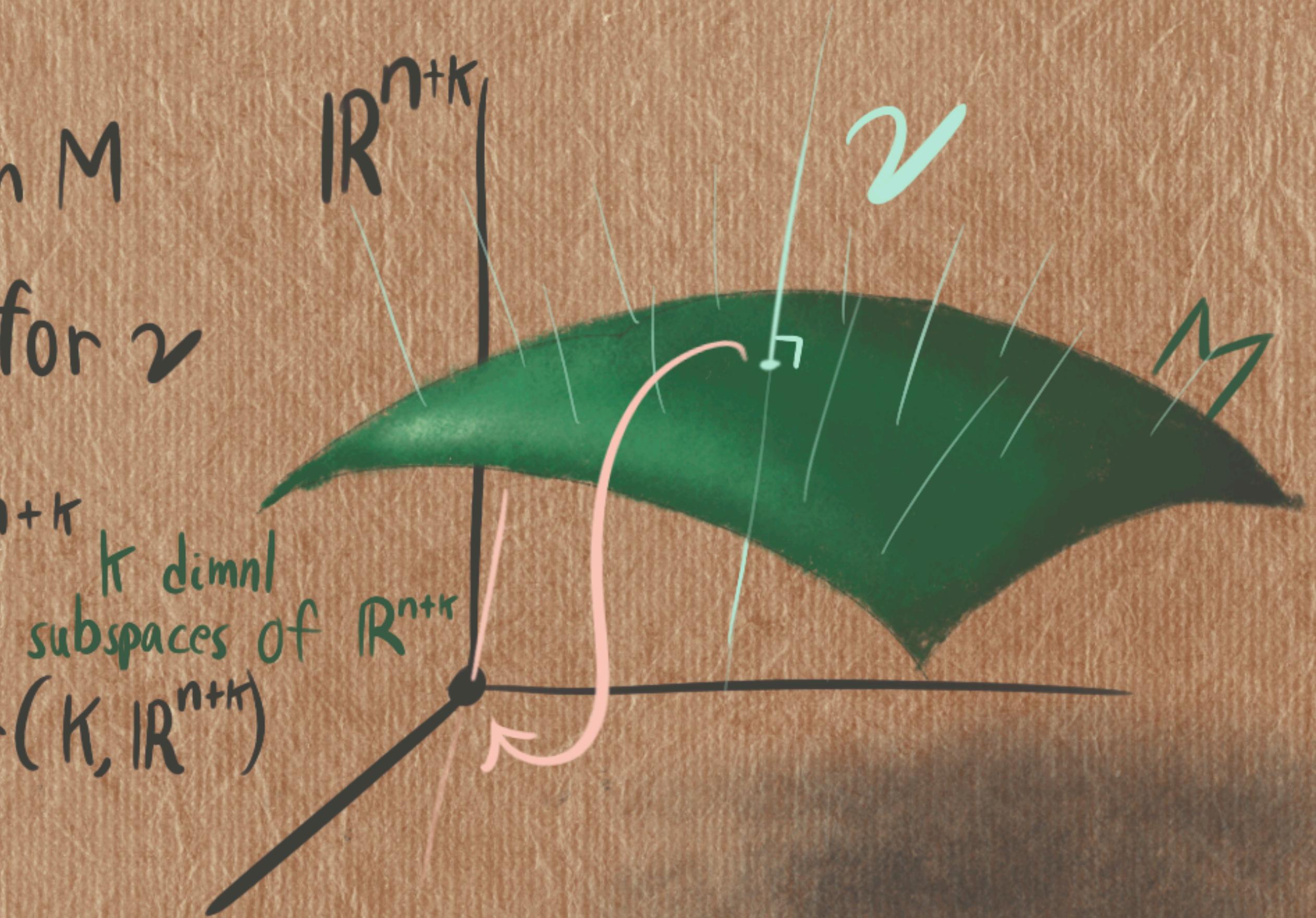
\mathcal{V}_p canonically associated to pt in $\text{Gr}(k, \mathbb{R}^{n+k})$

\Rightarrow map $f: M \rightarrow \text{Gr}(k, \mathbb{R}^{n+k})$

Tautological k -bundle:

$$\mathcal{E}^k \longrightarrow \text{Gr}(k, \mathbb{R}^{n+k})$$

fiber @ p = subspace for p



$$\mathcal{V}_p = f^* \mathcal{E}_{f(p)}^k \quad \forall p \Rightarrow \mathcal{V} = f^* \mathcal{E}^k$$