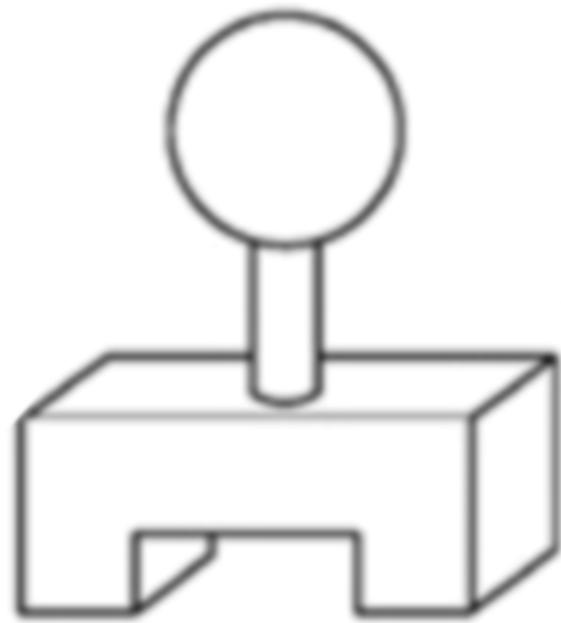


HOW WE SEE WITH GEOMETRY

Many cheerful facts 😊

5/12/23

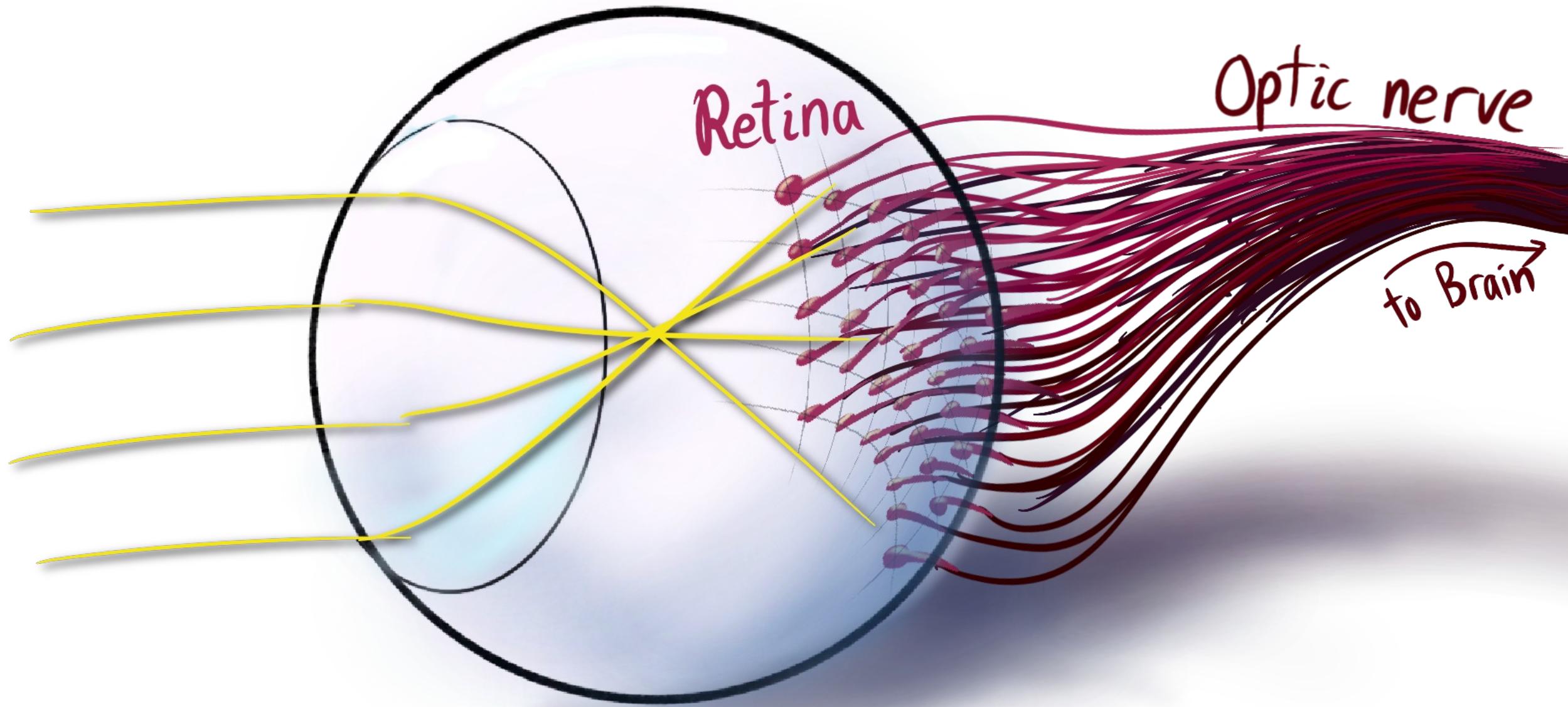
is this:



The same as this?

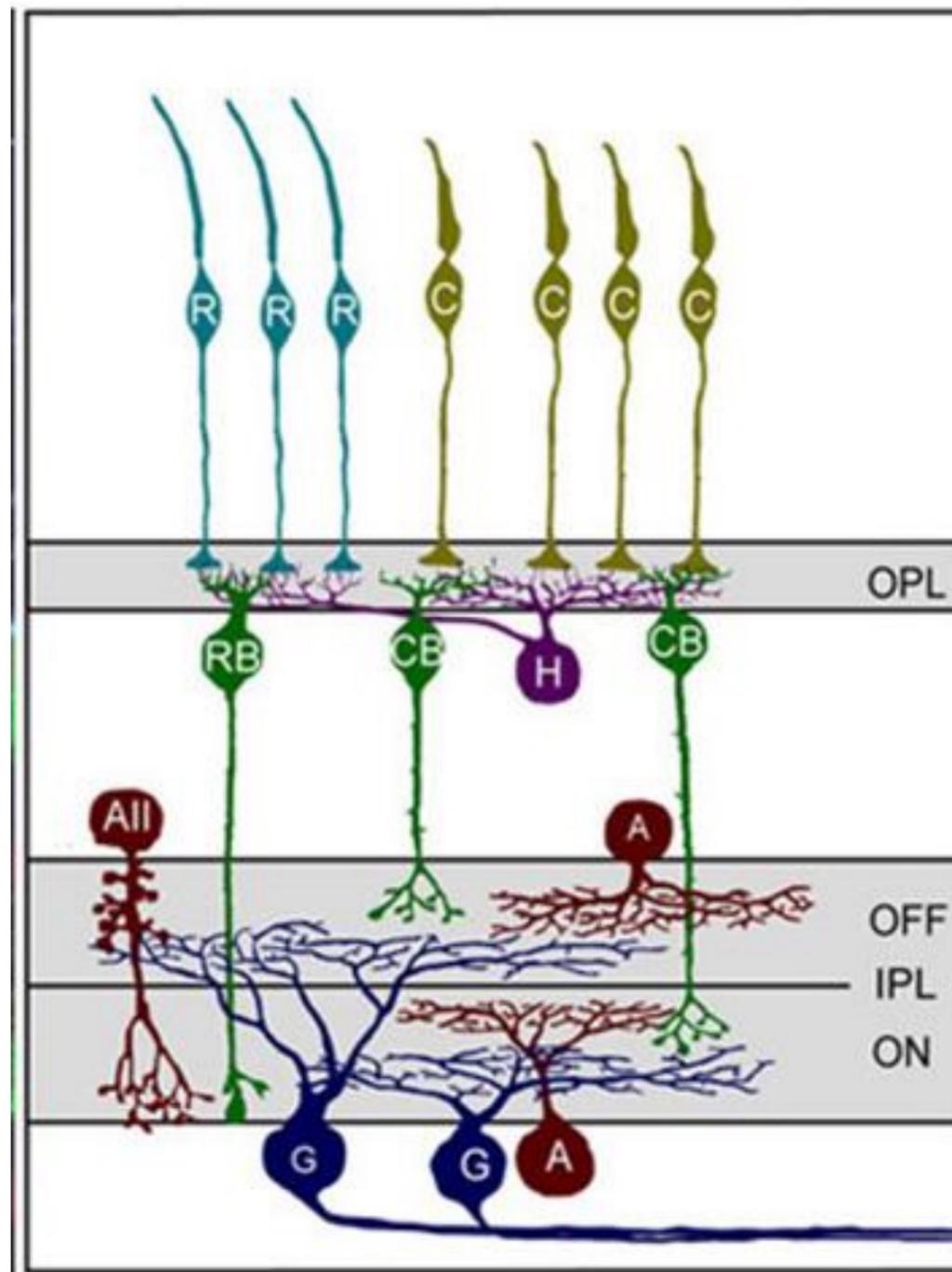


THE RETINA

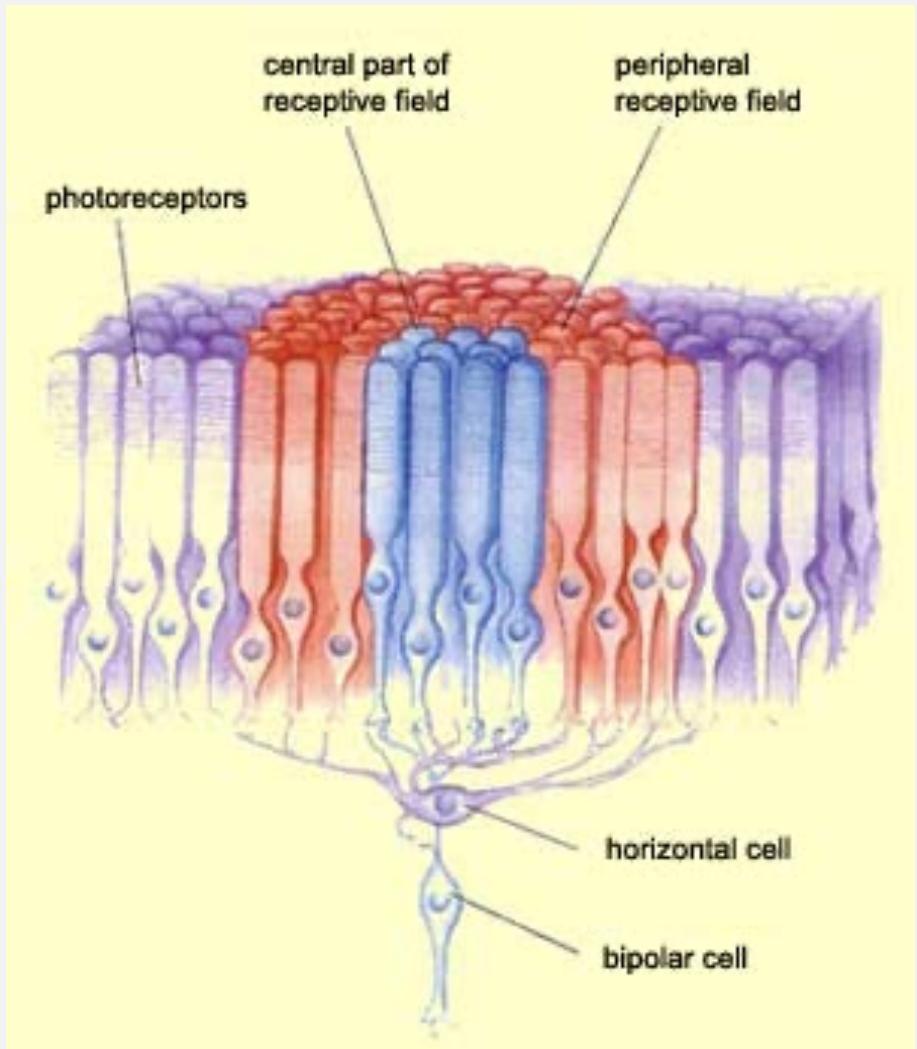


Light capturers

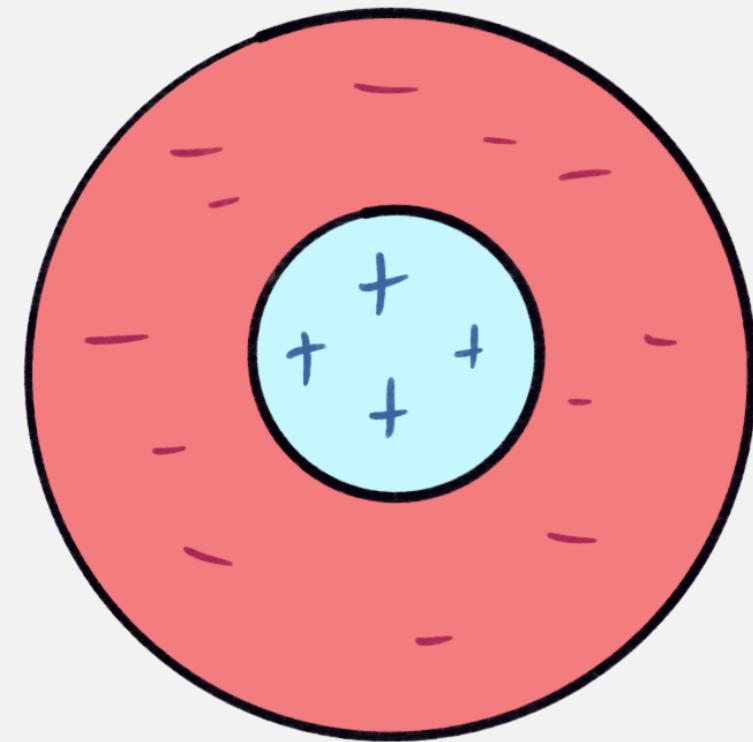
Nerve cells



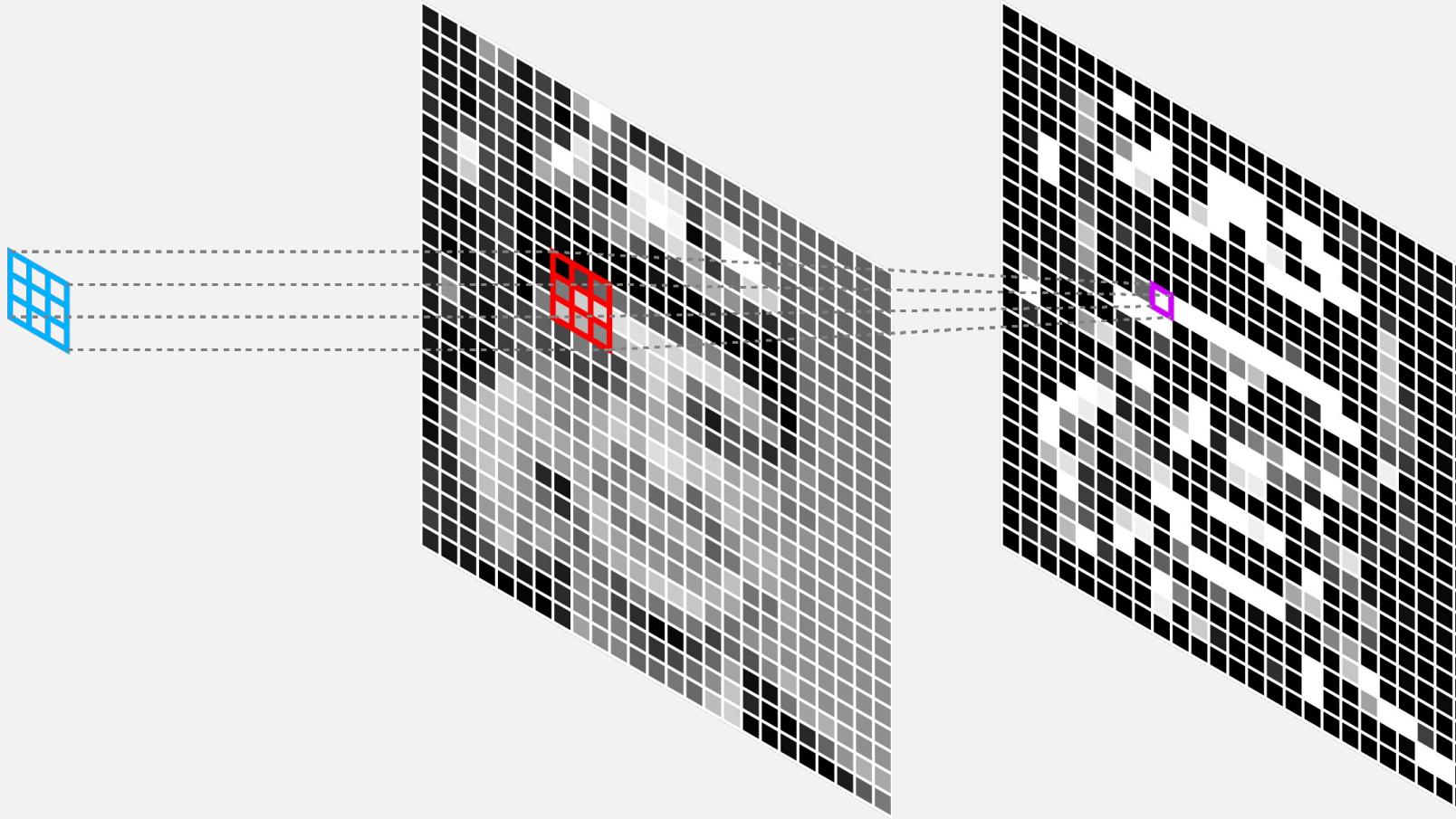
LATERAL INHIBITION



Response of neuron
to light



Causes Sharpening



| | | |
|----|----|----|
| -1 | -1 | -1 |
| -1 | 8 | -1 |
| -1 | -1 | -1 |

Kernel

| | | |
|-----|-----|-----|
| 45 | 81 | 87 |
| 194 | 203 | 215 |
| 164 | 116 | 131 |

Input

255*

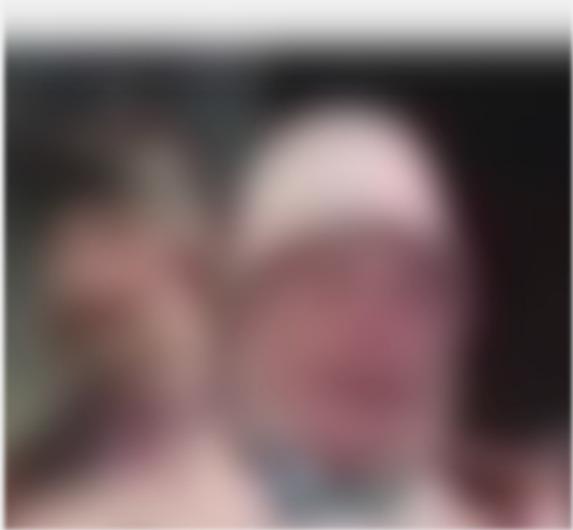
* = $\max(255, 657)$

Output

<https://gregorygundersen.com/blog/2017/02/24/cnns/>

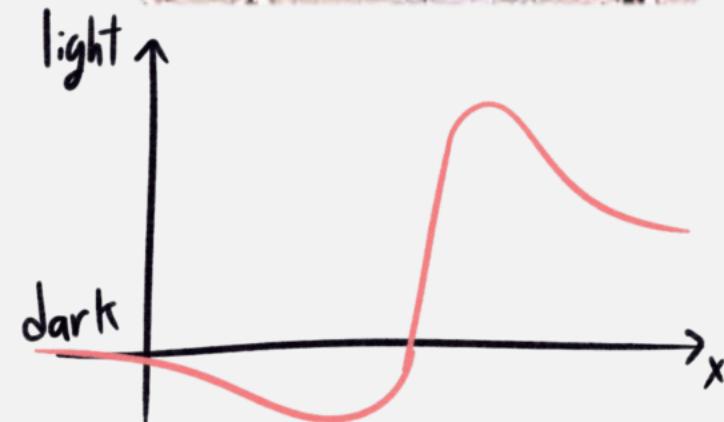
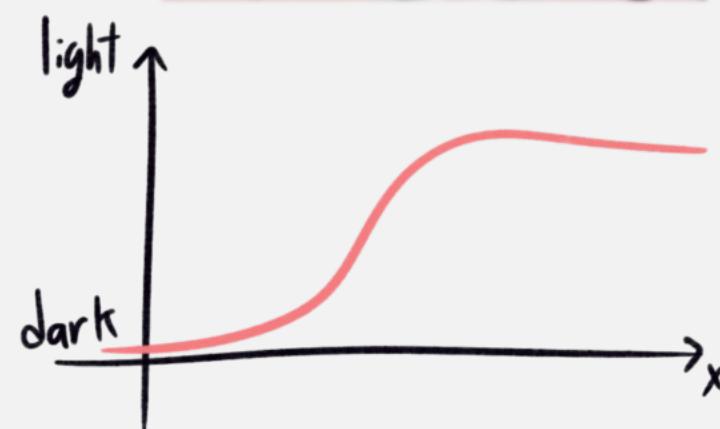
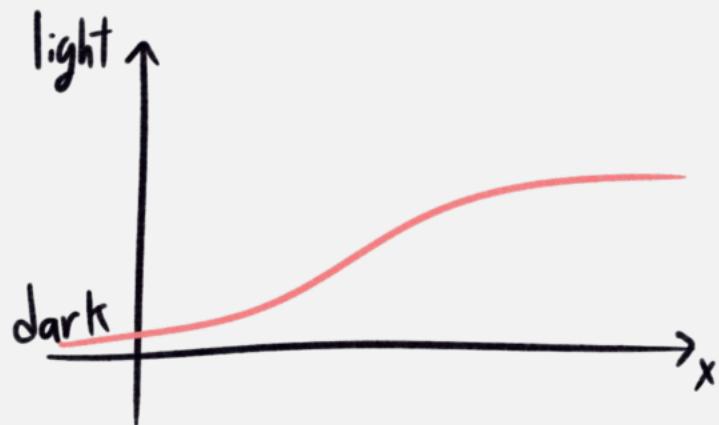
$$\begin{bmatrix} 0 & .1 & 0 \\ .1 & .6 & .1 \\ 0 & .1 & 0 \end{bmatrix}$$

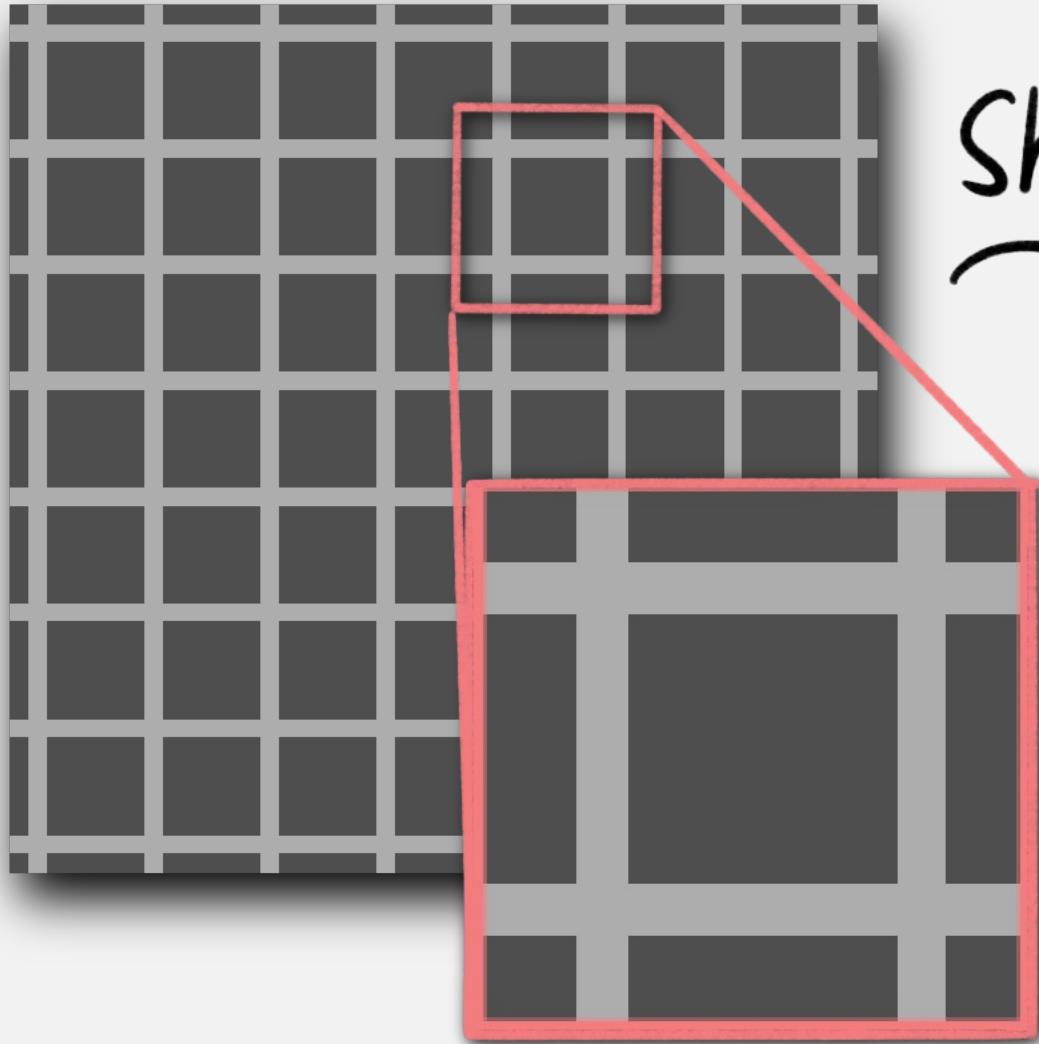
Blur



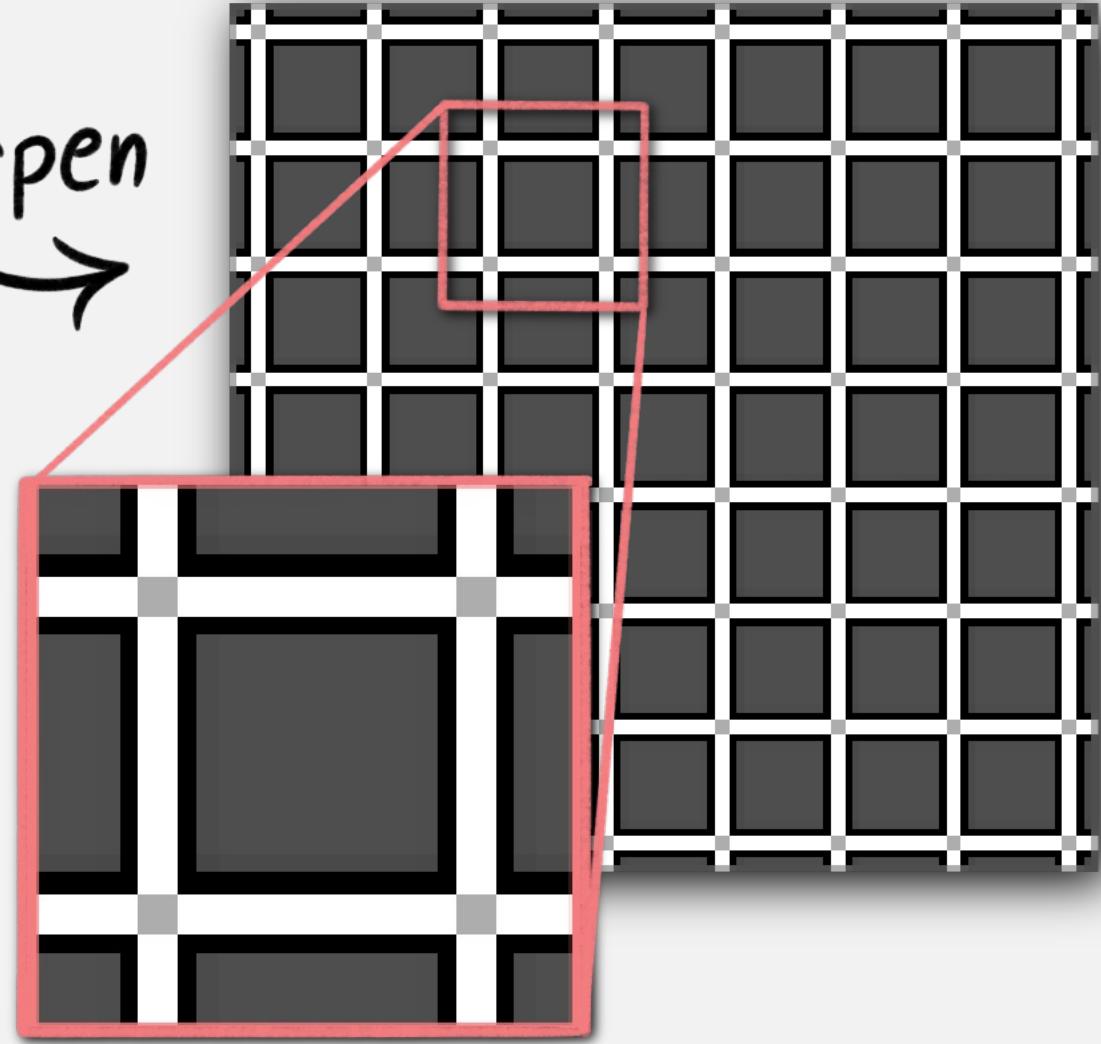
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Sharpen

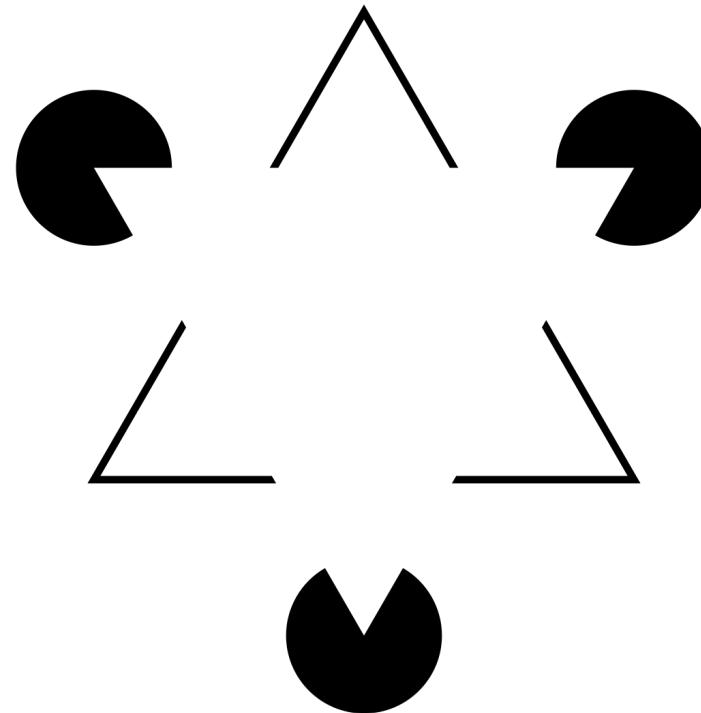


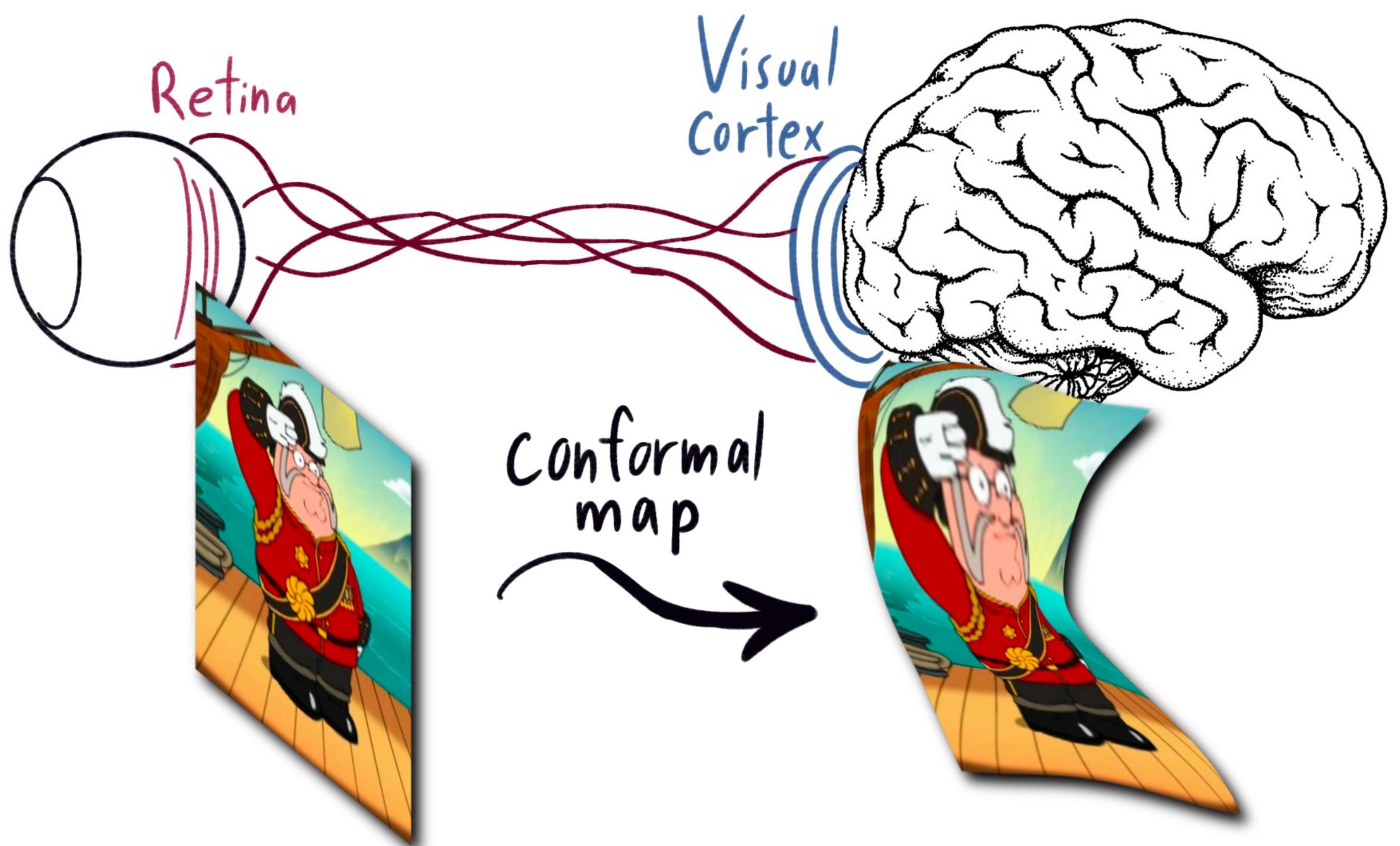


Sharpen

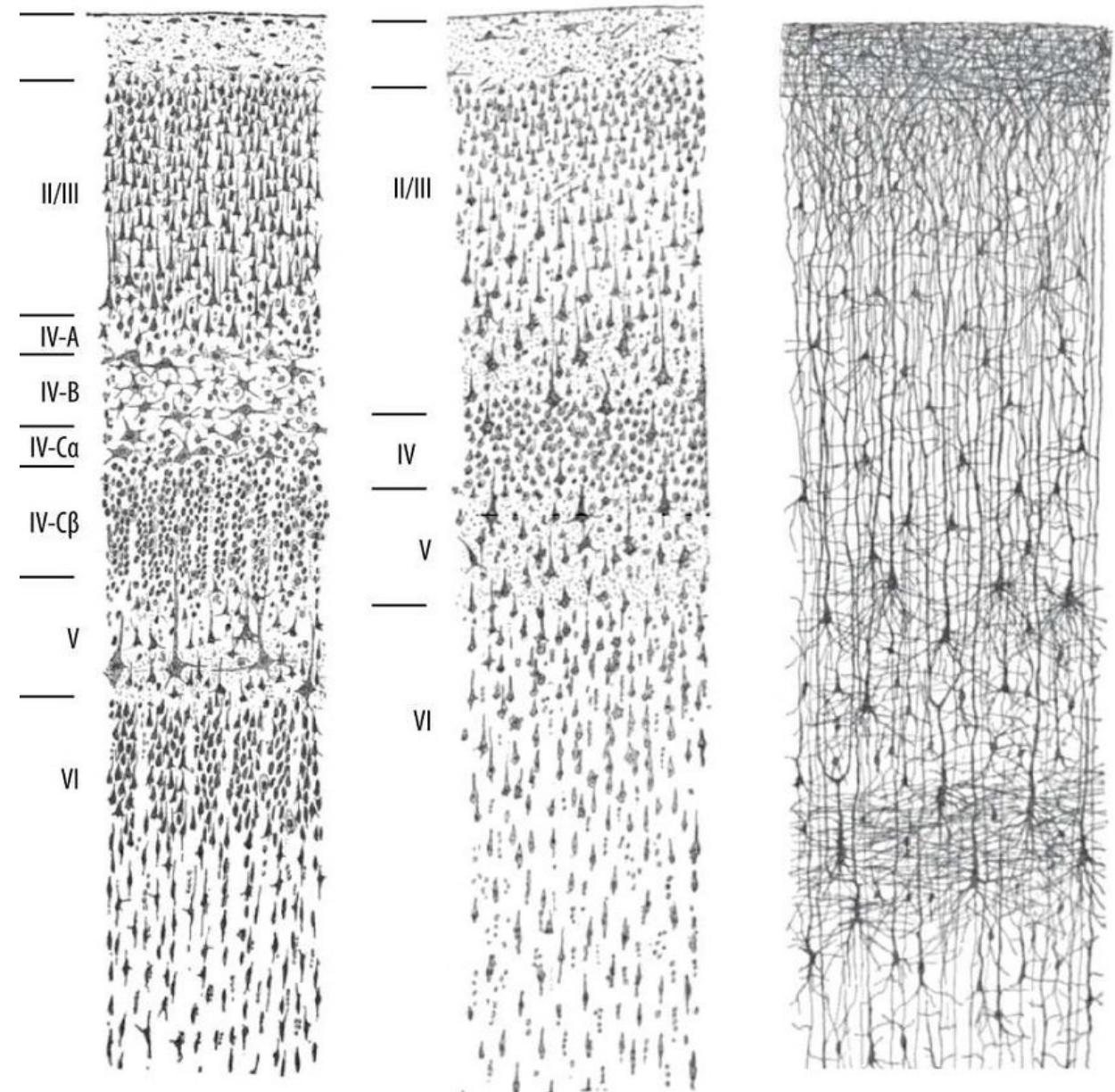


CONTACT GEOMETRY IN THE VISUAL CORTEX

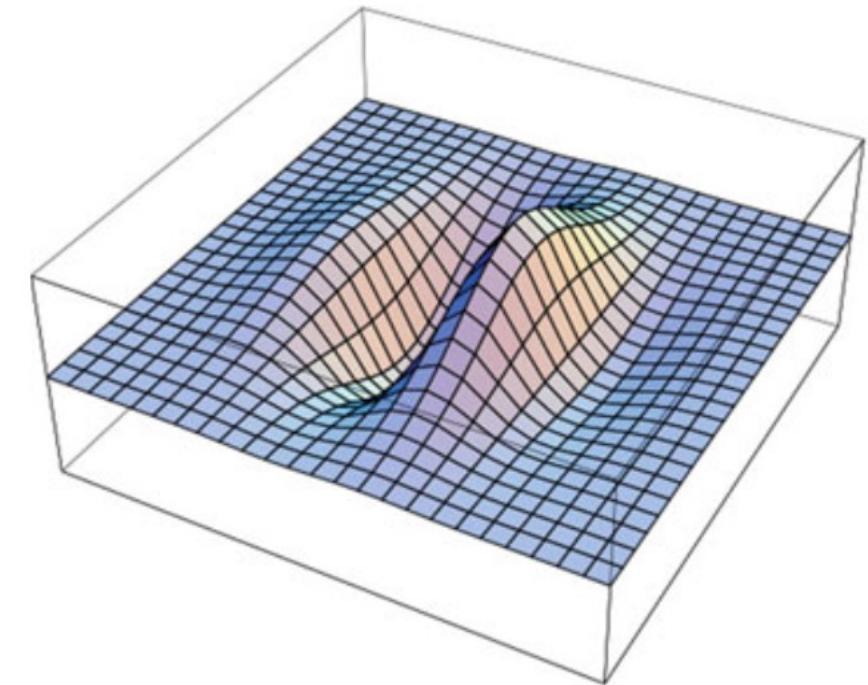
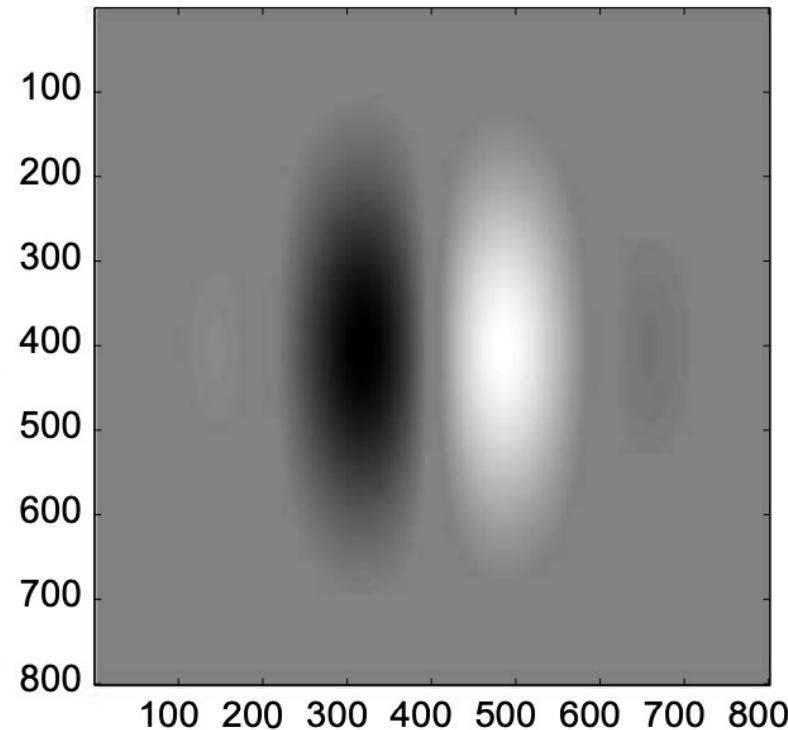
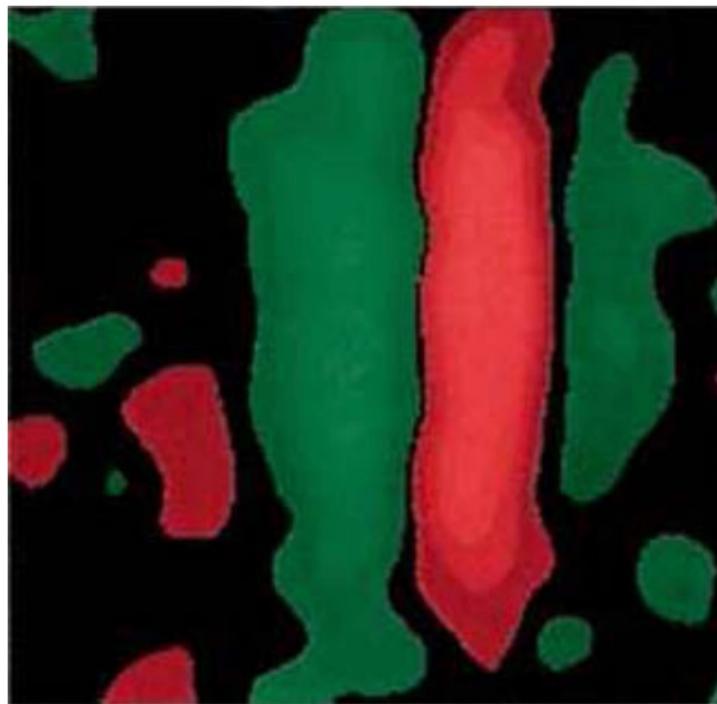




VISUAL CORTEX CROSS SECTION



Response of neuron in visual cortex to light

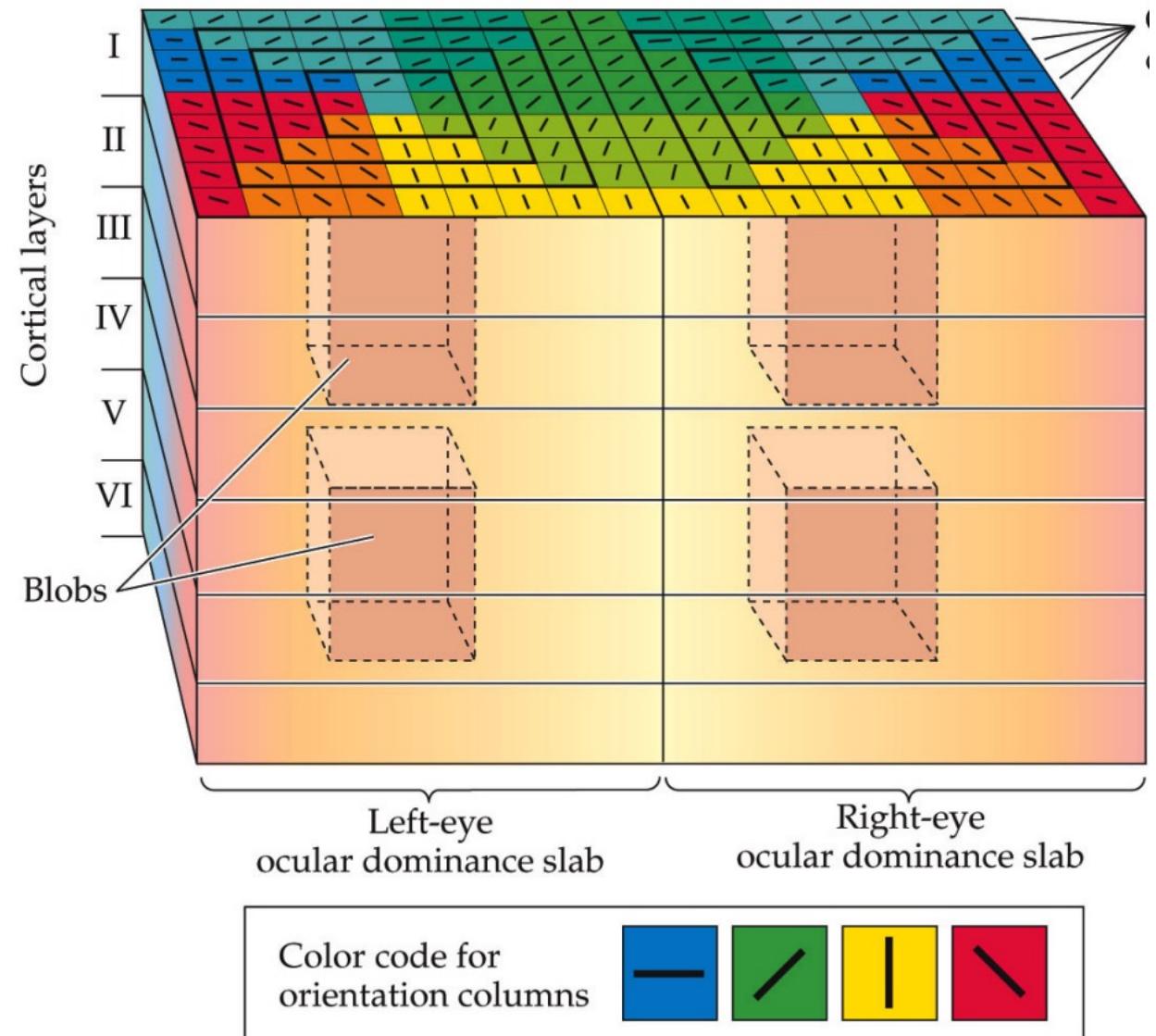
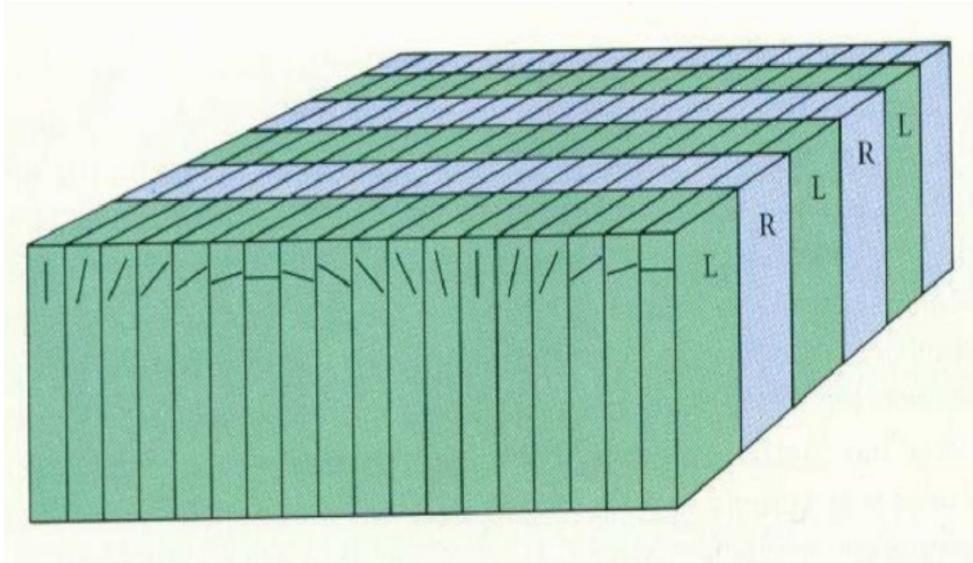


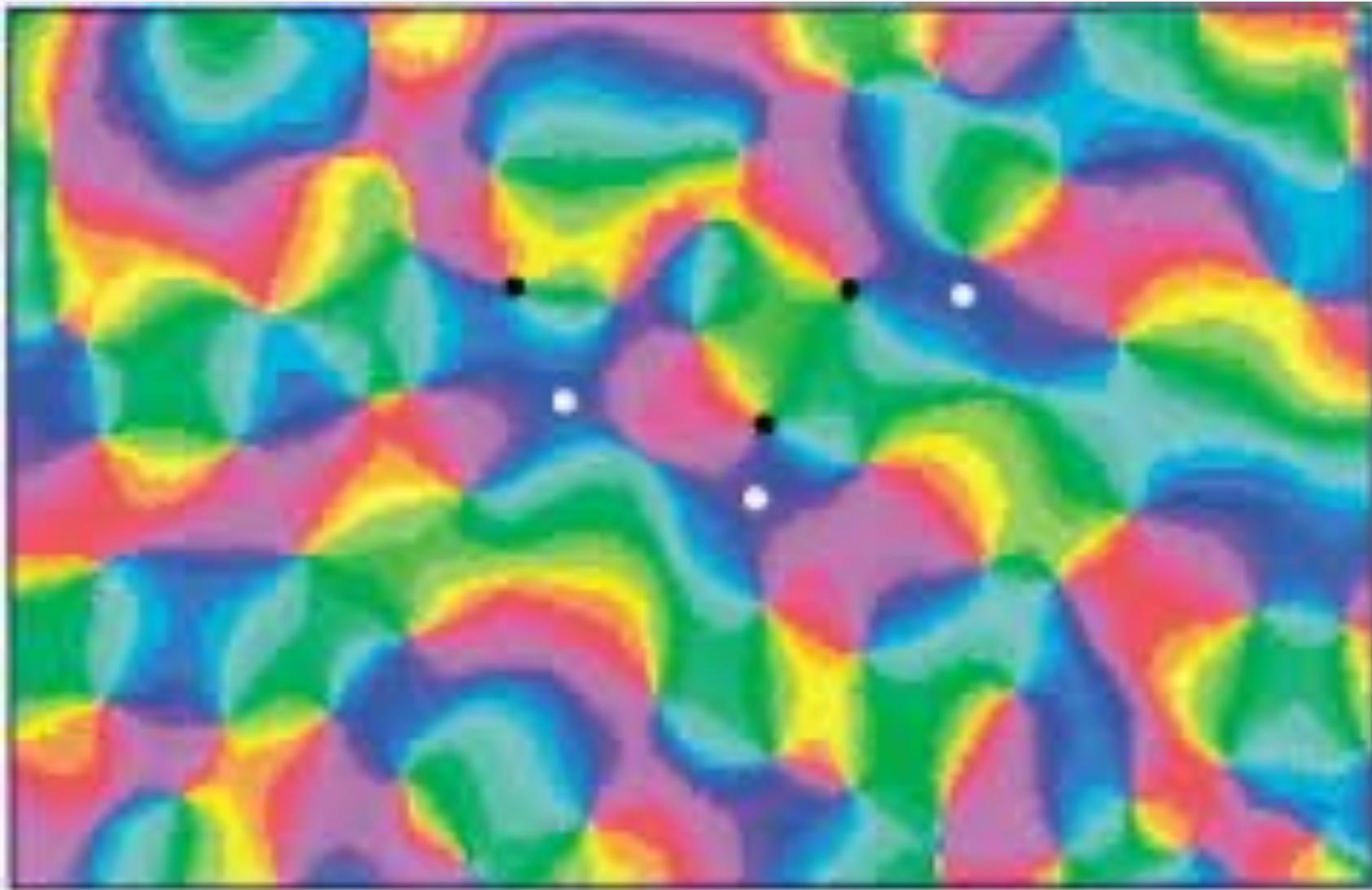
Neurons Perform Edge Detection



Hypercolumns

Microcolumns



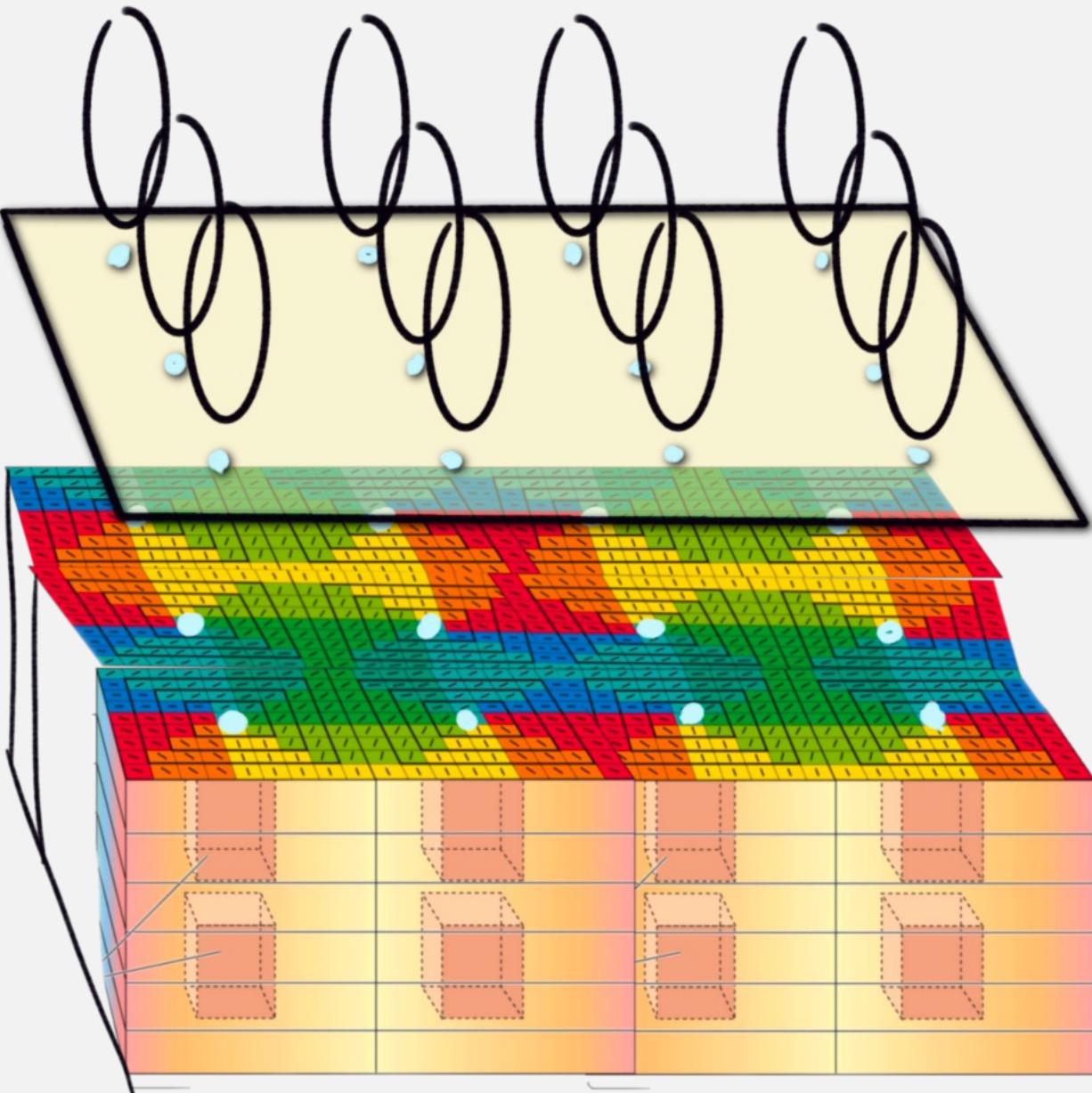


1mm

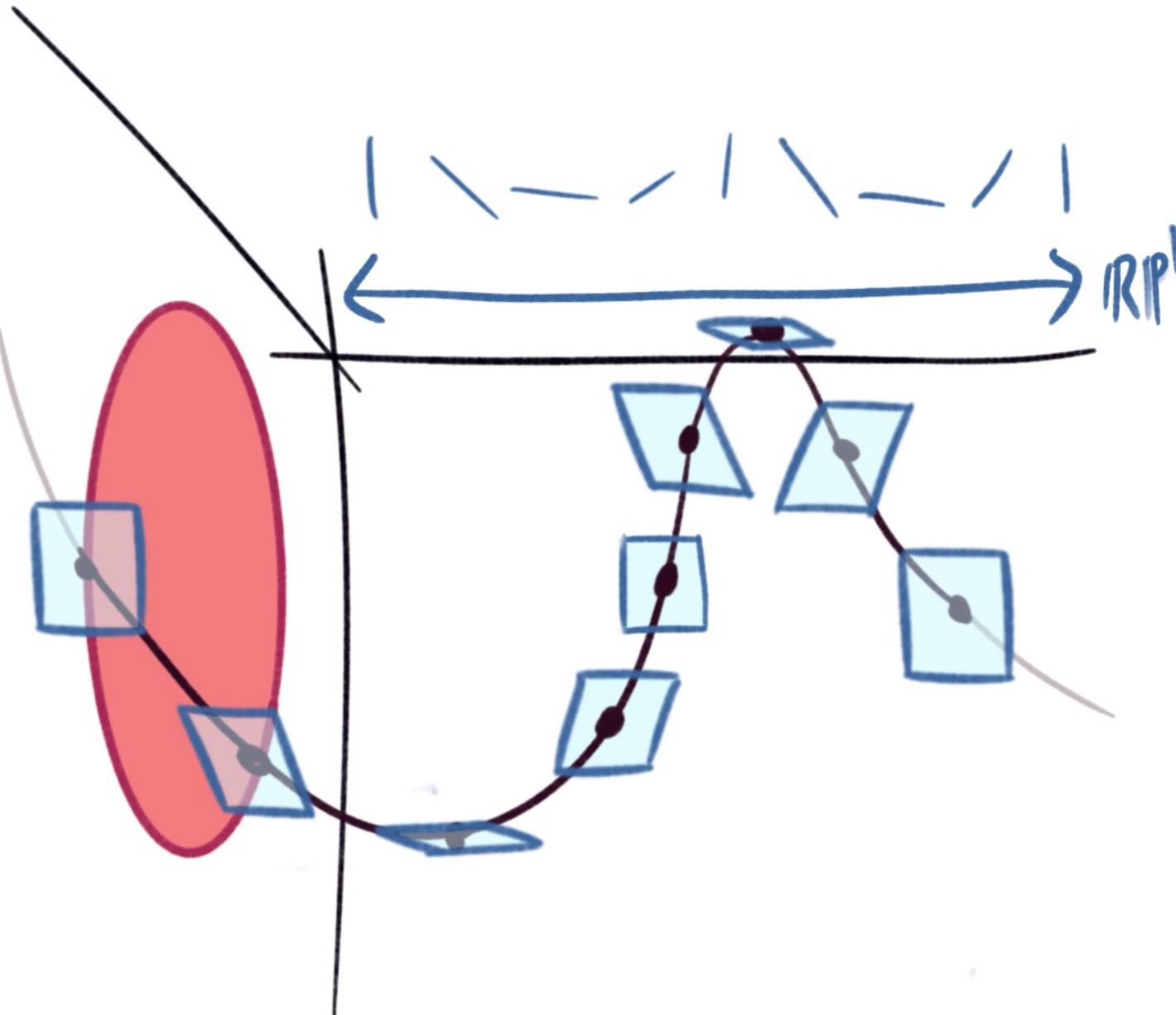


$$\mathbb{R}^2 \times \mathbb{RP}^1$$

↓

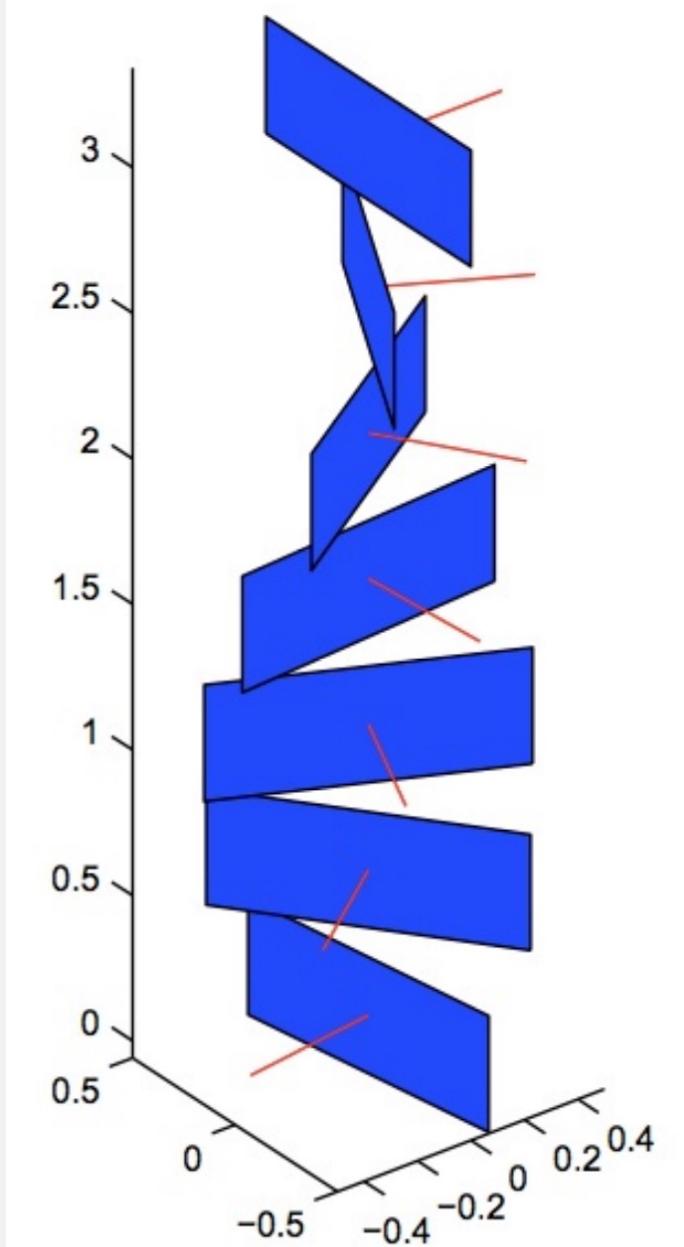
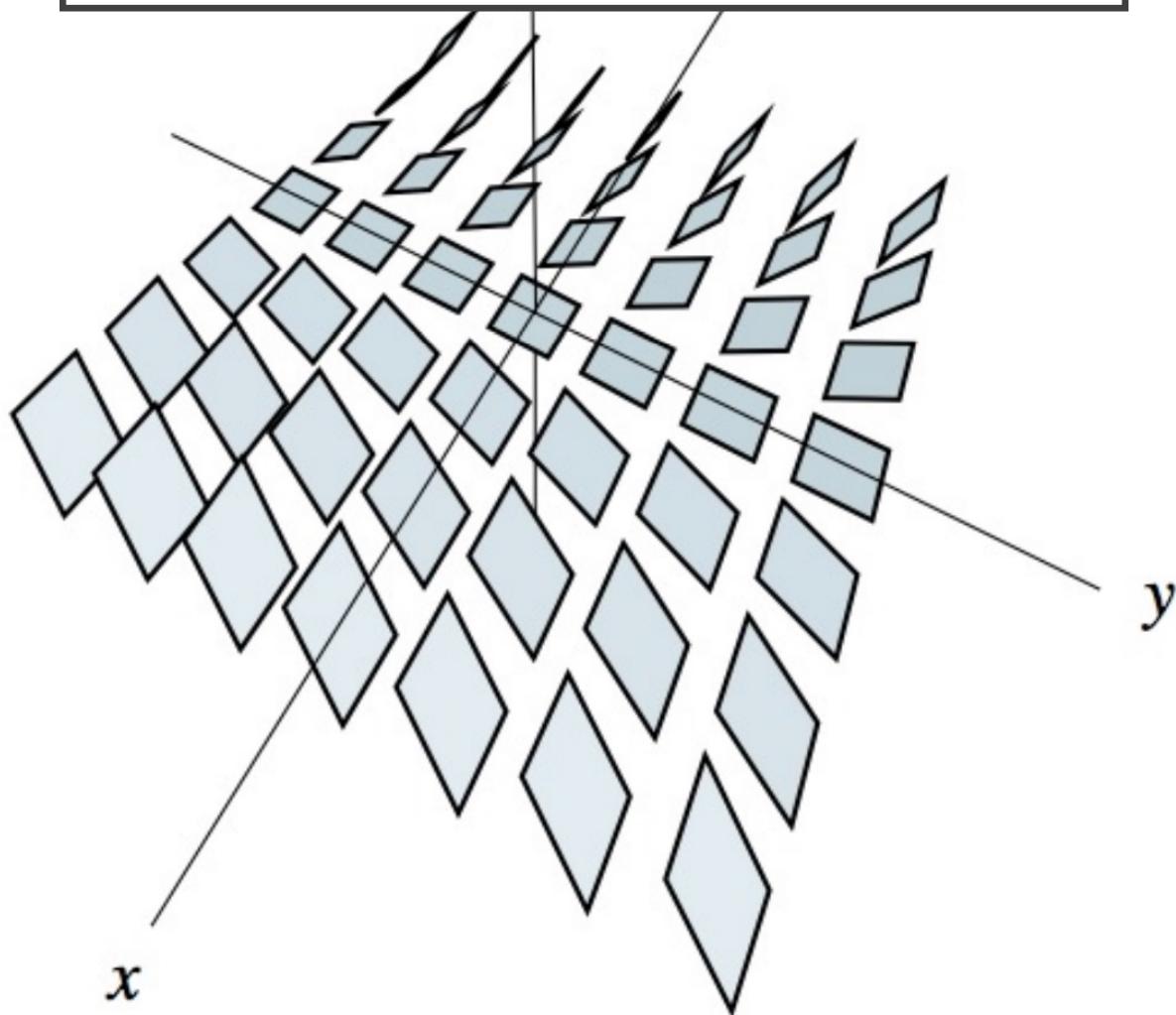
$$\mathbb{R}^2$$


Visual
field

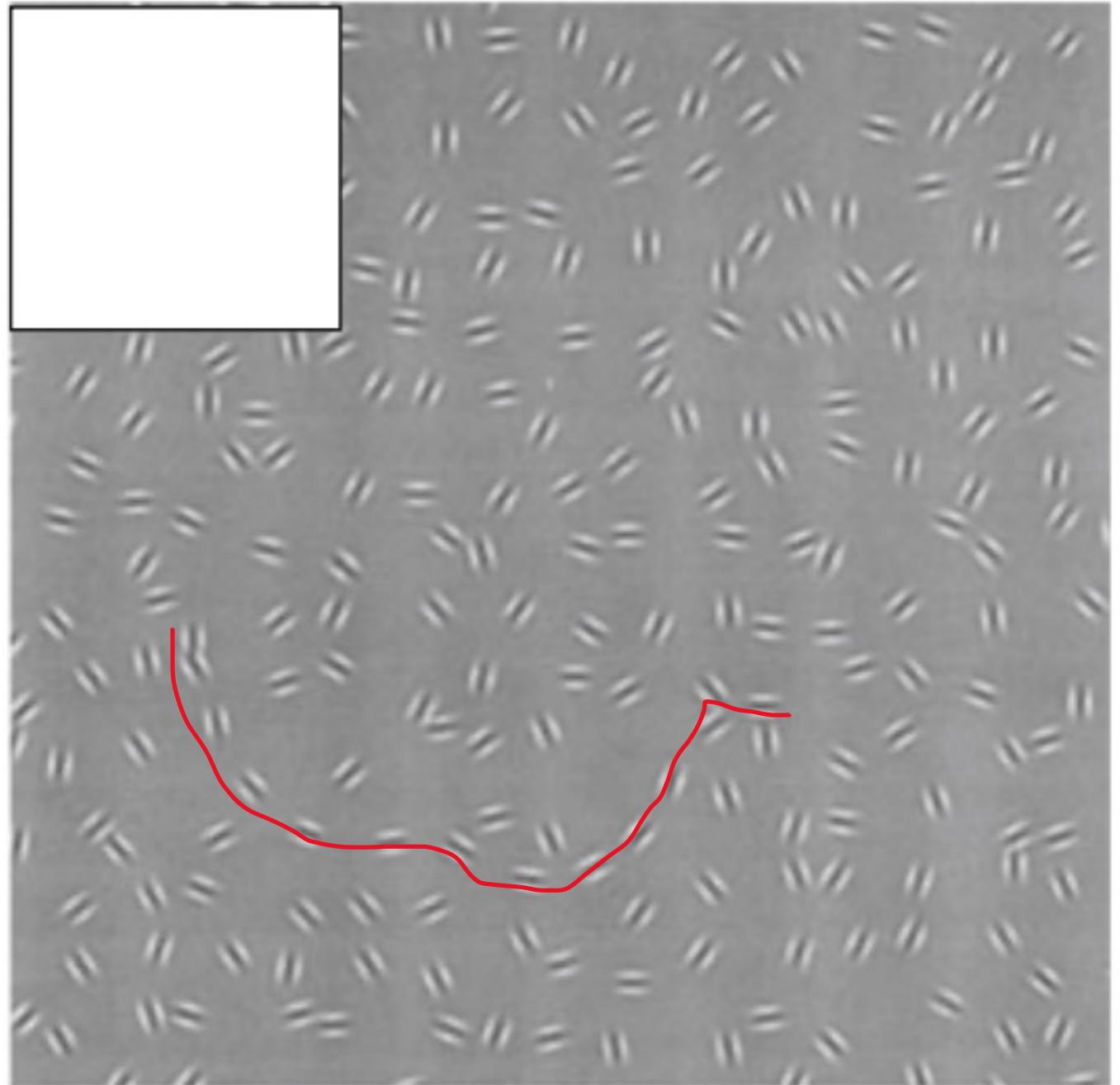


Path Lifting
from \mathbb{R}^2 to
 $\mathbb{R}^2 \times \mathbb{R}^1$

CONTACT STRUCTURE



Contact structure
Provides Connection



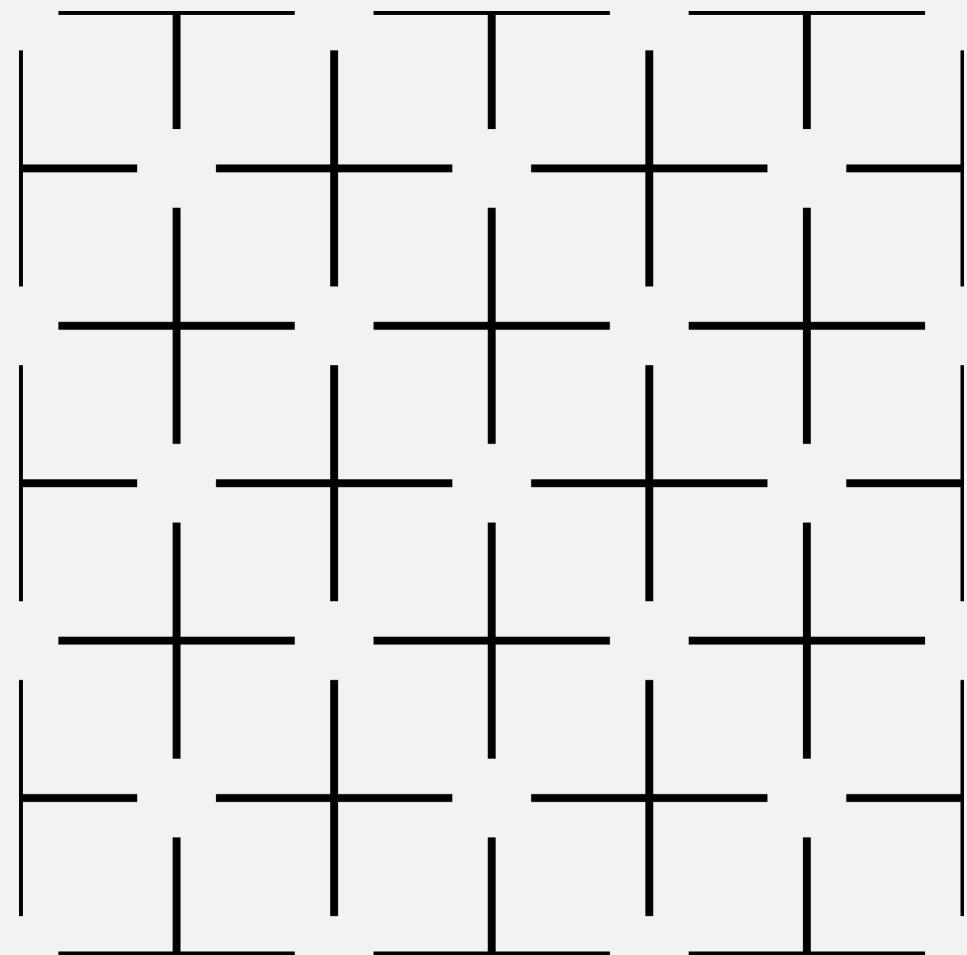
Illusory contour

=

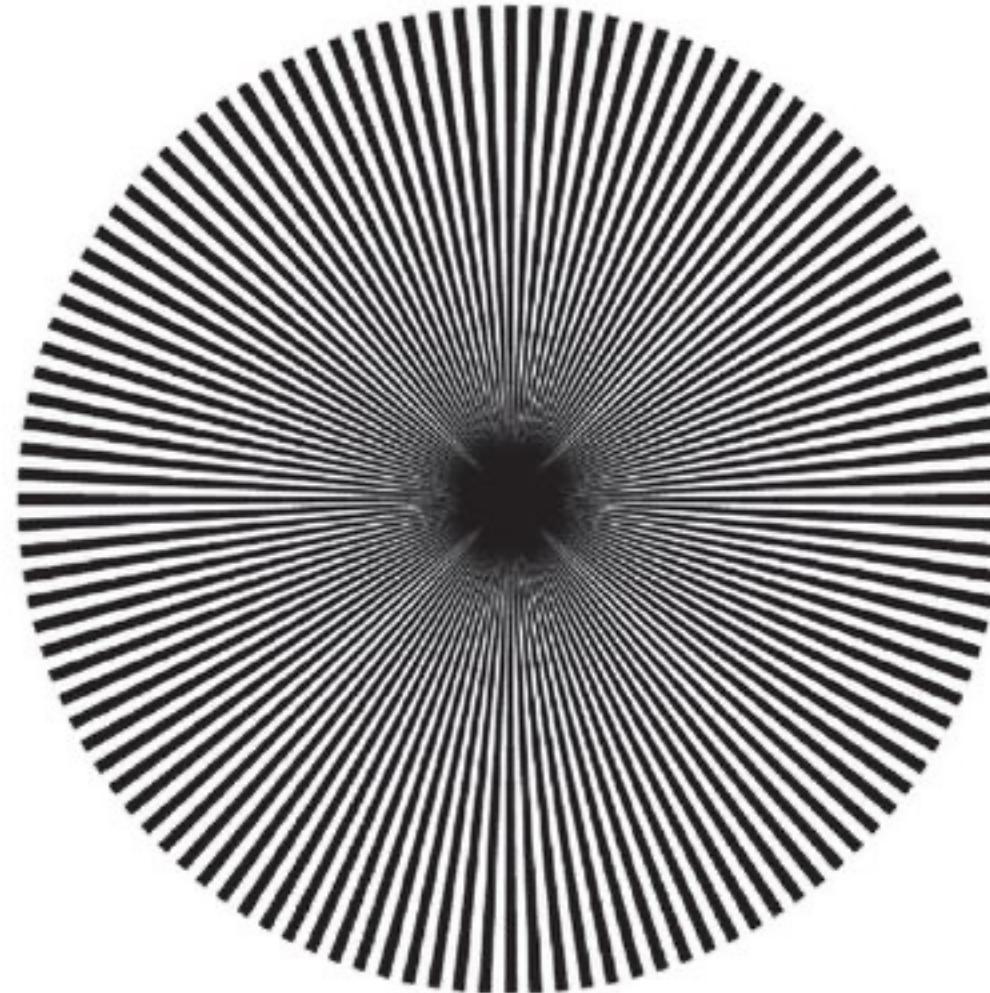
Minimal path tangent to contact
planes

=

“Legendrian Geodesic”



SYMMETRIES OF THE VISUAL CORTEX



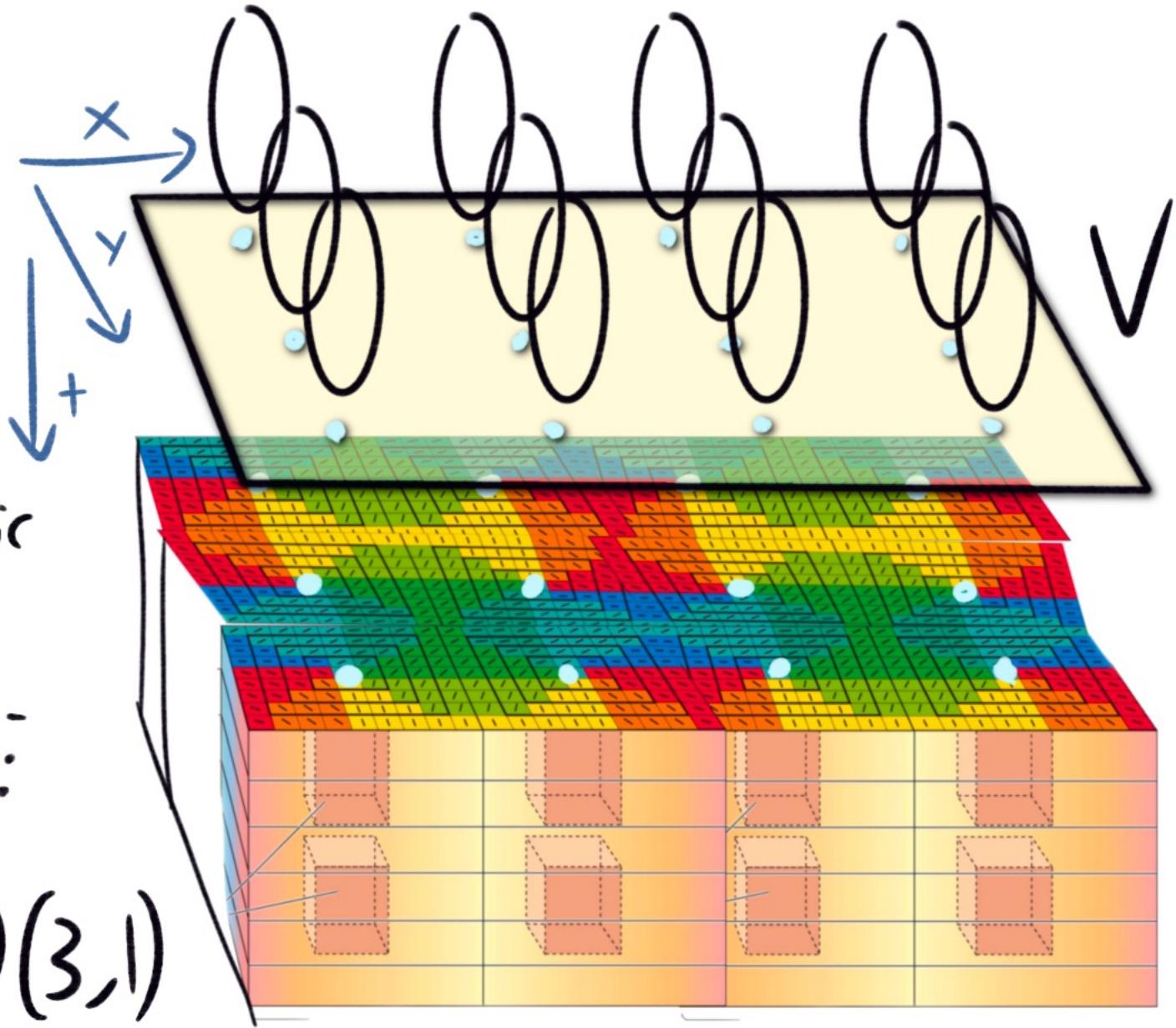
Full visual cortex

is $\underbrace{V \times \text{IRP}^1}_{x,y,\theta} \times \underbrace{\text{IR}}_{+}$

carries lorentzian metric

Symmetries are Angle-preserving transforms:

Conformal group $CO(3,1)$

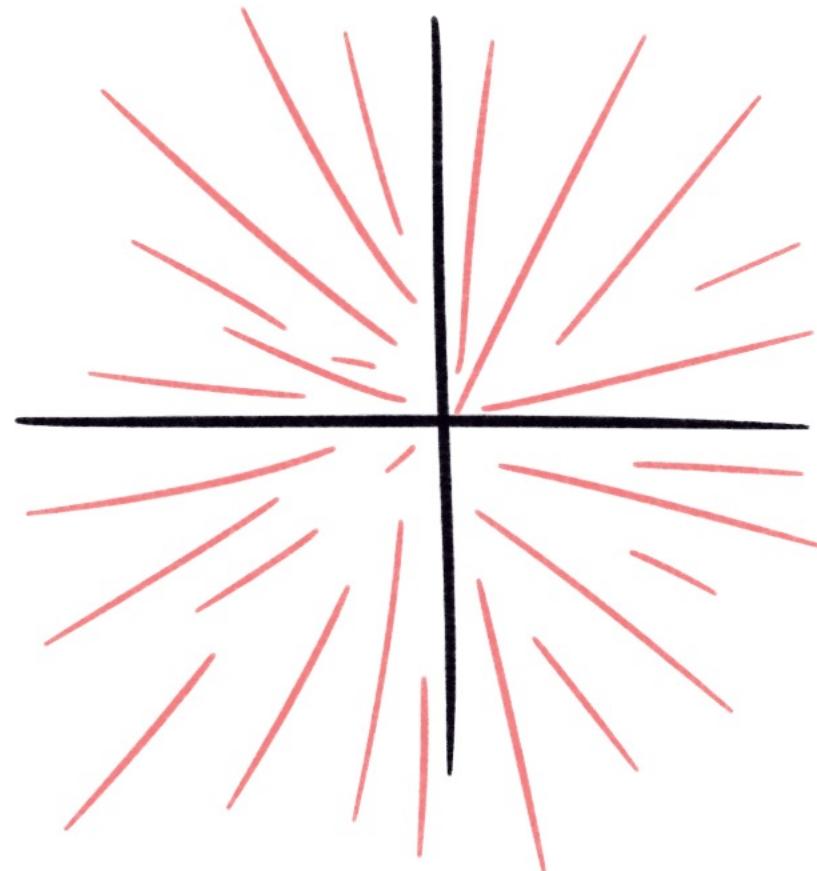


CORRESPONDENCE BETWEEN THE VARIOUS PERCEPTUAL CONSTANCES AND
LIE TRANSFORMATION GROUPS

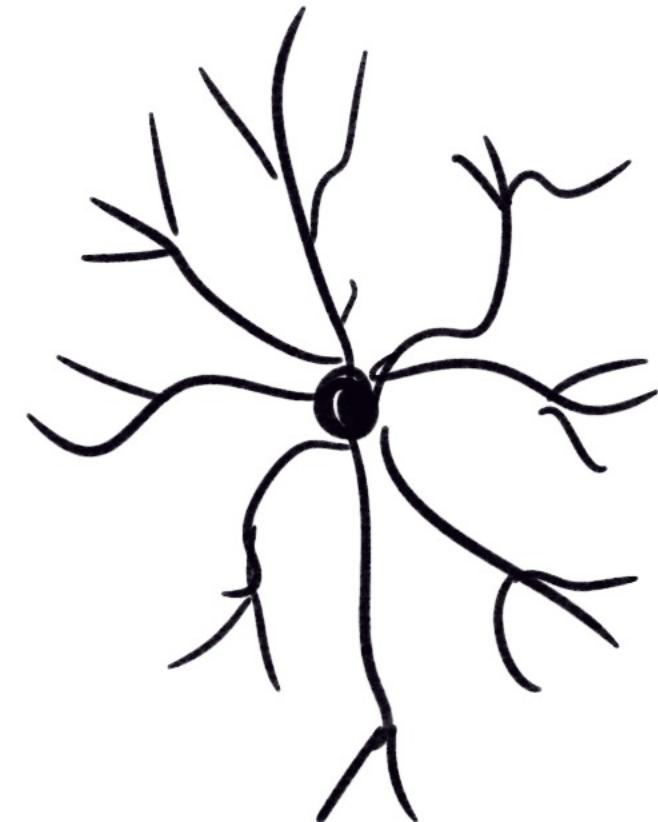
| Perceptual invariance | Lie transformation group |
|-----------------------------------|---|
| A. Shape constancy | A. Affine, or Special Linear, Group |
| a. Location in the field of view | a. Horizontal and vertical translations |
| b. Orientation | b. Rotation |
| c. Binocular vision | c. Pseudo-Euclidean (or hyperbolic) rotations |
| d. (Form memory) | d. (Time translations) |
| B. Size constancy | B. Dilatation group |
| | a. Spiral effects |
| C. Motion | C. Two-dimensional Lorentz group |
| D. (Efferent binocularity) | D. (Hyperbolic rotations in plane-time*) |
| E. (Circulating ("motor") memory) | E. (Rotations in plane-time*) |

Scaling \longleftrightarrow Scale invariance

Generated by $x\partial_x + y\partial_y$

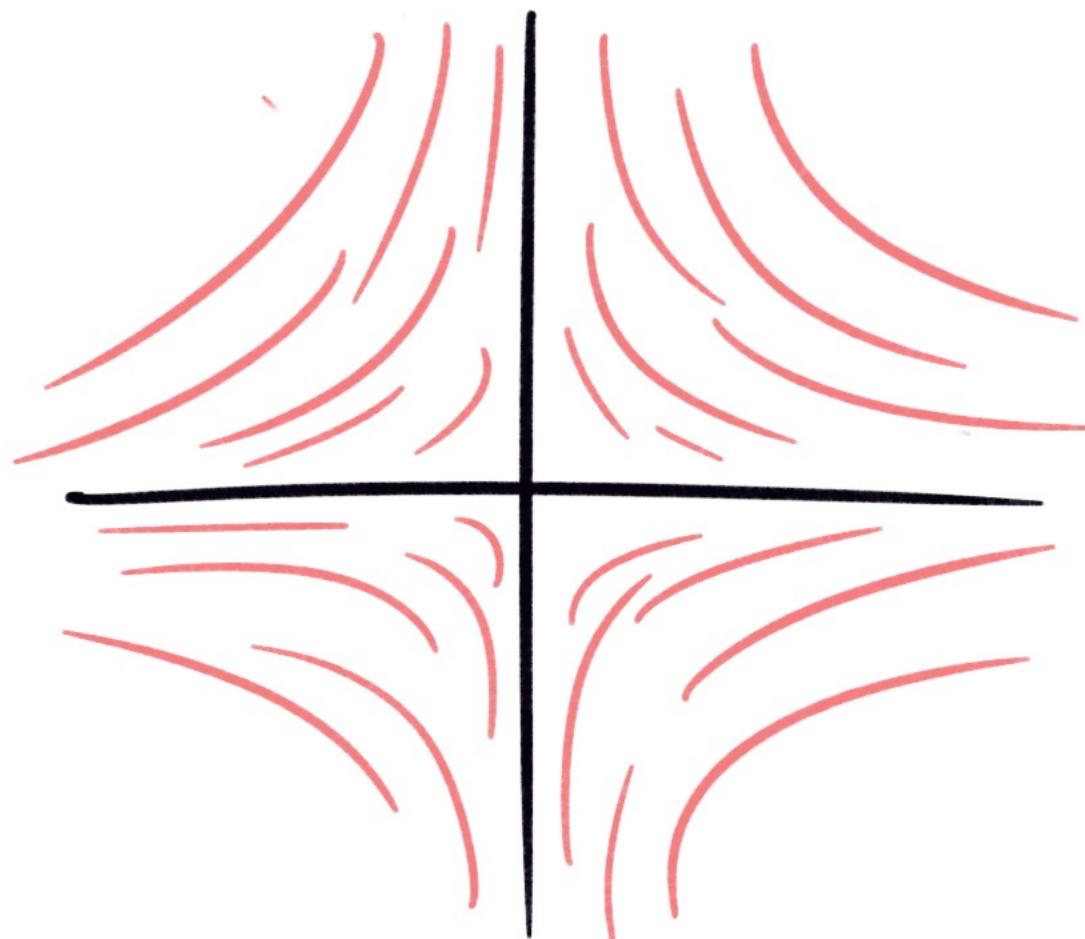


Controlled by
stellate neuron

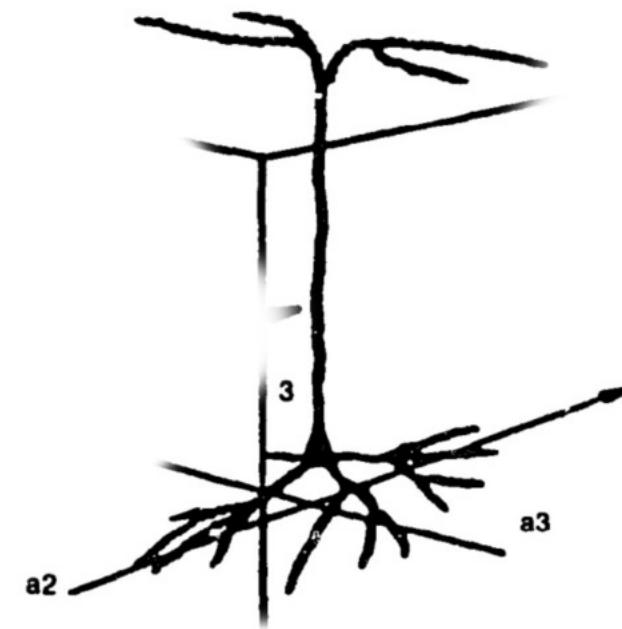


Hyperbolic Rotations \Leftrightarrow Binocular invariance

Generated by $x\partial_x - y\partial_y$

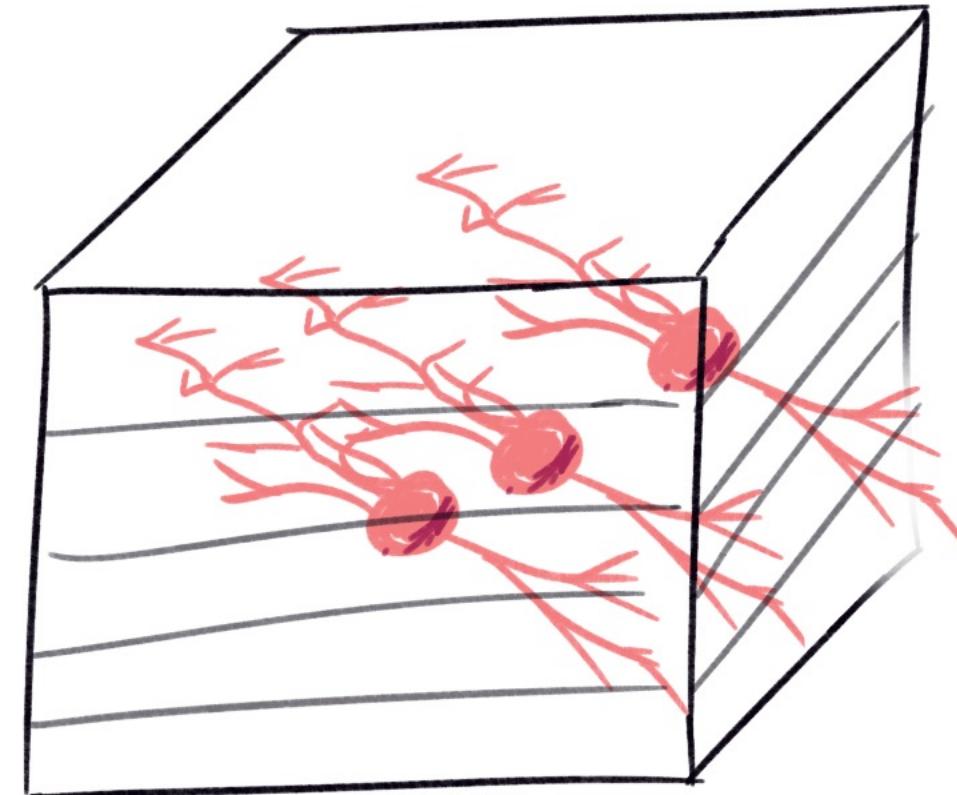
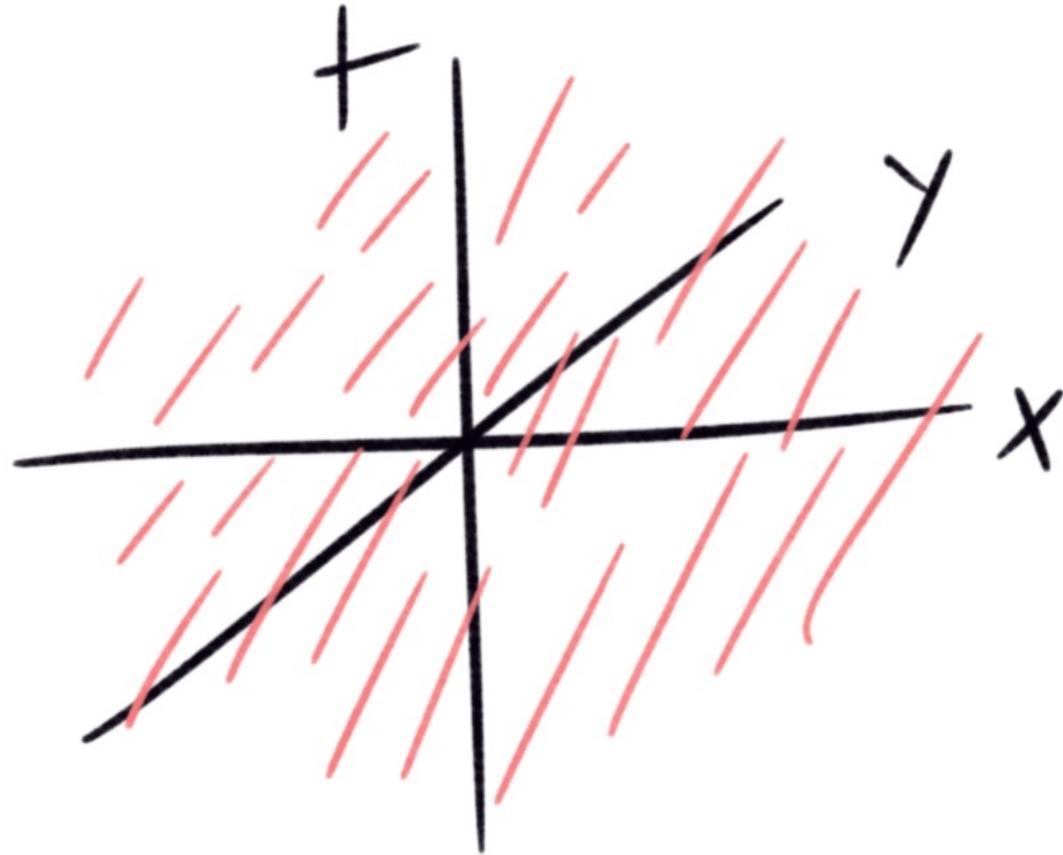


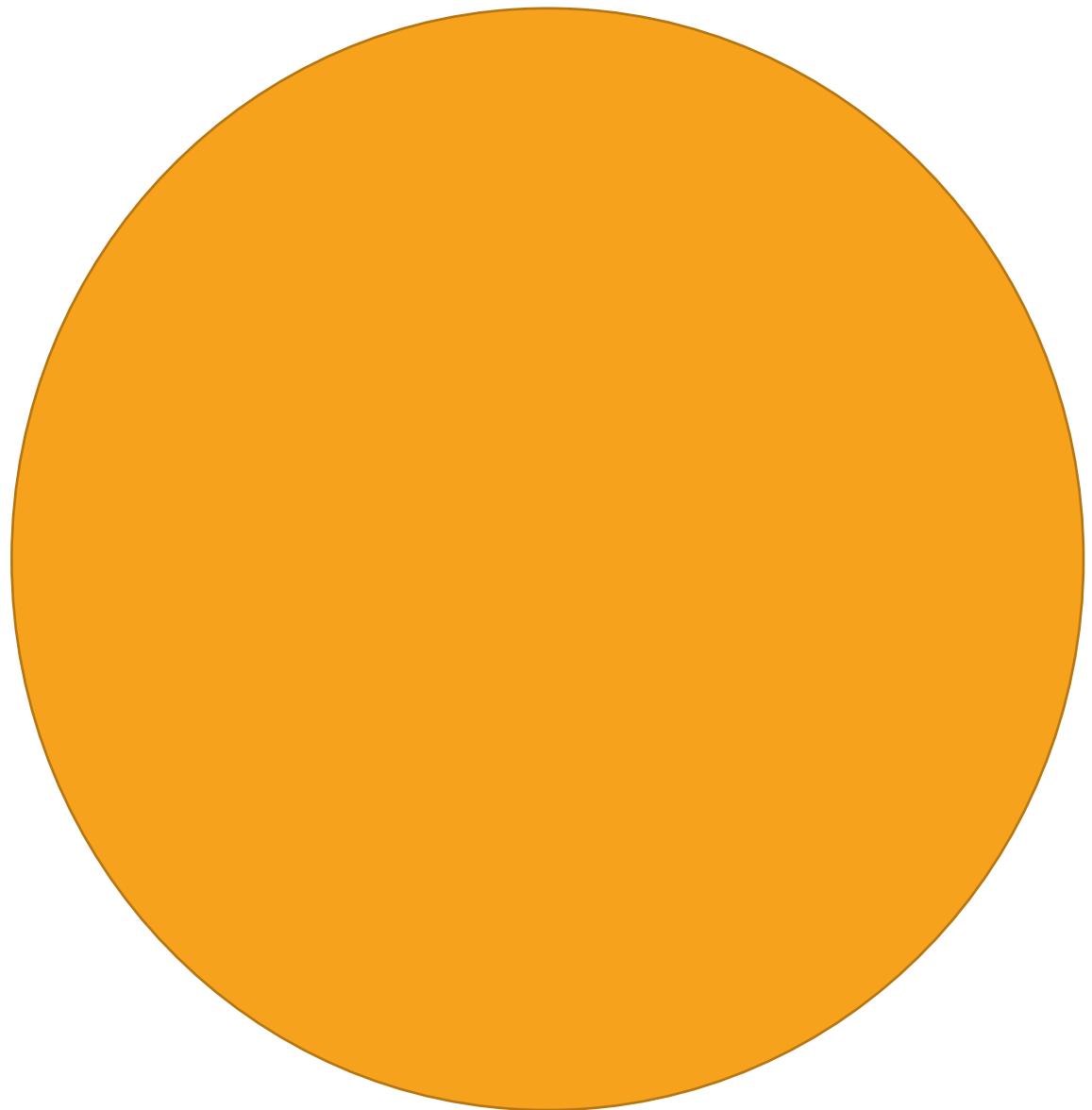
Controlled by
Pyramidal neuron



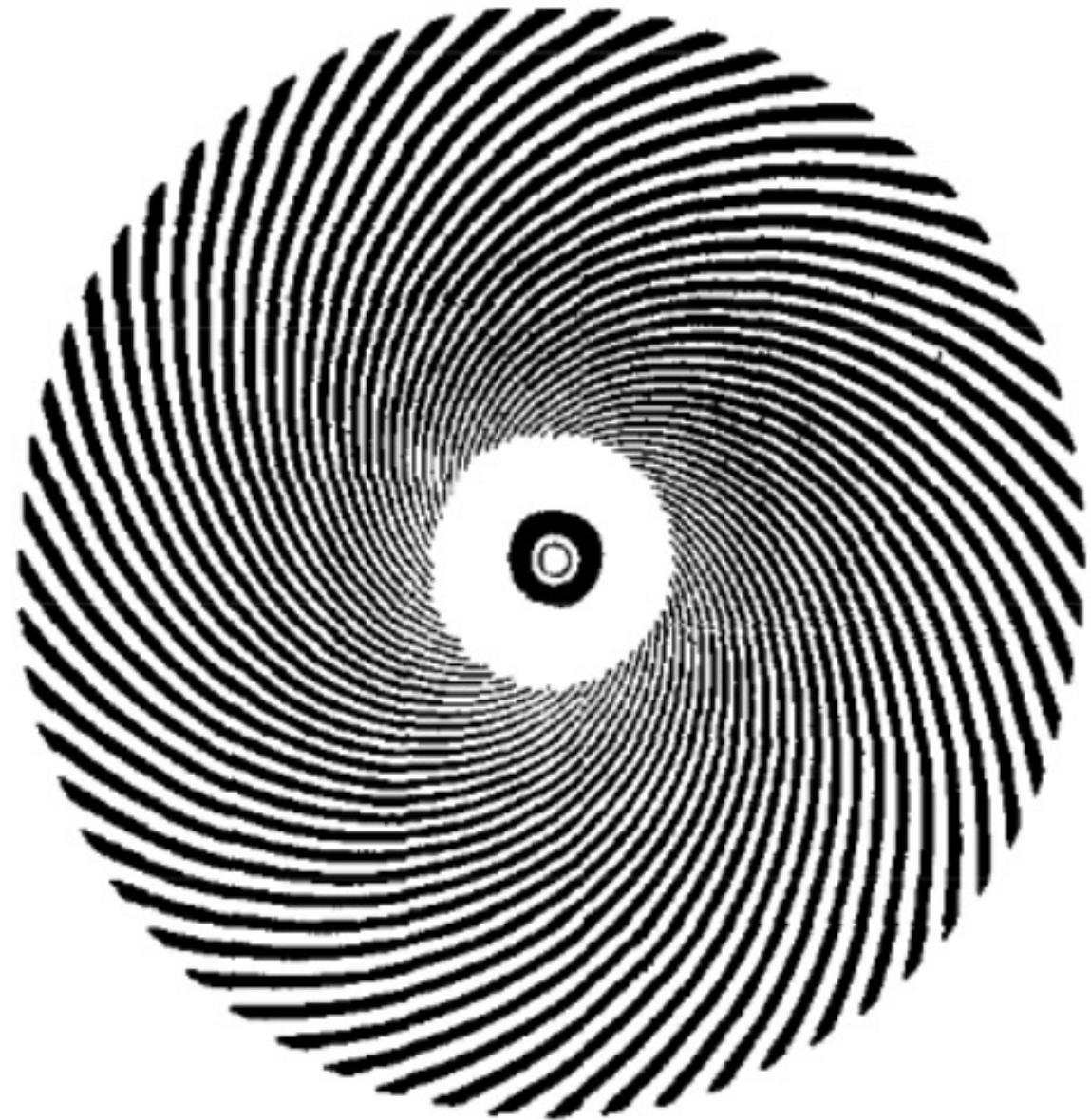
Translation w/ time \iff Motion ih variance

Generated by $\partial_x + \partial_x$



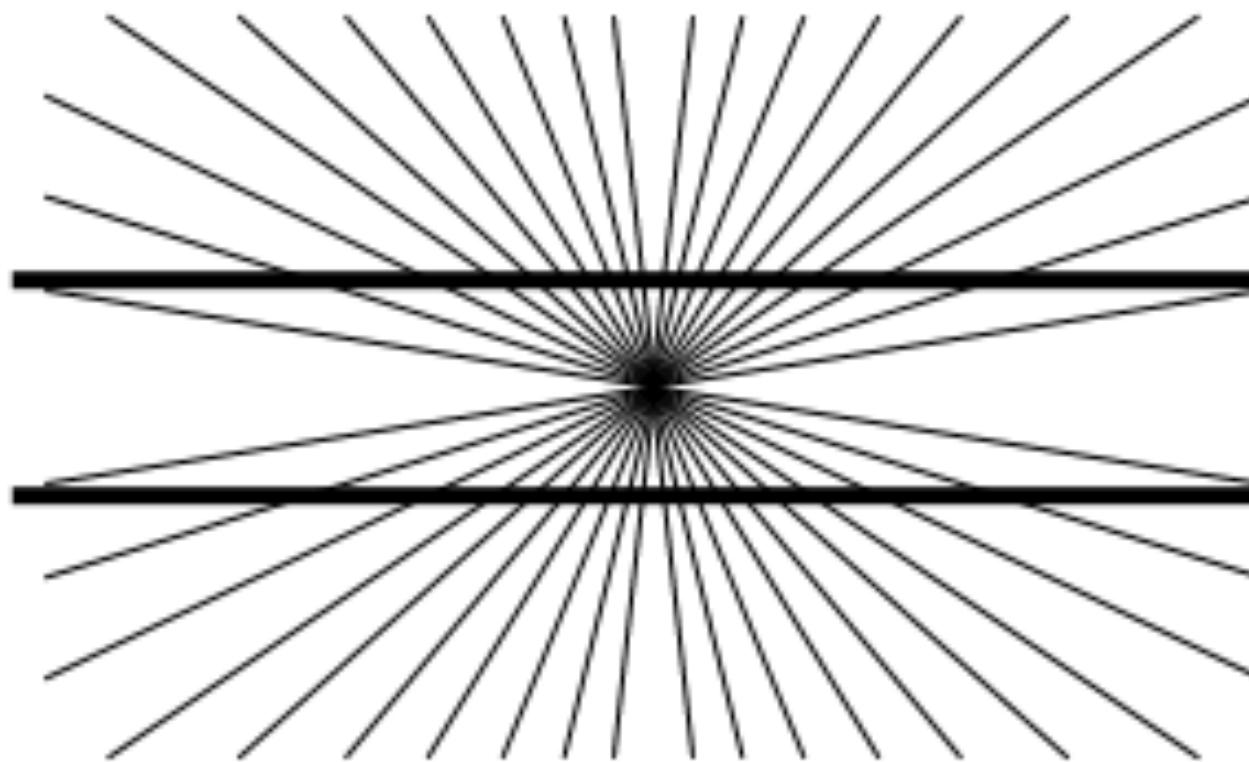


Color afterimage



Shape afterimage

Shape afterimages add like vector fields



NOW CLOSE YOUR EYES

CHP for arbitrary base spaces, and it therefore follows that the cortical contact bundle is a fibre bundle in the sense of Hurwicz as well. Finally [42], “the image of a fibration in the sense of Serre under the singular complex functor is a fibration in the sense of Kan.” The latter is general enough to handle any aspect of information-processing psychology, and the presence in cognitive phenomena of the simplicial functor (the category of simplicial objects) has been argued at length in [29] and [43].

Invariance in the Kan fibration case, which is embodied in such higher cognitive faculties as conscious thought, plans, long-term behavioral sequences, etc., apparently corresponds to Freyd’s “Theorem” [44]: *an elementary property on categories is invariant up to equivalence types of categories iff it is a diagrammatic property.*” Thus “chasing around the diagram” in “trains of thought” is more basic than “logical,” conscious thought. Intuition precedes insight and inspiration and certainly the laborious processes of conscious thought itself.

REFERENCES

- Main reference: “The Visual Cortex is a Contact Bundle”, by W. Hoffman
 - <http://www.its.caltech.edu/~matilde/VisualCortexContactBundle.pdf>
- “The Lie algebra of visual perception” by W. Hoffman
 - <https://www.sciencedirect.com/science/article/pii/0022249666900058>