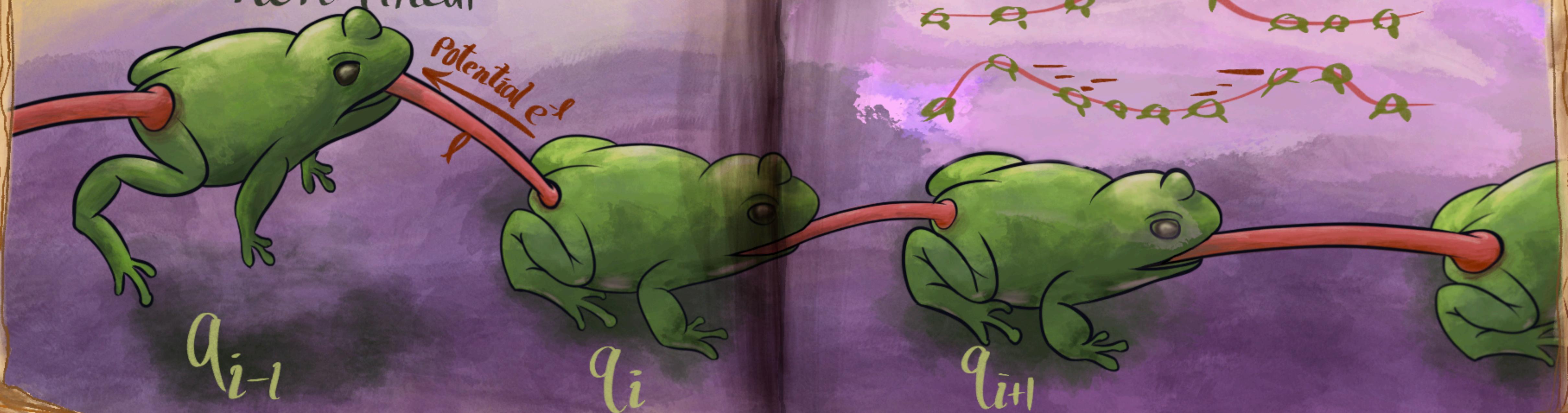


Toda Lattice

N toads on a Line

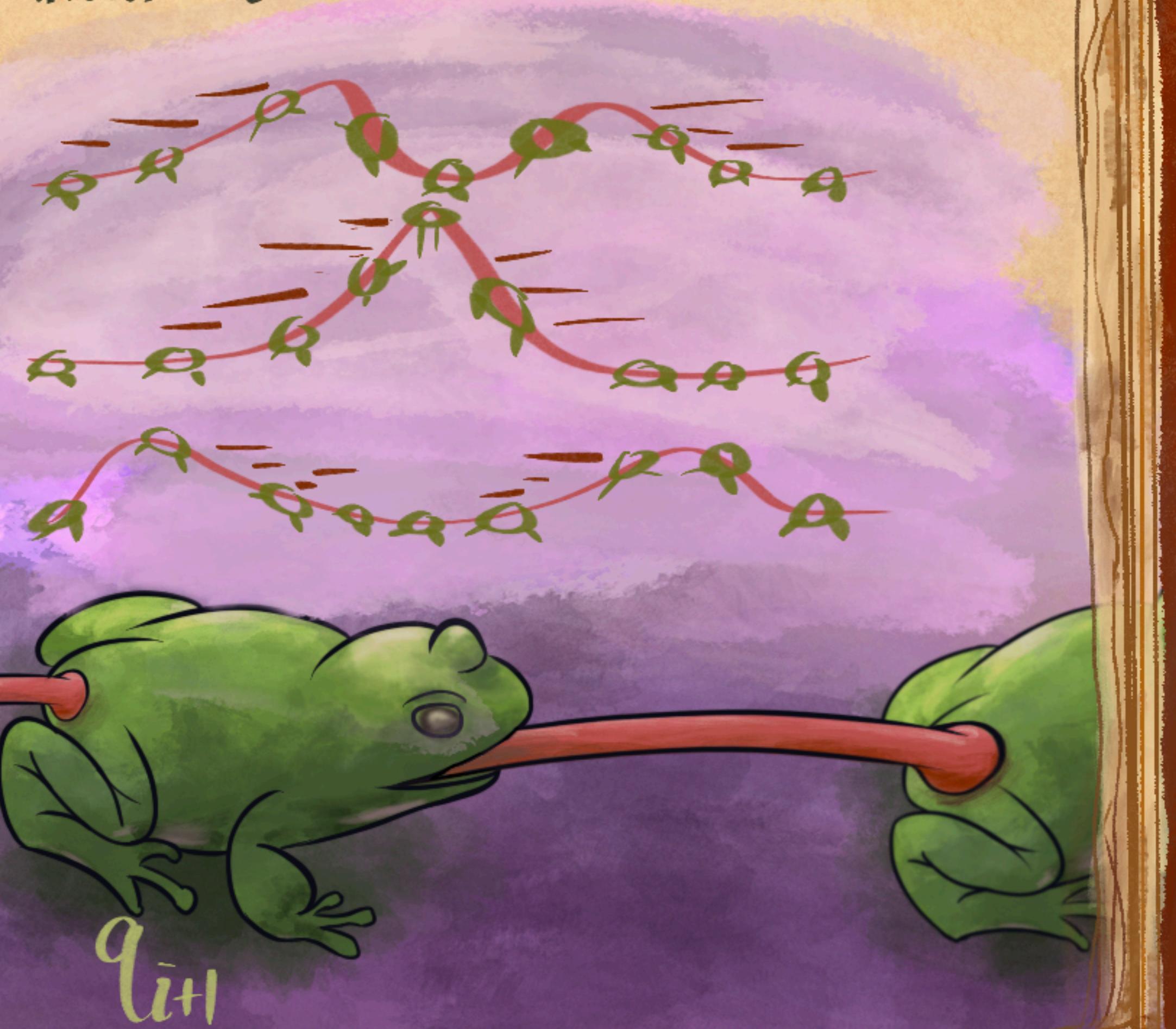
$$\mathcal{H} = \sum p_i^2 + e^{q_{i+1} - q_i}$$

non-linear



has Solitons which cross w/o interacting
splits into sum of solitons

Completely integrable: conserved quantity
= amount of n^{th} soliton



Lax pair

write toda lattice E.O.M as

$$\dot{L} = [A, L]$$

then $L(t) = V L V^{-1}$, $V = \exp(A t)$

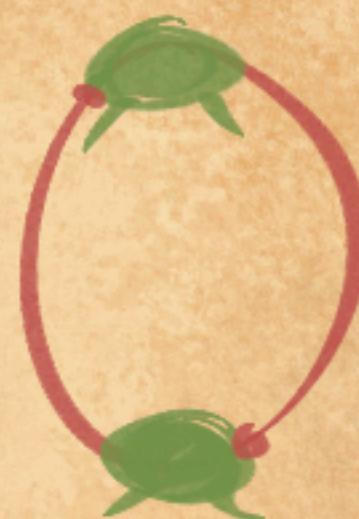
so, spectrum constant!

Eigenvalues = conserved quantities

Eigenvectors = 'angles'

Important example:

$N=2$ periodic



natural 1-parameter family

$$L(z) = \begin{bmatrix} p_1 & e^{\bar{q}} + \frac{1}{z} e^{-\bar{q}} \\ e^{\bar{q}} + z e^{-\bar{q}} & p_2 \end{bmatrix} \quad \bar{q} = \frac{q_1 - q_2}{2}$$

$$A(z) = \begin{bmatrix} 0 & e^{\bar{q}} - \frac{1}{z} e^{-\bar{q}} \\ -e^{\bar{q}} + z e^{-\bar{q}} & 0 \end{bmatrix} \quad z \in \mathbb{C}$$

Solitons = e.vects of L (2 of them)
classified by e.vals, determined by I.C.s

evolution: travel around circle linearly
 \Rightarrow linear flow around $U(1)^2$ \Downarrow