

$$2: \Omega_2 = \mathbb{Z}_2$$

surface classification:

$$\mathbb{RP}^2 \# \mathbb{RP}^2 \# \cdots \# \mathbb{RP}^2$$



WTS

$$\begin{cases} \mathbb{RP}^2 \text{ not boundary} \\ \mathbb{RP}^2 \# \mathbb{RP}^2 \text{ boundary} \end{cases}$$

suppose $\partial M = \mathbb{RP}^1$

$$2M = M \sqcup M/\partial M \text{ closed}$$

Closed,
odd-dim $\Rightarrow 0$

$$\cancel{\chi(2M) = 2\chi(M) - \chi(\partial M)} \Rightarrow \chi(\partial M) = 2\chi(M)$$

meyer-vatoris

$$\text{But, } \chi(\mathbb{RP}^2) = 1 \text{ odd!}$$

even! \times

