

# Lax pair

write toda lattice E.O.M as

$$\dot{L} = [A, L]$$

then  $L(t) = V L V^{-1}$ ,  $V = \exp(A t)$

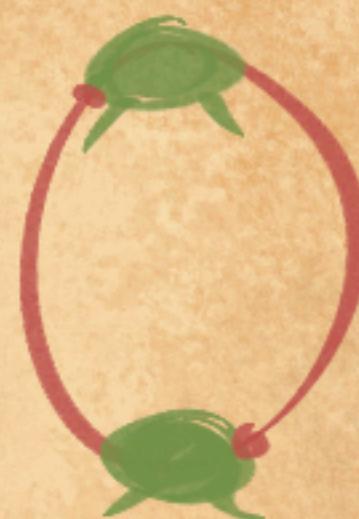
so, spectrum constant!

Eigenvalues = conserved quantities

Eigenvectors = 'angles'

Important example:

$N=2$  periodic



natural 1-parameter family

$$L(z) = \begin{bmatrix} p_1 & e^{\bar{q}} + \frac{1}{z} e^{-\bar{q}} \\ e^{\bar{q}} + z e^{-\bar{q}} & p_2 \end{bmatrix} \quad \bar{q} = \frac{q_1 - q_2}{2}$$

$$A(z) = \begin{bmatrix} 0 & e^{\bar{q}} - \frac{1}{z} e^{-\bar{q}} \\ -e^{\bar{q}} + z e^{-\bar{q}} & 0 \end{bmatrix} \quad z \in \mathbb{C}$$

Solitons = e.vects of  $L$  (2 of them)  
classified by e.vals, determined by I.C.s

evolution: travel around circle linearly  
 $\Rightarrow$  linear flow around  $U(1)^2$   $\Downarrow$

# Chapter 4

## Spectral cemetery

