Thom's Theorem:

$$\begin{tikzcd} \mathbb{R}^{n+k+1}_+ = S^{n+k+1} \arrow[r] \arrow[d, leftrightarrow, "\cong"] & Th(\nu \oplus 1) \arrow[r] \arrow[d, leftrightarrow, "\cong"] & MO(k+1) \arrow[d, leftrightarrow, "\nu \to \nu \oplus 1"] \\ \Sigma S^{n+k} \arrow[r] & \Sigma Th(\nu) \arrow[r] & \Sigma MO(k) \end{tikzcd}$$

$\Sigma MO(k) \longrightarrow MO(k+1) \Rightarrow$ pre-spectrum!

$\pi_k(MO) = \varinjlim \pi_{n+k}(MO(n))$ well defined

$$\boxed{\Omega_k \cong \pi_k(MO)}$$

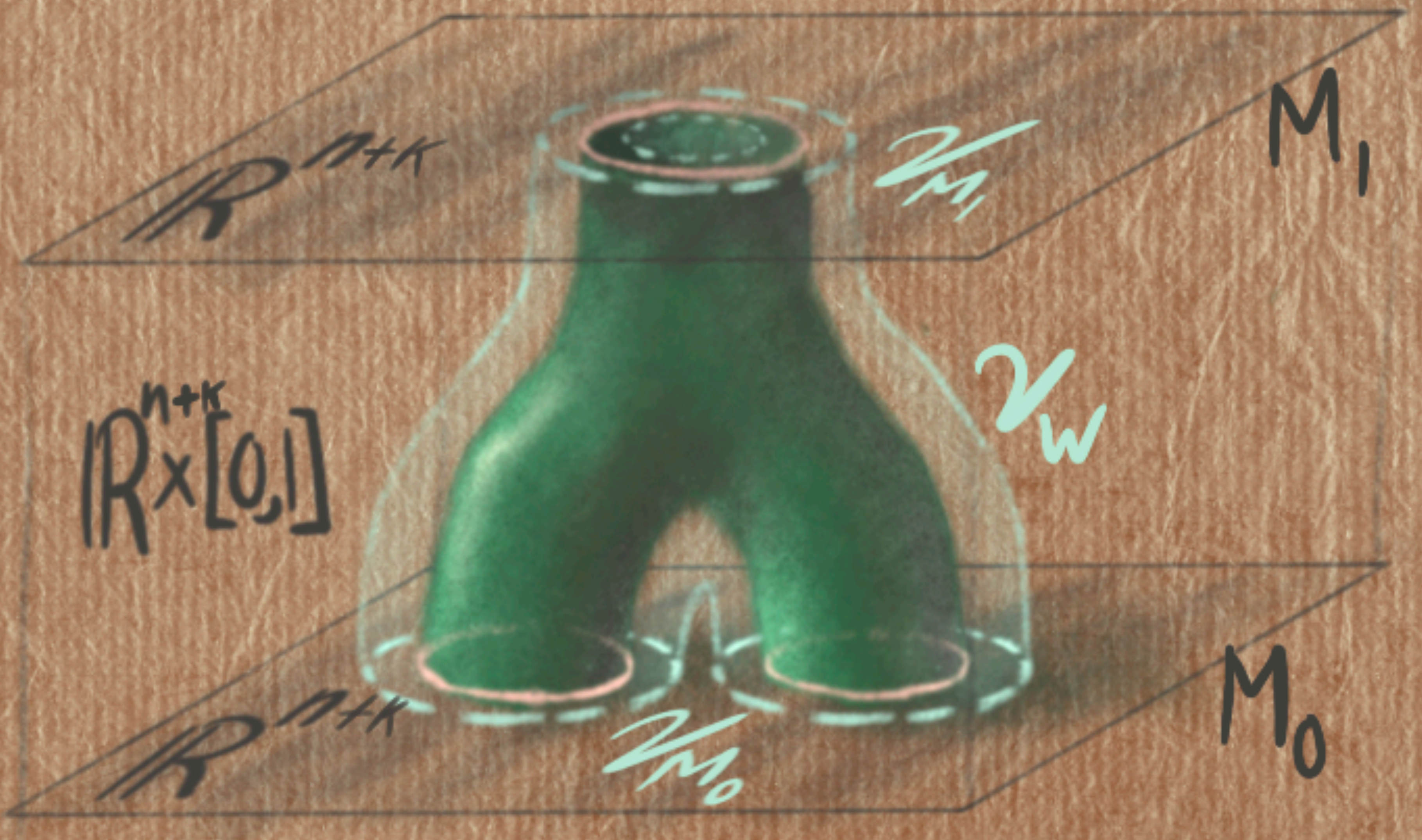
$$\begin{tikzcd} \Omega_k \arrow[r, bend left, "\alpha"] \arrow[r, phantom, "\alpha\beta = \beta\alpha = id"] & \pi_k(MO) \arrow[l, bend left, "\beta"] \end{tikzcd}$$

$\alpha: \Omega_K \longrightarrow \pi_K(MO)$

$i \in \{0,1\}$

cobordism: $\partial W = M_0 \sqcup M_1$ $E(\nu_{M_i}) = E(\nu_W)|_{\mathbb{R}^{n+k} \times i}$

P.T(M_i) is P.T$(W)|_{S^{n+k} \times i}$ $\Rightarrow$ W gives homotopy P.T$(M_0) \rightarrow$ P.T(M_2)



Group homomorphism:

$\alpha[M \sqcup N] \quad = \quad \alpha[M] + \alpha[N]$