

Thom spectrum of X

$$MO_K(X) = MO(K) \wedge X_+$$

$$\begin{aligned}\Omega_K(X) &= \lim_{n \rightarrow \infty} \pi_{n+K}(MO_n(X)) \\ &= \pi_K(MO(X))\end{aligned}$$

General cobordism Theory

Cobordisms w/ extra structure

Tangential structures: lifts $M \xrightarrow{\quad} BO(k)$

e.g. oriented cobordism: For $\partial W = M \sqcup N$,
demand W have orientation s.t. $\partial W = M - N$

Framed cobordism: n pointwise-L.I sections of ν (i.e - trivialization)

\Rightarrow tubular nbhd $E(\nu) = M \times D^n$, $Th(\nu) = M \times S^n$

Pontryagin-Thom map: $\mathbb{R}^{n+k+1} \xrightarrow{\quad} M \times S^n \xrightarrow{\quad} S^{n+k} \xrightarrow{\quad} S^n$
defines element of $\pi_n S$!

theory	spectra
cobordism	M_0
oriented cobordism	MSO
spin cobordism	$MSpin$
framed cobordism	MFr
cowbordism	M_{00}

$$\Omega_k^{Fr} \cong \pi_k S^*$$