

Result of renormalization:

$$\|dA\|^2 \quad \text{topological}$$

$U(1)$ gauge theory: $\mathcal{L} = \frac{1}{g(u)^2} S dA \wedge *dA + \Theta(u) S dA \wedge A + \dots$
1-form A

combine into $\chi(u) = \Theta(u) + \frac{i}{g^2(u)}$ "electric"
"magnetic"

strengths $g(u), \Theta(u)$ depend on a parameter

"space of vacua" columb branch

possible zero-energy configurations

(i.e zero curvature, etc)

$$u \in \mathbb{C}$$



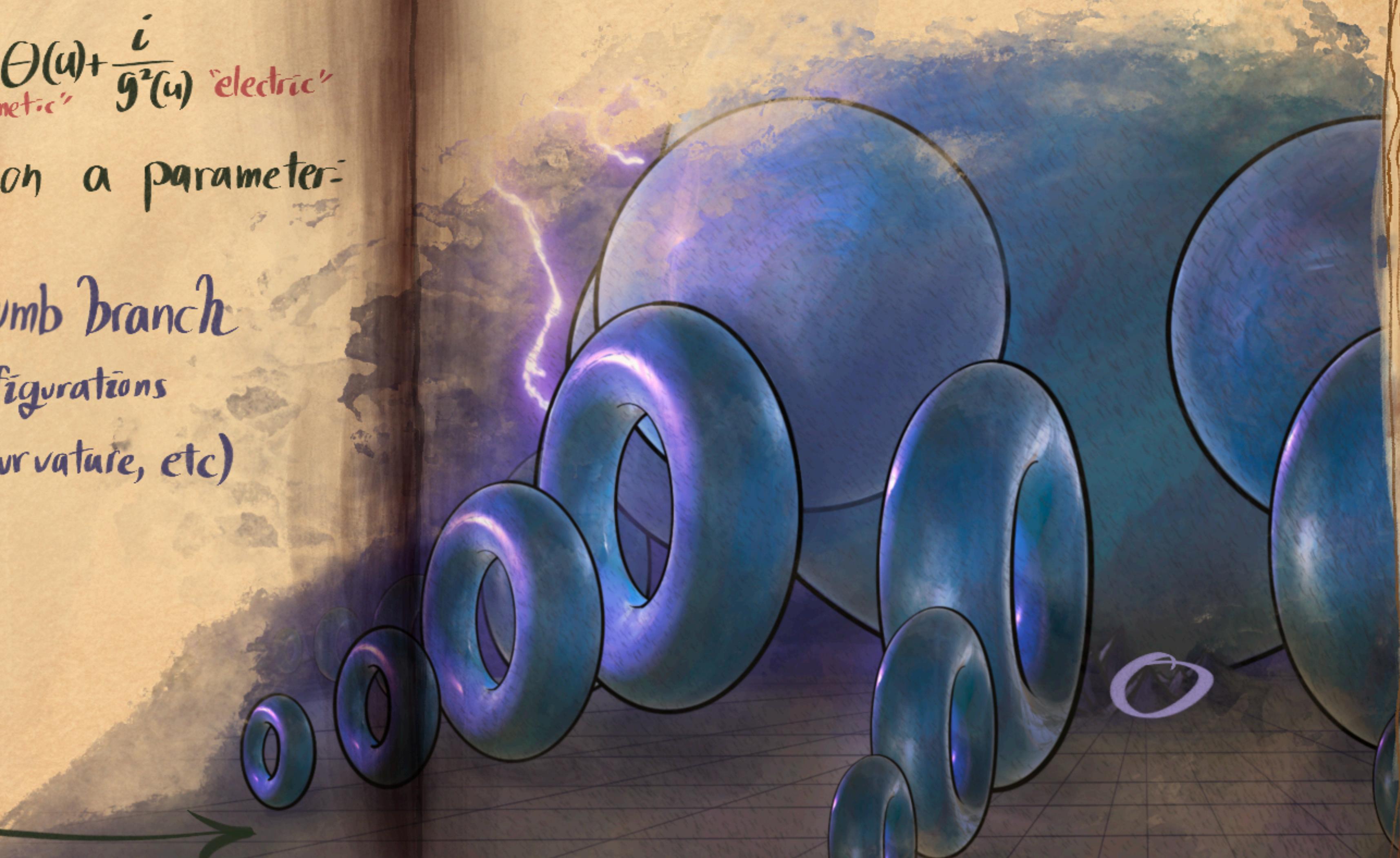
$$\tau \in \{\mathbb{C} \mid \operatorname{Im} \tau > 0\}$$

$$\text{Symmetries: } \tau \mapsto \tau + 1$$

$$\tau \mapsto -\frac{1}{\tau}$$

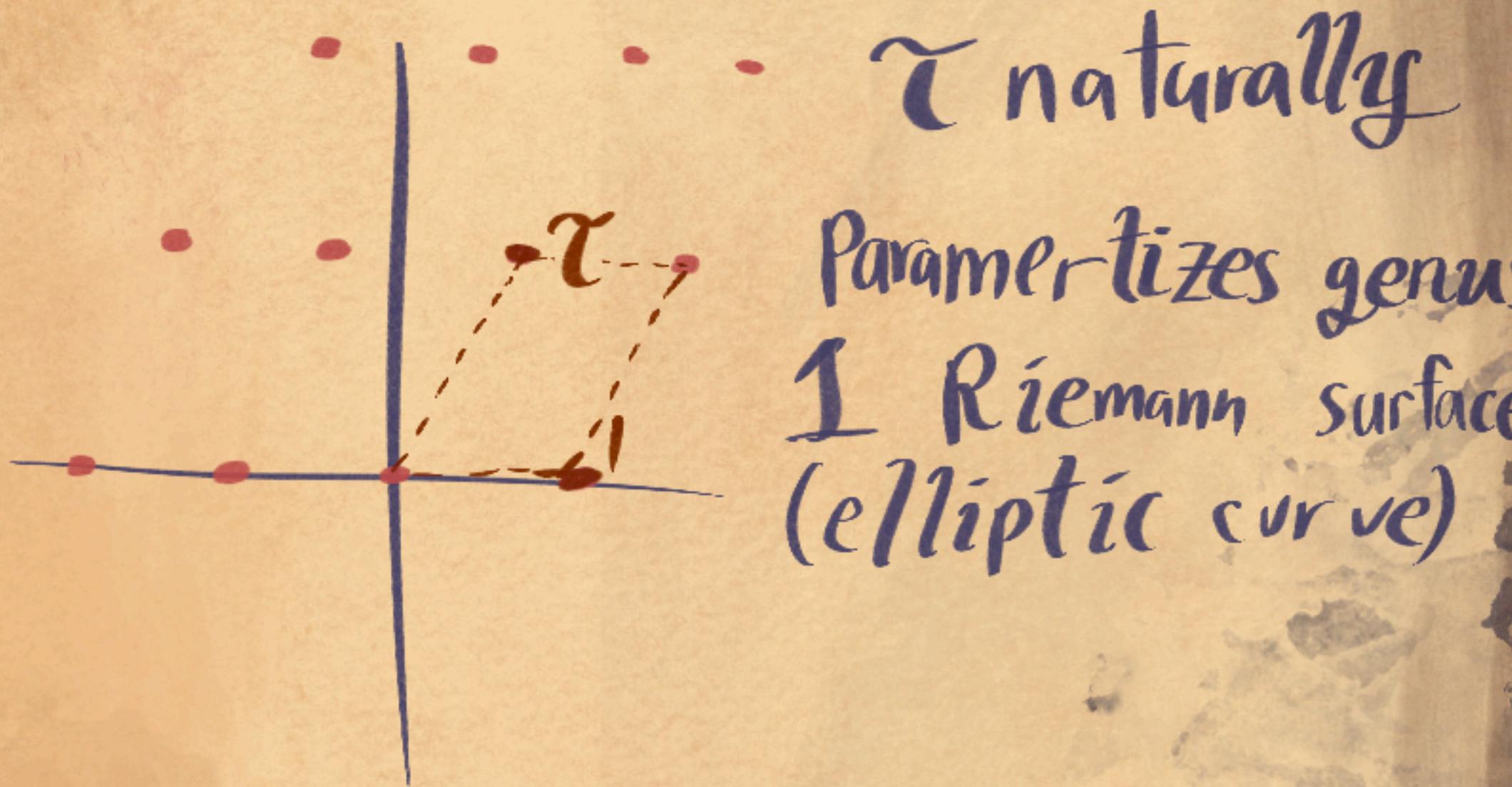
$$(\theta=0 \Rightarrow g \mapsto \frac{1}{g})$$

electric-Magnetic
duality



$\tau \mapsto \tau +$ generates $SL(2, \mathbb{Z})$ action
 $\tau \mapsto -\frac{1}{\tau}$

$\{1, \tau\}$ forms a lattice!!



$\Rightarrow \tau(u)$ gives elliptic curve fibration over $\mathbb{C}!$

Over zero: renormalization breaks \Rightarrow fibers degenerate

carries natural Kähler structure
fiber tori Lagrangian

InTegrable!

