



Hyperbolic Matter thru Higgs bundles

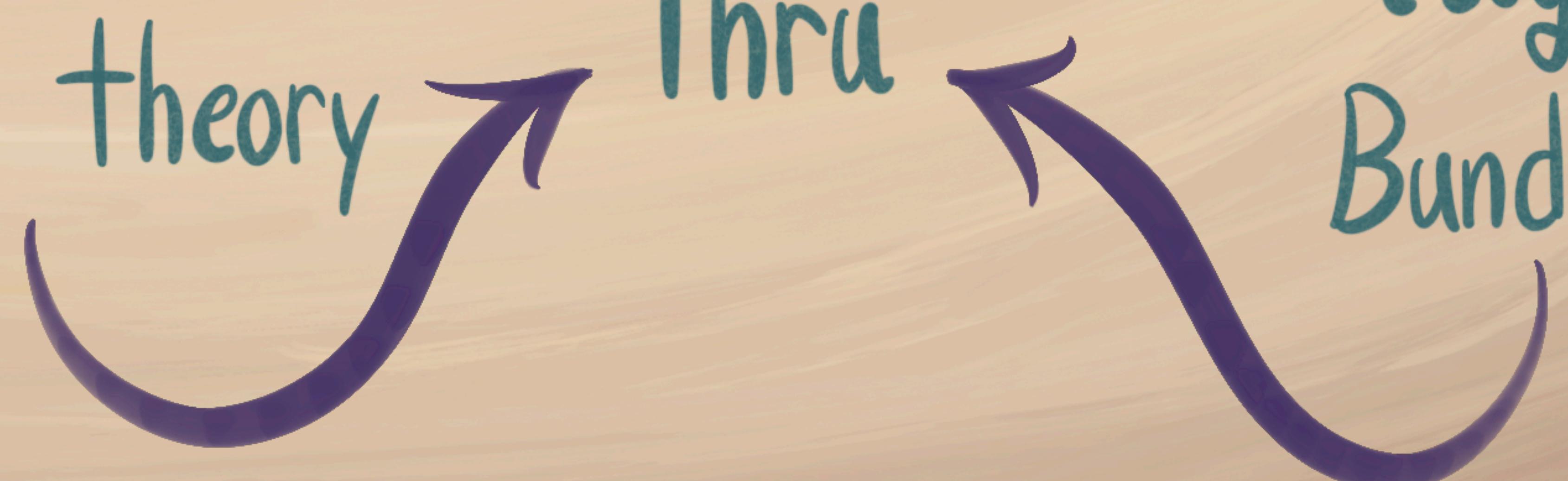
elliot Kienzle
U of Maryland

Outline

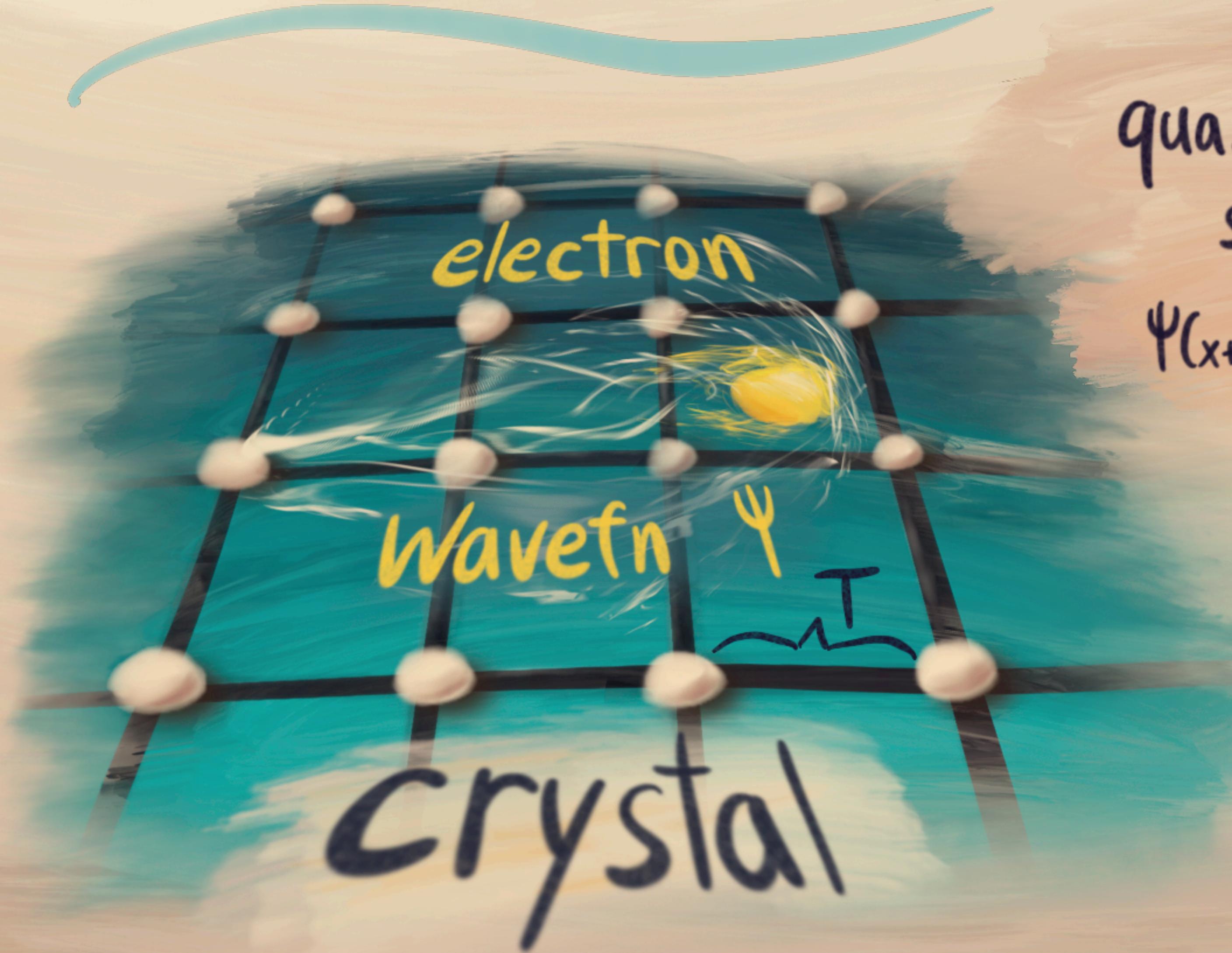
Hyperbolic
Band theory

thru

Higgs
Bundles



Condensed Matter

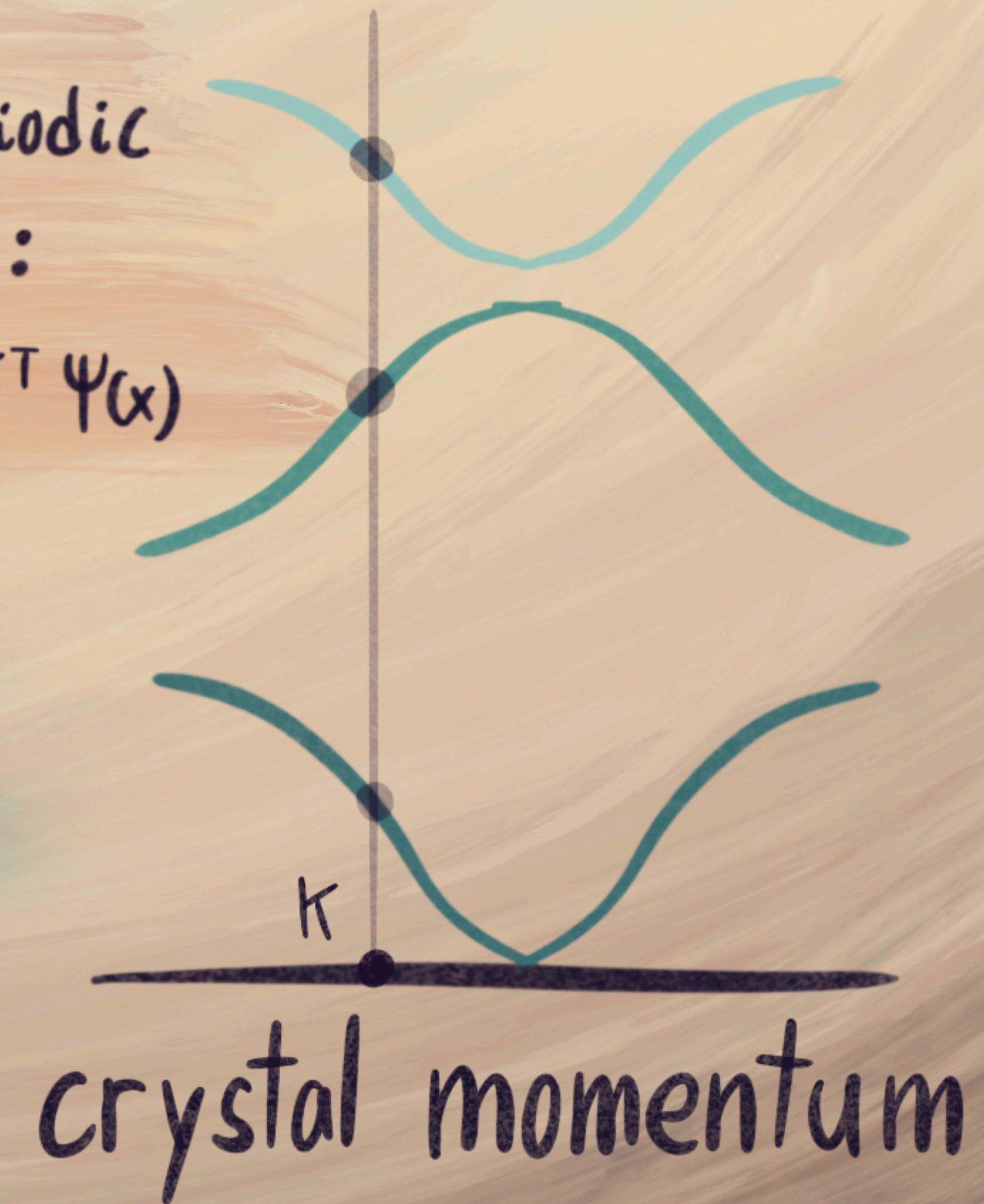


Band theory

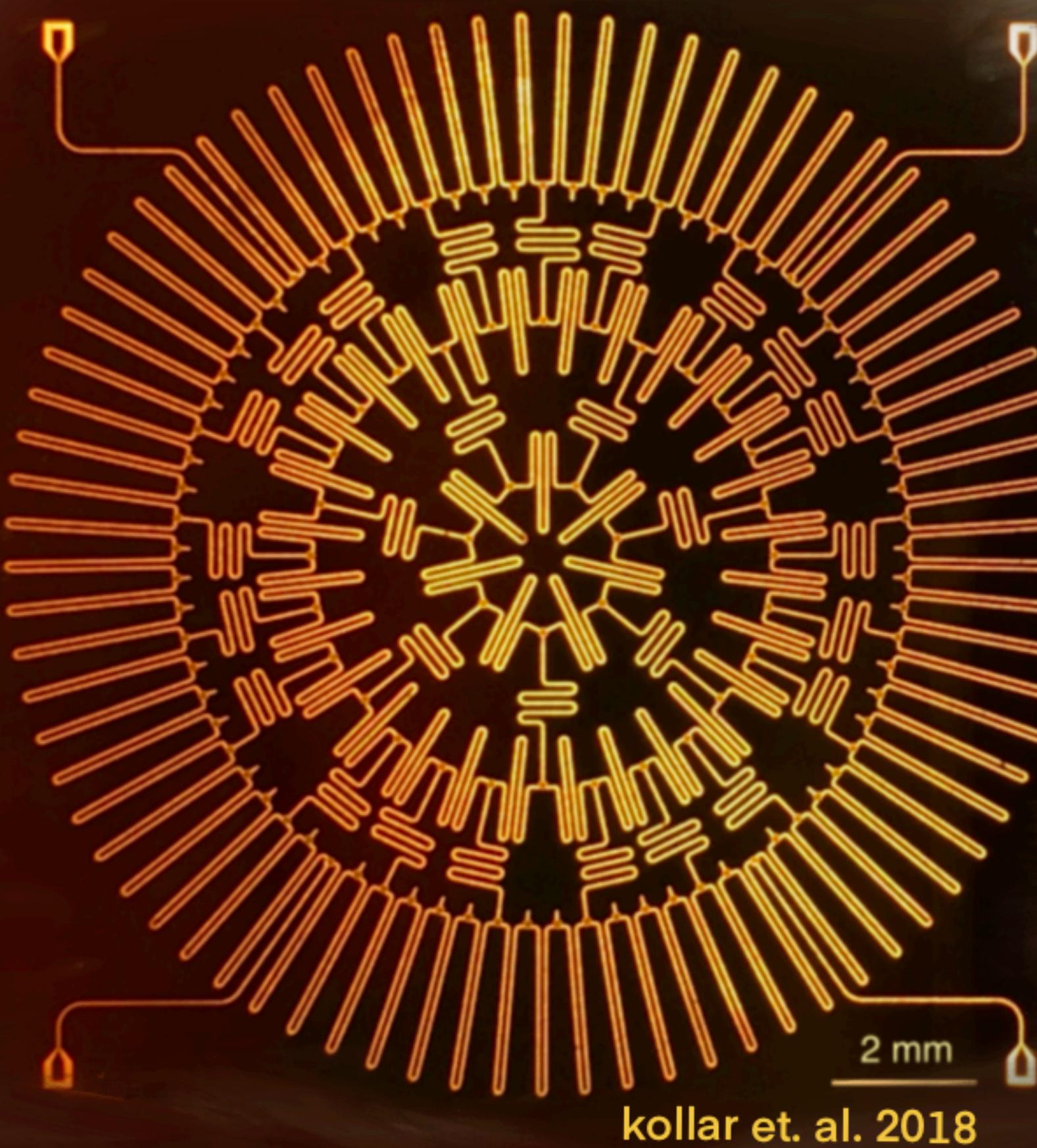
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quasiperiodic
states:

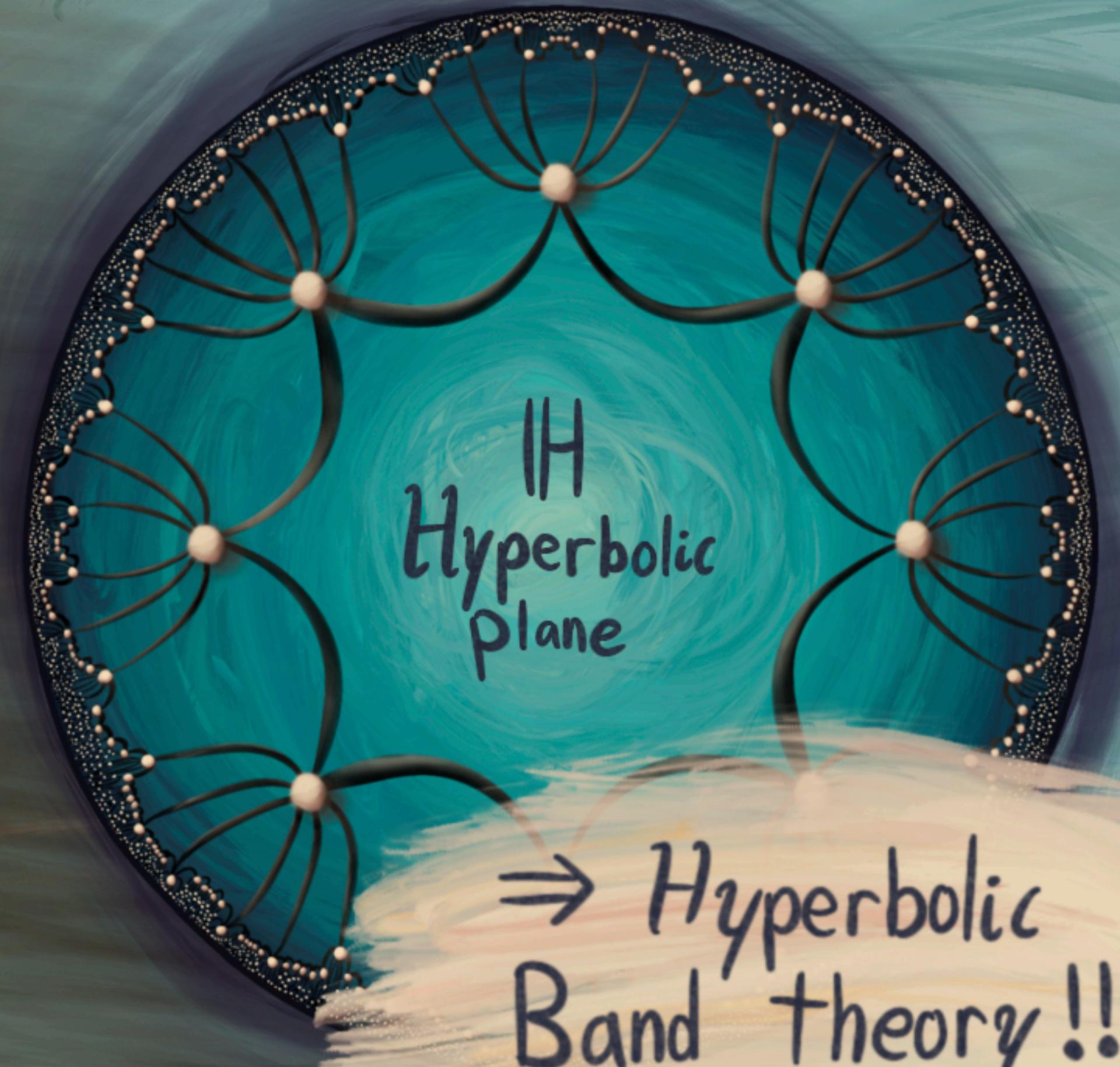
$$\Psi(x+T) = e^{i k T} \Psi(x)$$



Hyperbolic Matter



discrete
model



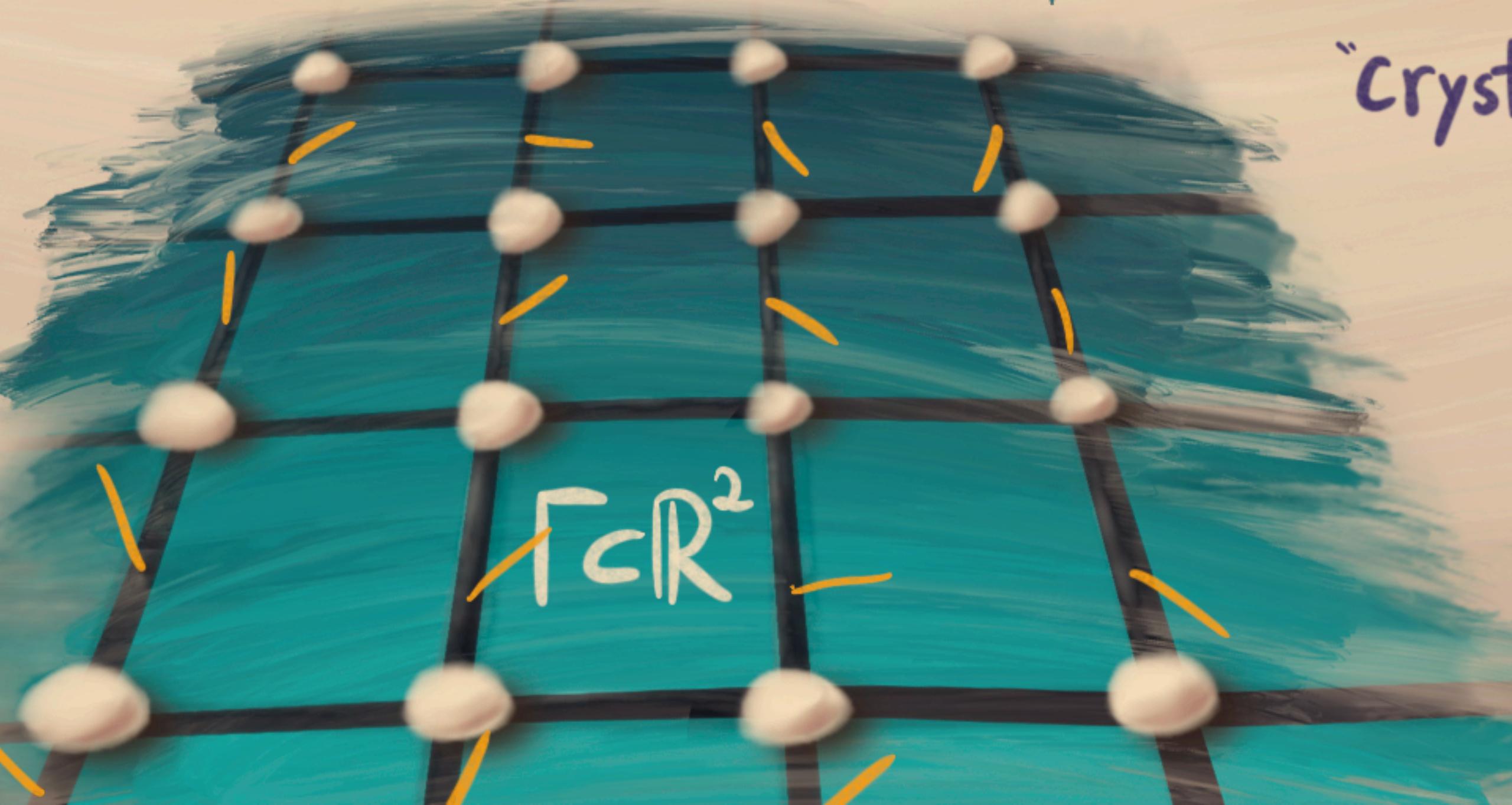
$H = \Delta + V$ periodic ? spectrum?

$T_{\gamma \in \Gamma}$ translation

$$T_{\gamma_2 \gamma_1} = T_{\gamma_1 + \gamma_2} = T_{\gamma_1} T_{\gamma_2}$$

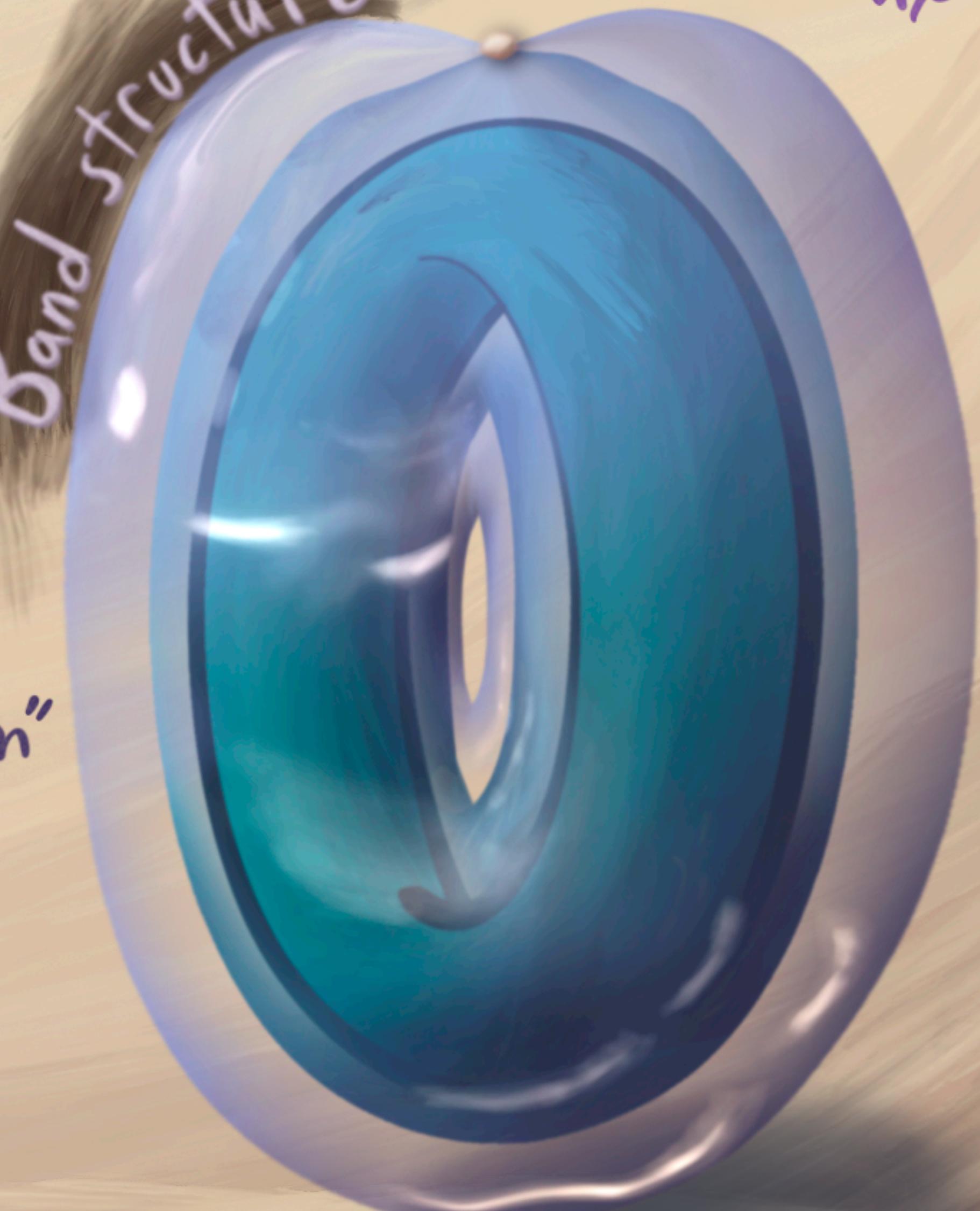
eigenspace: $\mathcal{H}_\rho : \Gamma \rightarrow U(1) = \{\Psi \mid T_\gamma \Psi = \rho(\gamma) \Psi\}$

$$HT_\gamma = T_\gamma H \Rightarrow H : \mathcal{H}_\rho \rightarrow \mathcal{H}_\rho$$



"crystal momentum"
 ρ

Graph spectrum $H|_{\partial P_\rho^5}$



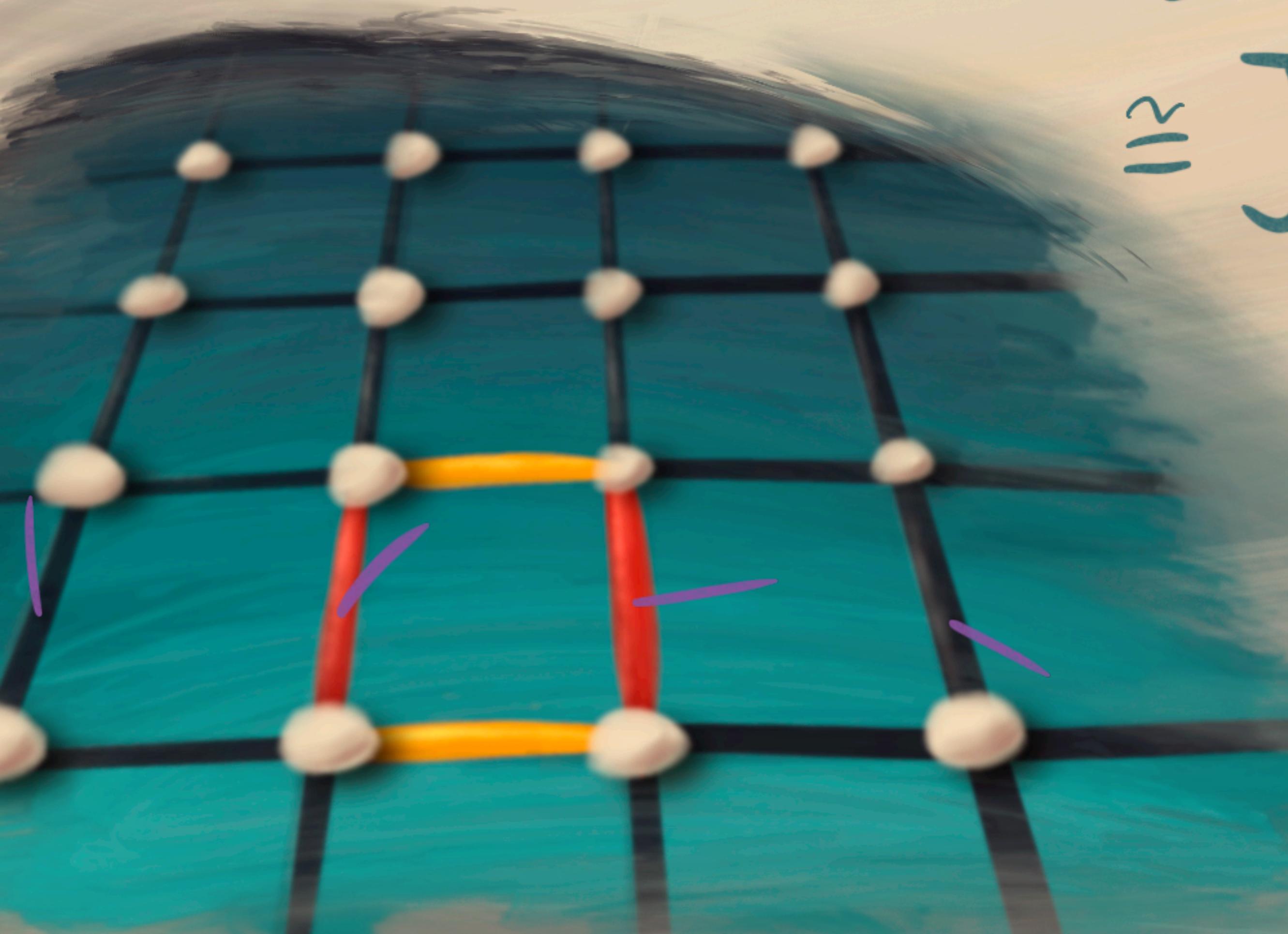
$$\text{Hom}(\mathbb{Z}^2, U(1)) = U(1) \times U(1)$$

Euclidean⁶ crystal



$\text{Hom}(\mathbb{Z}^2, U(1)) \cong \{\text{Holomorphic Line bundles}\}$

$\cong \text{Jac}(\Sigma)$



Hyperbolic Bloch

Theorem

T_γ unitary rep.
 \Rightarrow splits into irreps.

$$\begin{array}{ccc} \Gamma & \xrightarrow{T} & \mathcal{H} \\ \Gamma & \xrightarrow{\rho} & V_\Gamma \end{array}$$

n-dimnl
"multiplet"

$$\mathcal{H} = \bigoplus_{\text{Hom}_{\text{irr}}(\Gamma, U(n))} \mathcal{H}_\rho$$

T_γ are symmetries $\Rightarrow H: \mathcal{H}_\rho \rightarrow \mathcal{H}_\rho$

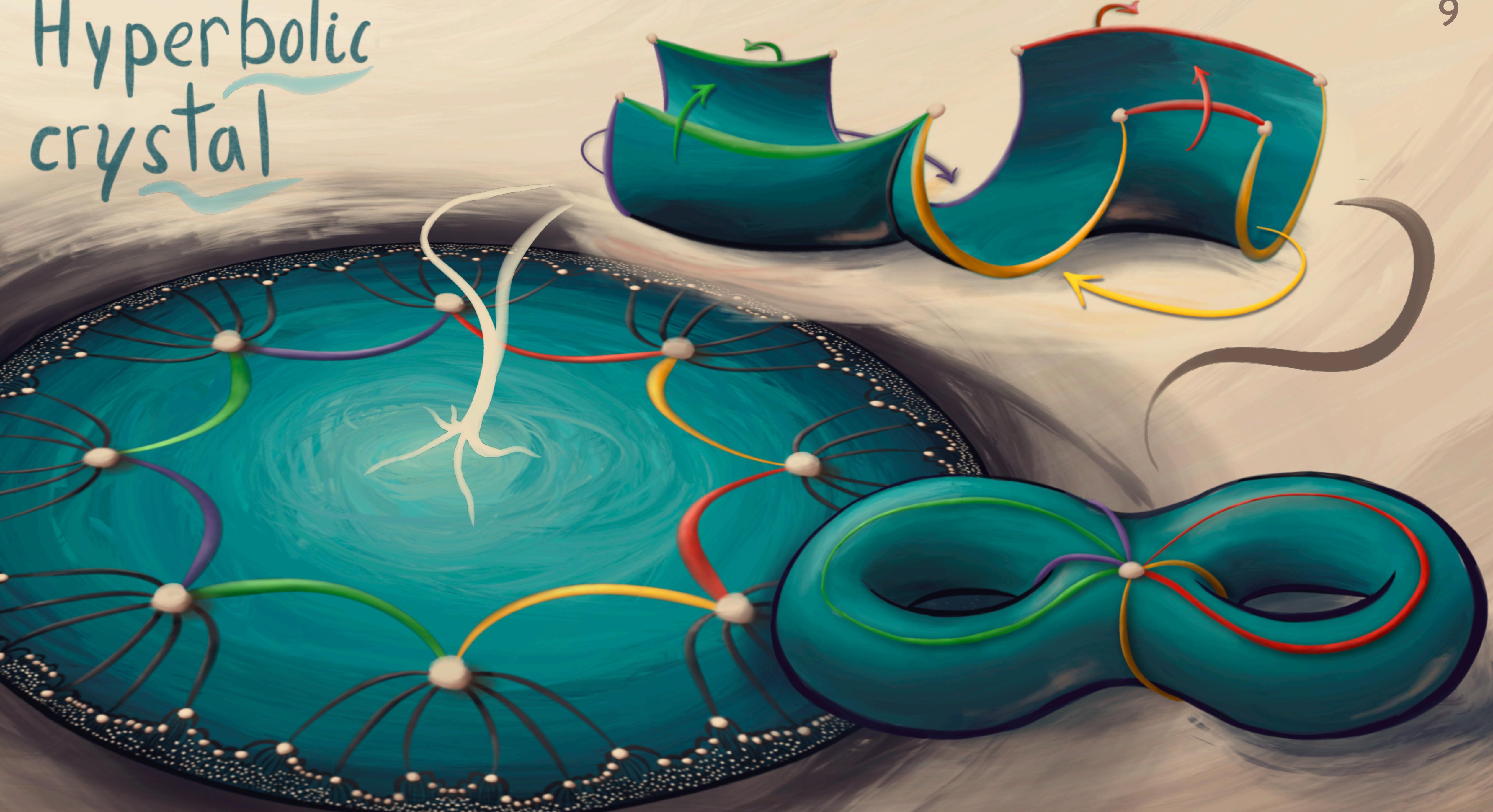
Band structure $\text{Spec}(H|_{\mathcal{H}_\rho})$ over $\text{Hom}_{\text{irr}}(\Gamma, U(n))$

$\Gamma \subset \text{Isom}(H^2)$ nonabelian δ 8



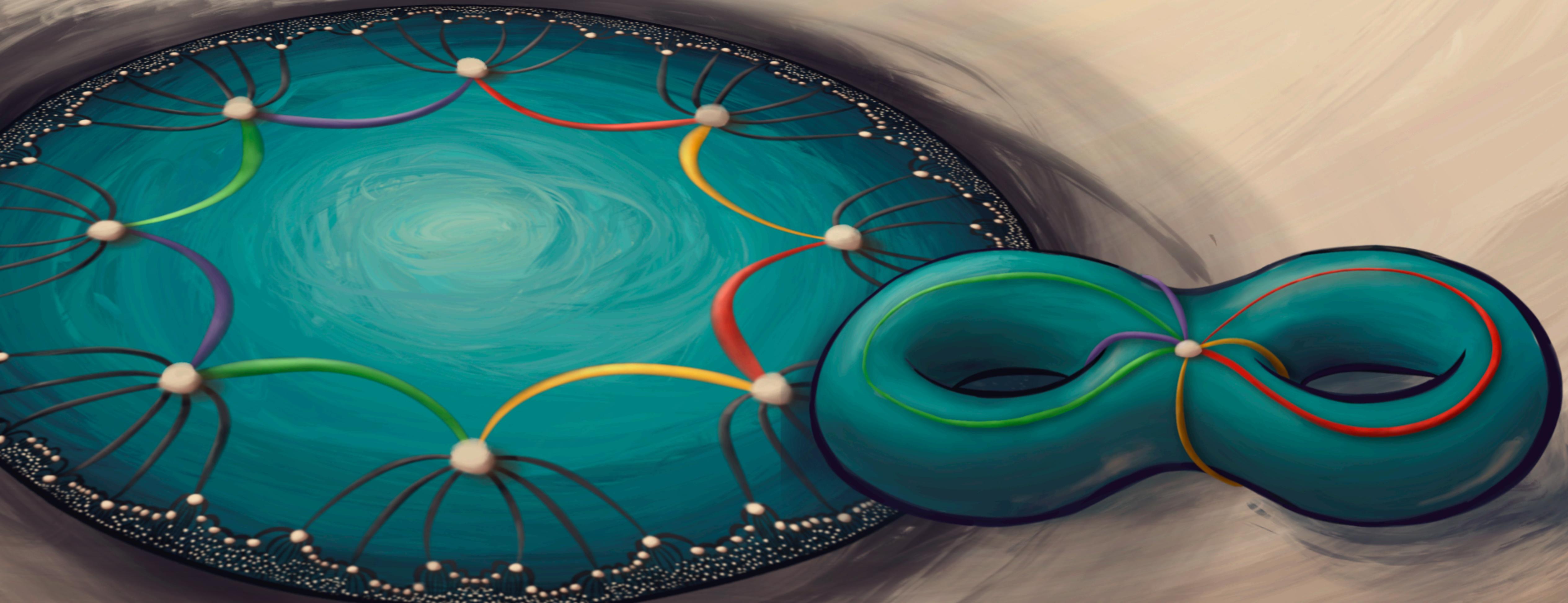
Hyperbolic crystal

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Narayanan- Seshardi

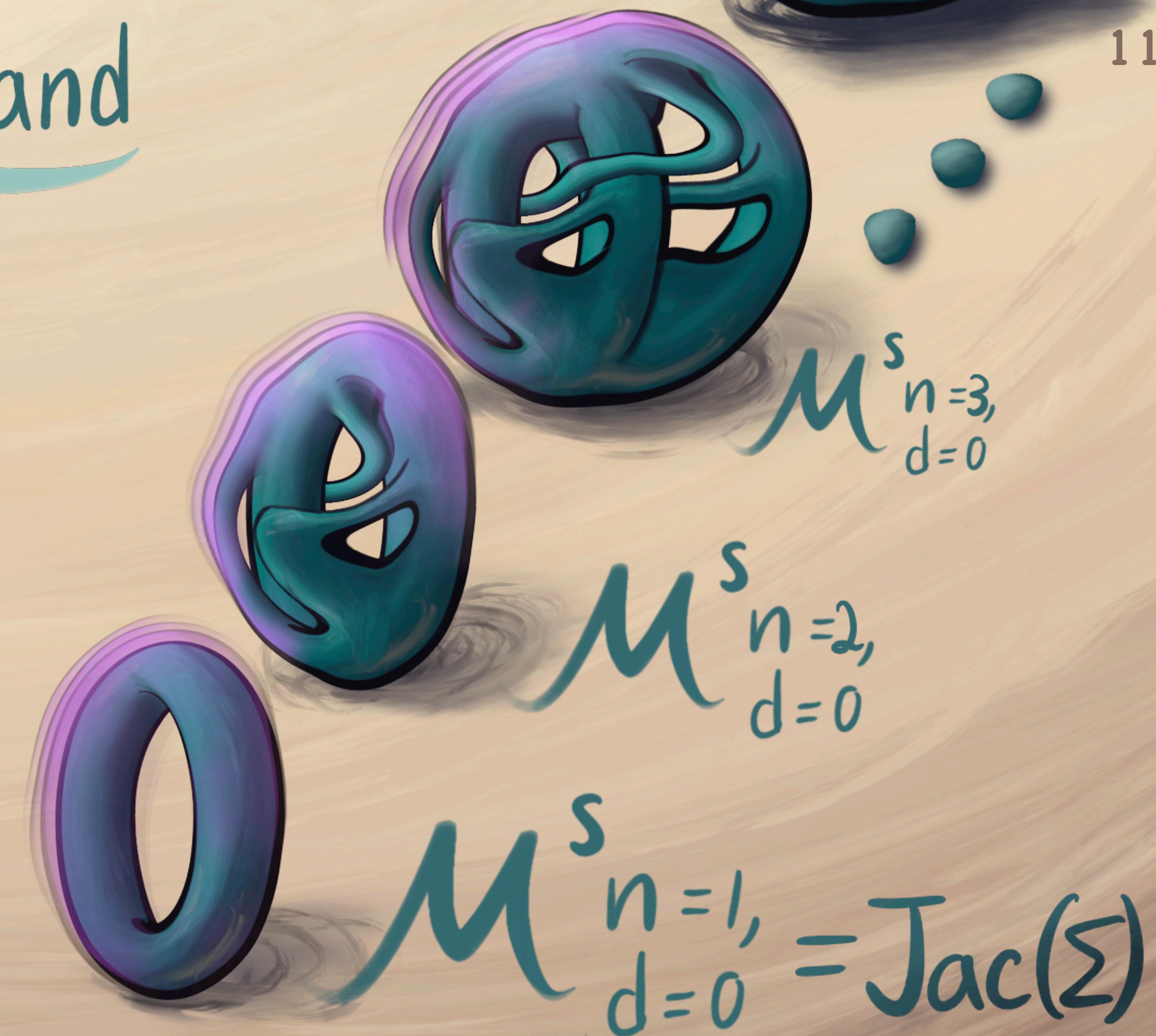
$$\text{Hom}_{\text{irr}}(\Gamma, U(n)) \cong \left\{ \begin{array}{l} \text{stable bundles on } \Sigma \\ \text{rank. } n \text{ deg. } 0 \end{array} \right\} \cdot \mathcal{M}^s_{d=0}^n$$



Hyperbolic Band Structure

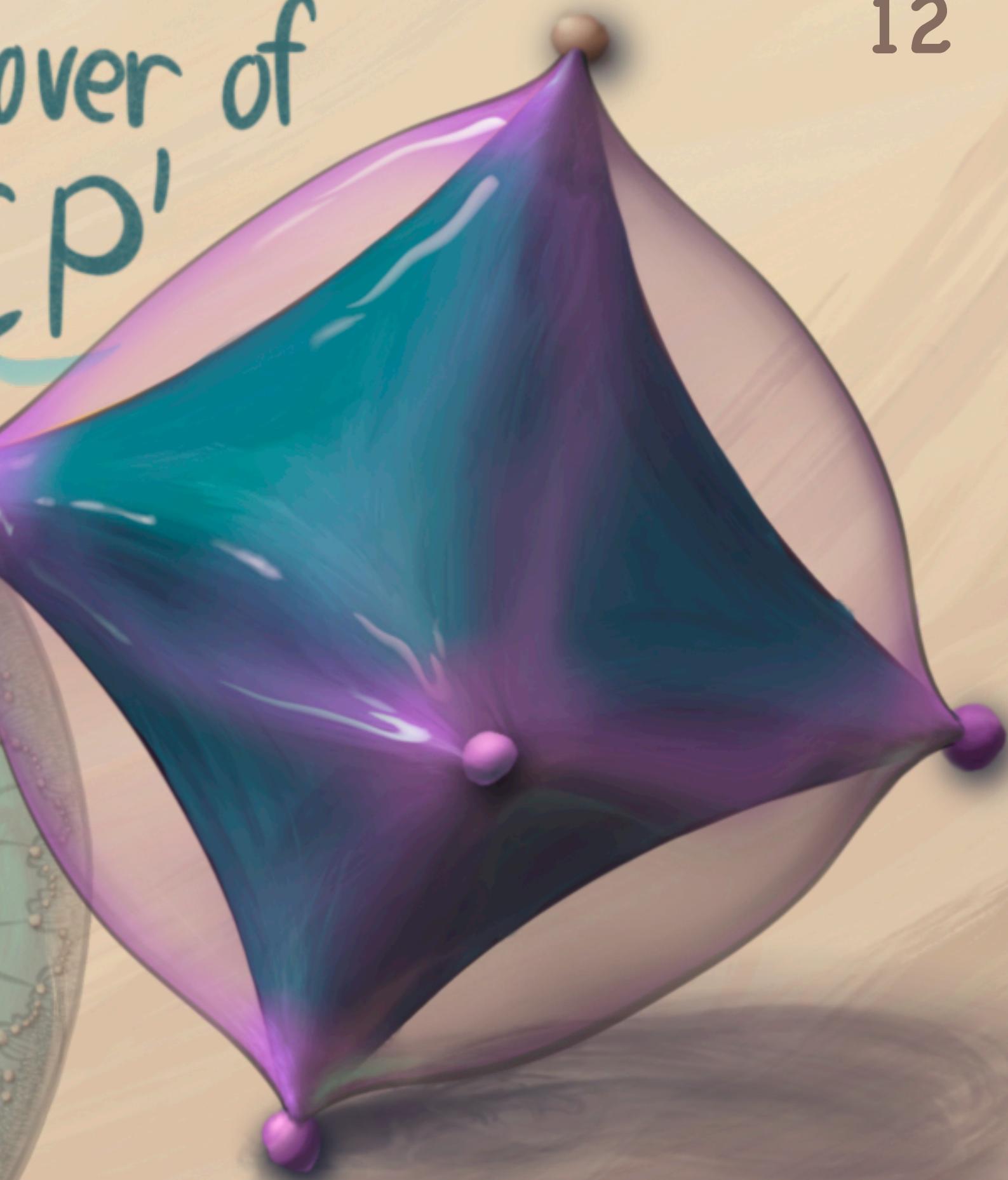
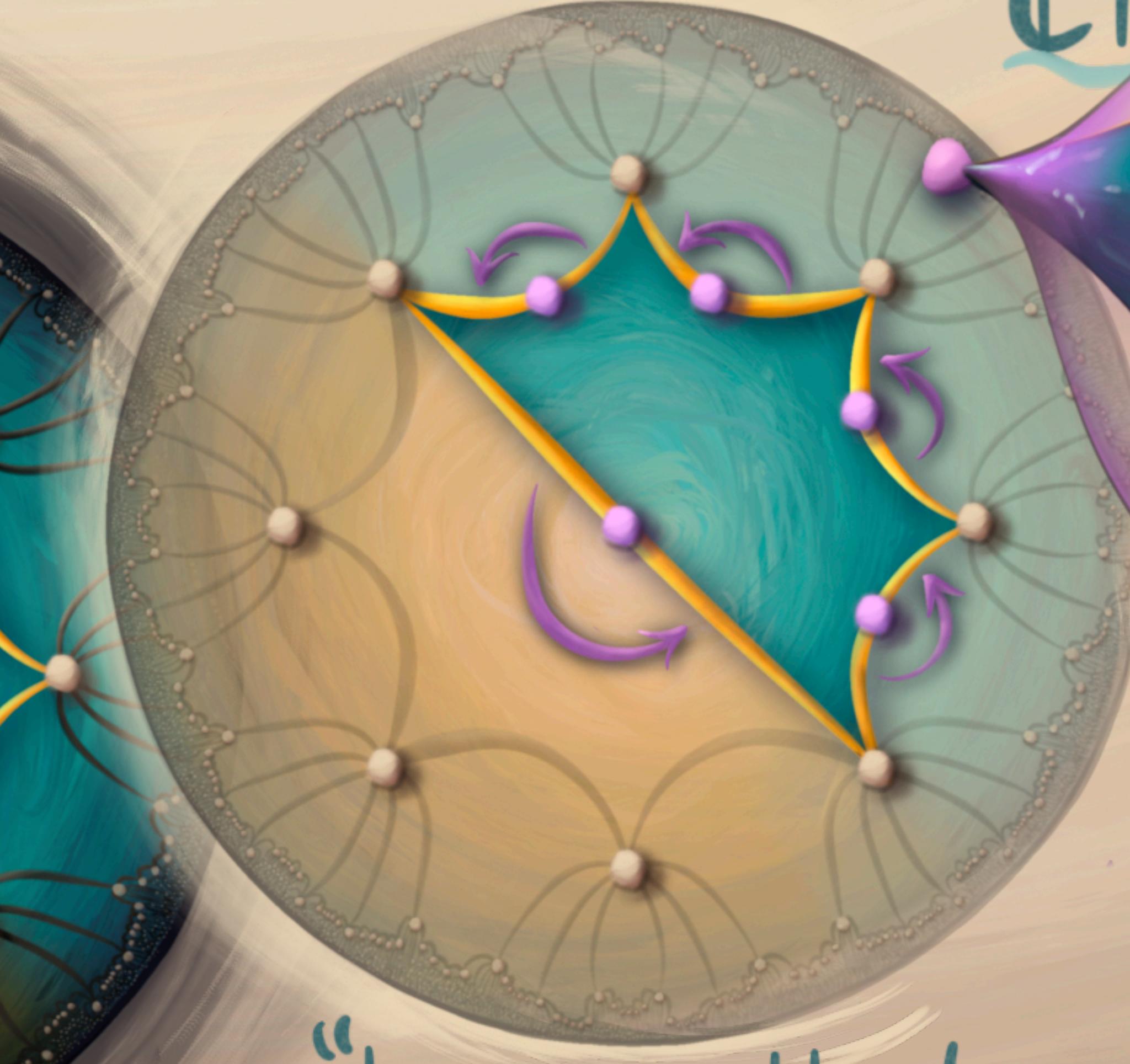
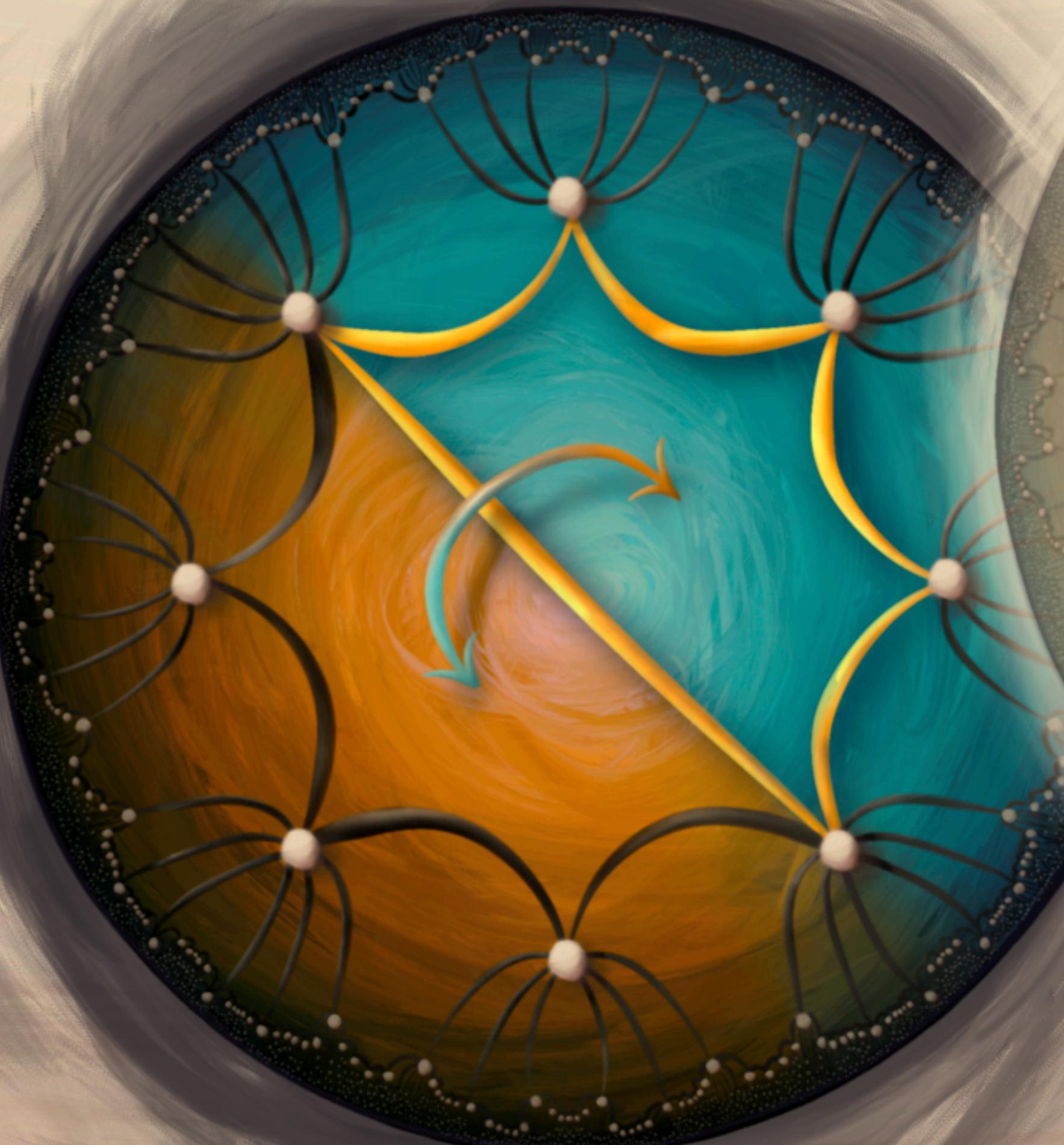
$$H = \bar{\partial}_E^* \bar{\partial}_E + V$$

Self-Adjoint ✓
 Elliptic ✓
 Compact mfd ✓



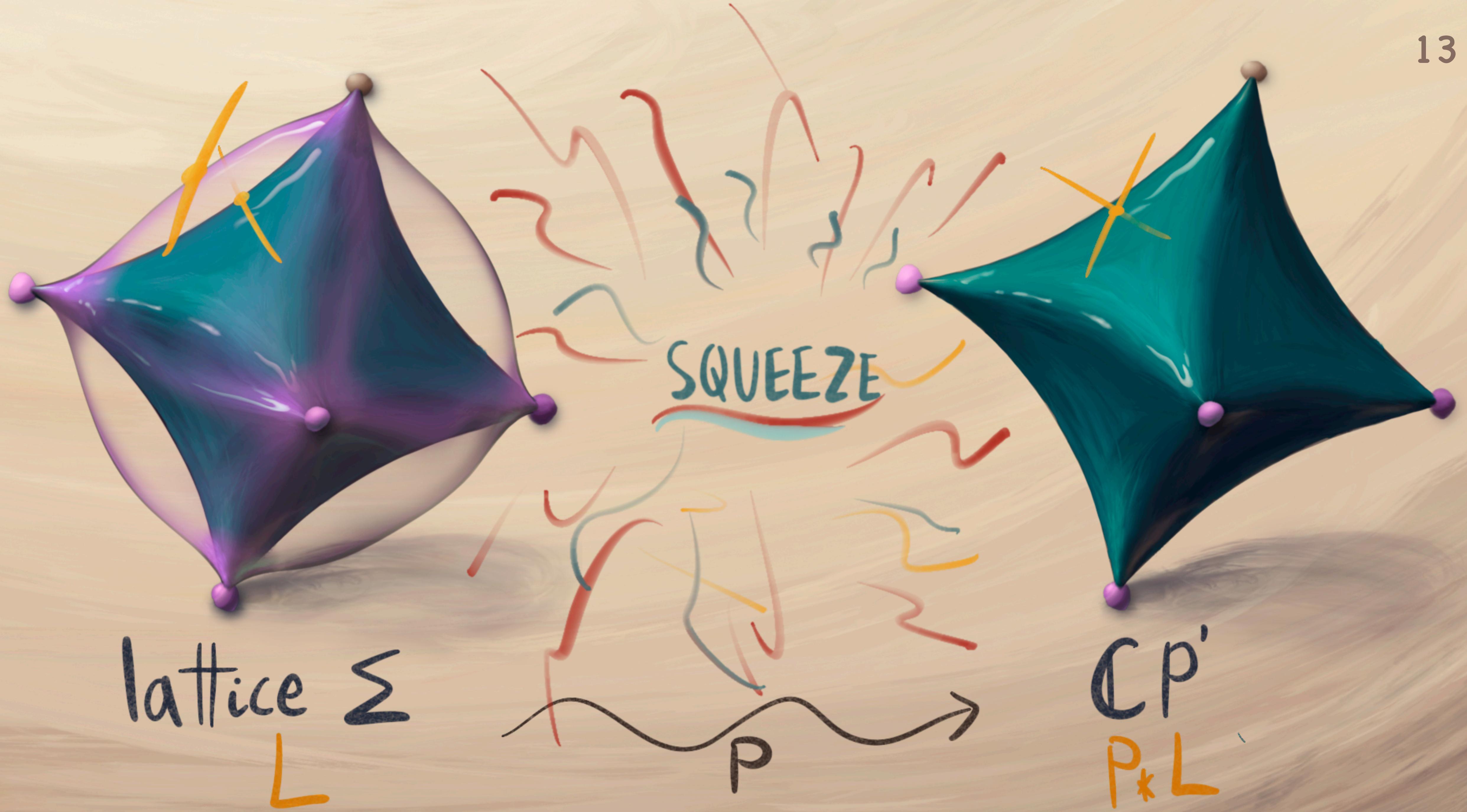
Spatial inversion
symmetry:

Double cover of
 $\mathbb{C}\mathbb{P}^1$



“hyperelliptic involution”

cone points



Higgs bundles Crystal data \iff spectral data¹⁴

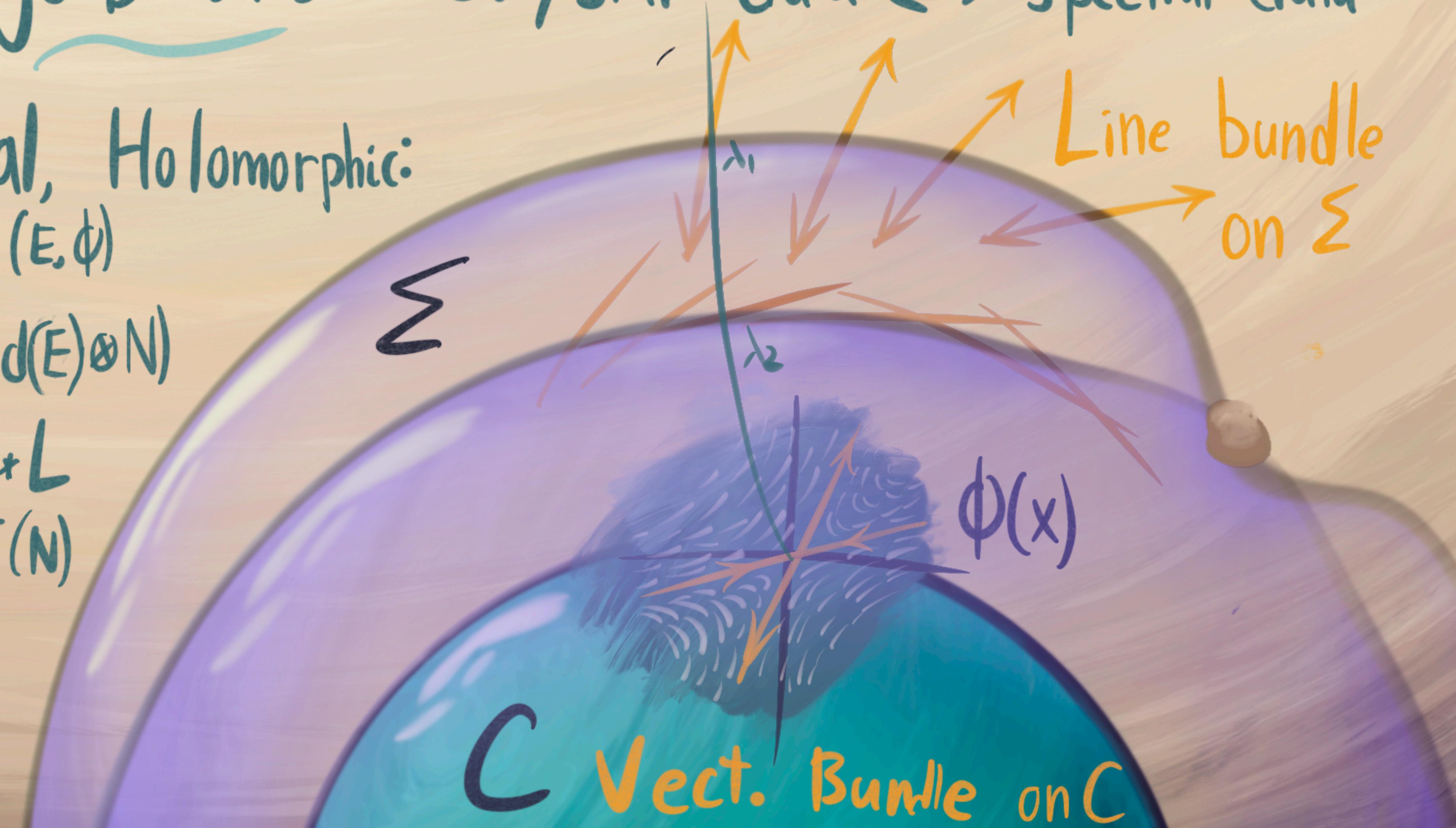
Global, Holomorphic:

Pair (E, ϕ)

$\phi \in H^0(\text{End}(E) \otimes N)$

$E = \pi_* L$

$\lambda_1, \lambda_2 \in \Gamma(N)$



Pushforward operator

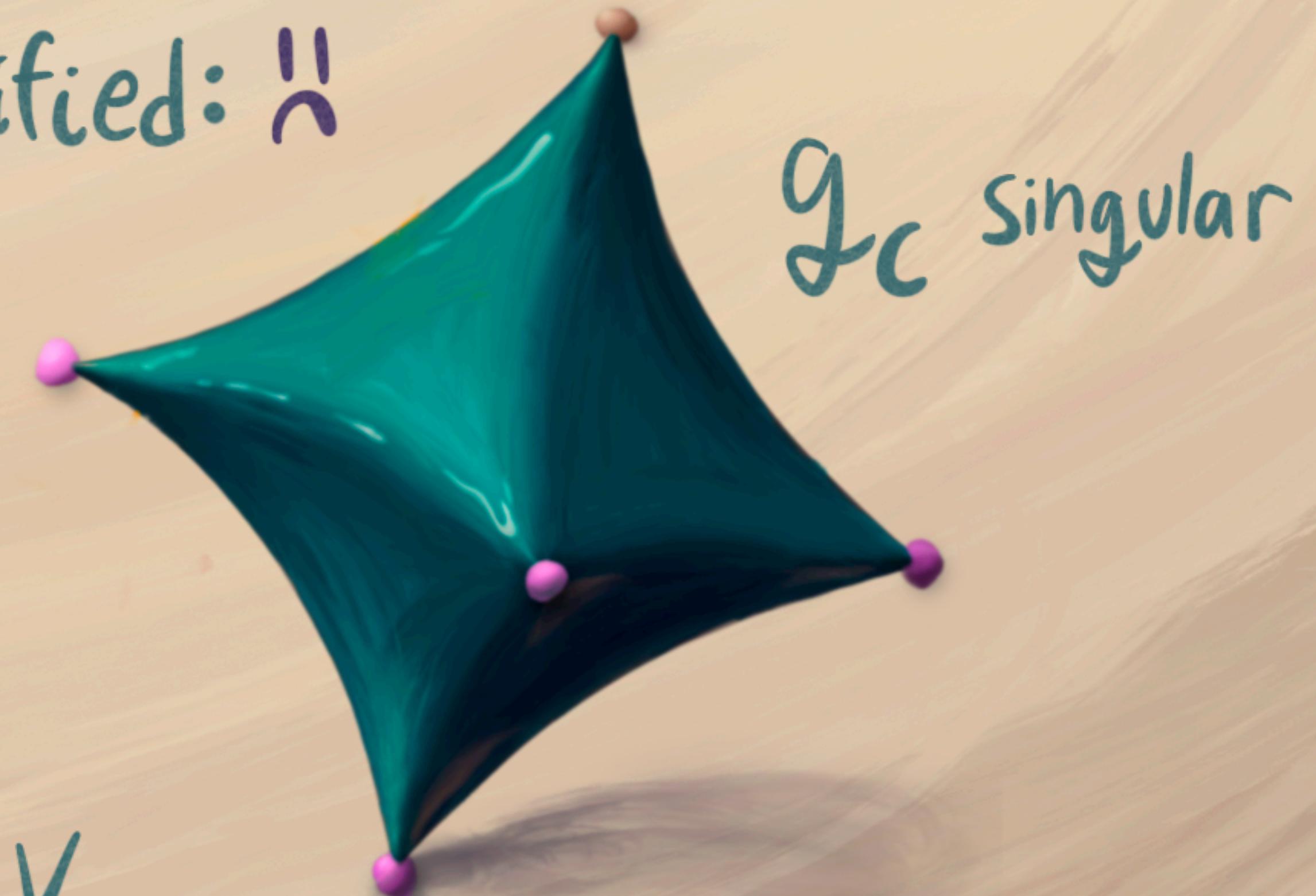
$$\Gamma(L) \xrightarrow{\bar{\partial}_L^* \bar{\partial}_L + V} \Gamma(L)$$

$\downarrow P_*$

$$\Gamma(E) \xrightarrow{??} \Gamma(E)$$

unramified: $P_* H_L = H_E$

ramified: \vdash



$$P_* \bar{\partial}_L := \bar{\partial}_E$$

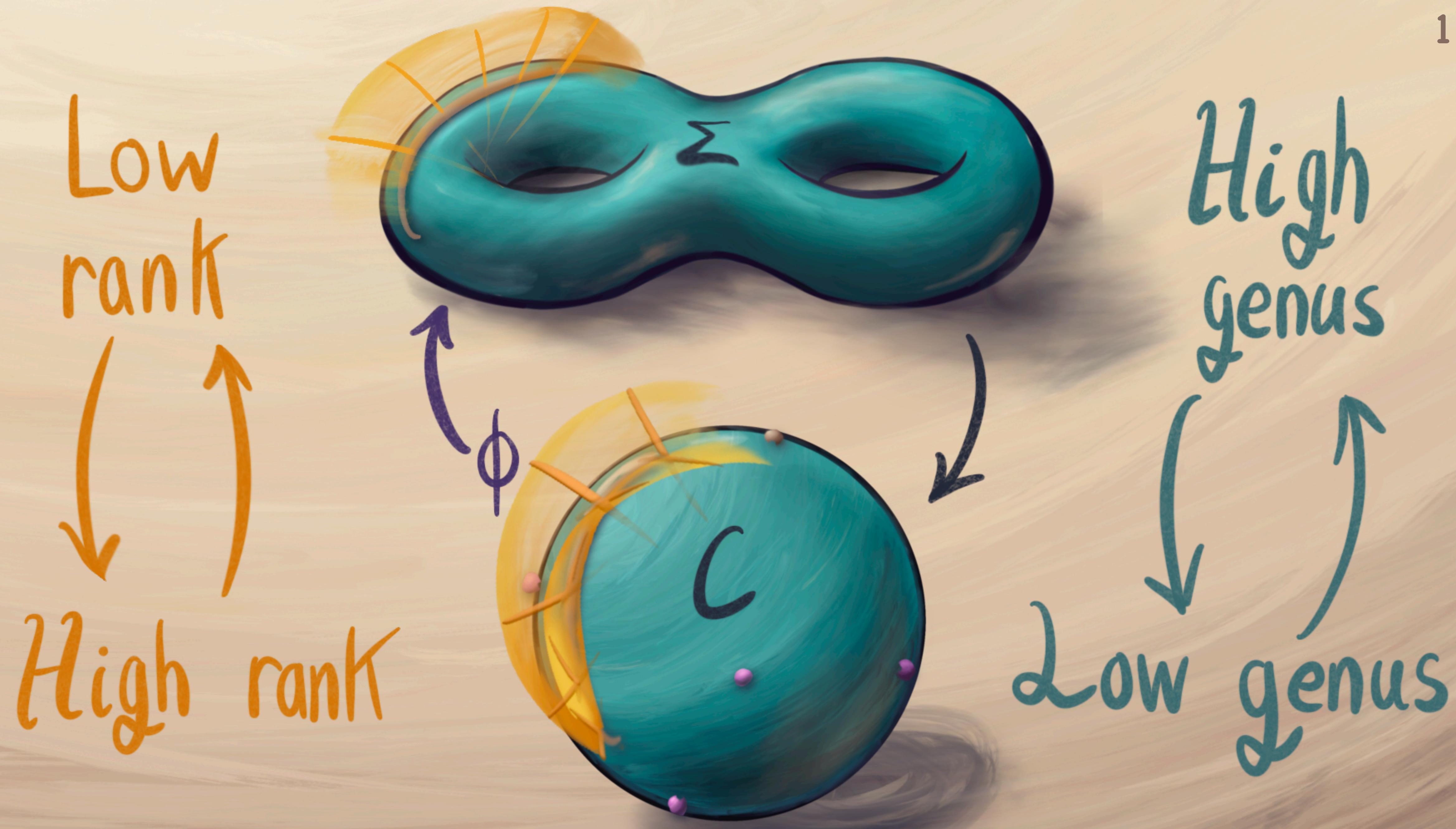
$$P_* V = V \cdot \text{Id}$$

when V
lifts from C

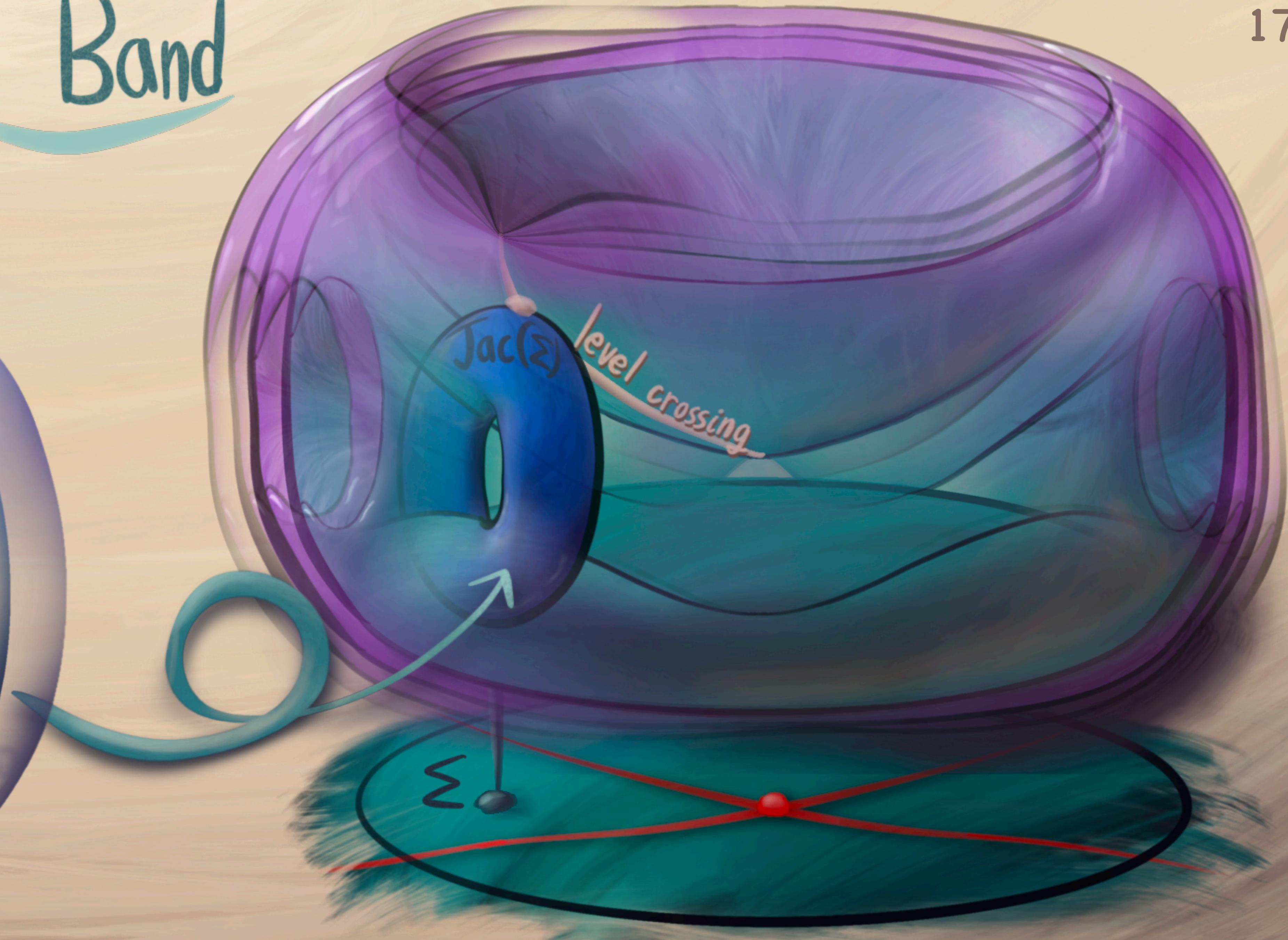
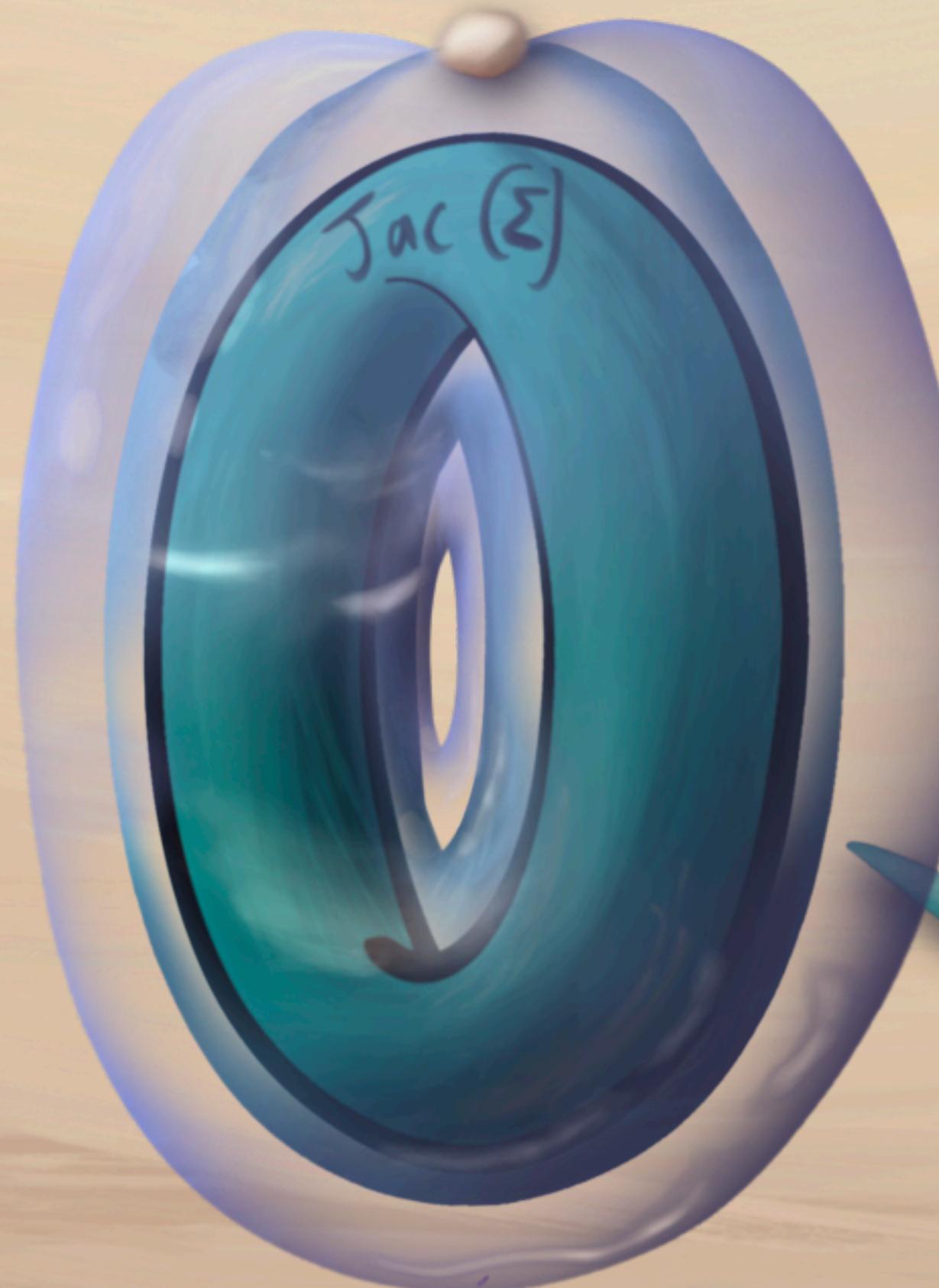
$$P_* \bar{\partial}_L^* = \bar{\partial}_E^*$$

$$\text{when } g_{\Sigma} = P^* g_C$$

Imposes B.C.s



Universal Band Structure



Complex momenta

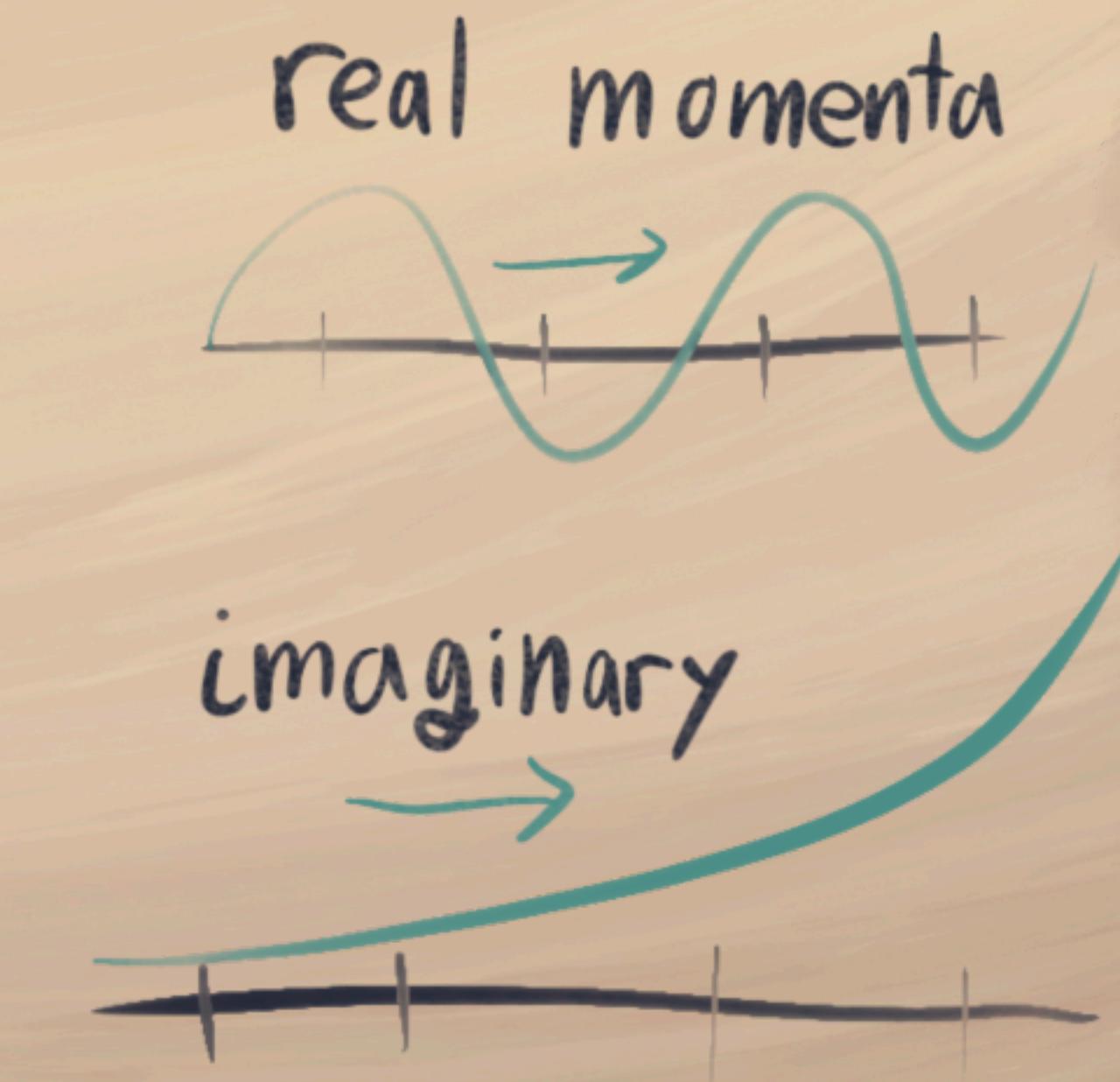
in AG-land, we must not suffer \mathbb{R}
 \Rightarrow allow non-Hermitian H

$$\text{Hom}_{\text{irr}}(\Gamma, \text{U}(n)) \rightsquigarrow \text{Hom}_{\text{irr}}(\Gamma, \text{GL}(n, \mathbb{C})) \xrightarrow{i\infty}$$

$$\mathcal{M}^s(\Sigma, \overset{\text{rank}=n}{d=0})$$

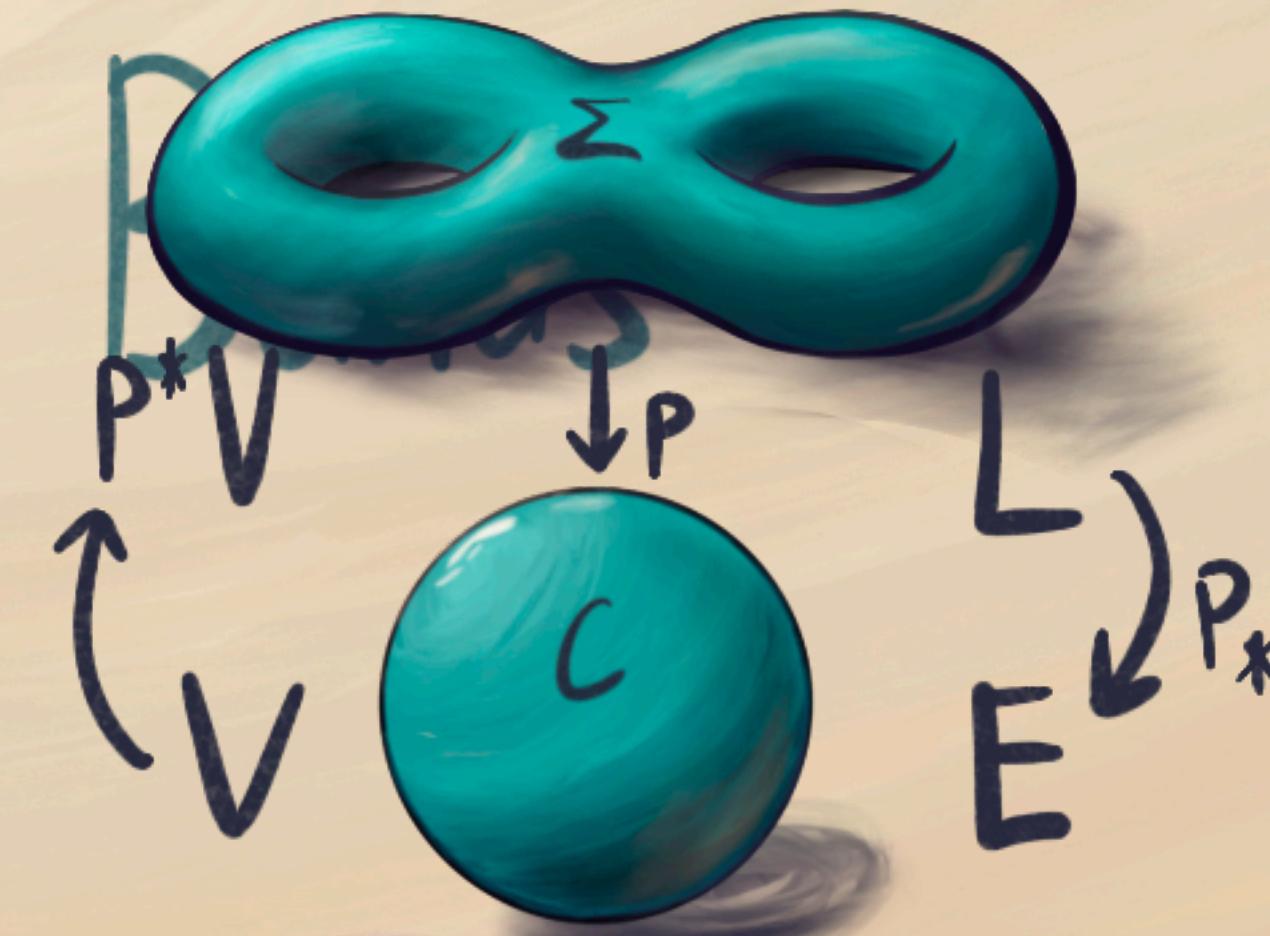
$$\mathcal{M}^{\text{Higgs}}(\Sigma, \overset{\text{rank}=n}{d=0})$$

Holo.	Flat
$\bar{\partial}_E$	$d + iA$
$(\bar{\partial}_E, \phi)$	$d + i(A - i(\phi + \phi^+))$



Complex Momenta

$$(\bar{\partial}_{E^+} \phi + \phi^+)^\ast (\bar{\partial}_{E^+} \phi + \phi^+) + V$$



Crystal Moduli

$$\bar{\partial}_L^\ast \bar{\partial}_L + p^\ast V$$

Complex



Daydream:

this diagram commutes

