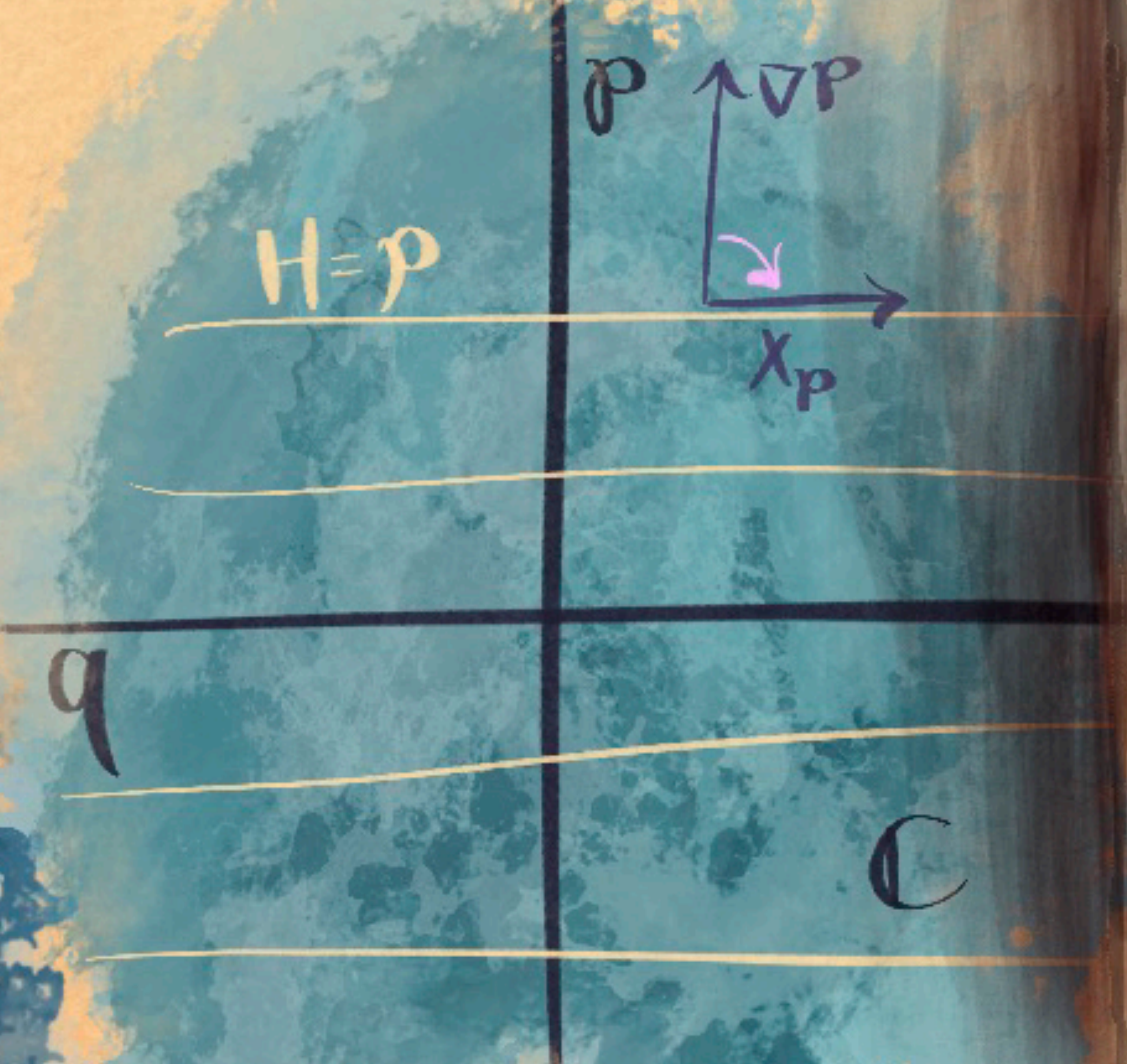
Hamiltonian mechanics

phase space: Kähler mfld M

Hamiltonian $H: M \to \mathbb{R}$ "$\times\, i$"

flow under H is $X_H = J \cdot \nabla H$

eg: 1D dynamics, $X_p = J \cdot (0,1) = (1,0)$



Symmetries

Y symmetry vect. field:

preserves H	preserves M
$Y(H)=0$	$Y = X_{H'}$

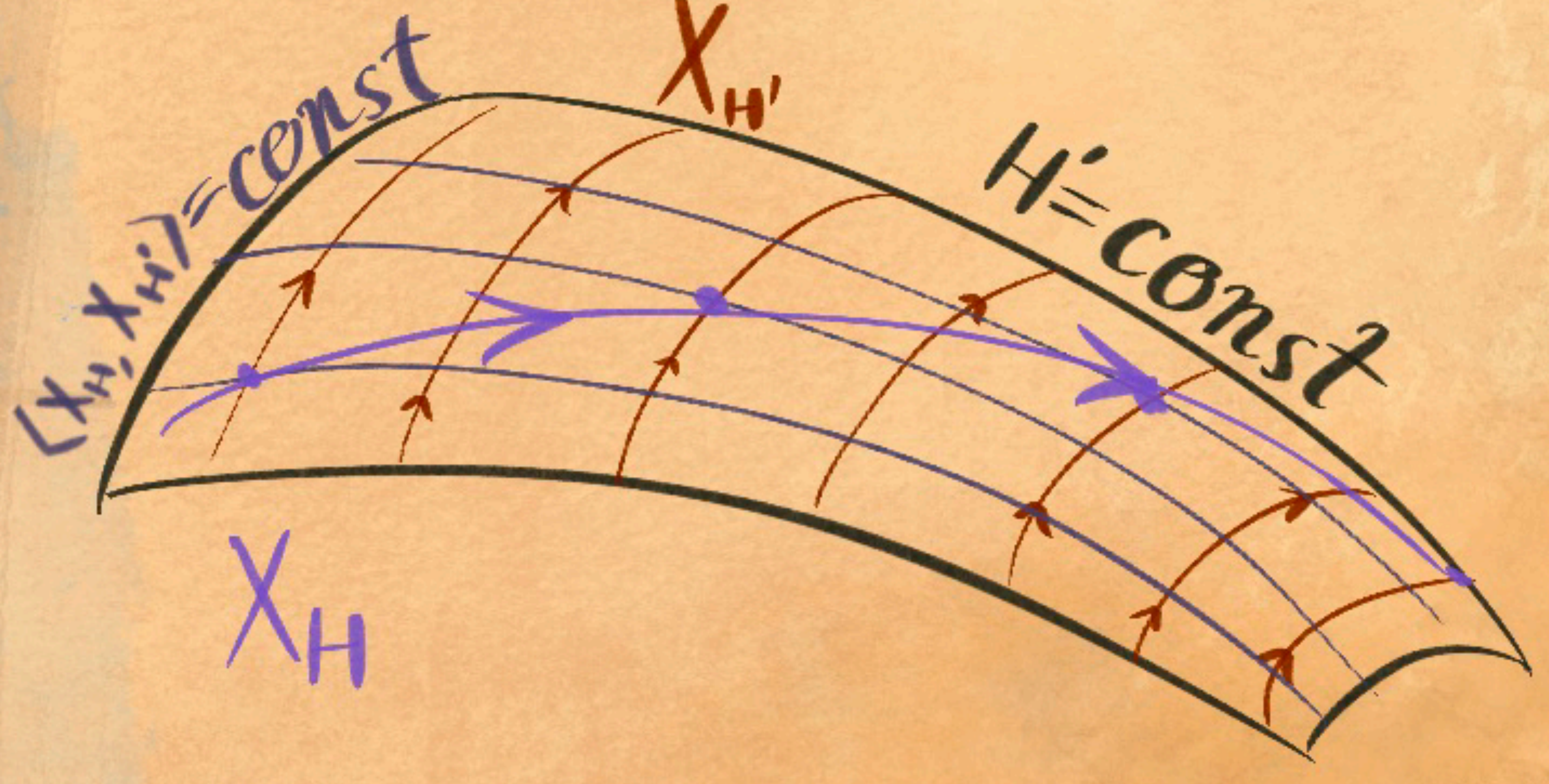
Noether's Thm:

H' conserved!

$$0 = X_{H'}(H) = \langle \nabla H, J\nabla H' \rangle = \langle -J\nabla H, \nabla H' \rangle = -X_H(H') = 0$$

symmetry acts double!

1) H' const 2) $\langle X_H, X_{H'} \rangle = \text{const}$



Louiville integrabillity

Maximal symmetry

N mutually commuting symmetries

$H_1, \dots, H_n$ w/ $[X_{H_i}, X_{H_j}] = 0$

$X_{H_i}(H) = 0 \ \forall i \Rightarrow X_H$ lives on $\bigcap_i \{H_i = \text{const}\}$ $=$ n-fold $\mathcal{L}$

$\{X_{H_i}\}$ span $T\mathcal{L} \Rightarrow \langle X_H, X_{H_i} \rangle = \text{const}$ fixes X_H!

e.g on $\mathbb{R}^3$, p_x, p_y, p_z conserved $\Rightarrow X_H$ const. velocity.

$\mathcal{L}$ lie grp w/ abelian Lie alg $\{X_{H_i}\}$

$\Rightarrow \mathcal{L} = U(1)^p \times \mathbb{R}^q$

compactness $\Rightarrow$ $\mathcal{L} = U(1)^n$

X_H linear flow