Stable normal bundles

$M^k \hookrightarrow \mathbb{R}^{n+k}$ n shouldn't matter!! as long as it's big enough

$\mathbb{R}^{n+k} \hookrightarrow \mathbb{R}^{n+k+1}$ induces $\nu \to \nu \oplus 1$ trivial line bundle

$n \gg 1 \Rightarrow$ space of embedings is connected $\Rightarrow$ all normal bundles iso. "stable normal bundle"

$(X \times I)_+ = X_+ \wedge I_+ = X_+ \wedge S^1 = \Sigma X \Rightarrow Th(\nu \oplus 1) = \Sigma\, th(\nu)$

$\nu \to \nu \oplus 1 \Rightarrow BO(k) \to BO(k+1)$

Thom's Theorem:

$$\begin{tikzcd}
\mathbb{R}^{n+k+1}_{+} = S^{n+k+1} \arrow[r] \arrow[d, leftrightarrow, "\cong"] & Th(\nu \oplus 1) \arrow[r] \arrow[d, leftrightarrow, "\cong"] & MO(k+1) \arrow[d, leftrightarrow, "\nu \to \nu \oplus 1"] \\
\Sigma S^{n+k} \arrow[r] & \Sigma Th(\nu) \arrow[r] & \Sigma MO(k)
\end{tikzcd}$$

$\Sigma MO(k) \longrightarrow MO(k+1) \Rightarrow$ pre-spectrum!

$\pi_k(MO) = \varinjlim \pi_{n+k}(MO(n))$ well defined

$$\Omega_k \cong \pi_k(MO)$$

$$\begin{tikzcd}
\Omega_k \arrow[rr, bend left, "\alpha"] \arrow[rr, phantom, "\alpha\beta = \beta\alpha = id"] & & \pi_k(MO) \arrow[ll, bend left, "\beta"]
\end{tikzcd}$$