Louiville integrabillity

Maximal symmetry

N mutually commuting symmetries

$H_1, \ldots, H_n$ w/ $[X_{H_i}, X_{H_j}] = 0$

$X_{H_i}(H) = 0 \;\forall i \Rightarrow X_H$ lives on $\bigcap_i \{H_i = \text{const}\}$ $\parallel$ n-fold $\mathcal{L}$

$\{X_{H_i}\}$ span $T\mathcal{L} \Rightarrow \langle X_H, X_{H_i} \rangle = \text{const}$

fixes X_H!

e.g on $\mathbb{R}^3$, p_x, p_y, p_z conserved $\Rightarrow X_H$ const. velocity.

$\mathcal{L}$ lie grp w/ abelian Lie alg $\{X_{H_i}\}$

$\Rightarrow \mathcal{L} = U(1)^p \times \mathbb{R}^q$

X_H linear flow

compactness $\Rightarrow$

$\mathcal{L} = U(1)^n$

symplectic form $\omega(X,Y)=\langle X, J\,Y\rangle$

$[X_{H_i}, X_{H_j}]=0 \Leftrightarrow \underline{\omega|_{\mathcal{L}}=0}$

$\mathcal{L}=U(1)^n$ Lagranian torus

Phase space is a Lagrangian Torus fibration

Base = {possible $\vec{H_i}$} fiber = $\vec{H}'(\vec{b})$

(degenerates in places)