$\beta: \pi_k(MO) \longrightarrow \Omega_k$

When does $f: S^{n+k} \longrightarrow MO$ come from P.T?

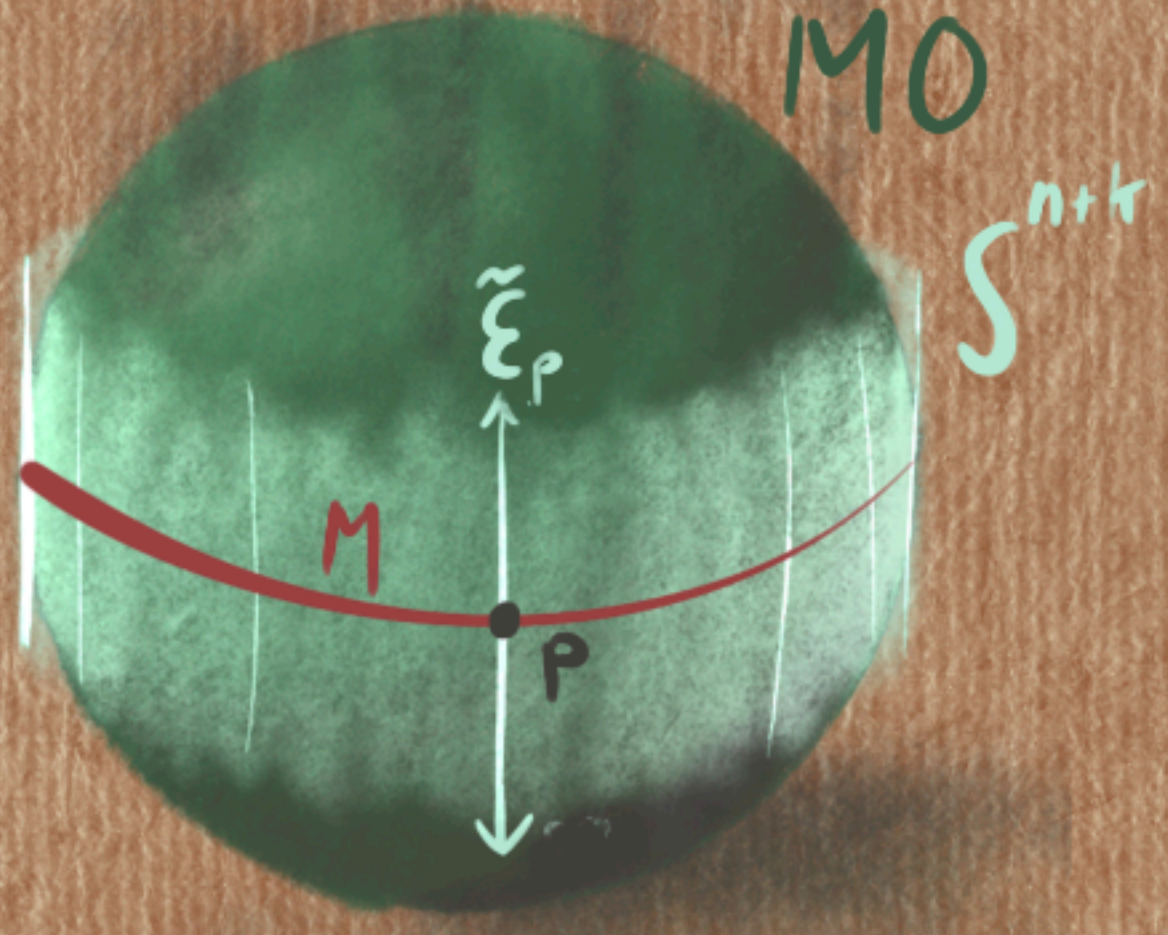
0-section of $\tilde{\varepsilon}^k$

Take $S^{n+k} \supset M = f^{-1}(\theta)$ say S^{n+1} intersects θ transversly: $T_p\theta \oplus T_pS^{n+k} = T_pMO$

normal bundle

dimension count: $T_pS^{n+k} = T_pM \oplus \tilde{\varepsilon}_p$

So, P.T∘classifying sends $p \in M$ to $p \in Gr$
& sends $T_pM^{\perp}$ to $\tilde{\varepsilon}_p$. i.e, it sends M to f!



S^{n+k} compact $\Rightarrow$ lies in some $Th(Gr(k, \mathbb{R}^{n+k}))$

Thom transversality transversality is generic!

$\Rightarrow \exists\, \tilde{f} \in [f]$ w/ $\alpha[f^{-1}(\theta)] = [f]$

Q.E.D!

Thom spectrum of X

$$MO_k(X) = MO(k) \wedge X_+$$

$$\Omega_k(X) = \lim_{n\to\infty} \pi_{n+k}(MO_n(X))$$
$$= \pi_k(MO(X)$$