Universal bundle

every k-bundle is pullback of $\varepsilon^k \to Gr(k,\mathbb{R}^{n+k})$ for some n

Glue all $Gr(k,\mathbb{R}^{n+k})$ together:

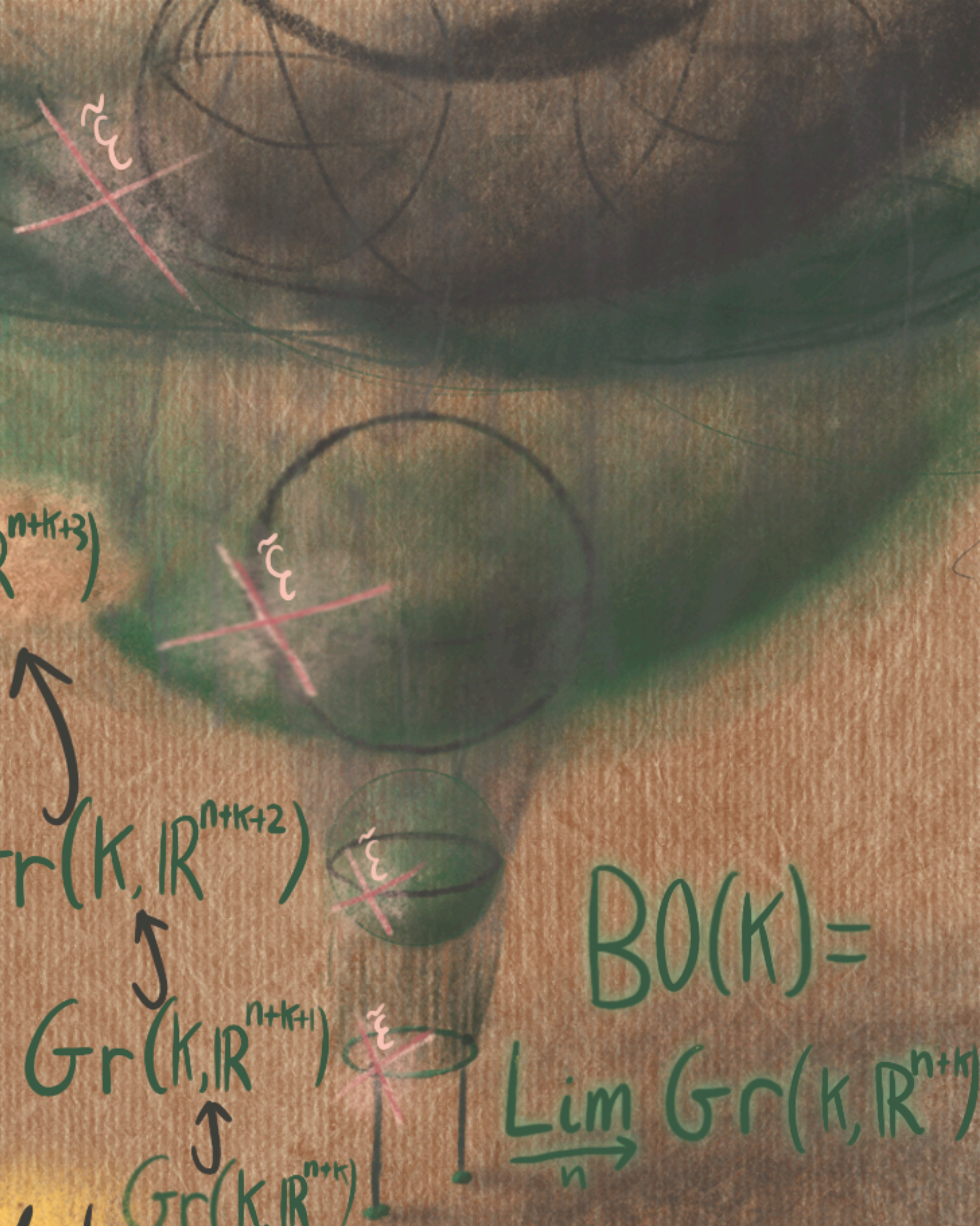
$\mathbb{R}^{n+k} \hookrightarrow \mathbb{R}^{n+k+1}$ induces $Gr(k,\mathbb{R}^{n+k}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+1})$

$BO(k) :=$ telescoped mapping cylinder of inclusions

$Gr(k,\mathbb{R}^{n+k}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+1}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+2}) \hookrightarrow Gr(k,\mathbb{R}^{n+k+3}) \hookrightarrow \cdots \hookrightarrow Gr(k,\mathbb{R}^{n+k}) \to$

$BO(k) = \varinjlim_{n} Gr(k,\mathbb{R}^{n+k})$

$\tilde{\varepsilon}^k \to BO(k)$



every k-bundle is a pullback of $\tilde{\varepsilon}^k$!!

induced by classifying map $f: X \to BO(k)$

invariants of $[X, BO(k)] \leftrightarrow$ invariants of V Chern-Weil theory

cohomology of $BO(k)$ = charactistic Classes

Collapse everything outside ν
(set distance to 'infinity')

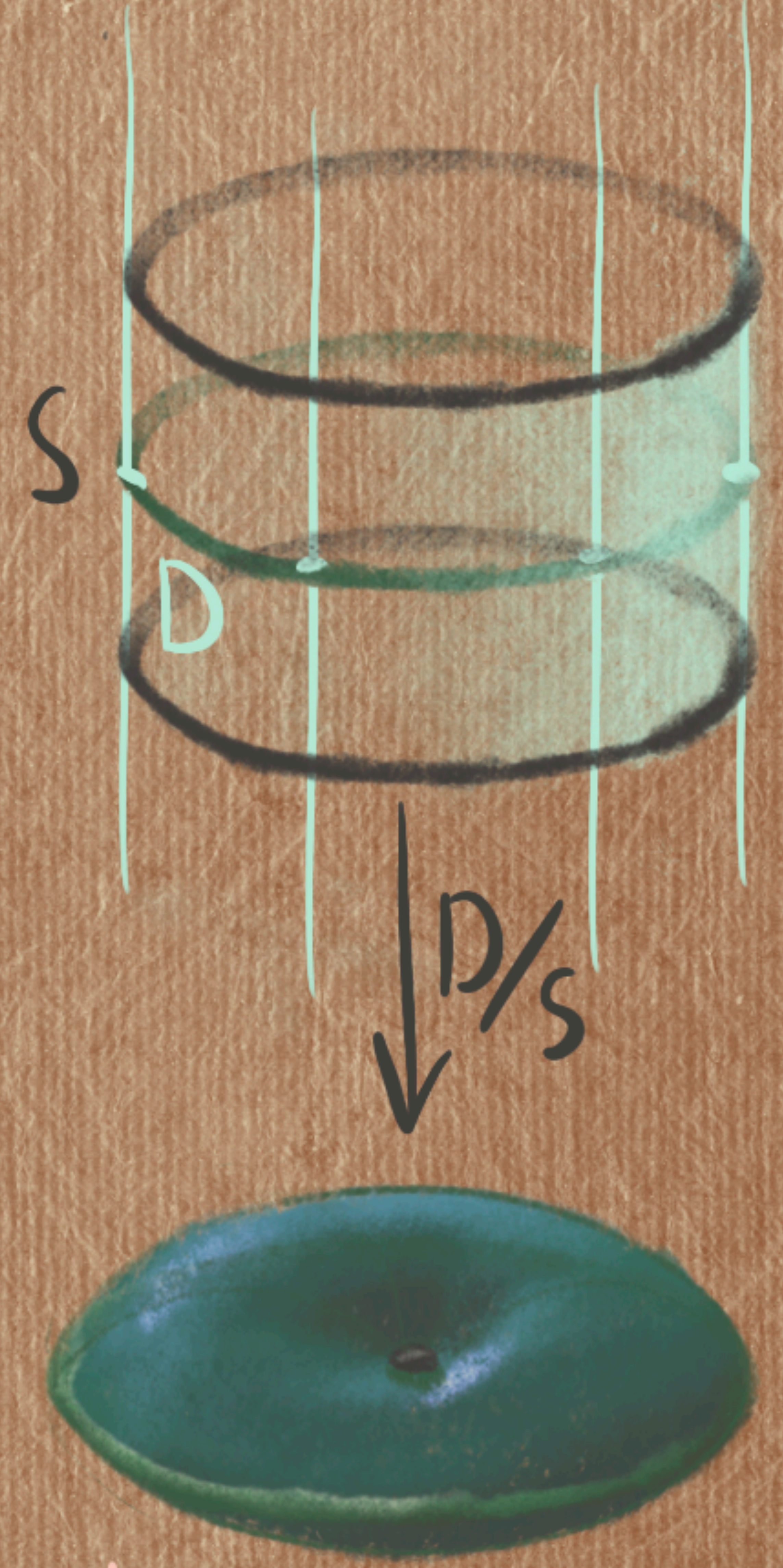
$\Rightarrow$ 1-pt compactification of ν

Thom space $Th(\nu) = \nu_+$

1-pt compactification is functorial:

$$\begin{array}{ccc} \nu & \longrightarrow & \tilde{\varepsilon}^k \\ \downarrow & & \downarrow \\ Th(\nu) & \longrightarrow & Th(\tilde{\varepsilon}^k) = MO(k) \end{array}$$

$$\nu \underset{\text{open}}{\hookrightarrow} \mathbb{R}^{n+k} \Rightarrow \underset{\substack{\parallel \\ S^{n+k}}}{\mathbb{R}^{n+k}_+} \longrightarrow \underset{\substack{\parallel \\ Th(\nu)}}{\nu_+}$$



Thom space