

# **Introduction to Chess Endgames**

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# Introduction

The endgame is the final part of a chess game, which occurs when there are relatively few pieces left on the board. As the number of pieces decreases further we eventually reach theoretical endgames: those endgame positions which have names and well established evaluations and strategies. The goal of this book is to explain the most basic and important theoretical endgames.

The material is organized in such a way that the easiest and most important material is in the earliest chapters, so I recommend that you study the chapters in order. However, there is no reason why you need to study the entire book right away. For example, a beginner might want to study only the first few chapters initially, while a 2000+ player should be able to handle the entire book easily. However, your rating is less important here than what you are comfortable with and interested in. If the material starts to seem overly difficult as you go through the book, then it might make sense to put it aside for a while and come back to it at some later point.

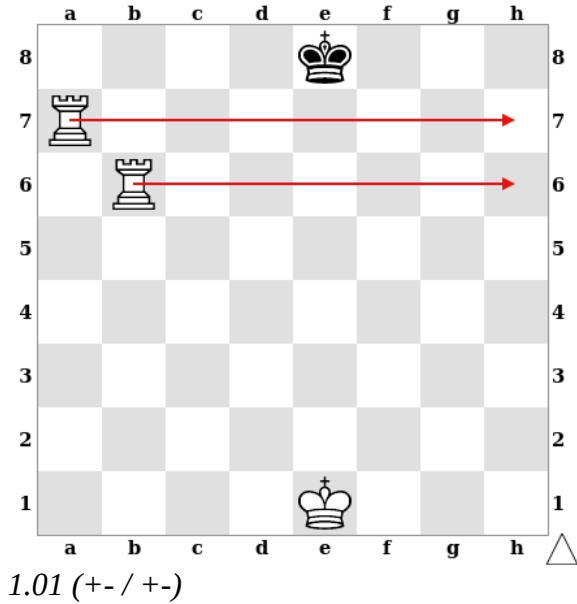
Each chapter includes two important sections after the main instructional section. First, there is a short section that provides a conclusion to the chapter. These conclusions sometimes recommend further practice and sometimes summarize the most important rules and results from the chapter. In particular, they often provide guidance about what portion of the material is really worth memorizing. Second, there are exercises with solutions. These exercises are designed to be representative of the material covered in the chapter, rather than to be especially challenging.

There is a popular perception of endgames as being somewhat boring. I have consistently found the opposition to be true; endgames seem to me to have some of the most interesting, surprising, and clever ideas of any part of chess. I hope that you come to agree with this, and that you find this book enjoyable as well as useful.

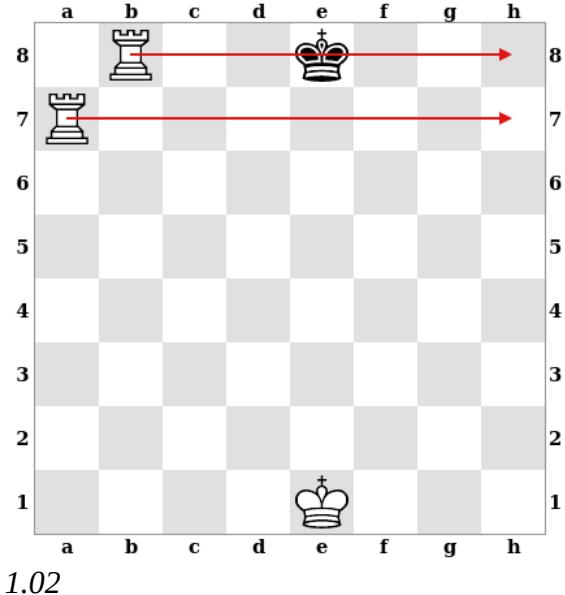
# 1 – Overkill Checkmates

There are a few important checkmate patterns that occur when one side has multiple major pieces (queens and rooks) on the board and the other side has only a king. Knowing these patterns can help you win games more quickly and avoid unnecessary stalemates.

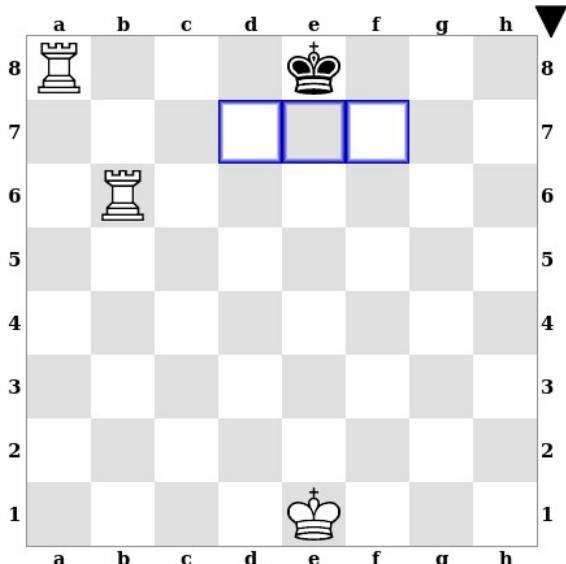
## Ladder Mate



Here White's rooks control the sixth and seventh ranks. White has checkmate in one move with **1.Rb8#**, leading to the next diagram.

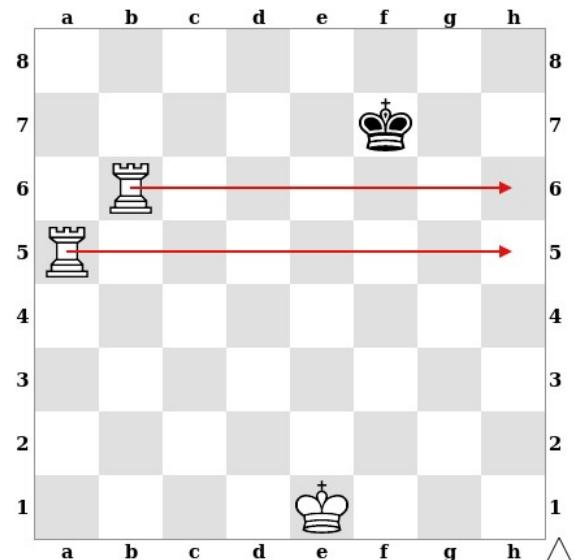


Notice that the rook on a7 is preventing Black's king from escaping to the seventh rank.



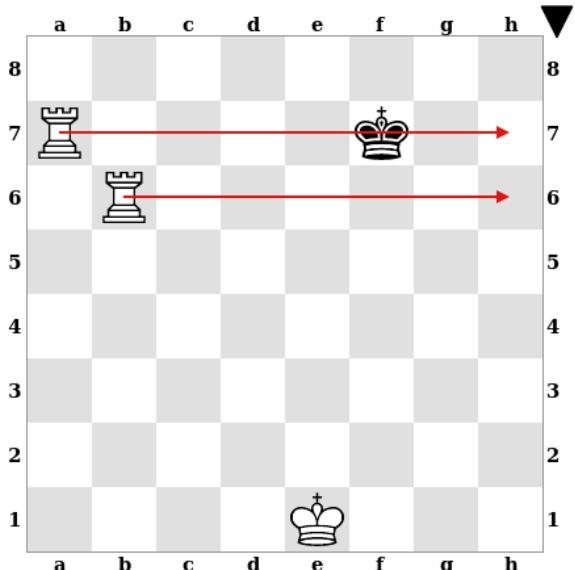
1.03 (\* / +-)

It would not have been checkmate if White had played 1.Ra8+ in diagram 1.01. In this case White would have lost control of the seventh rank, giving Black's king escape squares.



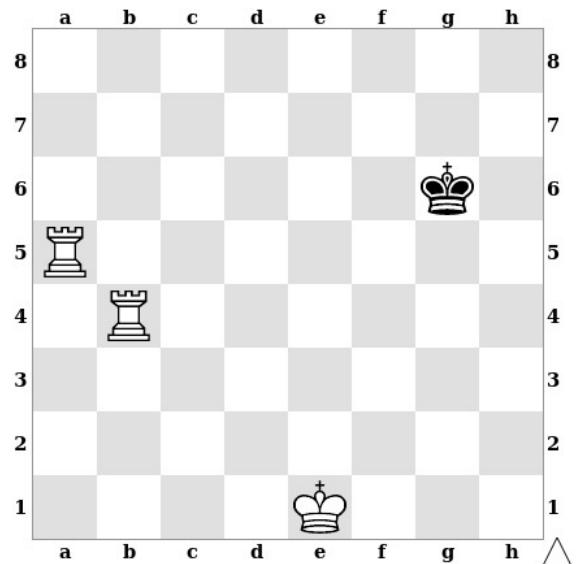
1.04 (+- / +-)

Here White has mate in two moves beginning with **1.Ra7+**, leading to the next diagram. Again, it would be a mistake to play 1.Rb7+, allowing Black to move his king back to the sixth rank.



1.05 (\* / +-)

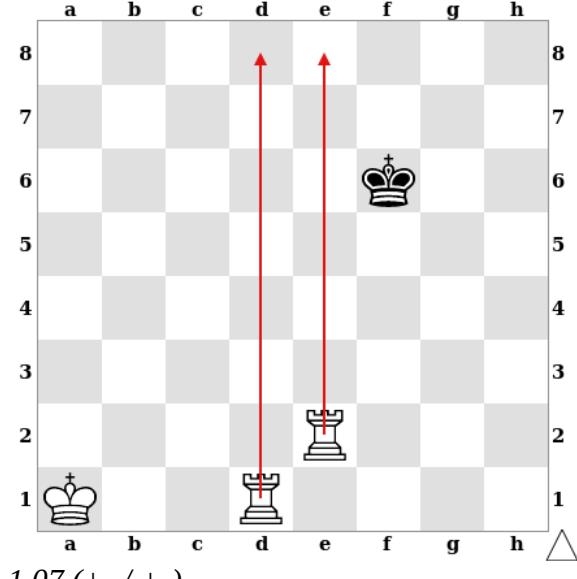
Notice that the rook on b6 covers the sixth rank, so that Black's king is forced to move to the eighth rank. After 1...Ke8 we arrive back at diagram 1.01 and White has 2.Rb8#.



1.06 (+- / +-)

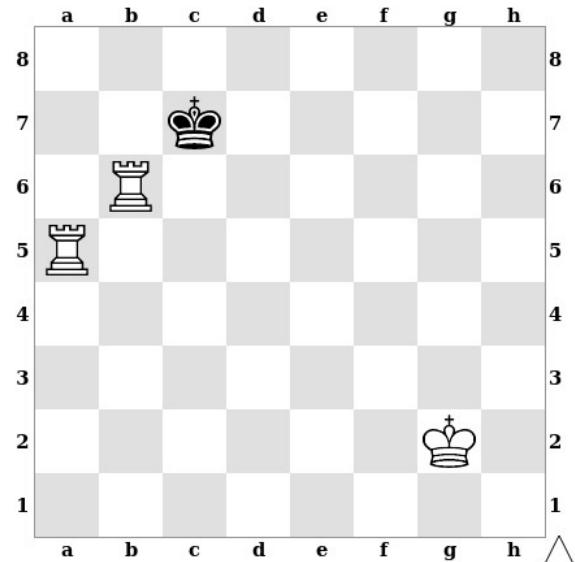
With Black's king further back, White can use a series of checks to drive him to the edge of the board:

**1.Rb6+ Kf7 2.Ra7+ Ke8 3.Rb8#.**



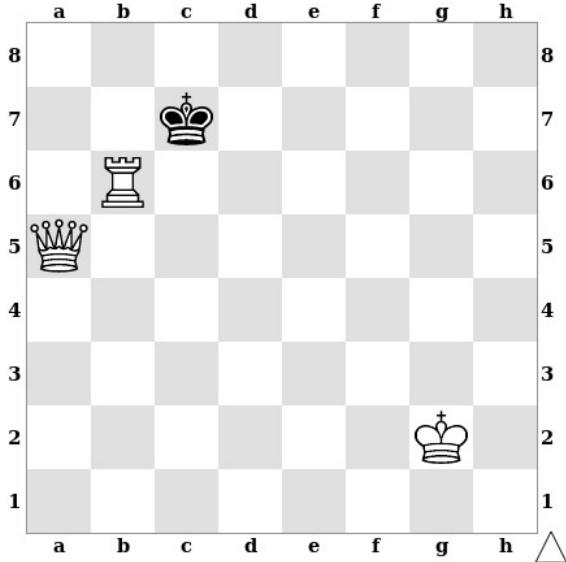
1.07 (+- / +-)

It is sometimes faster to drive the king in a different direction. Here White mates in three with **1.Rf1+ Kg5  
2.Rg2+ Kh4 3.Rh1#.**



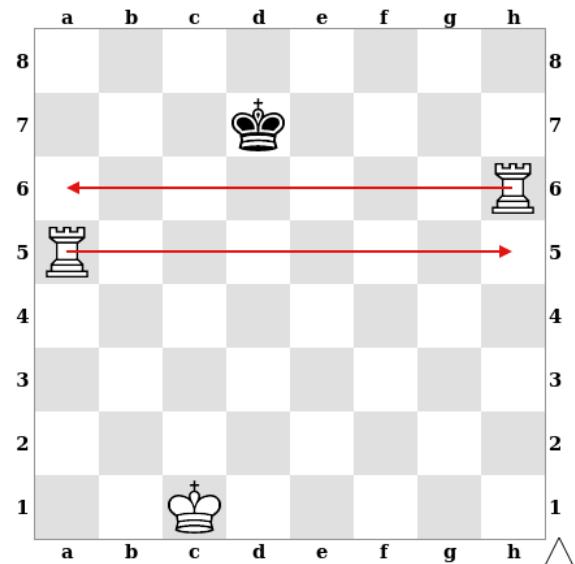
1.08 (+- / +-)

In some cases the king is too close to the rooks for the mate to continue normally. However, this problem can be solved by simply swinging one or both of the rooks to the other side of the board before continuing. In this position White can mate in four:  
**1.Rh6 Kb7 2.Rg5 Kc7 3.Rg7+ Kd8 4.Rh8#.**



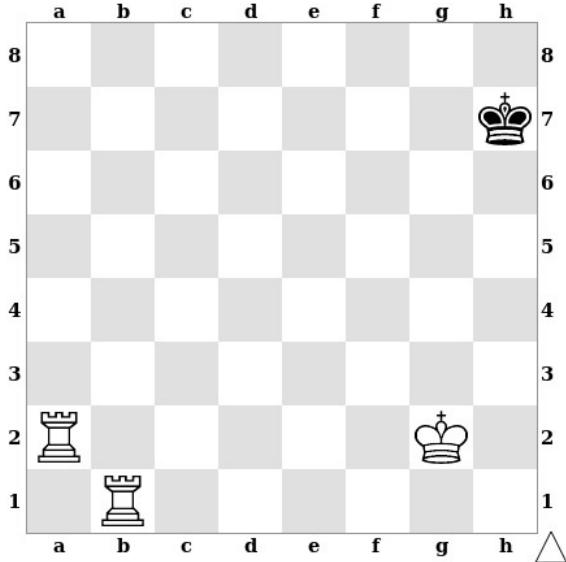
1.09 (+- / +-)

When one of the pieces is a queen it makes things even easier. The queen guards the rook so it isn't necessary to swing the pieces to the other side of the board before continuing. White mates in two with **1.Qa7+ Kc8 2.Rb8#.**



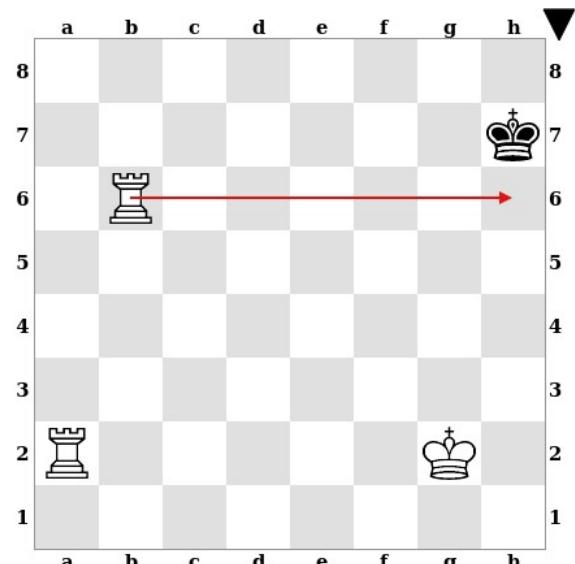
1.10 (+- / +-)

It isn't necessary for the rooks to be on the same side of the board as one another. Here White mates in two with **1.Ra7+ Kc8 2.Rh8#.**



1.11 (+- / +-)

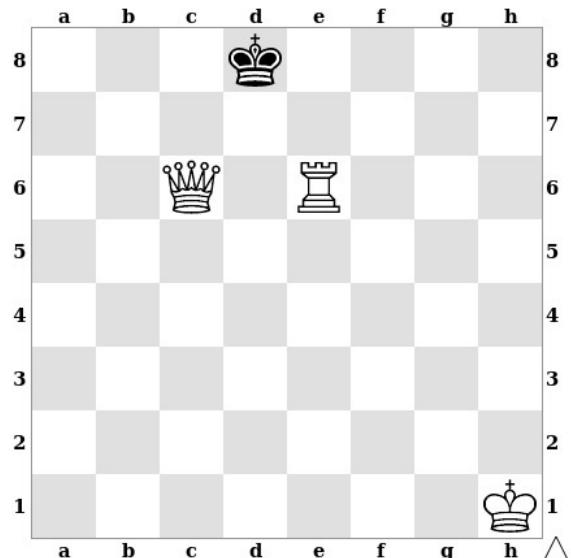
In many cases the fastest mate does not begin with check. The important thing is to cut off the king's escape route. Here White should begin with **1.Rb6** (or **1.Ra6**).



1.12 (+- / +-)

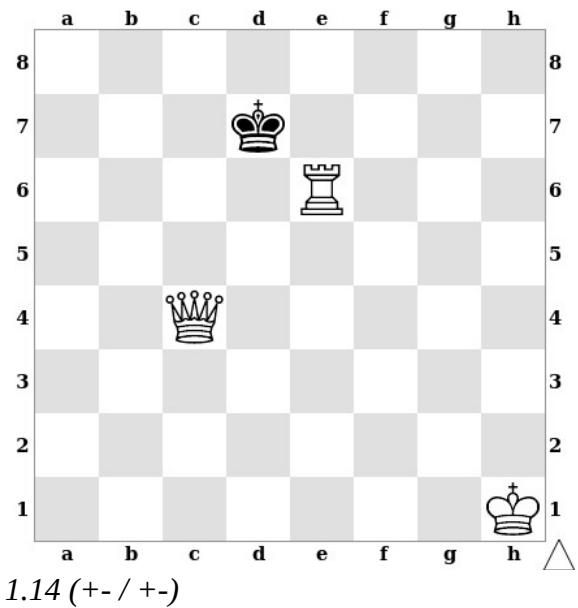
From b6 the rook controls the sixth rank so that Black's king cannot retreat down the board once the checks begin. White will continue with **2.Ra7+** and **3.Rb8#**.

## Staircase Mate

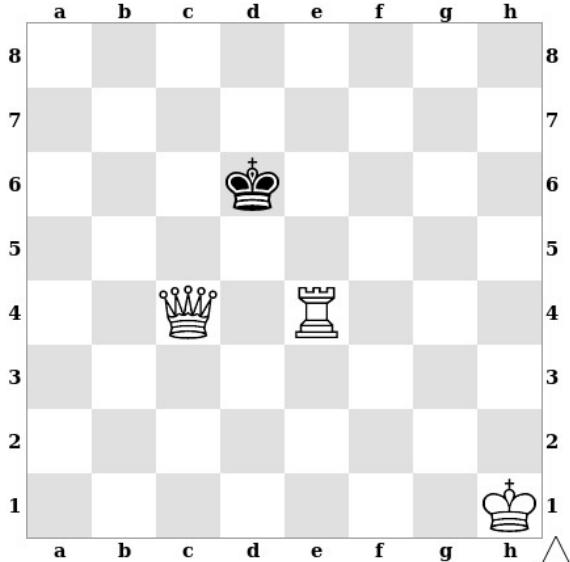


1.13 (+- / =)

Staircase mate is a variation on ladder mate involving a queen and rook. In this position, White has mate in one move with 1.Re8#.

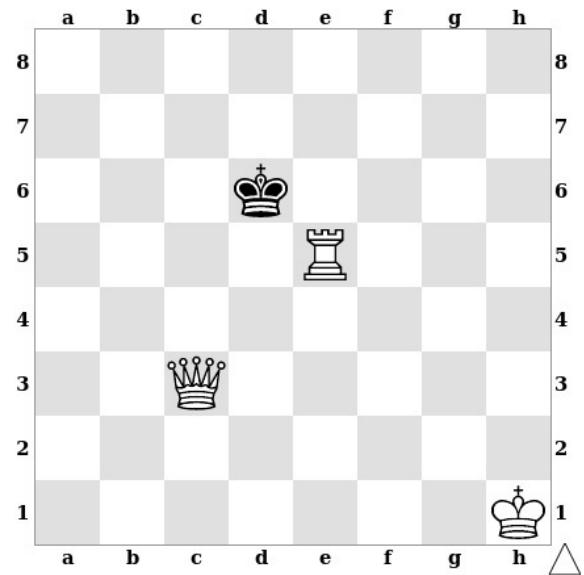


1.14 (+- / +-)



1.15 (+- / +-)

Here White can mate in three moves: **1.Re6+ Kd7  
2.Qc6+ Kd8 3.Re8#.**



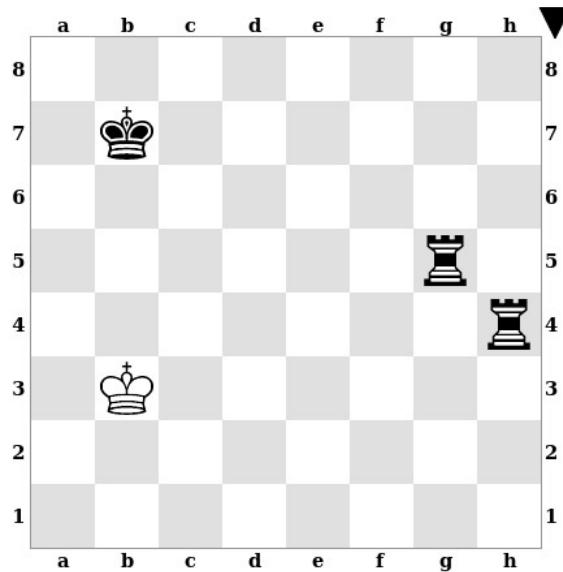
1.16 (+- / +-)

This position features a slight variation on the previous sequence. White again mates in three with **1.Qc5+ Kd7 2.Re7+ Kd8 3.Qc7#.**

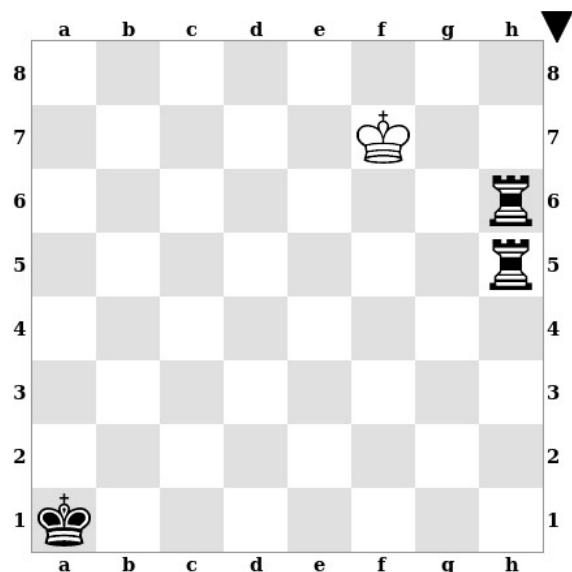
## **Conclusion**

In this chapter we looked at a couple of important mating patterns with heavy pieces. The important thing with these patterns is to be able to see them quickly and easily, so make sure to do the exercises.

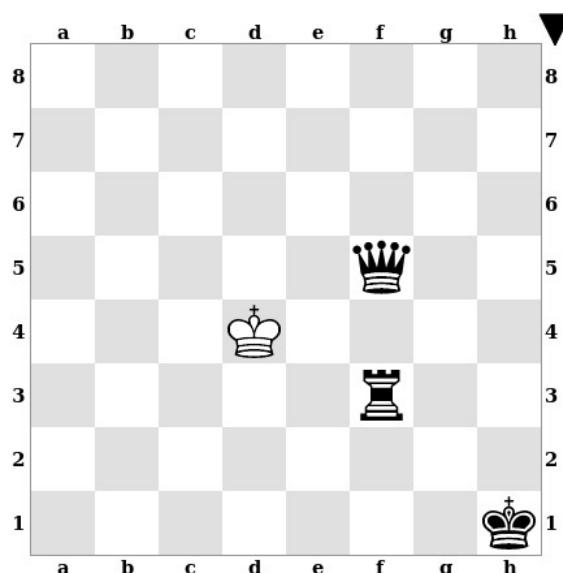
## Exercises



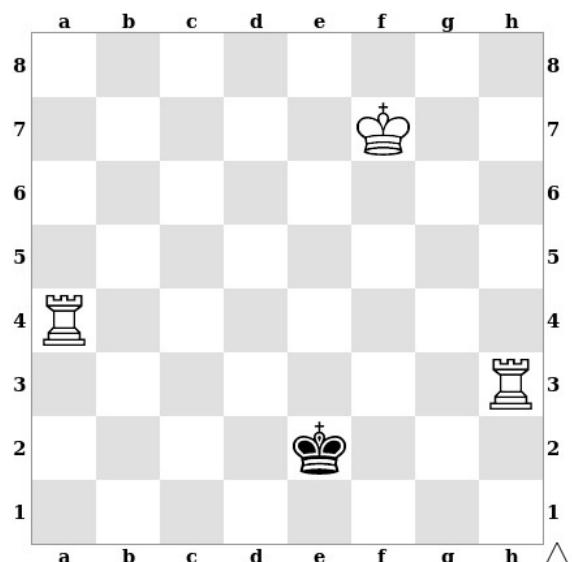
E1.01 (B)



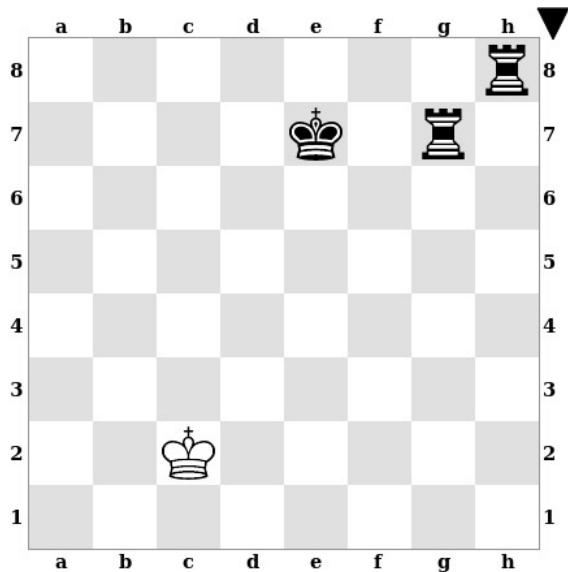
E1.02 (B)



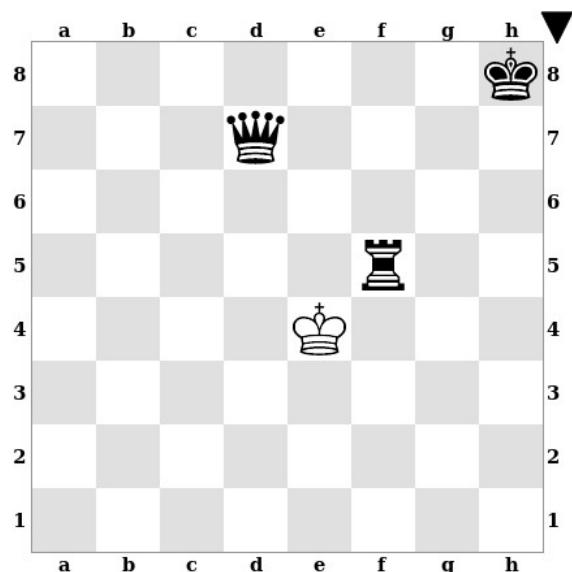
E1.03 (B)



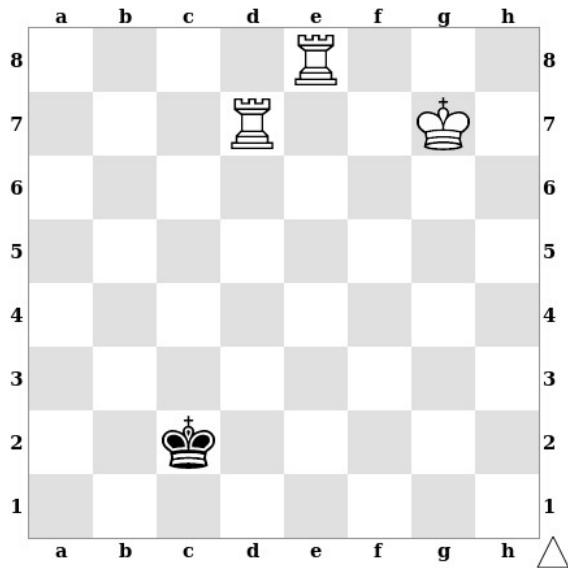
E1.04 (W)



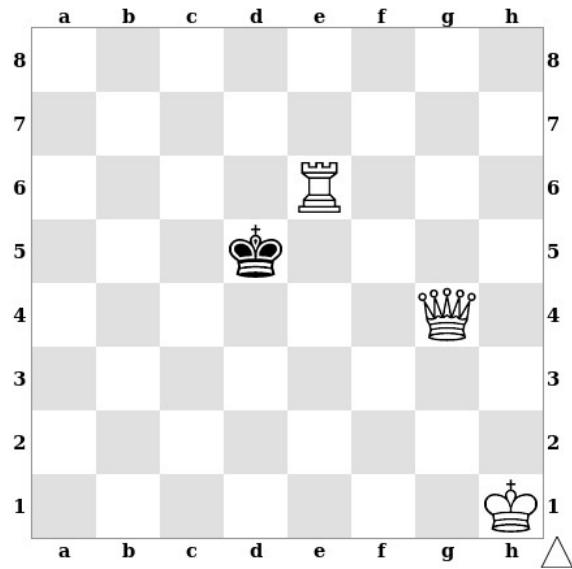
E1.05 (B)



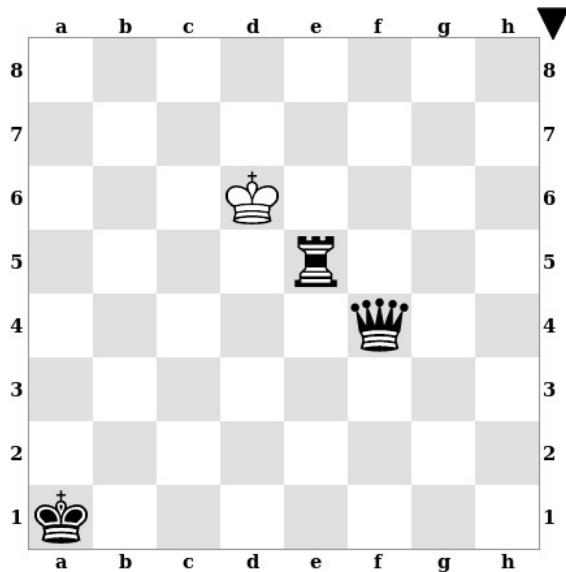
E1.06 (B)



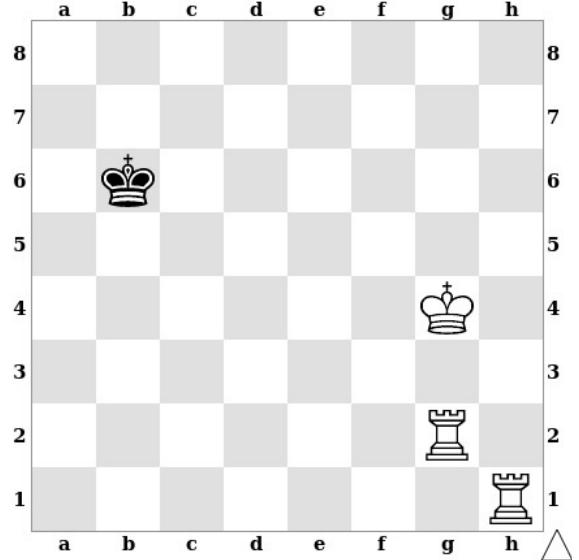
E1.07 (W)



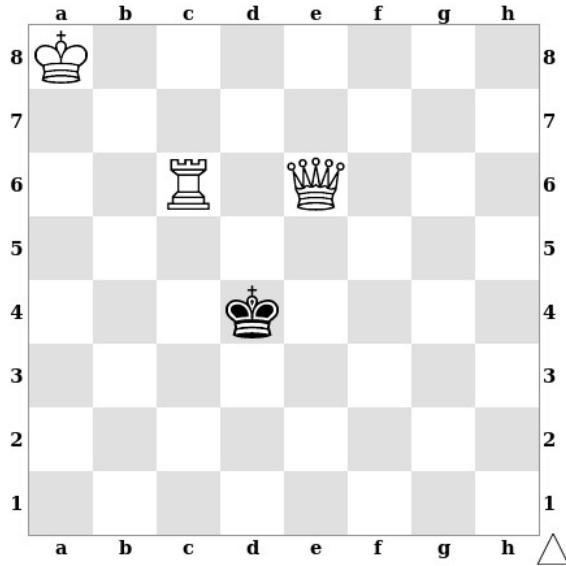
E1.08 (W)



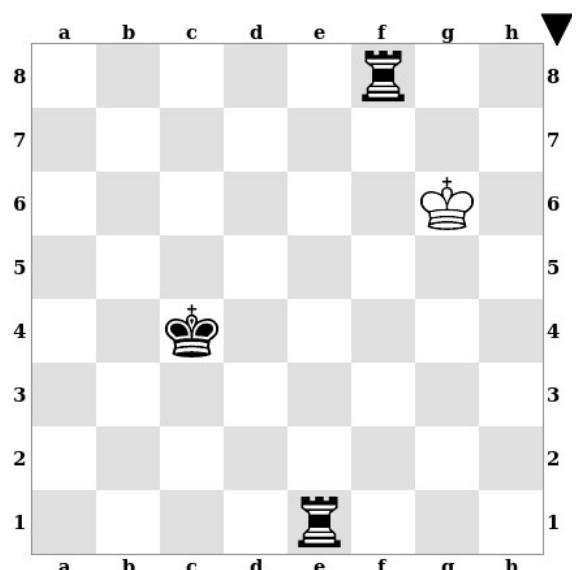
E1.09 (B)



E1.10 (W)



E1.11 (W)



E1.12 (B)

## Solutions

### E1.01

1...Rg3+ 2.Kc2 Rh2+ 3.Kd1 Rg1#

### E1.02

Black needs to rearrange his rooks before continuing. 1...Ra5 (or 1...Rb5, or 1...Ra6 etc.) 2.Kg7 Rb6 3.Kf7 Ra7+ 4.Ke8 Rb8#

### E1.03

1...Rd3+ 2.Kc4 Qd5+ 3.Kb4 Rb3+ 4.Ka4 Qb5#

### E1.04

1.Ra2+ Kf1 2.Rh1#

### E1.05

1...Rh3 (or 1...Rg3) cutting off the king's escape first. 2.Kd2 Rg2+ 3.Ke1 Rh1#

### E1.06

1...Qd5+ 2.Ke3 Rf3+ 3.Ke2 Qd3+ 4.Ke1 Rf1#

### E1.07

1.Rc8+ Kb3 2.Rb7+ Ka4 3.Ra8#

### E1.08

1.Qe4+ Kc5 2.Rc6+ Kb5 3.Qc4+ Ka5 4.Ra6#

### E1.09

1...Qf6+ 2.Kd7 Re7+ 3.Kd8 Qf8#

### E1.10

1.Rc1 (or 1.Rc2) 1...Kb5 2.Rb2+ Ka4 3.Ra1#

### E1.11

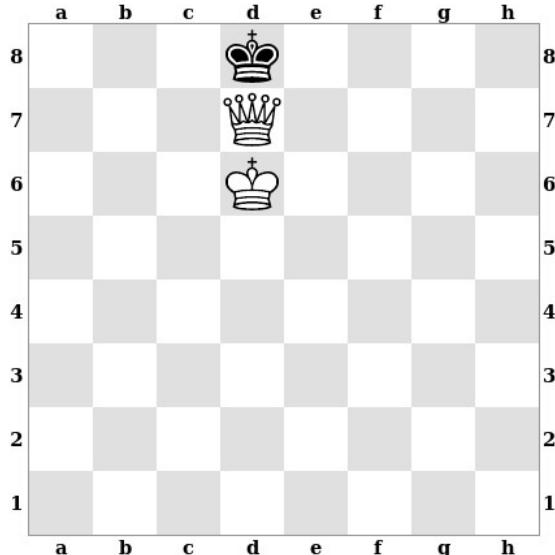
1.Rc4+ Kd3 2.Qe4+ Kd2 3.Rc2+ Kd1 4.Qe2#

**E1.12**

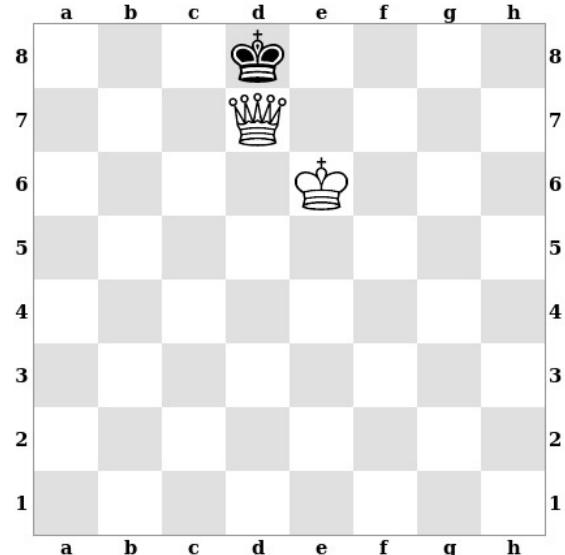
1...Rg1+ (or 1...Rf2) 2.Kh7 Rf2 3.Kh6 Rh2#

## 2 – King and Queen Mate

A queen and king together can force checkmate against a lone king. The checkmate can only be achieved when the weaker side's king is on the edge of the board (the a-file, h-file, first rank, or eighth rank); thus the first step is to drive the weaker side's king to an edge. Both the stronger side's queen and king need to be involved in the final checkmate; the queen cannot checkmate alone.

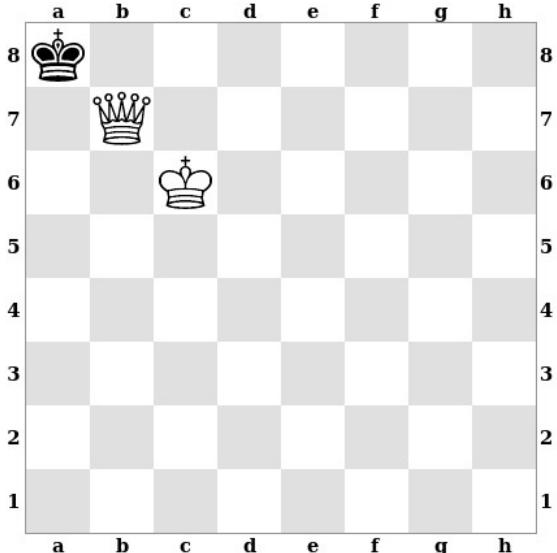


2.01

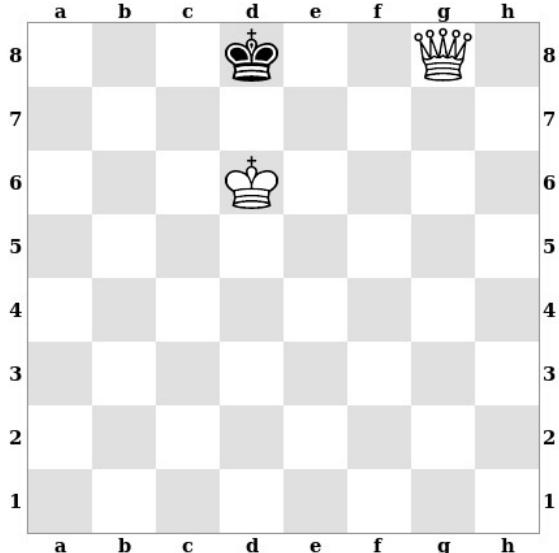


2.02

The first diagram above shows a typical *support mate* in which White's queen stands directly in front of Black's king while White's king guards the queen. The second diagram shows a slight variation on the same pattern. Even with White's king off-center from Black's the support mate still works.

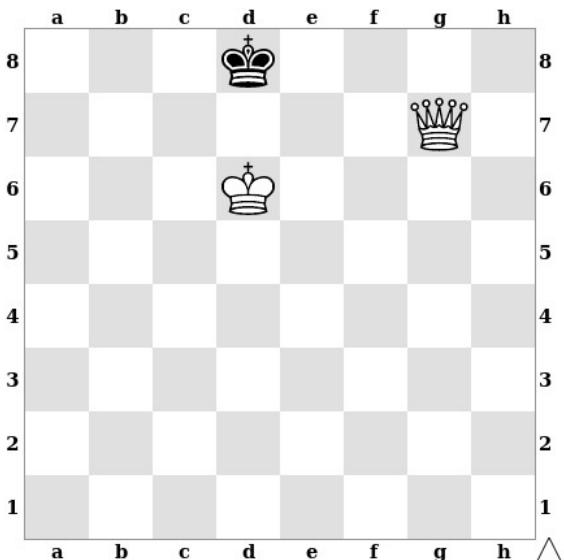


2.03



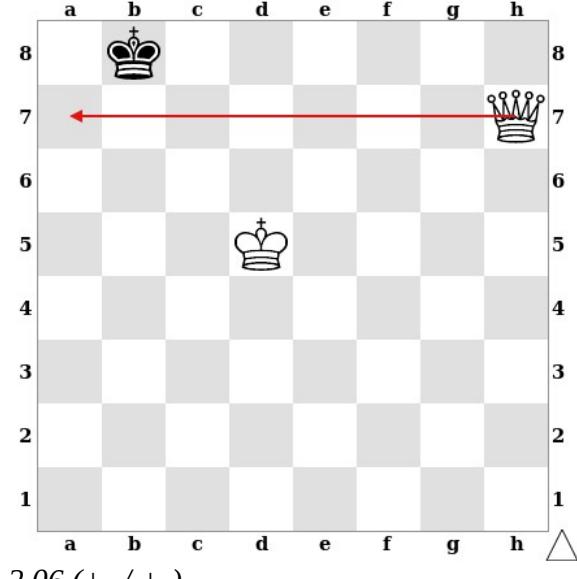
2.04

Diagram 2.03 shows a third support mate pattern, much like the first two. Diagram 2.04 shows something quite different, however. Here White has created checkmate by checking Black's king along the edge, while White's king controls all of the seventh rank escape squares and prevents Black's king from moving forward.



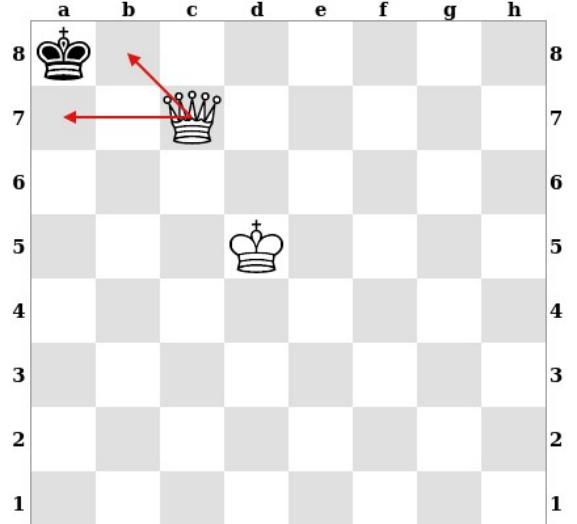
2.05 (+- / +-)

In this position White has four different moves that produce checkmate: the support mate 1.Qd7# and mate along the edge with any of 1.Qh8#, 1.Qg8#, or 1.Qf8#.



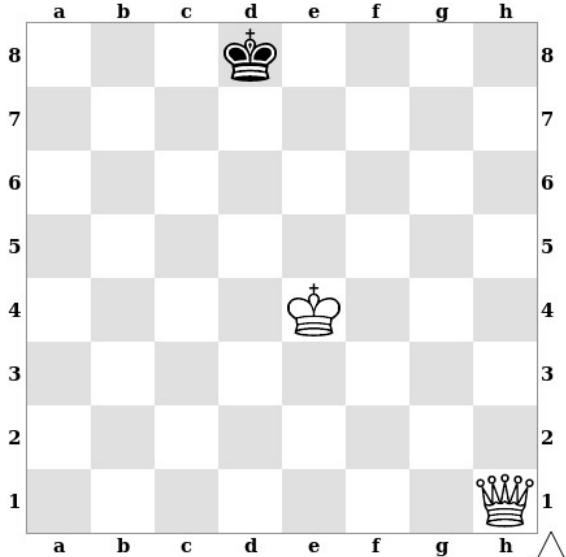
2.06 (+- / +-)

In this position White's queen has trapped Black's king on the edge of the board by controlling the entire seventh rank. White now needs to bring his king closer before checkmating: **1.Kc6 Ka8 2.Qb7#**. If Black instead plays **1...Kc8**, then White has a number of mates such as **1.Qc7#**.



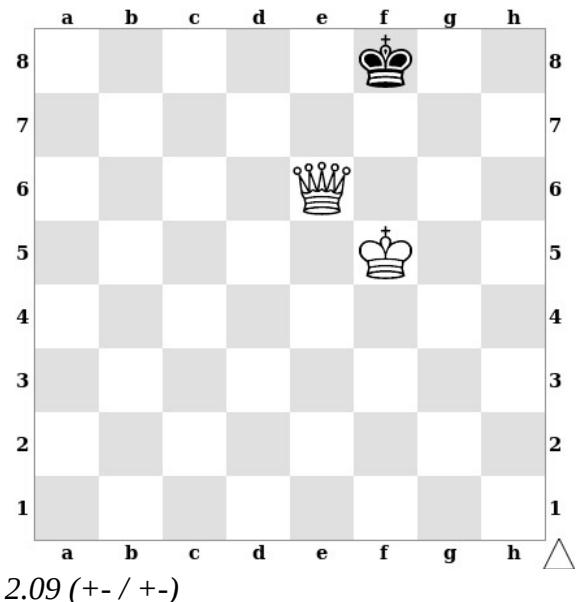
2.07 (+- / =)

Here White's queen again traps Black's king on the edge, but White needs to be careful. After a careless move like **1.Kc6?** = the game would end in stalemate since Black's king has no moves available. White should instead move his queen back to give Black's king some room. Of course, he should also keep Black's king trapped on the edge of the board. He might play **1.Qh7 Kb8**, reaching diagram 2.06.



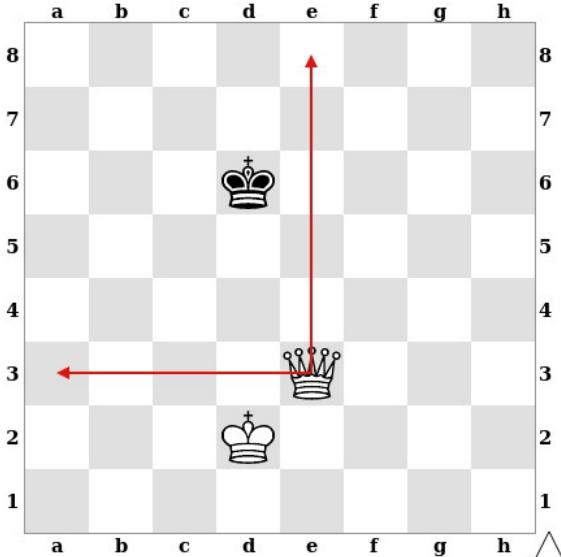
2.08 (+- / +-)

Whenever the weaker side's king is on the edge of the board the next step for the stronger side is to trap it there. Here White's fastest win starts with **1.Qh7**, controlling the seventh rank and trapping Black's king on the eighth rank. With Black's king unable to leave the edge of the board White has time to bring his king in and mate. The game might end **1...Kc8 2.Kd5 Kb8 3.Kc6 Ka8 4.Qb7#**.



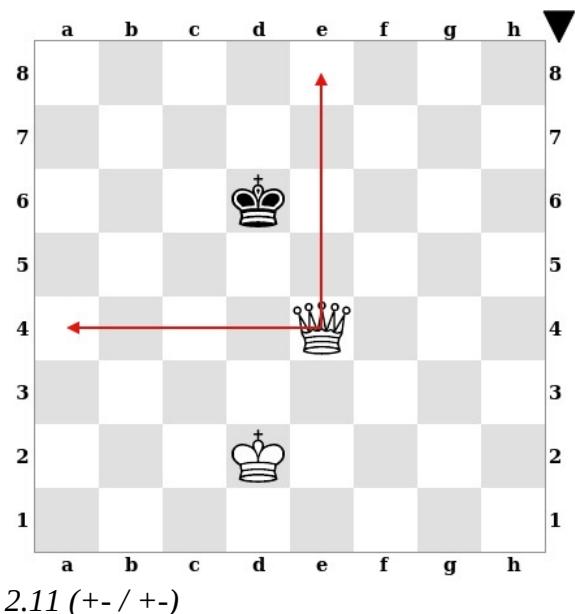
2.09 (+- / +-)

In this position, similarly, White should start by trapping the king on the edge with **1.Qd7**. It is important that White not play **1.Kf6?** = which is stalemate. After **1.Qd7** the game could continue **1...Kg8 2.Kg6 Kf8 3.Qf7#**.



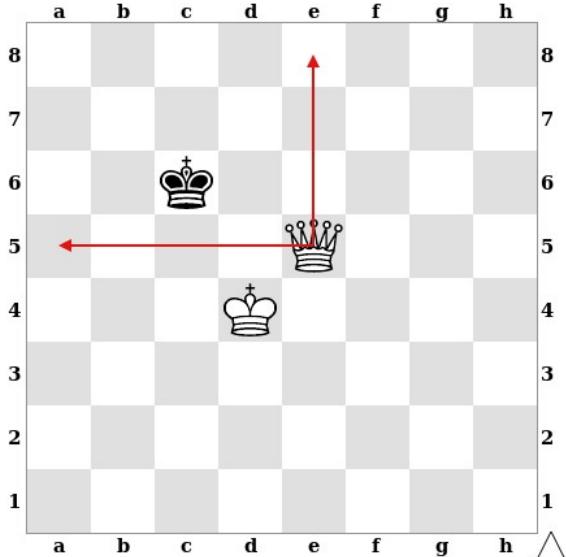
2.10 (+- / +-)

From a typical starting position, the first step in mating with king and queen is to drive the weaker side's king to the edge of the board. Notice that in this position White's queen controls the third rank and e-file. This creates a box that Black's king cannot leave. White will make this box smaller whenever possible without letting Black's king escape. This will eventually force Black's king to the edge of the board. White can start with **1.Qe4**, beginning a process that we will follow through the remaining diagrams.



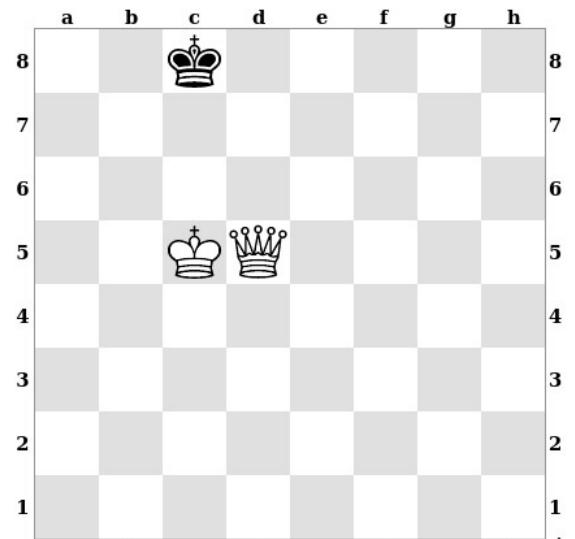
2.11 (+- / +-)

The box is now smaller and Black's king is still trapped inside. After **1...Kc5** White has no way to make the box smaller without letting Black's king out. When this is the case White should move his king closer to Black's king. Play might continue **2.Kc3 Kd6 3.Kd4 Kd7 4.Qe5 Kc6**.



White can now make the box even smaller with 5.Qd5+ when play might proceed 5...Kc7 6.Kc5 Kc8.

2.12 (+- / +-)



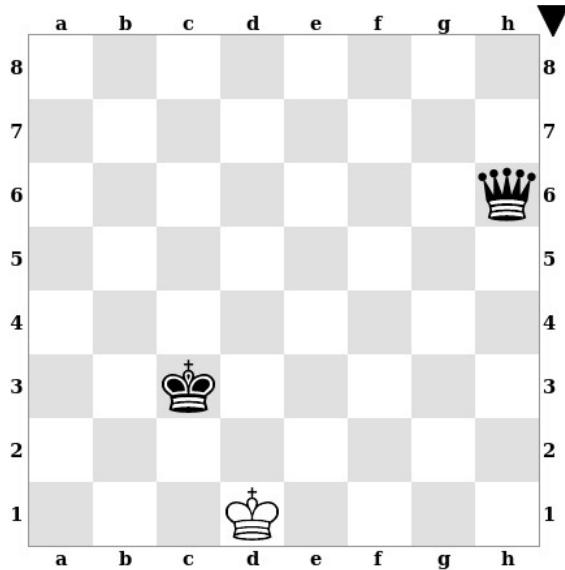
2.13 (+- / +-)

This is an important moment in the procedure. Black's king has reached the edge of the board, so White should forget entirely about the box and should instead find some move to trap Black's king on the edge. He can do this with 7.Qf7 when the game might end 7...Kb8  
8.Kb6 Kc8 9.Qc7#.

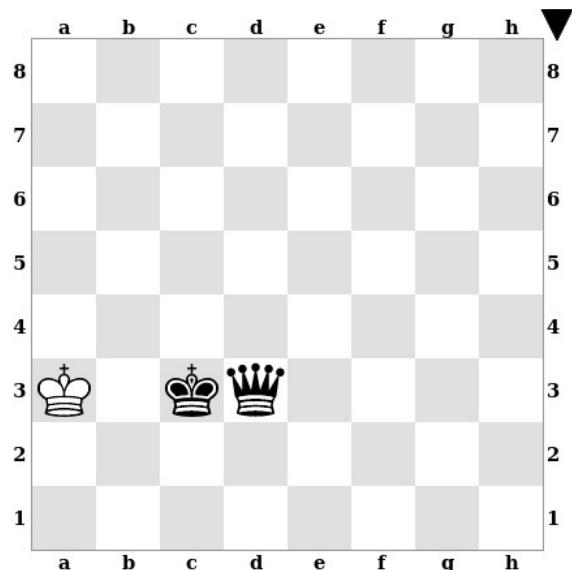
## **Conclusion**

While it is very basic, the king and queen checkmate is easily the most useful endgame in chess. Reading this chapter is a good way to get started, but this is one endgame that demands a lot of actual practice. Practice from different starting positions against a coach, a friend, or a chess engine. Try setting up the pieces in different ways so that you sometimes drive the king up, sometimes down, and sometimes to the side.

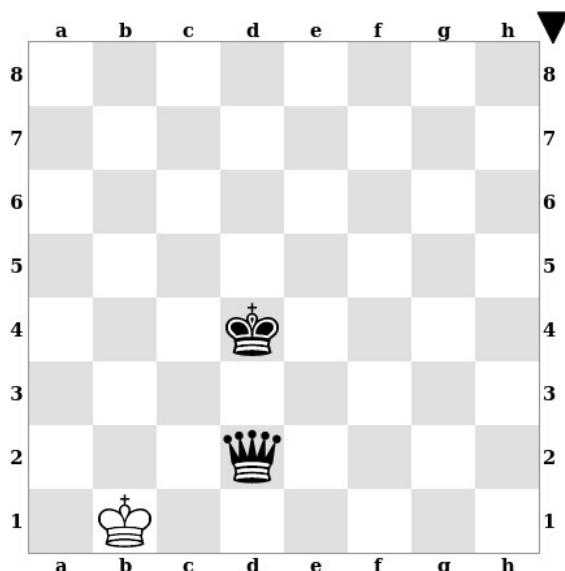
## Exercises



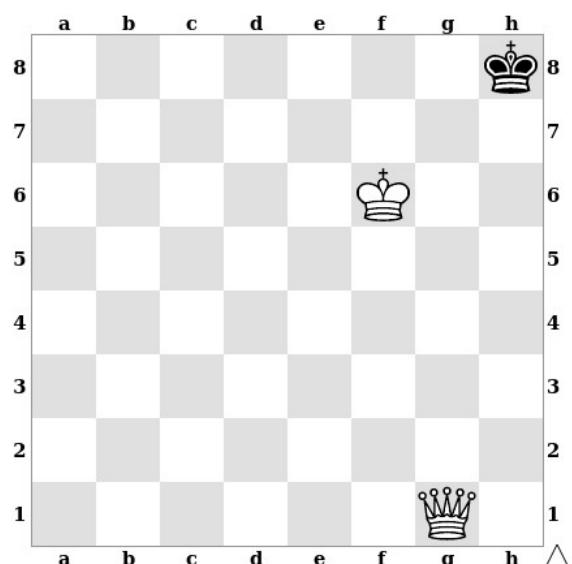
E2.01 (B)



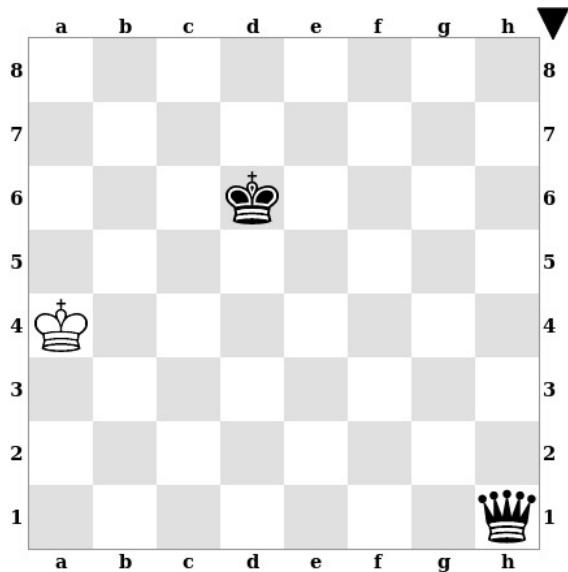
E2.02 (B)



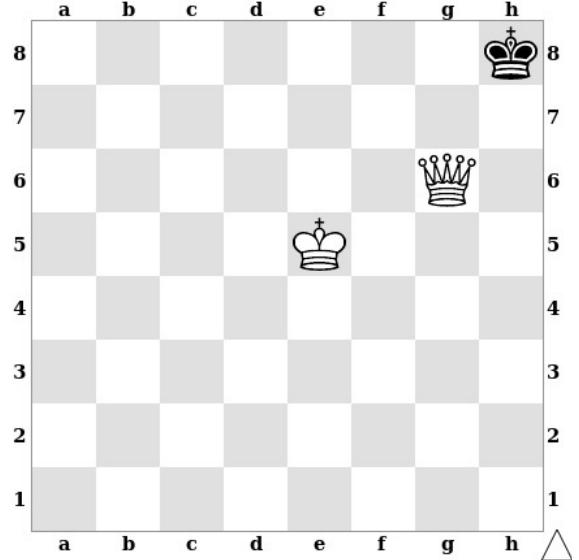
E2.03 (B)



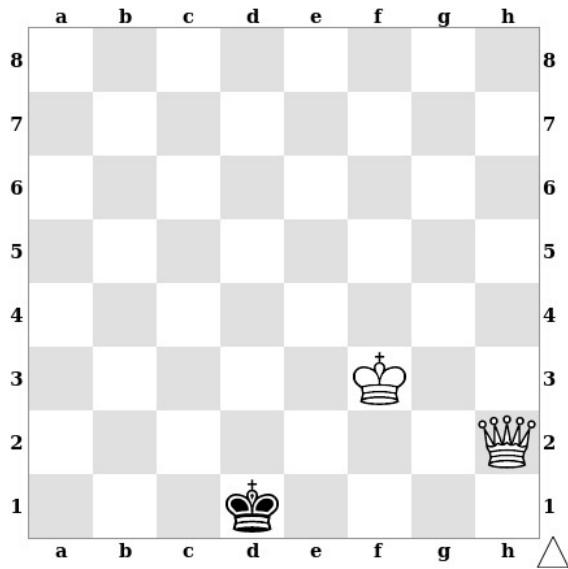
E2.04 (W)



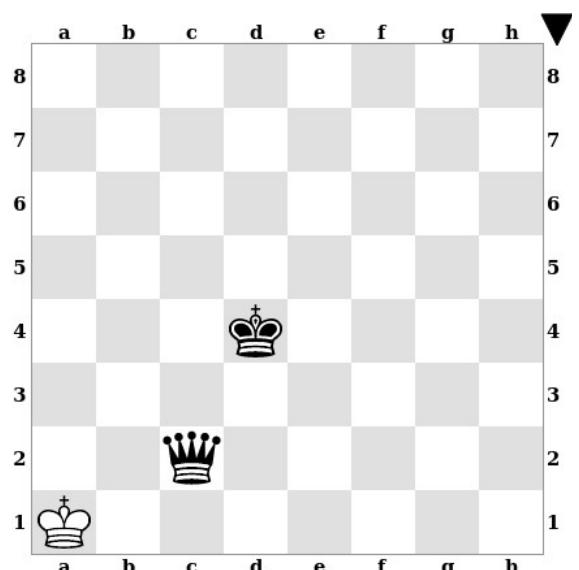
E2.05 (B)



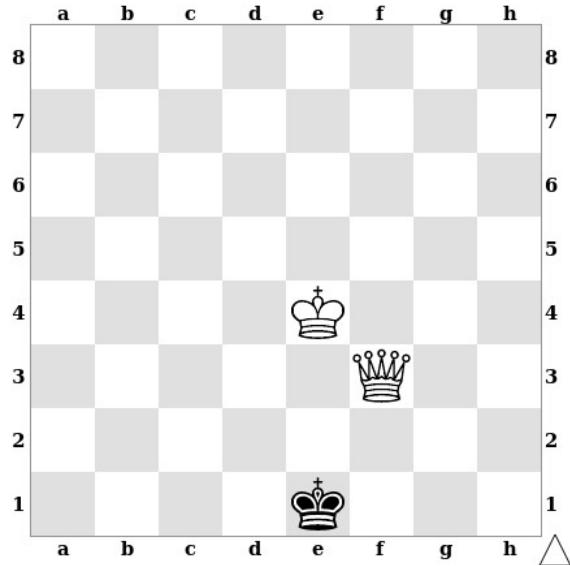
E2.06 (W)



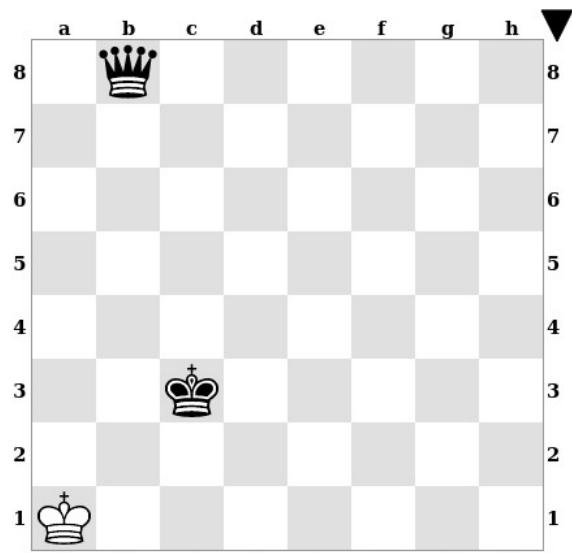
E2.07 (W)



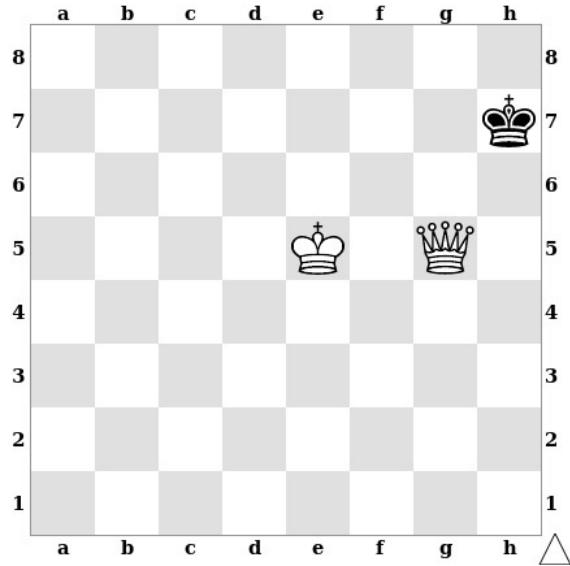
E2.08 (B)



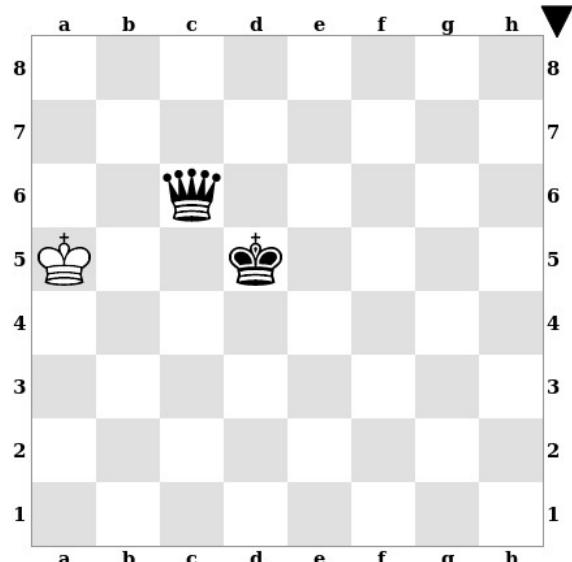
E2.09 (W)



E2.10 (B)



E2.11 (W)



E2.12 (B)

## Solutions

### E2.01

1...Qd2#

### E2.02

1...Qa6#

### E2.03

1...Kc3 2.Ka1 Qb2#

### E2.04

1.Qg7#

### E2.05

1...Qb1 (or 1...Qb7), trapping the king on the edge. 2.Ka3 Kc5 3.Ka4 Qb4#

### E2.06

1.Qg5 (or some other queen on the g-file, but not 1.Kf6? = stalemate) 1...Kh7 2.Kf6 Kh8 3.Qg7#

### E2.07

1.Ke3 is fine, but fastest is 1.Qb2 Ke1 2.Qe2#

### E2.08

1...Qd2 (or another queen move on the second rank, but not 1.Kc3? =) 2.Kb1 Kc3 3.Ka1 Qb2#

### E2.09

1.Qg2 (not 1.Ke3? = or 1.Kd3? =) 1...Kd1 2.Kd3 Kc1 3.Qc2#

### E2.10

1...Qb2#

### E2.11

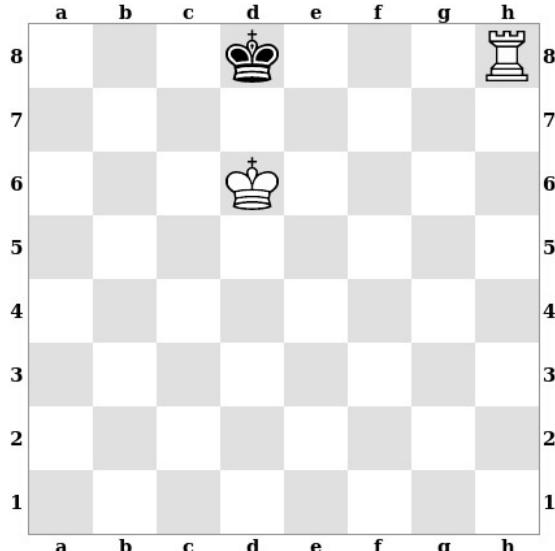
1.Kf6 Kh8 2.Qg7#

**E2.12**

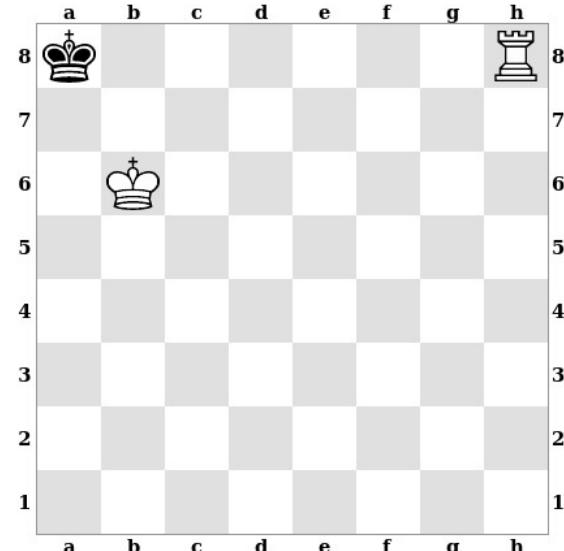
1...Qb7 (not 1...Kc5? = or 1...Kc4? =) 2.Ka4 Kc4 3.Ka3 Qb3#

### 3 – King and Rook Mate

A king and rook can also force checkmate against a lone king. The process is nearly the same as with a king and queen, but there are two important differences. The first difference is simply that the rook has fewer ways to deliver checkmate than the queen. The second difference involves the need for a powerful concept that is important in many other endgames as well.

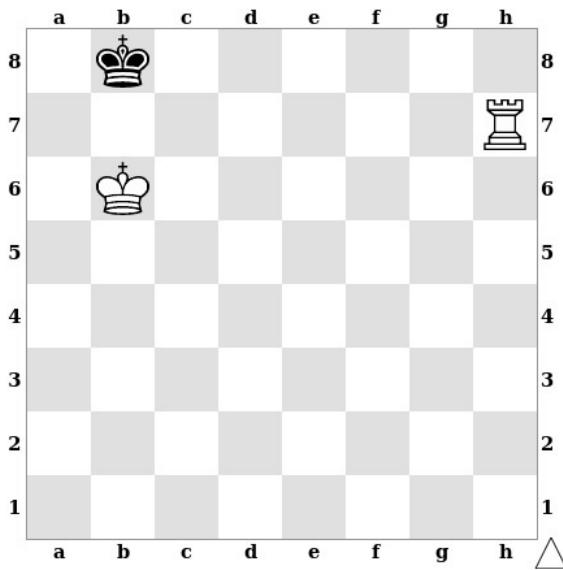


3.01



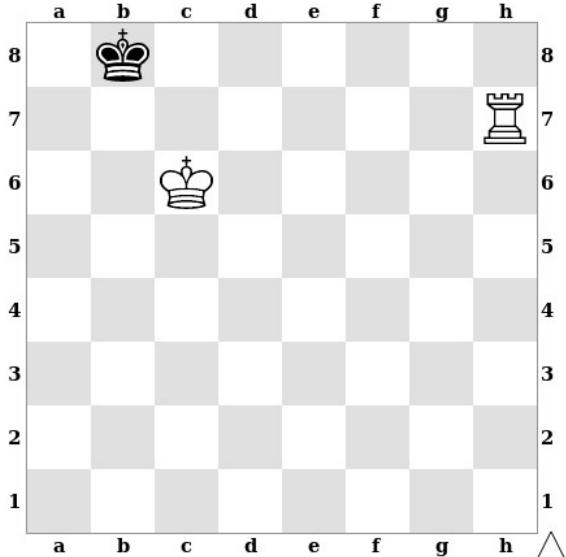
3.02

The above diagrams show the two possible ways for a king and rook to checkmate another king. The rook can't perform a support mate on an empty board so both mates involve a check along the edge of the board. In order for this check to be mate the kings usually need to be lined up, as in diagram 3.01. However, with the weaker side's king in the corner the mate in 3.02 is possible.



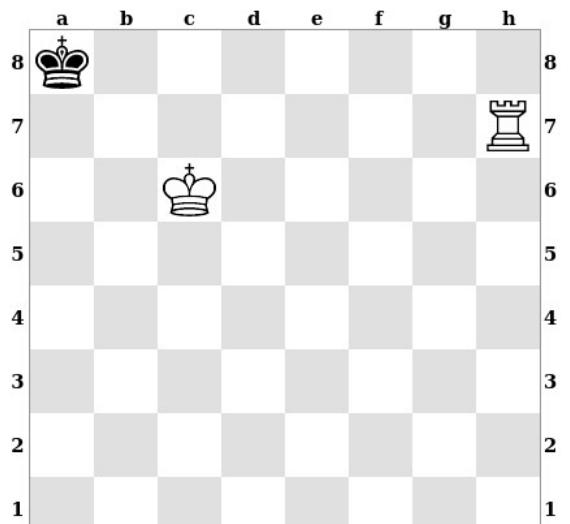
3.03 (+- / +-)

Here White has checkmate in one move with **1.Rh8#**. If his rook were a queen, White would also have **1.Qb7#**, but with a rook the move **1.Rb7+** is not mate.



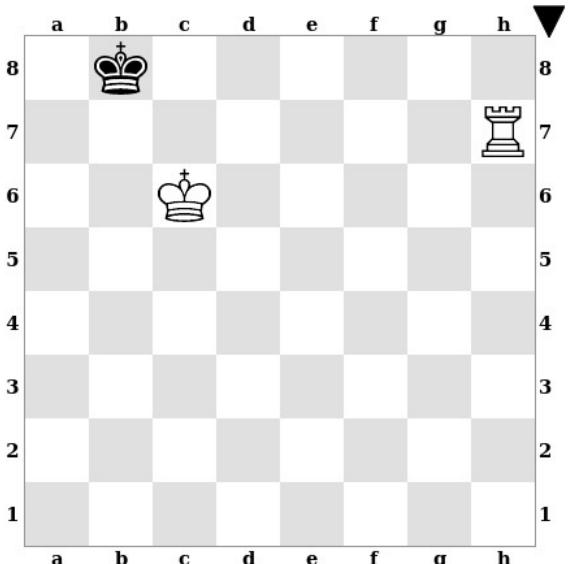
3.04 (+- / +-)

In this position White does not have mate in one move available. The move 1.Rh8+ is not mate because Black can play 1...Ka7. The problem is that the two kings are not lined up and Black's king is not in the corner either. It seems logical then to play 1.Kb6 in order to line the kings up; but Black can then play 1...Kc8 when the kings are no longer lined up and there is no mate. It turns out that this position is quite tricky. We will return to it below.



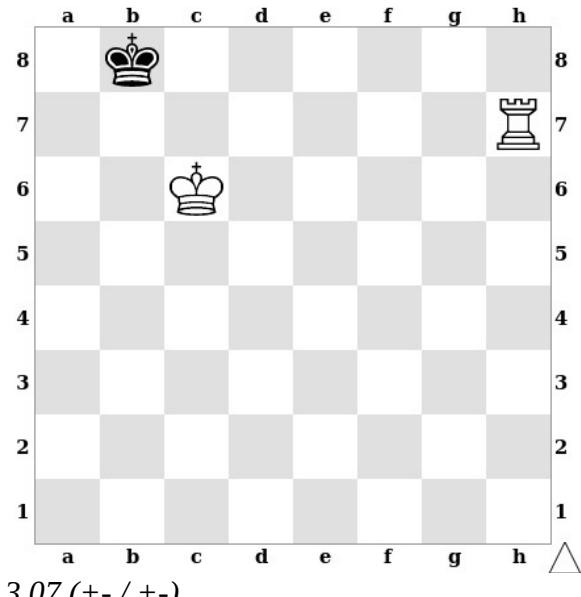
3.05 (+- / +-)

Here, again, White doesn't have mate in one with 1.Rh8+ because of 1...Ka7. However, White can force mate in two moves with **1.Kb6 Kb8 2.Rh8#**.



3.06 (+- / +-)

This position is the same as the tricky one in diagram 3.04 except that Black now moves first instead of White. This turns out to make the position very easy. If Black plays 1...Kc8, then White mates with 2.Rh8#. If Black instead plays 1...Ka8, we have diagram 3.05; White then mates in two with 2.Kb6 and 3.Rh8#.

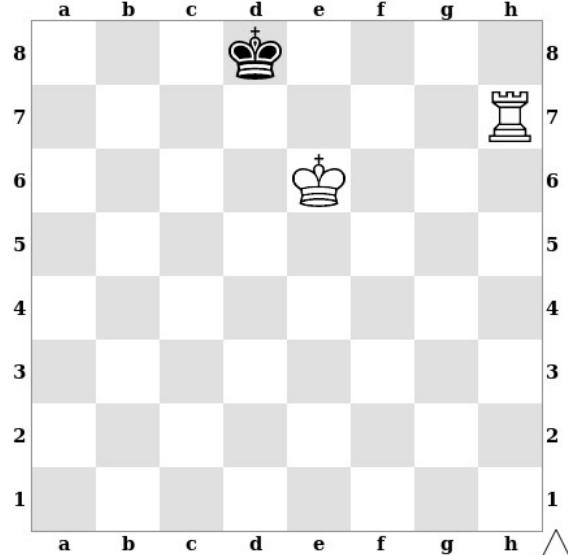


3.07 (+- / +-)

Here is 3.04 with White to move again. Neither 1.Rh8+ Ka7 nor 1.Kb6 Kc8 accomplish anything. However, the previous example showed that Black loses quickly if he moves first. The key idea is for White to make a move simply to make it Black's turn, forcing him to move first. White can do this with 1.Rg7 when Black must play 1...Kc8 2.Rg8# or 1...Ka8 2.Kb6 Kb8 3.Rg8#.

As it turns out, White can only win in 3.07 by forcing Black to make a committal move. As a result, if the defender were allowed to skip his turn, a king and rook could not win by force against a lone king. Of course the rules of chess don't actually allow this.

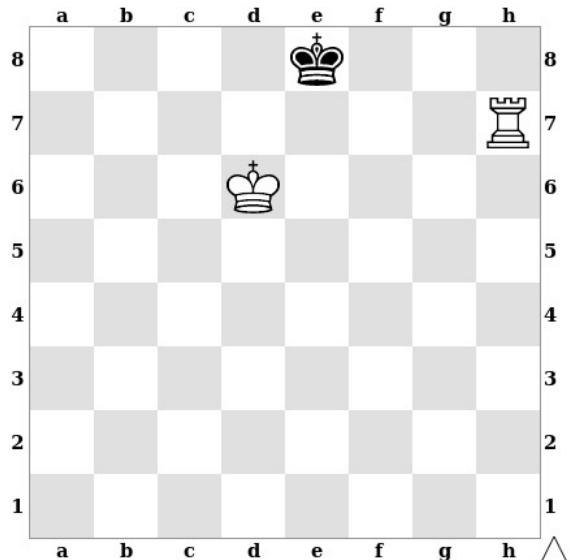
In fact, this is our first example of *zugzwang*, a situation in which a player's best move would be to skip his turn if this were allowed. In the endgame this normally means that a player loses a position that he could draw if he were allowed to "pass." In diagram 3.06 Black is in zugzwang, while in 3.07 White can only win by putting Black in zugzwang. Putting the opponent in zugzwang is often accomplished by playing a *waiting move*, a move that changes as little as possible about the position. In 3.07, 1.Rg7 is a better waiting move than 1.Kb6 since the latter changes White's king position and slows down the mate after 1...Kc8.



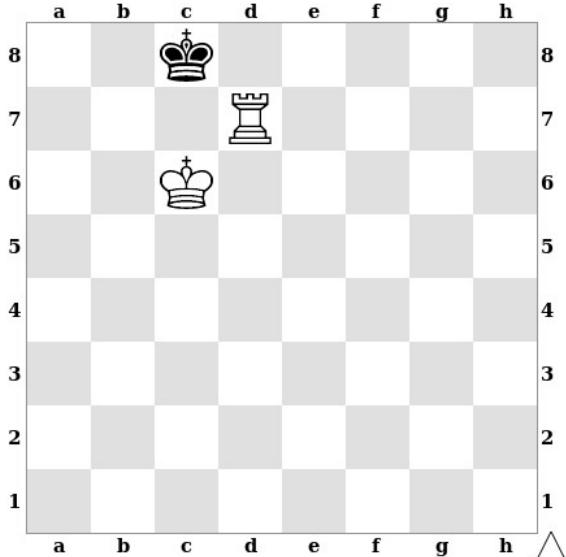
3.08 (+- / +-)

A waiting move is necessary whenever the kings aren't lined up yet. Here White again wins with 1.Rg7. Now Black has to choose between 1...Ke8, allowing mate with 2.Rg8#, and moving his king closer to the corner with 1...Kc8. In that case White plays 2.Kd6 when Black faces the same choice again. Black can hold out longest by moving to the corner: 2...Kb8 3.Kc6 Ka8 4.Kb6 Kb8 5.Rg8#.

Again the kings aren't lined up and a waiting move is necessary. However 1.Rg7 doesn't work well here because of 1...Kf8, when the rook is attacked and must move again. Instead White should begin with 1.Ra7. Now Black's king can only approach the rook by moving in front of White's king, allowing mate (1...Kd8 2.Ra8#). Black's only other choice is to run for the corner, leading to a familiar sequence: 1...Kf8 2.Ke6 Kg8 3.Kf6 Kh8 4.Kg6 Kg8 5.Ra8#. This example shows that the best waiting move in these positions is generally one that puts White's rook on the opposite side of White's king from Black's king.

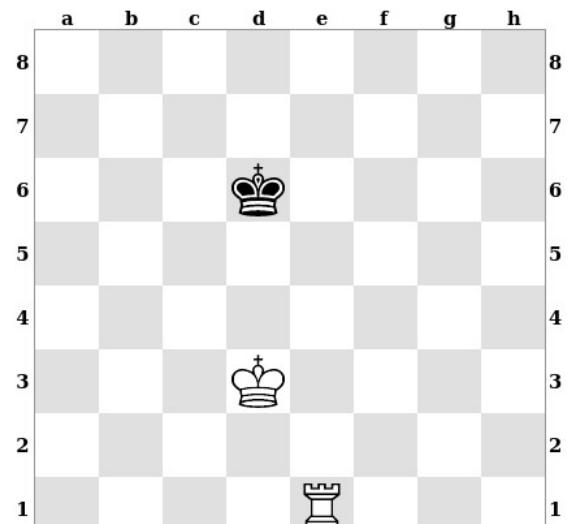


3.09 (+- / +-)



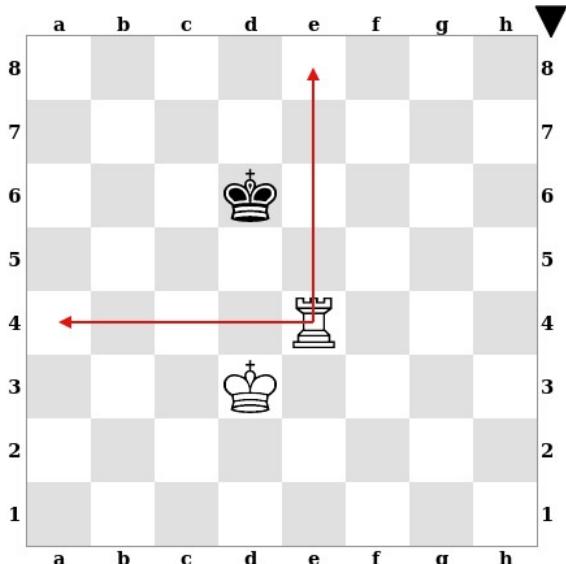
3.10 (+- / +-)

The rook sometimes needs to make a couple of moves to get things started. Here the kings are lined up, but mate isn't yet possible because White's rook is too close to Black's king ( $1.Rd8+?$  =  $Kxd8$ ). To begin with White needs to give his rook enough room to check, so he plays **1.Rh7**. However, after **1...Kd8** the kings are no longer lined up and Black's king is coming to harass the rook. White's best waiting move is to swing to the other side of his king with **2.Ra7**. Now **2...Kc8** allows **3.Ra8#**, and **2...Ke8 3.Kd6** etc. is the standard chase into the corner.



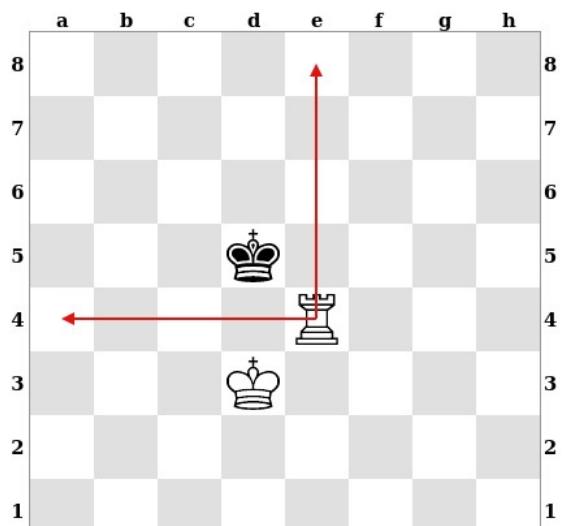
3.11 (+- / +-)

The process of driving Black's king to the edge is much the same with a rook as with a queen. Here White begins with **1.Re4**.



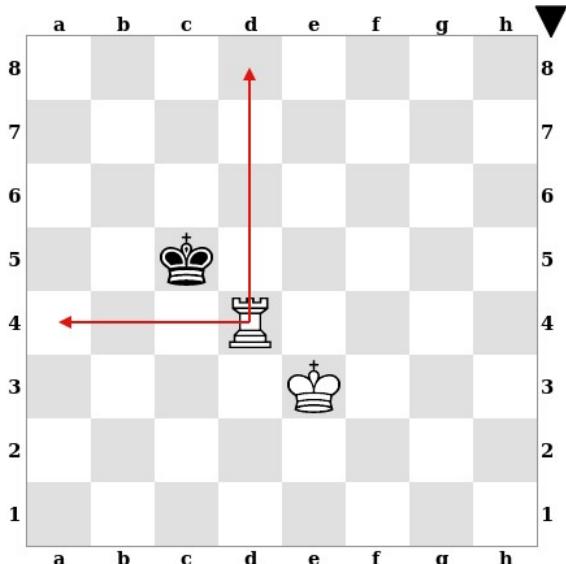
3.12 (+- / +-)

Black's king is now trapped in a box, just as in the queen mate. However, after 1...Kd5 we see an important difference between these two mates.



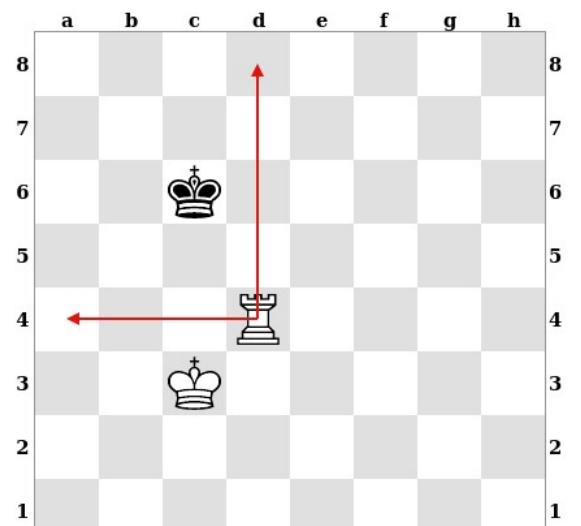
3.13 (+- / +-)

It is now White's turn, but none of his available moves accomplish anything. Any rook move makes the box larger or allows Black's king to escape. There is also no move for White's king that improves its position. It turns out that White again needs to use zugzwang to make progress. The only suitable waiting move is **2.Ke3**, since this at least doesn't make White's position worse. Now every move for Black allows White to start making progress again. 2...Kd6 allows 3.Kd4 when White's king has moved forward. Instead, 2...Kc5 allows 3.Rd4.



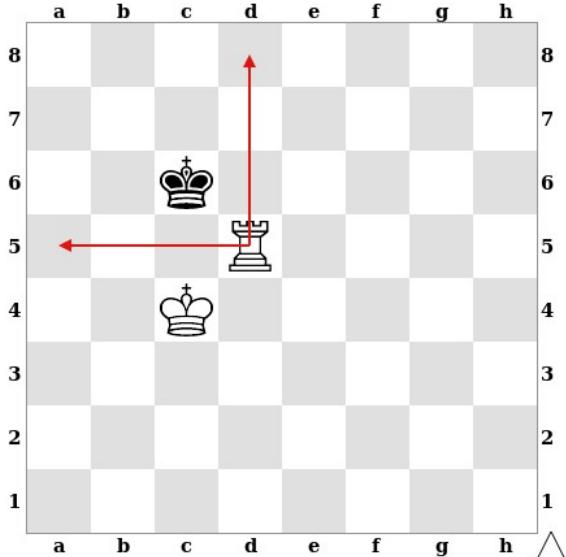
The box is now smaller and Black's king is still trapped inside. After 3...Kc6 White improves his king position with 4.Kd3 Kc5 5.Kc3 Kc6.

3.14 (+- / +-)



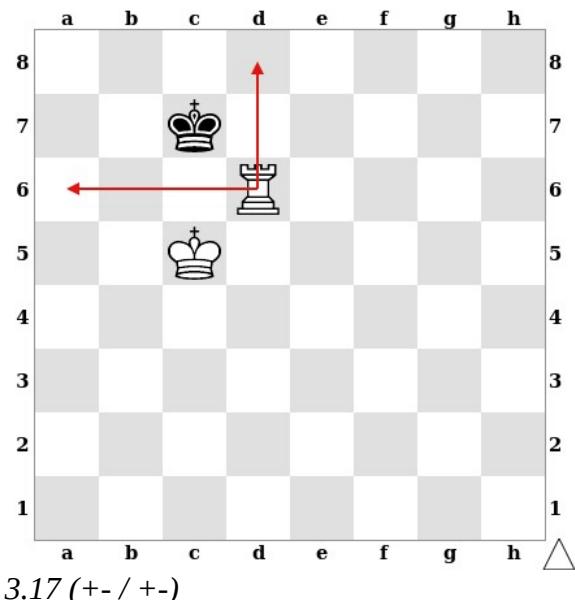
3.15 (+- / +-)

Now after 6.Kc4 Kb6 7.Rd5 Kc6 we reach the next diagram.



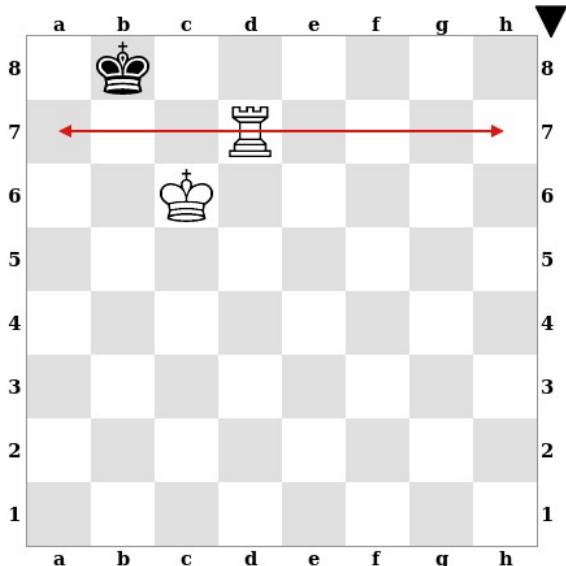
3.16 (+- / +-)

Again we have a position in which White has no productive moves and must put Black in zugzwang to make progress. After **8.Kd4** Black can choose between **8...Kb6 9.Rc5**, when the box shrinks further, and **8...Kc7 9.Kc5 Kb7 10.Rd6 Kc7**.



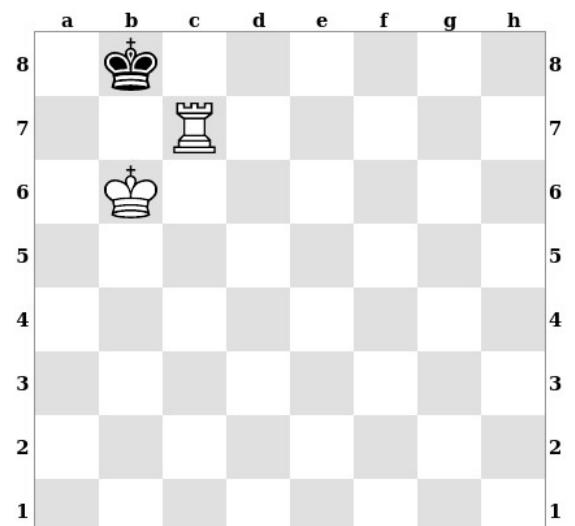
3.17 (+- / +-)

One again White needs to use zugzwang to make progress: **11.Kd5 Kc8 12.Kc6 Kb8 13.Rd7**.



3.18 (+- / +-)

Black's king is now trapped on the edge of the board and White will soon mate as in earlier examples. The game might end with 13...Kc8 14.Rh7 Kd8 15.Ra7 Ke8 16.Kd6 Kf8 17.Ke6 Kg8 18.Kf6 Kh8 19.Kg6 Kg8 20.Ra8#.



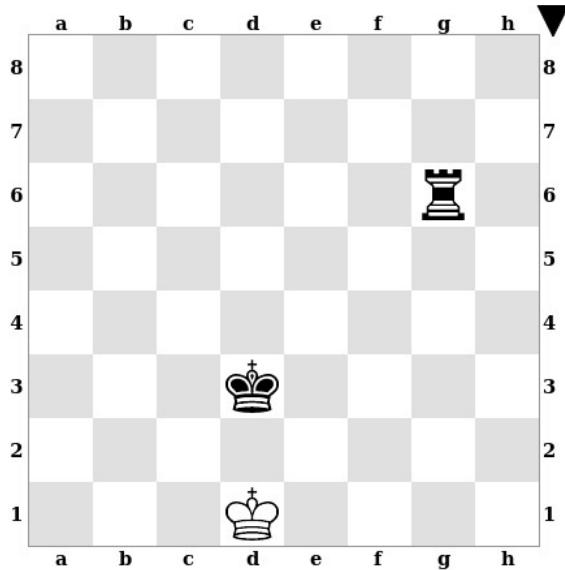
3.19 (+- / +-)

Finally, it is worth knowing about a little trick that can sometimes be used to mate more quickly. Here White could play 1.Rh7 and begin arranging the sort of mate shown in previous examples; however, a more clever approach is to play the waiting move 1.Rc6 (or almost any other rook move on the c-file) when Black is in zugzwang and is forced to play 1...Ka8 2.Rc8#.

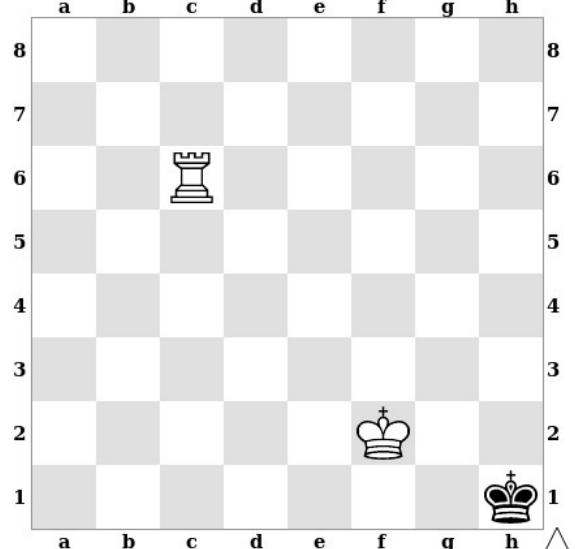
## **Conclusion**

The king and rook checkmate is much less important than the king and queen checkmate, simply because it comes up less often. However, it is still simple enough to be worth mastering. The new challenge in this endgame is the need to understand zugzwang, which is a concept worth learning, given its frequent occurrence throughout all chess endgames. Much like the king and queen checkmate, the most important thing to do with the king and rook mate is to practice it until it seems easy.

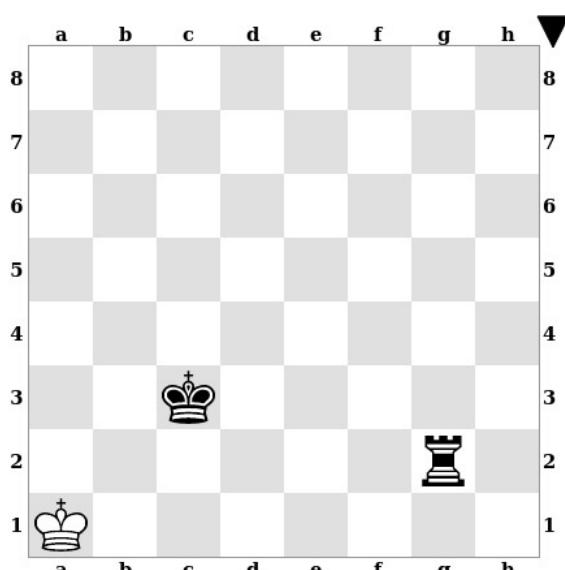
## Exercises



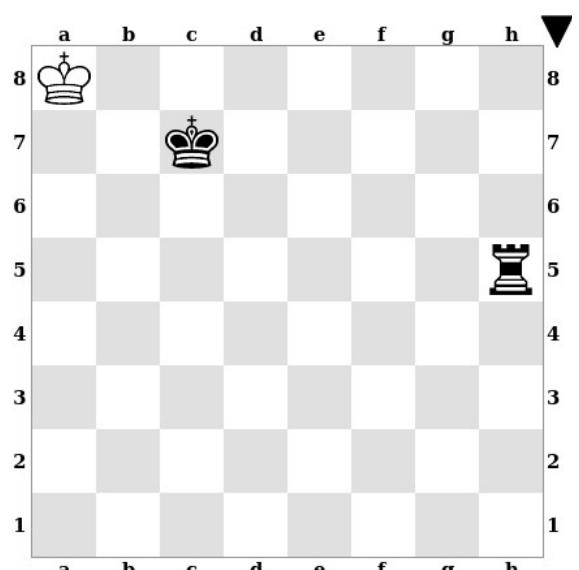
E3.01 (B)



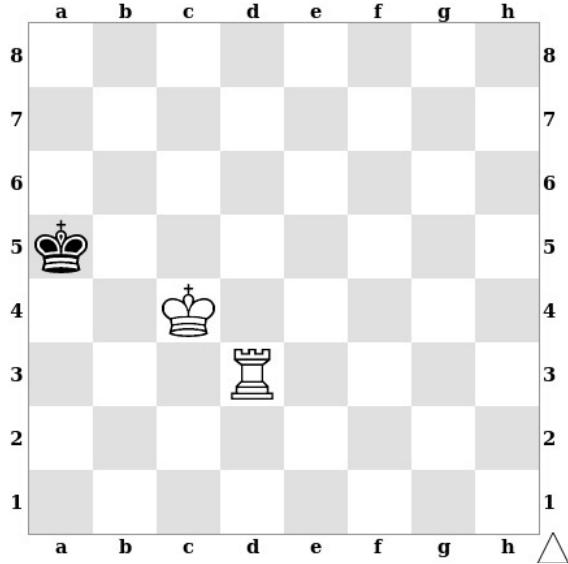
E3.02 (W)



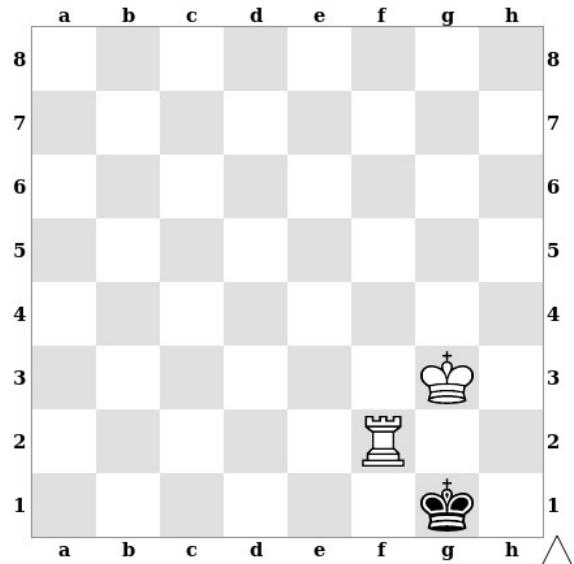
E3.03 (B)



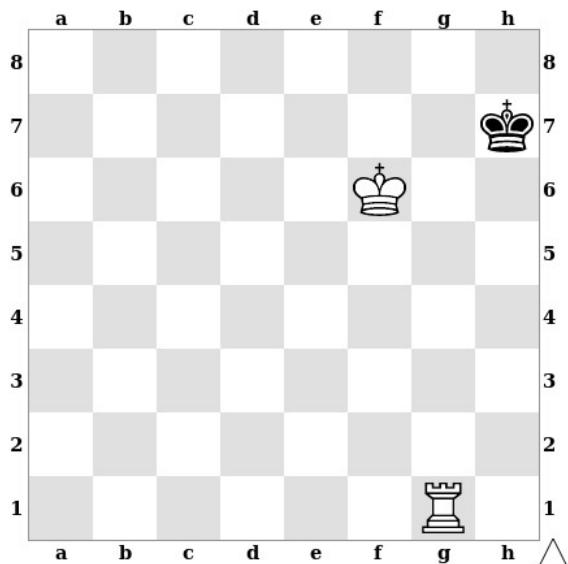
E3.04 (B)



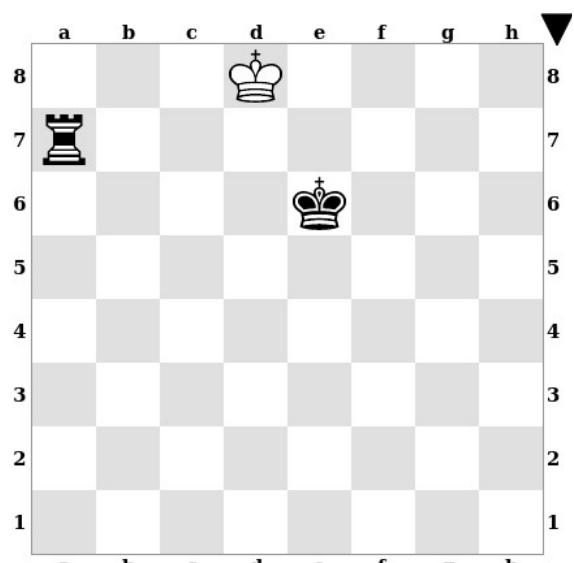
E3.05 (W)



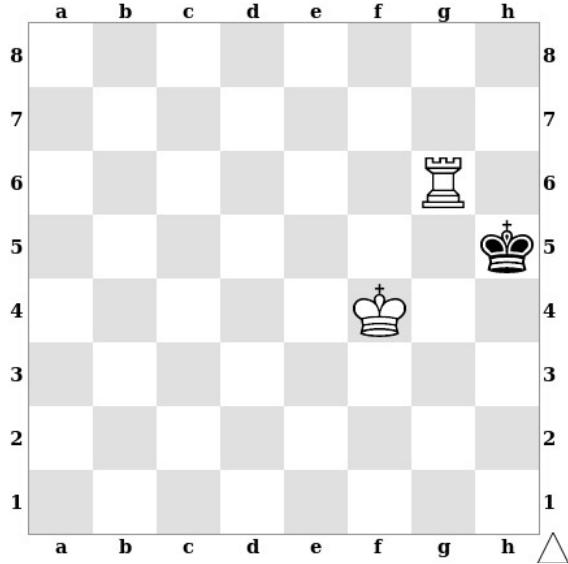
E3.06 (W)



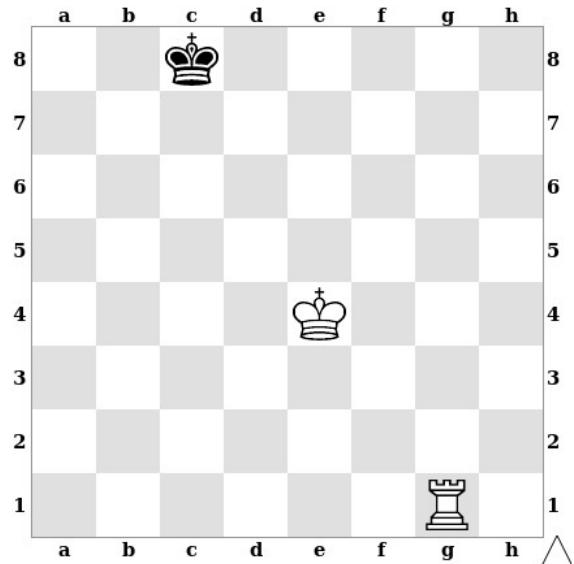
E3.07 (W)



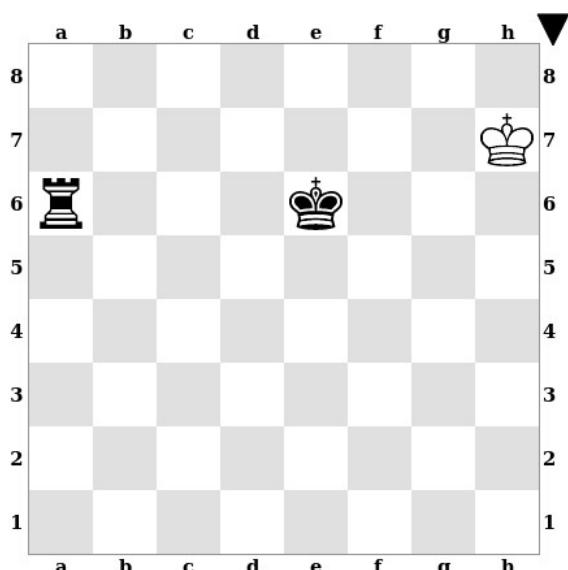
E3.08 (B)



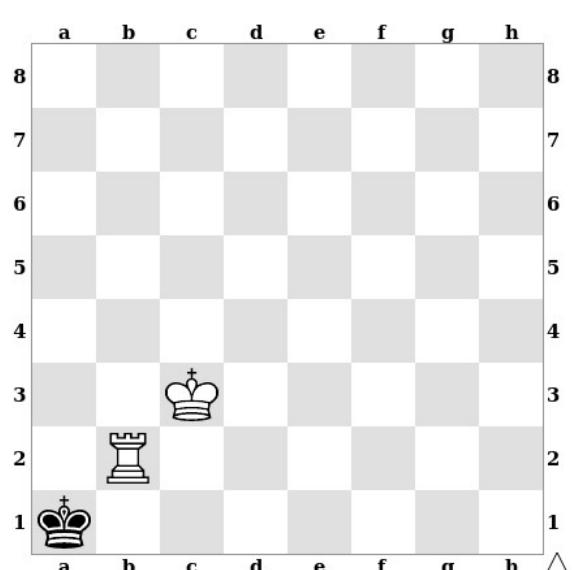
E3.09 (W)



E3.10 (W)



E3.11 (B)



E3.12 (W)

## Solutions

### E3.01

**1...Rg1#**

### E3.02

**1.Rh6#**

### E3.03

**1...Kb3 2.Kb1 Rg1#**

### E3.04

**1...Ra5#**

### E3.05

**1.Rd6 Ka4 2.Ra6#**

### E3.06

**1.Rf3** (or another rook move on the f-file) **1...Kh1 2.Rf1#**

### E3.07

White has many ways to mate in three here, for example **1.Rg2 Kh8 2.Kf7 Kh7 3.Rh2#** or **1.Rh1+ Kg8 2.Rh2 Kf8 3.Rh8#**. However, **1.Kf7 Kh6** is much slower.

### E3.08

**1...Rh7** (or some other tempo move that puts the rook on the other side of the kings. Slower is **1...Rb7 2.Kc8**) **2.Kc8 Kd6 3.Kb8 Kc6 4.Ka8 Kb6 5.Kb8 Rh8#**

### E3.09

**1.Rf6** (or another rook move on the sixth rank) **1...Kh4 2.Rh6#**

### E3.10

**1.Rg7** trapping the king on the edge of the board. The game might end **1...Kd8 2.Ke5 Ke8 3.Ke6 Kd8 4.Rh7 Kc8 5.Kd6 Kb8 6.Kc6 Ka8 7.Kb6 Kb8 8.Rh8#**

**E3.11**

**1...Kf7 2.Kh8 Rh6#**

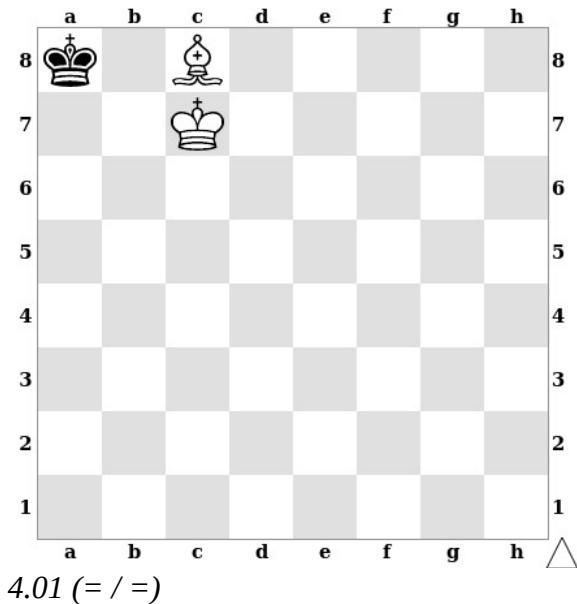
**E3.12**

**1.Rb3** (or 1.Rc2, but not a king move which would create stalemate) **1...Ka2 2.Kc2 Ka1 3.Ra3#**

## 4 – Other Pawnless Endings

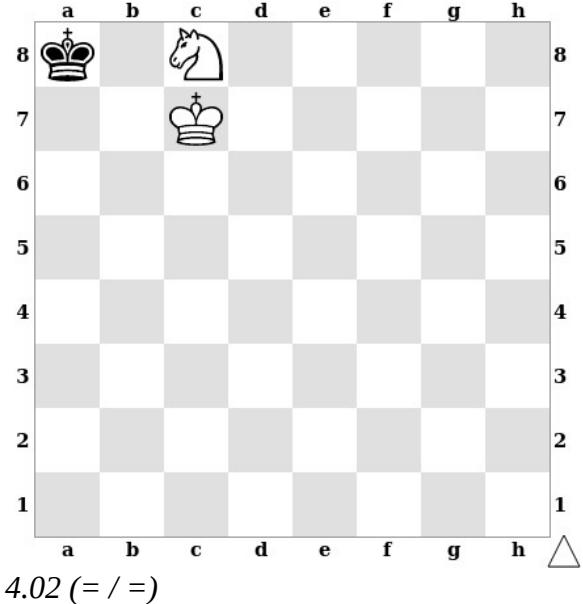
### Sufficient Material to Checkmate

It is possible to checkmate a lone king without a queen or rook; however, only certain combinations of minor pieces can do this. Minor piece checkmates can be quite tricky and come up in practical play only infrequently. For these reasons we will only focus here on learning which combinations of pieces can and cannot mate, rather than learning the actual checkmating techniques themselves.



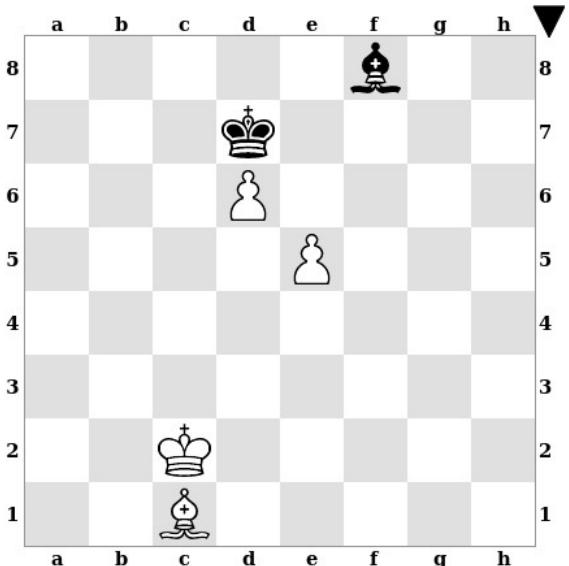
4.01 (= / =)

- A bishop and king cannot checkmate a lone king.  
Not only can checkmate not be forced, it is not possible to checkmate at all, even if the weaker side cooperates and plays the worst moves available.  
In this position White has progressed as far as possible toward mate: he has driven Black's king not only to the edge of the board, but to a corner and has closed in with his king and bishop. Still, 1.Bb7+ Ka7 is not checkmate. It would not change anything for White's king to be on b6; 1.Bb7+ Kb8 would not be checkmate either.



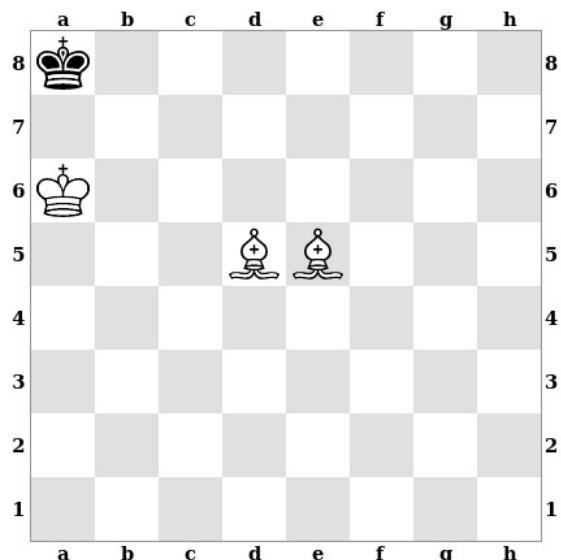
4.02 (= / =)

Similarly, a knight and king cannot checkmate at all against a lone king. Even with poor play from the lone king, checkmate simply isn't possible.

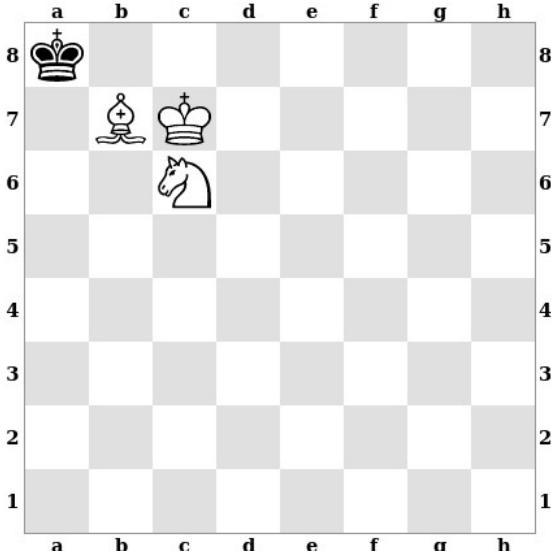


4.03 (+/-/=)

This knowledge can be put to practical use in important ways. For example, Black is two pawns down here and might easily lose. However, he has one way to draw the game: **1...Bxd6 2.exd6 Kxd6**. By sacrificing his bishop he eliminates White's last remaining pawns, leaving White with a useless extra bishop that cannot mate.



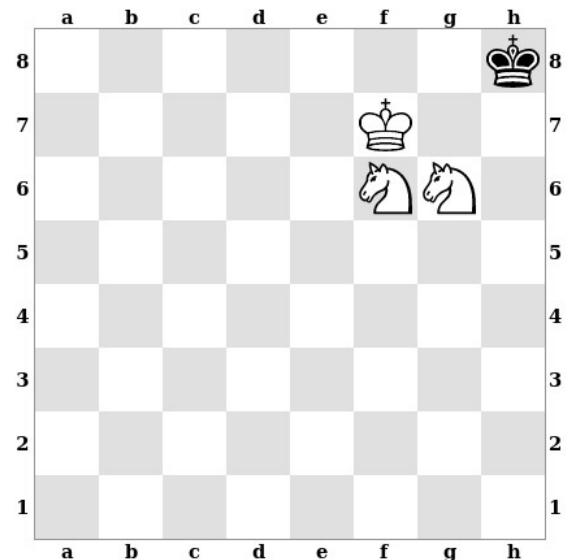
4.04



4.05

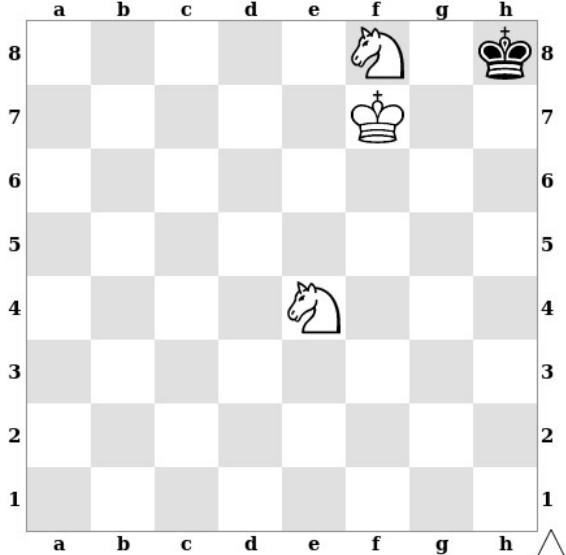
A bishop, knight, and king can also force checkmate against a lone king. This position shows a typical checkmate. To force mate, the weaker side's king normally needs to be forced into one of the two corners of the board that the bishop can attack. Here White has a light-squared bishop, so Black's king needs to be in the a8 corner or the h1 corner for mate to be forced.

The bishop and knight mate is notoriously difficult to learn and execute, with grandmasters having sometimes failed to perform it in serious games.



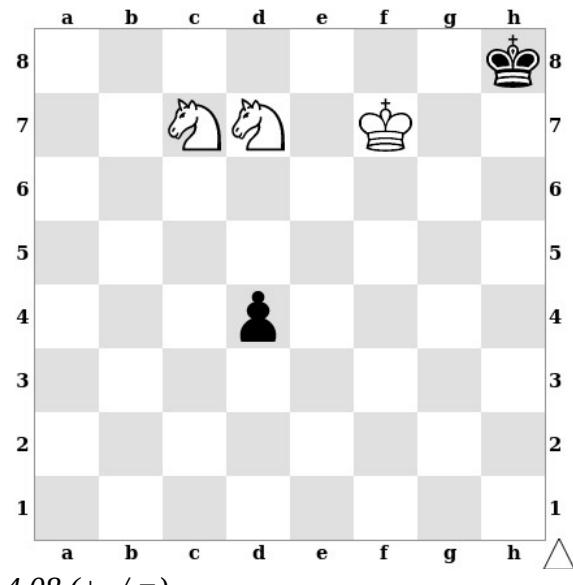
4.06

Two knights can achieve checkmate against a lone king. This position shows one possible checkmate.



4.07 (= / =)

However, while two knights can checkmate, they cannot actually force mate from most starting positions. In other words, the weaker side usually needs to make a mistake for checkmate to be possible. This position is an example of why mate can't be forced. First, 1.Ng6+ Kh7 2.Nf6+ Kh6 just drives Black's king away from the corner. White would like to play 1.Nf6 and 2.Ng6#; the problem is that 1.Nf6 creates an immediate stalemate.

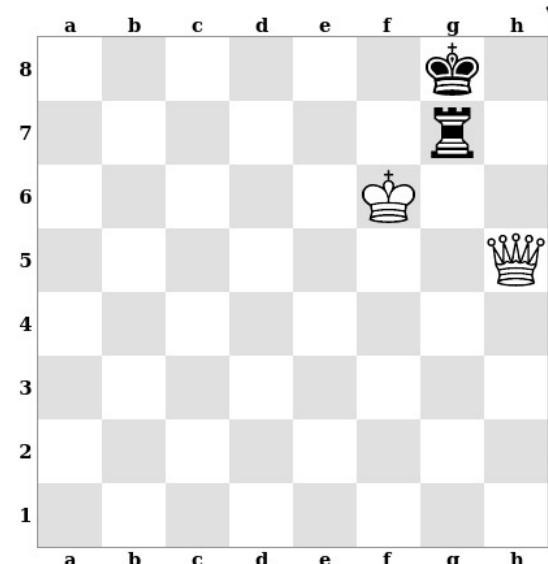


4.08 (+- / =)

Strangely, if the weaker side has a pawn this can cause him to lose. Without the black pawn on the board this position would be drawn. A move like 1.Nf8, for example, would be stalemate. However, because Black's pawn gives him available moves, White can trap the king without creating stalemate: **1.Nf8 d3 2.Ne8 d2 3.Nf6 d1=Q 4.Ng6#**. White allows Black to get a queen but checkmates him before the queen can cause any trouble.

## Pieces in Combat

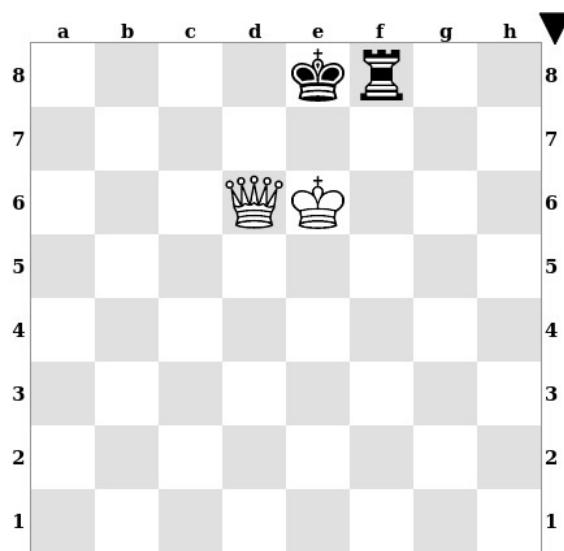
We will now look at a few important pawnless endgames in which both sides have pieces other than the king. These endgames are often quite tricky, so we will mostly be focused on summarizing the expected outcomes, rather than fully learning how to play the positions.



4.09 (+- / +-)

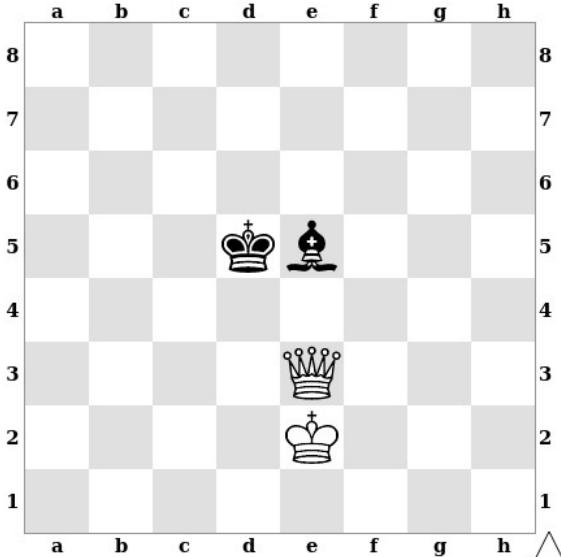
The endgame queen vs. rook is winning for the queen as long as the rook doesn't have an immediate trick to draw. However, the process of winning against good defense turns out to be extremely complicated. The good news is that most of your opponents won't play these positions perfectly either, so you will normally be able to win with the queen in an actual game.

This position gives a small taste of what should eventually go wrong for the side with the rook. Black is in zugzwang and is forced to move his rook somewhere (since 1...Kf8 2.Qh6 +- wins the rook). Every rook move allows some series of moves for the queen that win. For example, 1...Rg1 2.Qd5+ Kh7 3.Qe4+ Kh8 4.Qa8+ Kh7 (4...Rg8 5.Qh1#) 5.Qa7+ +-.



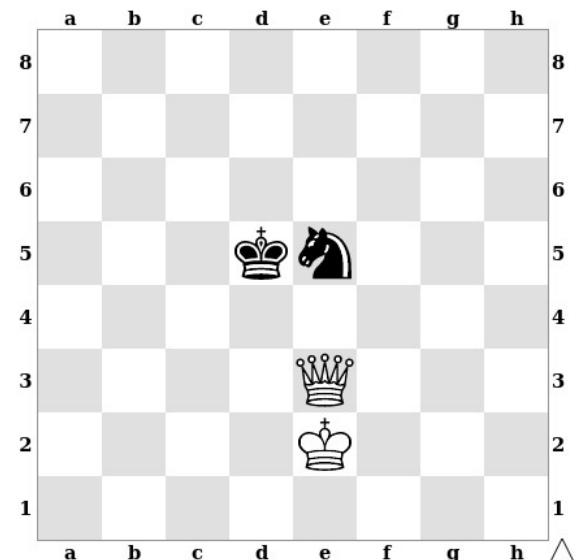
4.10 (+- / =)

If the attacker brings his pieces in carelessly it sometimes opens up stalemate tricks for the defender. Here Black can force a draw immediately with 1...Rf6+ =, leading to stalemate or a piece trade.



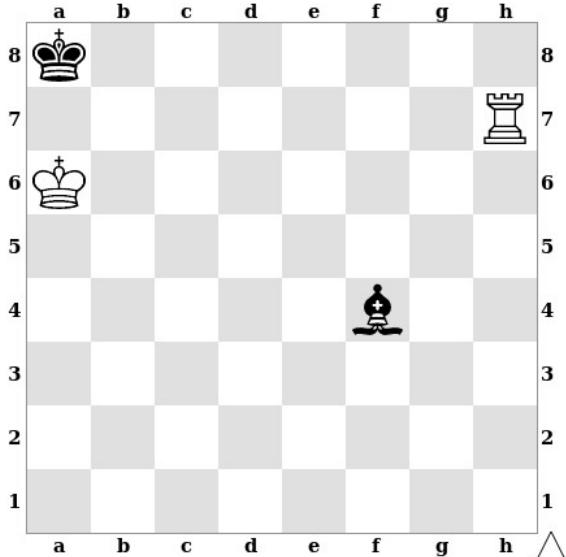
4.11 (+- / +-)

A queen wins easily against a bishop. An example mating sequence from this position goes **1.Qd3+ Ke6 2.Kf3 Bd6 3.Ke4 Ke7 4.Qd5 Bc7 5.Kf5 Bd8 6.Qe6+ Kg8 7.Kg6 Bh4 8.Qf7#**. Notice how White's pieces approach on light squares – the opposite color squares to those controlled by Black's bishop.



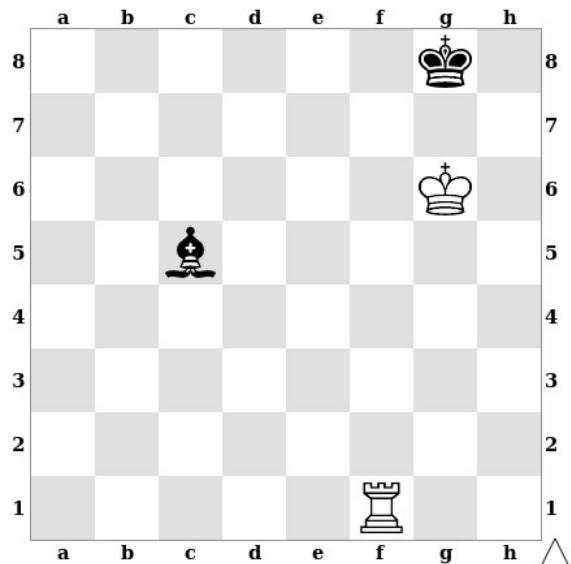
4.12 (+- / +-)

A queen also wins against a knight, though the process is a bit trickier than against the bishop. The stronger side should focus on pushing the defending king back to the edge of the board, limiting the knight's movement when possible, and, of course, not getting forked. A winning sequence from here might run **1.Qf4** (controlling the fourth rank and avoiding traps like **1.Kd2? Nc4+ =**) **1...Nc4 2.Kd3 Ne5+ 3.Kc3 Nc6 4.Qc4+ Kd6 5.Qb5 Ne5 6.Kd4 Nc6+ 7.Ke4 Ne7 8.Qe5+ Kd7 9.Qf6 Nc6 10.Kd5 Ne7+ 11.Kc5 Nc8 12.Qc6+ Kd8 13.Qb7 Ne7 14.Kd6 Nc8+ 15.Ke6 Nb6 16.Qxb6+ +-.**



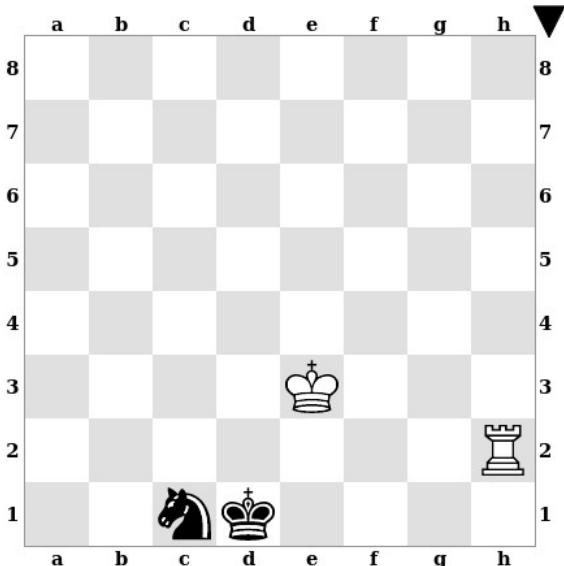
4.13 (= / =)

The endgame rook versus bishop is drawn from most starting positions. One useful tip for the defender is that if his king must be in a corner, it is best to be in a corner of the opposite color to the bishop. Here Black has a dark-square bishop and has correctly put his king in a light-square corner. White can play the scary looking **1.Rh8+**, but after **1...Bb8** Black is closer to being stalemated than checkmated. After **2.Rh7 Bf4** **3.Rf7 Bg3** **4.Rg7 Bf4 =**, White isn't making any progress.



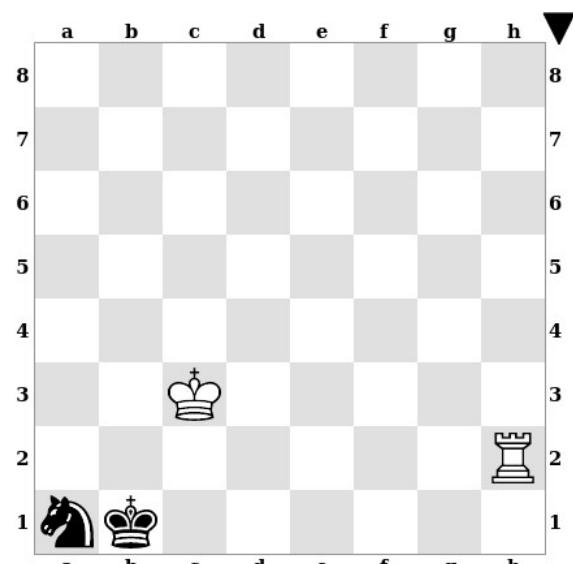
4.14 (+- / +-)

In some cases the rook can win, especially if the defending king is in the wrong corner. Here White wins with **1.Rc1 Bd6** **2.Rc8+ Bf8** **3.Rb8** ( zugzwang) **3...Kh8** **4.Rxf8#**.



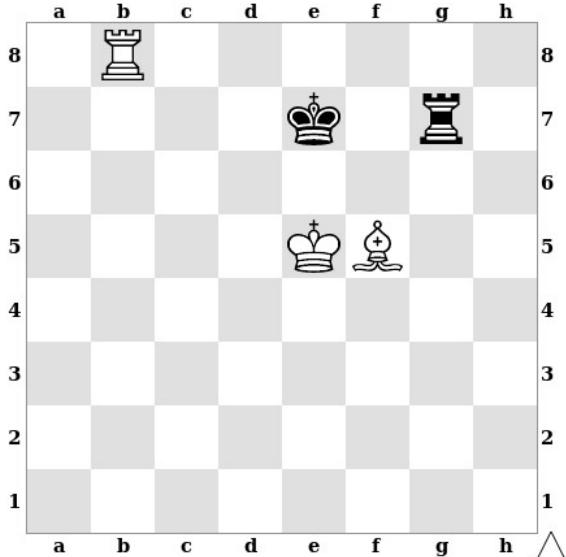
4.15 (= / =)

The endgame rook versus knight is also normally drawn. The most important goal for the defender is to keep the king and knight close together so that they can defend each other. Even having his pieces on the edge of the board is not usually a problem for the defender. Here Black draws after 1...Nb3 2.Rb2 Nc1 (2...Nc5? Is a typical mistake, allowing the king and knight to be separated. White now wins with 3.Rb6 +-, though the process is a bit tricky.) 3.Rd2+ Ke1 4.Rh2 Kd1 =.



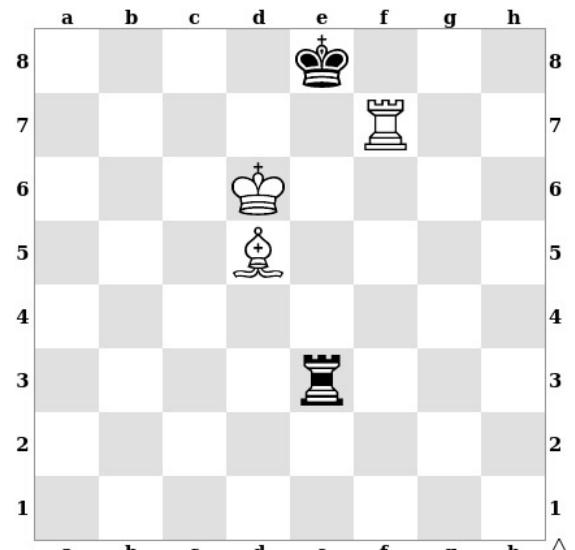
4.16 (+- / +-)

While the edge of the board is not itself dangerous, the knight gets into trouble in the corners of the board. In this position every move for Black immediately loses the knight or allows checkmate.



4.17 (= / =)

The endgame rook and bishop versus rook is also normally drawn. Both the defense and the typical winning methods are quite difficult. This position is a typical example of a drawn position. Black is using the *second rank defense*, and while his position looks scary he can hold the draw, e.g. 1.Rb7+ Kf8 2.Rb8+ Ke7 3.Be6 Rg5+ 4.Kf4 Rg7 =.



4.18 (+- / +-)

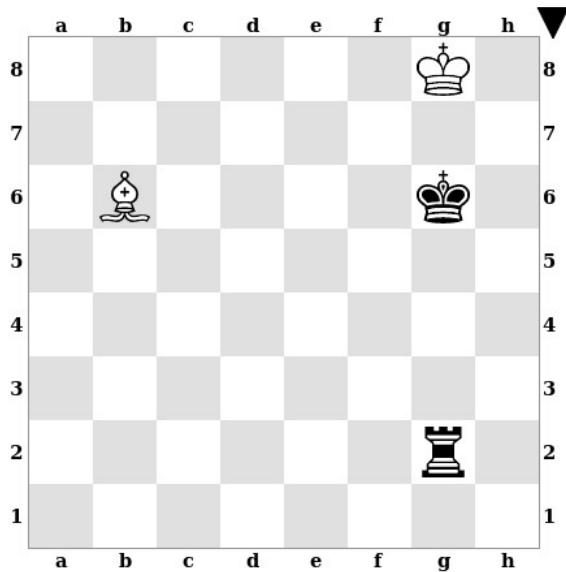
This is a simple example of a winning position. White wins with 1.Rf4 (threatening 2.Bc6+ and 3.Rf8+) 1...Kd8 (1...Rd3 2.Rg4 +-) 2.Be4 (threatening 3.Rf8#) 2...Ke8 3.Bc6+ Kd8 4.Rf8+ Re8 5.Rxe8#.

## Conclusion

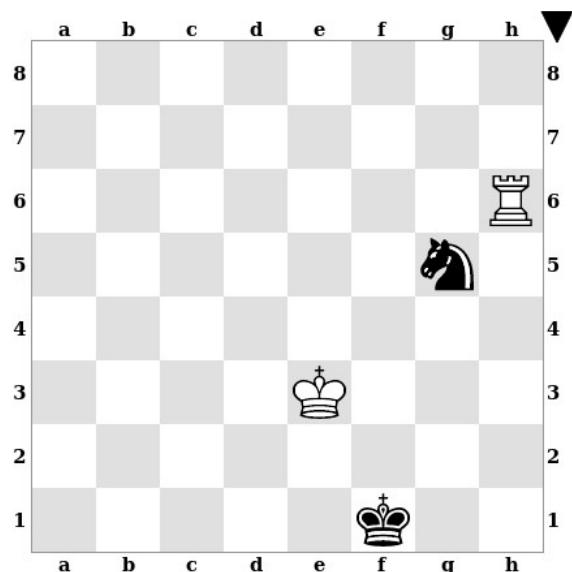
The most important information to remember from this chapter is the standard theoretical evaluation of each type of endgame. Here is a summary of these results. Results with an asterisk next to them have a significant number of exceptions to the normal evaluation.

Type	Result
Q vs. K	Win
R vs. K	Win
B vs. K	Draw (Automatic)
N vs. K	Draw (Automatic)
BB vs. K	Win
BN vs. K	Win
NN vs. K	Draw
Q vs. R	Win
Q vs. B	Win
Q vs. N	Win
R vs. B	*Draw
R vs. N	*Draw
RB vs. R	*Draw

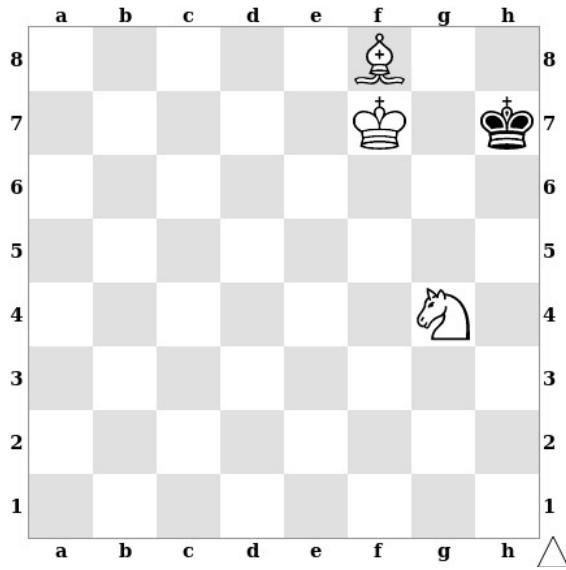
## Exercises



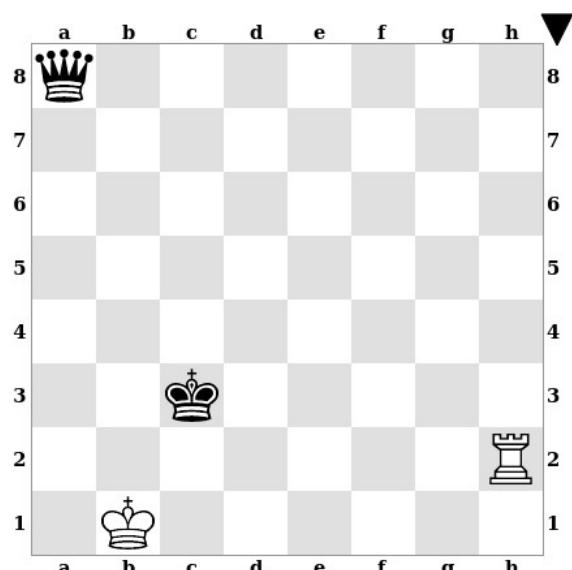
E4.01 (B)



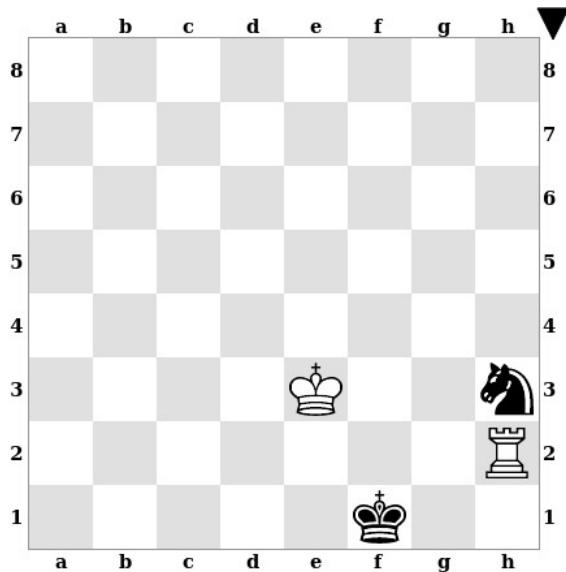
E4.02 (B)



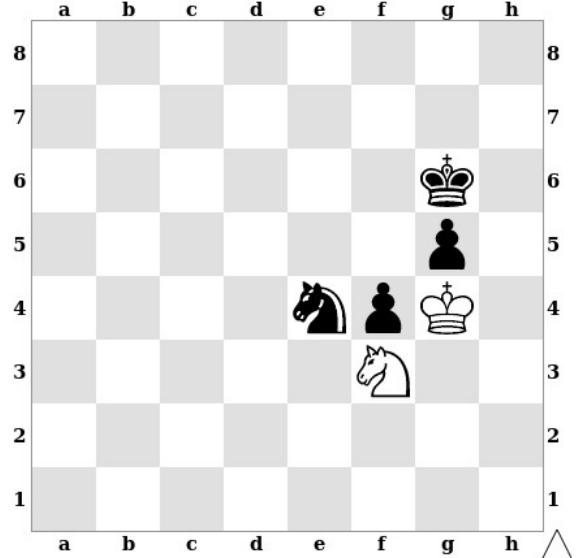
E4.03 (W)



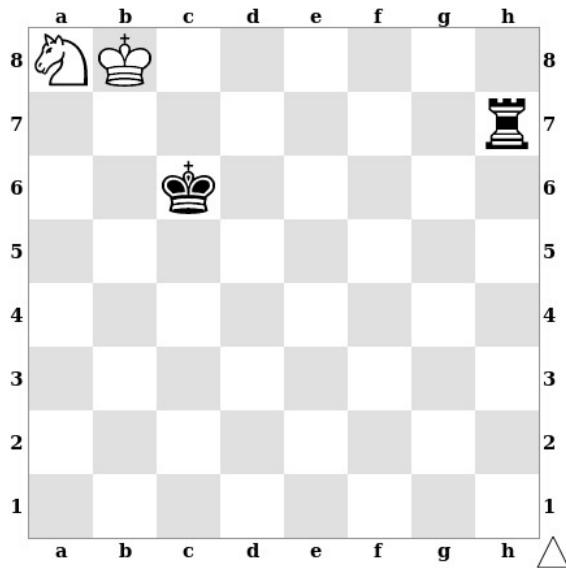
E4.04 (B)



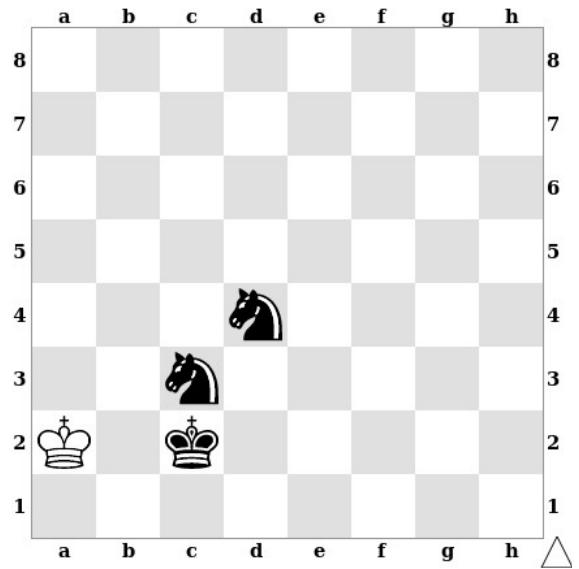
E4.05 (B)



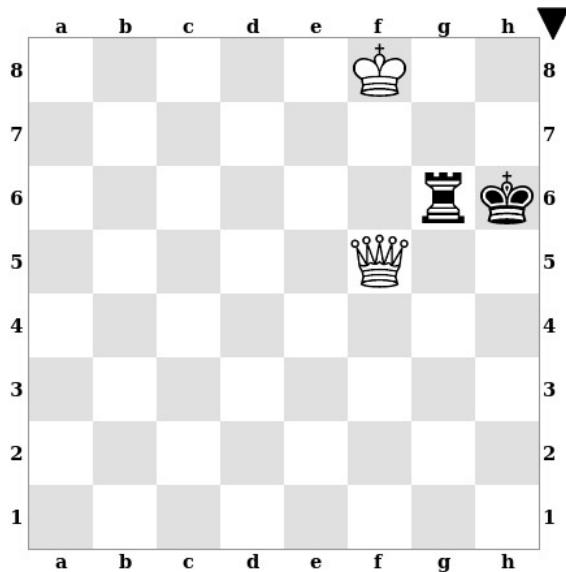
E4.06 (W)



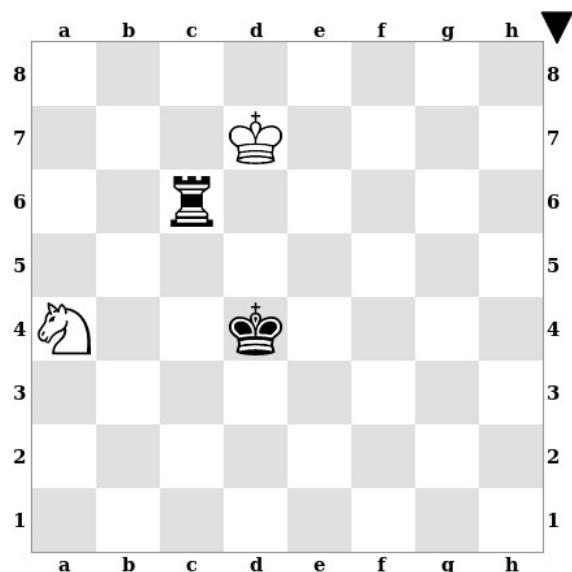
E4.07 (W)



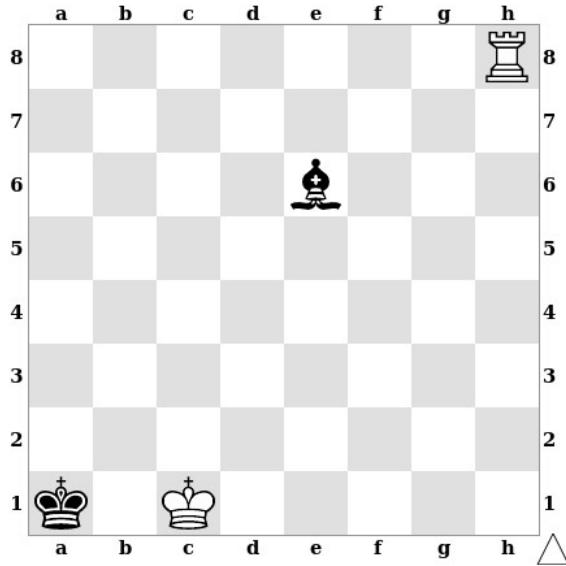
E4.08 (W)



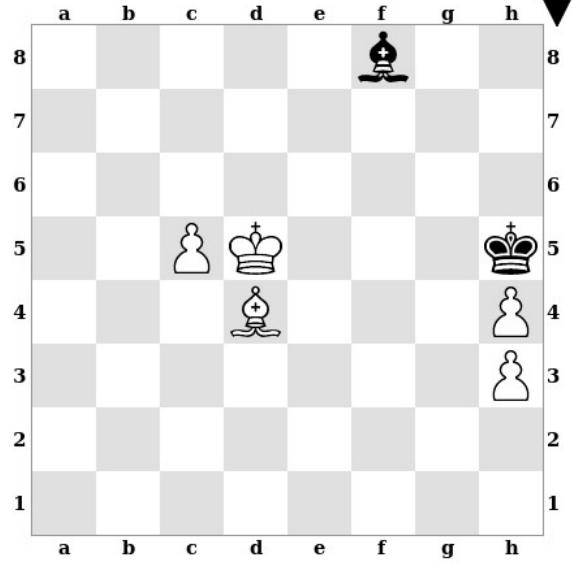
E4.09 (B)



E4.10 (B)



E4.11 (W)



E4.12 (B)

## Solutions

### E4.01

(-+) White's king is in the wrong corner. **1...Rb2 2.Bc7 Rc2 3.Bd6 Rc8+ 4.Bf8 Rb8 5.Kh8 Rxf8#**

### E4.02

(-+) The king and knight have become separated. **1...Kg2** (1...Nf7 2.Rf6+ +-; 1...Ke1 2.Rh1#)  
**2.Rg6** +-

### E4.03

(-+) **1.Nf6+ Kh8 2.Bg7#**

### E4.04

(-+) **1...Qb8+ 2.Ka2** (2.Rb2 Qxb2#) **2...Qxh2+ -+**

### E4.05

(=) **1...Ng1** Black must keep his pieces together. (1...Ng5? 2.Rh6 +-) **2.Rf2+ Ke1 3.Ra2 Kf1 =**

### E4.06

(=) **1.Nxg5** followed by capturing the other pawn, leaving Black without mating material.

### E4.07

(-+) The knight is stuck in the corner. **1.Nc7 Rxc7**

### E4.08

(=) **1.Ka3** (1...Ka1? 2.Nb3#)

### E4.09

(=) **1...Rg8+ 2.Ke7** (2.Kf7 Rf8+ =) **2...Rg7+ 3.Ke6** (3.Kf6 Rf7+ =) **3...Rg6+ 4.Ke5 Rg5 =**

### E4.10

(-+) The king and knight have become separated. **1...Ra6 2.Nb2 Ra2 3.Nd1 Rd2 -+**

### E4.11

(=) Black's king is in the right corner. **1.Ra8+ Ba2 2.Rb8 Be6 3.Rb6 Bf7 =**

#### **E4.12**

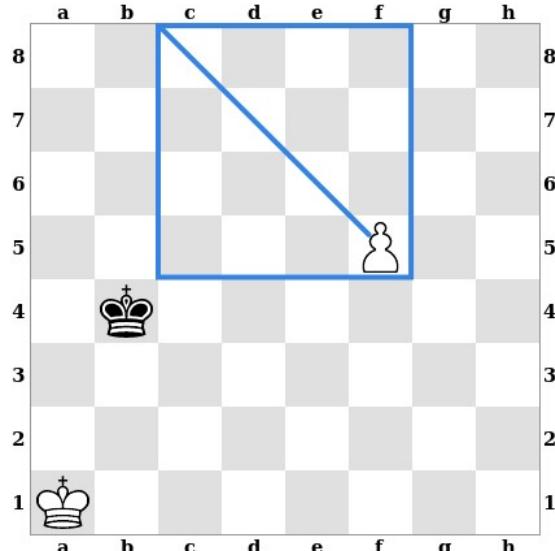
(=) 1...**Bxc5** Any other move allows 2.c6 when White is winning. Now Black can win the last of White's pawns. **2.Bf6** (2.Bxc5 Kxh4 3.Ke4 Kxh3 =) 2...**Bf2** 3.**Ke5** **Bxh4** 4.**Bg7** **Bg3+** 5.**Kf5 Kh3** =, followed by ...Kxh3.

## 5 – King and Pawn Versus King

Single pawn endgames seem as though they should be simple, but they actually have a lot of theory. The weaker side cannot win with just a king, so the question is whether the stronger side can win or the weaker side can hold a draw.

### Square of the Pawn

Some positions feature a race between the pawn and the weaker side's king. These races can be assessed through pure calculation, but there is a nice rule that makes things much easier.

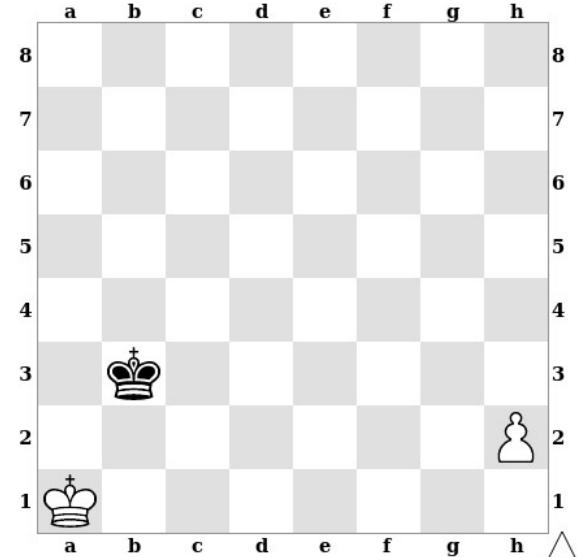


5.01 (+- / =)

**Rule 5.1 (Rule of the Square):** draw a square from the pawn to the promotion rank (you might start with the square's diagonal). If the weaker side's king can enter the square, then it will catch the pawn in time. If the pawn can move with the king already outside the square, then the pawn will safely promote.

If Black moves first, his king can enter the square and so he catches the pawn: 1...Kc5 2.f6 Kd6 3.f7 Ke7 4.f8=Q+ Kxf8 =.

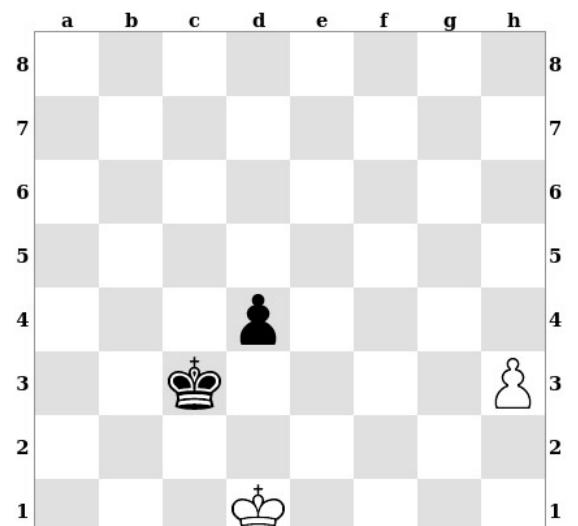
If White moves first, then his pawn gets to move with Black's king already outside the square and so it promotes safely: 1.f6 Kc5 2.f7 Kd6 3.f8=Q+ +-.



5.02 (+- / =)

One exception to the square rule occurs when the pawn is still on the second rank and can move two squares on its first move. Here Black's king appears to be in the pawn's square. However, after **1.h4 +-** Black's king can't step into the pawn's new square and thus can't catch the pawn.

For this reason, when a pawn is still on the second rank we should draw its square as though it were already on the third rank.



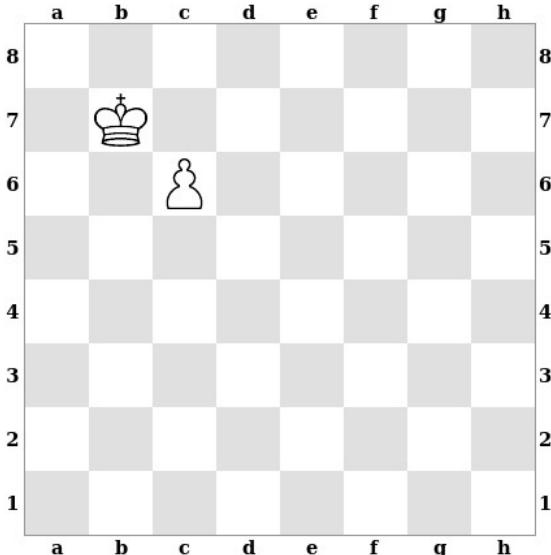
5.03 (+- / =)

The other exception occurs when there are obstacles in the king's path. Here Black's king is inside the square of White's pawn. However, after **1.h4 +-** there is no way for the king to get into the pawn's new square. White wins by simply pushing his pawn. Funnily, if Black were allowed to play **1...Kxd4**, capturing his own pawn, he would be able to draw.

## Key Squares (Concept)

The further theory of these endgames is divided according to whether or not the pawn is a *rook pawn* (a pawn on the a-file or h-file). We will start with non-rook pawns and then move on to rook pawns.

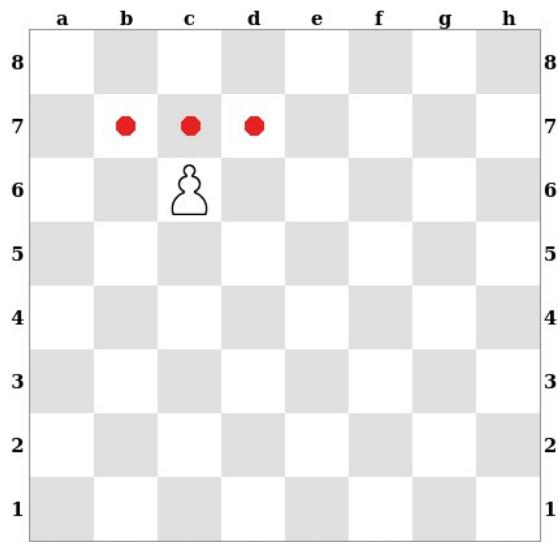
When the pawn cannot promote on its own, the outcome is decided by a struggle between the kings. There are certain squares, determined by the position of the pawn, that are of special importance in this struggle.



5.04

White's king is in an excellent position here. No matter where Black's king is and no matter who moves first, White will promote his pawn by playing 1.c7 and 2.c8=Q. What makes the position of White's king so powerful is that it controls the pawn's promotion square (c8) and the pawn's path to that square.

The b7 square is a *key square* of the c6 pawn because White is always winning if his king occupies b7, regardless of the other factors in the position. The primary issue in most single pawn endings is the fight for key squares.

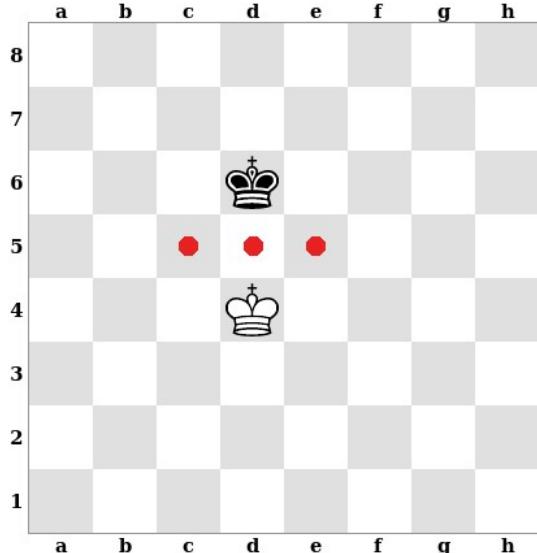


5.05

The d7 square is also a key square for the same reasons as b7: from here White's king controls the queening square and the path to it. If the White king were on c7, he would be able to move to at least one of b7 or d7, which means that c7 is also a key square of this pawn. We will learn more about key squares later, but first we need to learn another new concept.

## Opposition

When the kings fight for control of key squares, the result is often determined by a concept known as opposition. We will begin by looking at the opposition without any pawns on the board. Obviously, the game is drawn without a pawn, so we will imagine that White wins if his king can reach the marked squares in each diagram, and that Black draws if he can stop this.



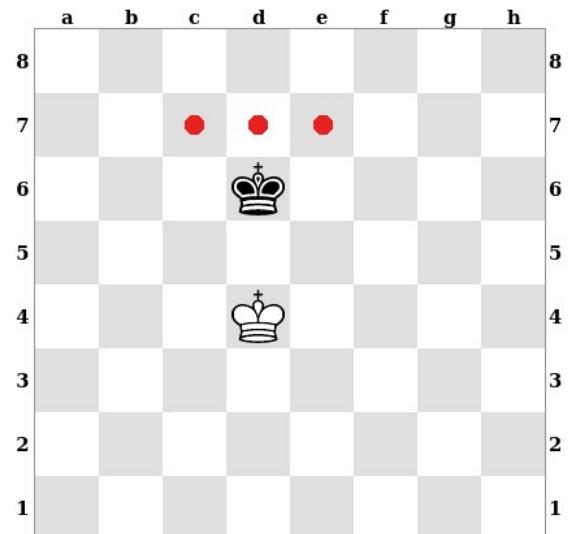
The kings here are in *direct opposition*, since they are on the same file with one square in between them.

If Black moves first, then White reaches one of the target squares: **1...Kc6 2.Ke5 +-** (or **1...Ke6 2.Kc5 +-**).

If White moves first, then Black can keep him off of the fifth rank altogether: **1.Ke4 Ke6 2.Kf4 Kf6 3.Ke4 Ke6 =**. If White tries moving backward, e.g. **1.Ke3 Ke5 =**, then he can never even get back to the fourth rank.

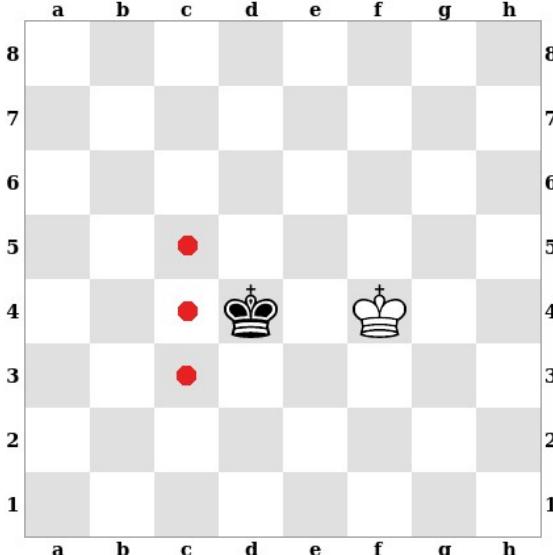
In situations in which it is important at all, opposition creates zugzwang. In other words, it is usually a disadvantage to have to move first when the kings are in opposition and an advantage to move second. For this reason we say that the player who moves second “has the opposition.”

5.06 (= / +-)



Having the target squares behind Black’s king makes no difference. If White has the opposition, he can force his way onto the squares by *outflanking*:

**1...Ke6 2.Kc5** (outflanking and threatening **3.Kc6**)  
**2...Kd7 3.Kd5 Kc6 4.Ke6 Kd8 5.Kd6 Ke8 6.Kc7 +-.**



5.08 (= / +-)

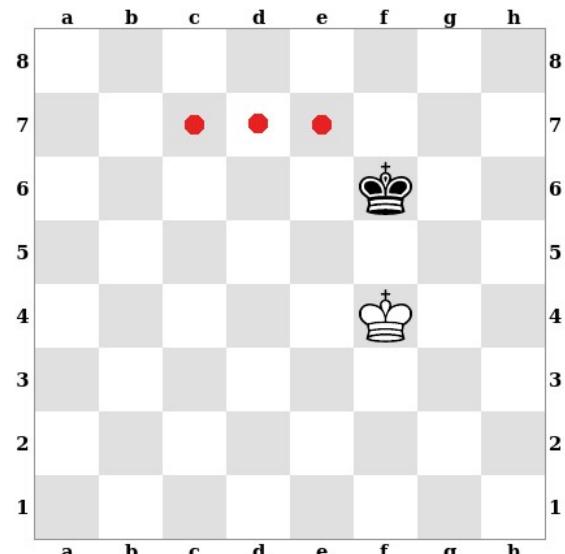
The kings here are in *horizontal opposition*. If Black has the opposition, then White never reaches the e-file: **1.Kf5 Kd5 2.Kf4 Kd4 =**.

If White has the opposition, he reaches the target squares: **1...Kd5 2.Ke3 (outflanking) Kc4 3.Ke4 Kc3 4.Kd5 Kb4 5.Kd4 Kb5 6.Kc3 +-.**

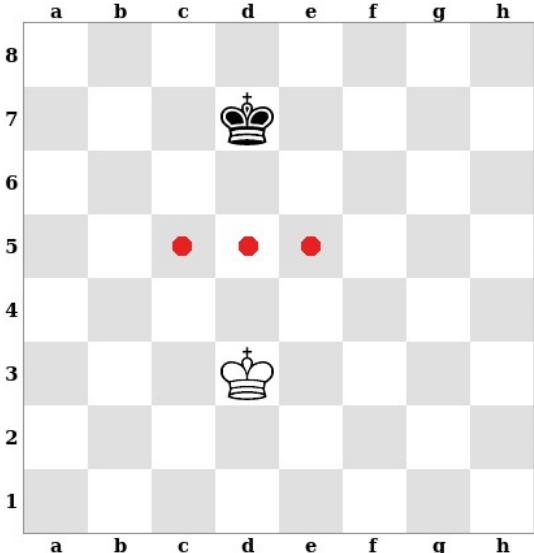
Outflanking allows the defender to take horizontal opposition, so the attacker should only outflank when his king is already lined up with a target square.

In this position, after **1...Ke6**, White should not outflank right away with **2.Kg5?**, allowing **2...Ke5 =**. Nor should he outflank after **2.Ke4 Kd6** with **3.Kf5?**, allowing **3...Kd5 =**.

Instead, he wins by going all the way to the d-file with opposition and then outflanking: **3.Kd4 Kc6 4.Ke5**. Now **4...Kc5** is pointless because of **5.Ke6** and **6.Ke7**. After **4...Kd7 5.Kd5 +-**, White will continue outflanking and reach the target squares.



5.09 (= / +-)



5.10 (= / +-)

Here the kings are in very distant horizontal opposition, with five squares between them. However, the ideas are the same as in the previous examples.

If Black has the opposition, then White can never reach the d-file, since this is halfway between the kings: **1.Kb4 Kf4 2.Kc5 Ke5 =**.

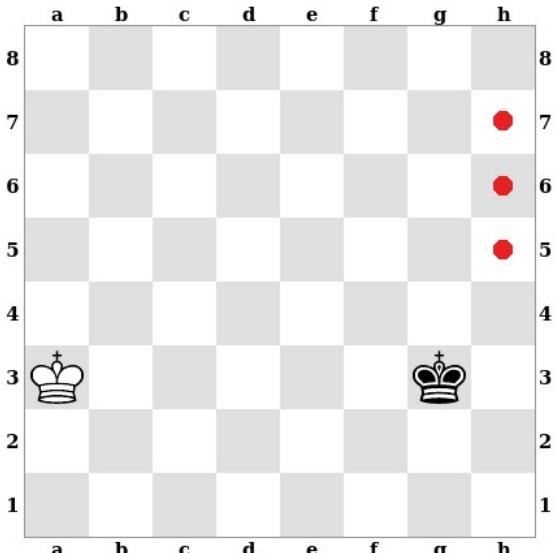
If White has the opposition, he reaches the target squares. First, he needs to take the opposition on the sixth rank: **1...Kg4 2.Ka4 Kg5 3.Ka5 Kg6 4.Ka6**. Now he begins outflanking and re-taking the opposition: **4...Kg7 5.Kb5** (outflanking) **5...Kf6 6.Kb6** (taking opposition; not 6.Kc6? Ke6 =) **6...Kf5 7.Kc7 Ke6 8.Kc6 +-**.

If Black instead began by moving his king forward, then White could move forward while taking opposition: **1...Kf4 2.Kb4 Ke5 3.Kc5 +-**.

In fact, the kings are in opposition whenever they are an odd number of squares apart on the same rank or file. When they are three or five squares apart, we say that they are in *distant opposition*.

If White has the opposition here, he can force his way onto the target squares: **1...Kc7 2.Ke4** (outflanking and threatening 3.Ke5) **2...Kd6 3.Kd4 +-**, with direct opposition.

If Black has the opposition, he can stop White from reaching the fifth rank: **1.Ke3 Ke7** (distant opposition) **2.Kf3 Kf7 3.Ke4 Ke6** = (direct opposition). If White instead plays 1.Ke2, then Black could play 1...Ke6 = or even 1...Ke8 =.

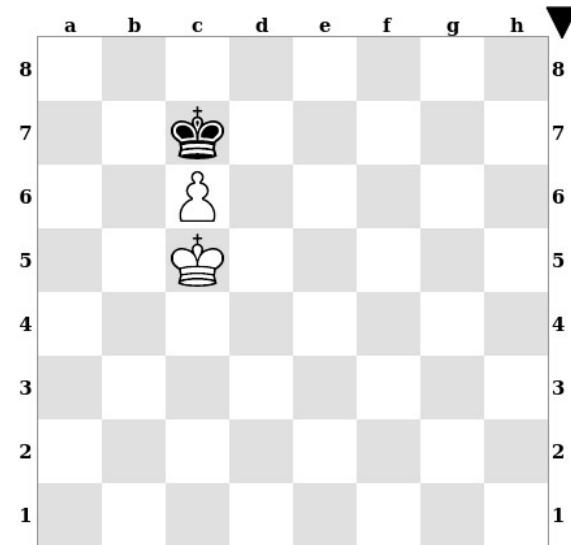


5.11 (= / +-)

Opposition is an interesting concept, but it is important not to overestimate its importance. On its own, opposition does not tell us anything about the evaluation of a position. It is extremely common for the stronger side to be unable to win despite having the opposition, or for the weaker side to lose despite having the opposition. In other words, there is no rule that says that opposition necessarily decides the game. In some cases opposition is decisive, but we will give specific rules about that as we go on.

## The Defensive System

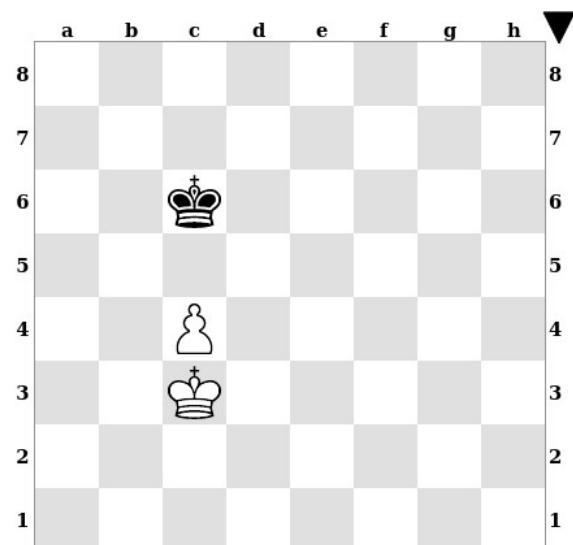
There is a standard sequence of moves that the weaker side uses to draw many positions. This involves moving back and forth on the next two squares in the pawn's path as long as possible, and then taking the opposition when it is no longer possible to stay on those two squares.



5.12 (= / =)

Here Black can draw using the standard defensive system. First, he moves back and forth on the next two squares in the pawn's path (c7 and c8) as long as possible: 1...Kc8 2.Kd5 Kc7 3.Kc5 Kc8 4.Kb6. Now that he is forced to do something else, Black takes the opposition: 4...Kb8 5.c7+. Finally, he moves back in front of the pawn: 5...Kc8 6.Kc6 = stalemate.

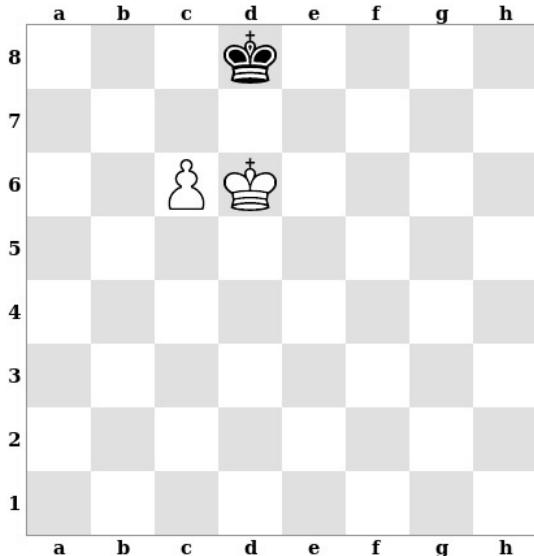
It is quite important that Black not play 1...Kd8? 2.Kd6 +- or 1...Kb8? 2.Kb6 +- . These both lead to losing versions of 5.14.



5.13 (= / =)

Here the pawn is less advanced, but Black can draw in the same way: 1...Kc5 2.Kd3 Kc6 3.Kd4 Kd6 4.c5+ Kc6 5.Kc4 Kc7 6.Kd5 Kd7 7.c6+ Kc7 =, reaching the previous diagram after 8.Kc5.

## Pawn on the Sixth Rank



5.14 (= / +-)

This position doesn't have a name, but it is quite important. White doesn't yet control the promotion square and the result depends on who moves first.

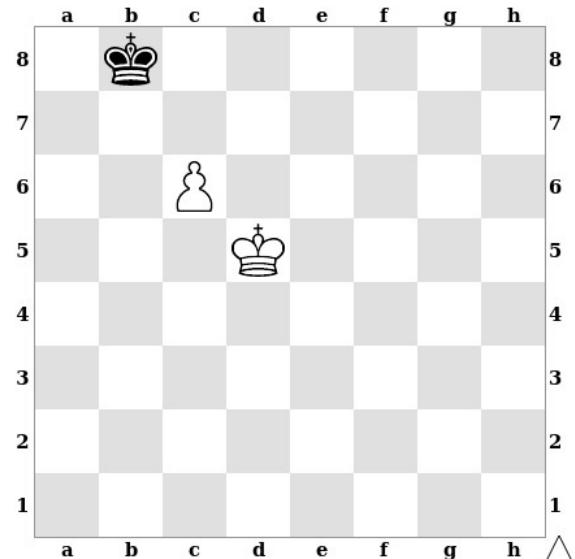
If Black moves first, then he loses: 1...Kc8 2.c7 Kb7 3.Kd7 +-.

If White moves first, then the position is drawn:

1.c7+ Kc8 2.Kc6 =. If White delays pushing his pawn, then Black draws using the standard defensive system:

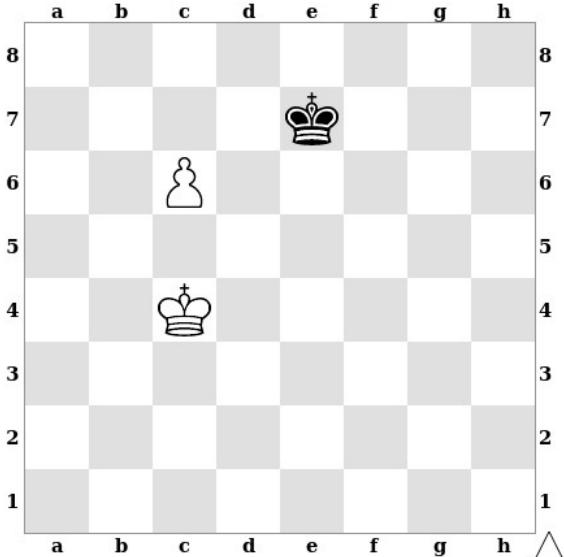
1.Kc5 Kc7 2.Kb5 Kc8 3.Kb6 Kb8 4.c7+ Kc8 5.Kc6 =.

There is a useful trick for remembering the result here. When the pawn moves to the seventh rank, the position is drawn if the pawn gives check and winning if it does not. Of course, this is only true if White doesn't yet control the promotion square.



5.15 (+- / =)

White can also win when the kings are on opposite sides of the pawn, as long as the pawn goes to the seventh rank without check: 1.Kd6 (threatening 2.Kd7) 1...Kc8 2.c7 Kb7 3.Kd7 +-.



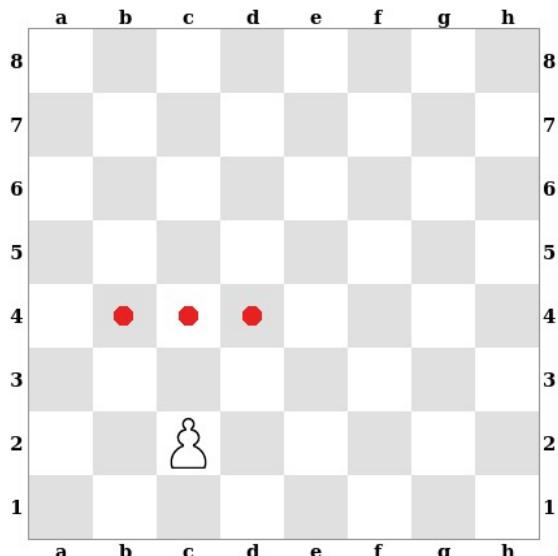
5.16 (+/-/=)

In some cases the stronger side can use a sequence of moves to force his way into a favorable version of  
 5.14: 1.Kd5 Ke8 2.Ke6 (2.Kd6? Kd8 =) 2...Kd8 3.Kd6  
**Kc8 4.c7 Kb7 5.Kd7 +-**.

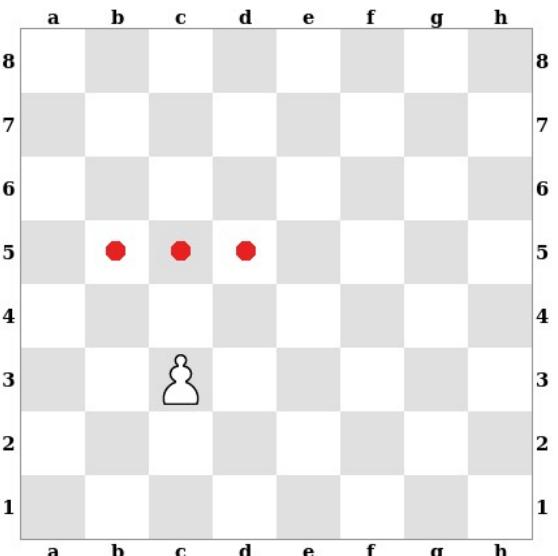
## Key Squares

It turns out that the stronger side's king position may be good enough to guarantee a win long before the king controls the promotion square. As a result, we can identify key squares of a pawn on any rank. White is always winning with his king on these squares, so long as Black can't immediately capture the pawn.

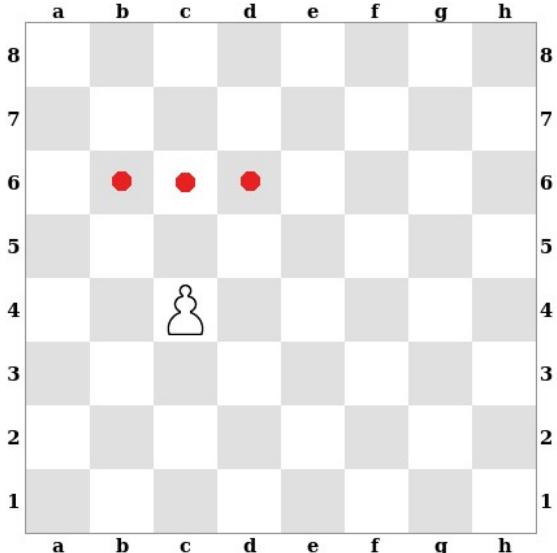
Here are the key squares of a white bishop pawn as it moves up the board.



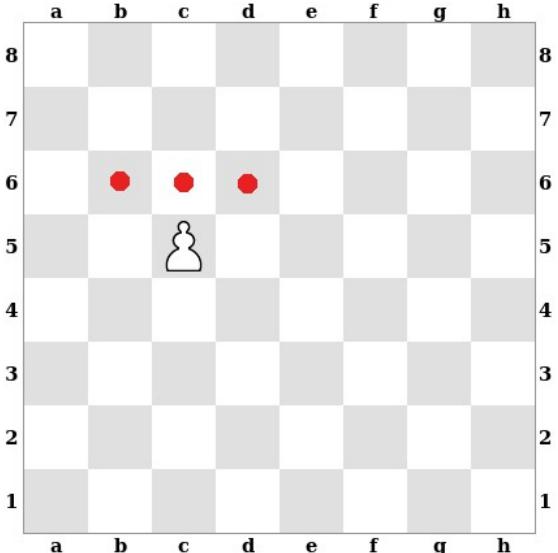
5.17



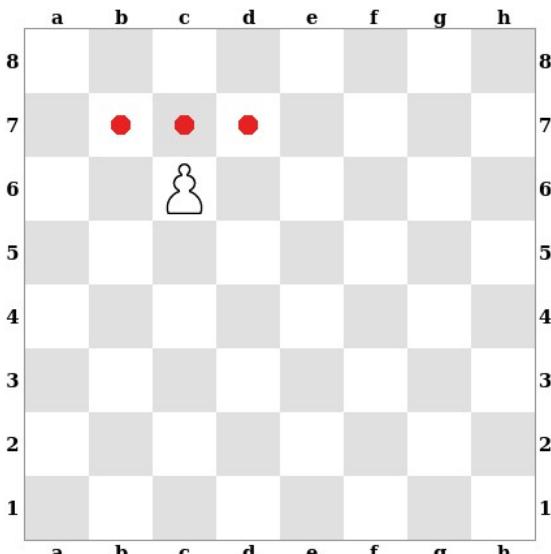
5.18



5.19



5.20

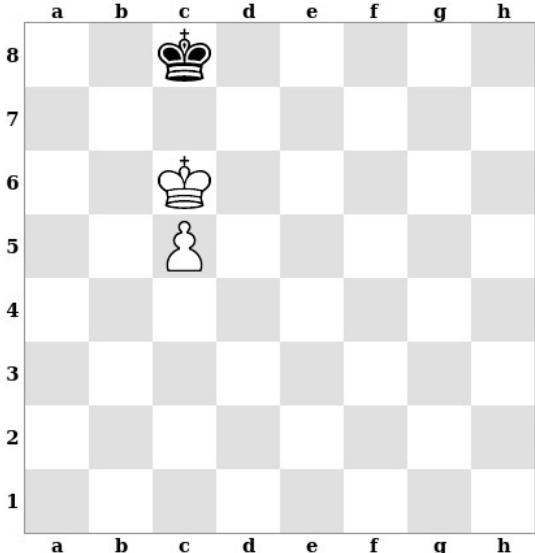


5.21

The key squares of a non-rook pawn are always the same, relative to the position of the pawn. In other words, the key squares of knight pawns, bishop pawns, and center pawns all work the same way.

A helpful observation is that key squares are two ranks ahead of the pawn when it is in its own half of the board and one rank ahead when the pawn is in the opponent's half of the board.

Pushing a pawn forward usually changes its key squares, which can completely change the evaluation of a position. For this reason, the stronger side should generally try to move his king some ways ahead of the pawn before pushing it.



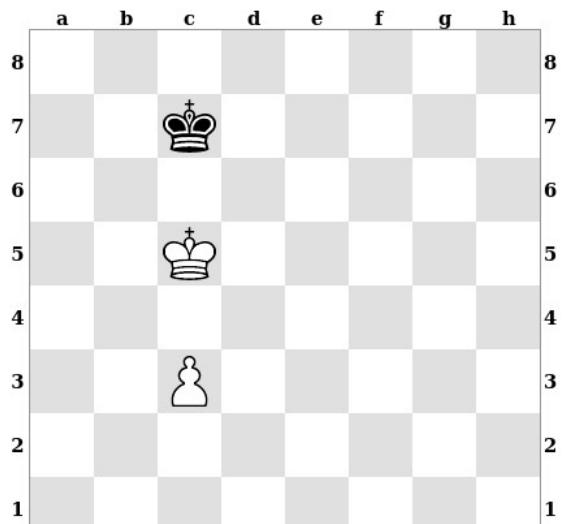
5.22 (+- / +-)

Here White's pawn is on the fifth rank and so the key squares are on the sixth rank. This means that White's king already occupies a key square and so he should win no matter who moves first.

With Black to move first, White wins by using the opposition to move his king further ahead of the pawn: **1...Kb8 2.Kd7 Kb7 3.c6+ Kb8 4.c7+ +-**.

If White moves first, he wins by getting a good version of 5.14: **1.Kb6 Kb8 2.c6 Kc8 3.c7 Kd7 4.Kb7 +-**.

Notice that White wins here whether or not he has the opposition initially. Occupying a key square is enough to guarantee a winning position, regardless of who has the opposition to begin with.



5.23 (+- / +-)

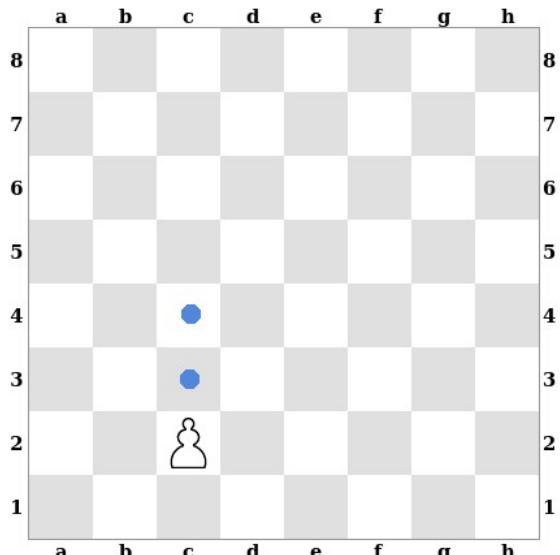
In this position White's pawn is on the third rank, so the key squares are on the fifth rank. Again, White's king already occupies a key square and so he should win in any case.

With Black to move first, White wins by using the opposition to get even further ahead of his pawn before pushing it: **1...Kb7 2.Kd6 Kc8 3.Kc6 Kb8 4.c4 Kc8 5.c5 Kb8 6.Kd7 +-**. If Black plays 2...Kb6, then White wins with **3.c4 Kb7 4.c5 +-**.

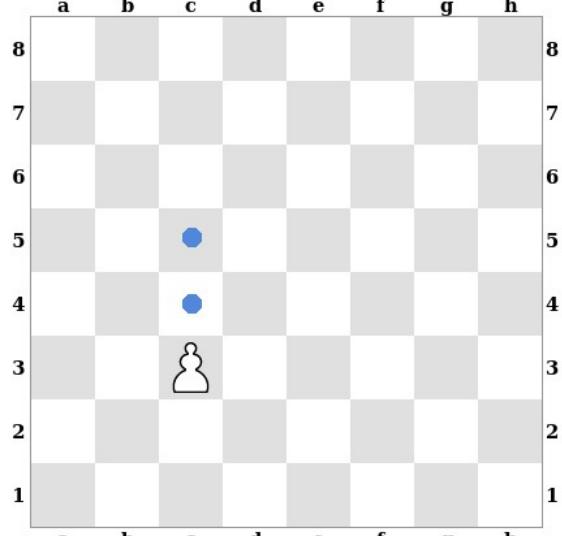
If instead White moves first, then he wins by using a pawn move to gain the opposition. Crucially, this still leaves his king ahead of his pawn: **1.c4 Kb7 2.Kd6 Kb6 3.c5+ Kb7 4.Kd7 +-**.

## Defensive Key Squares

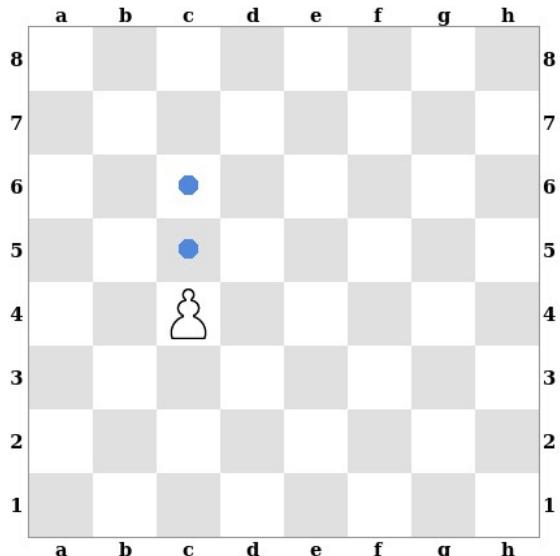
There are also squares that are significant for the weaker side. These squares aren't given a special name in most endgame books, but I find it natural to call them *defensive key squares*. If the weaker side's king can occupy one of these squares, then the position is always a draw, regardless of who has the opposition. Here are the defensive key squares of a bishop pawn as it moves up the board.



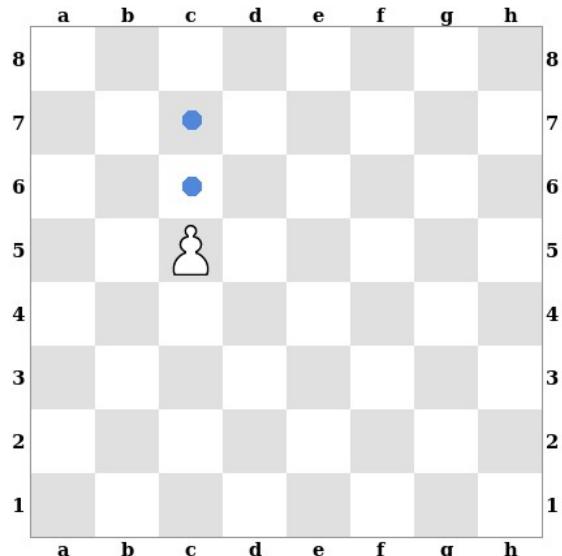
5.24



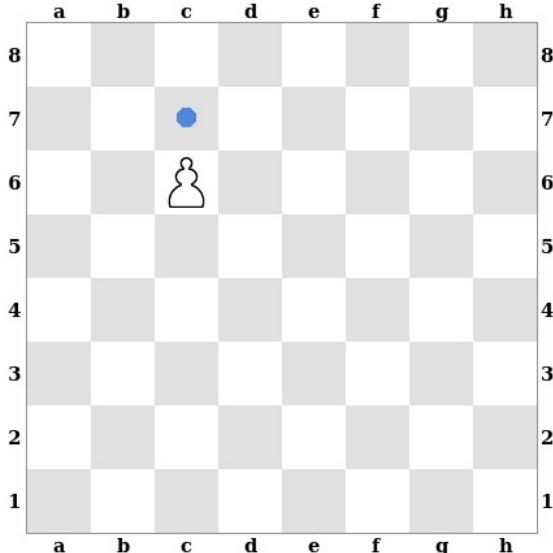
5.25



5.26

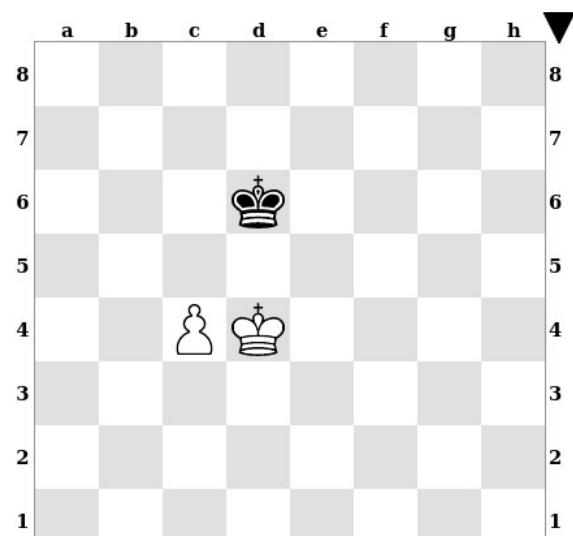


5.27



5.28

Defensive key squares are the same for all non-rook pawns, just like the stronger side's key squares. The pattern for defensive key squares is very simple: they are just the two next squares in the pawn's path, but not the eighth rank square for a sixth rank pawn. The exceptional nature of the eighth rank square is demonstrated by 5.14 with Black to move.



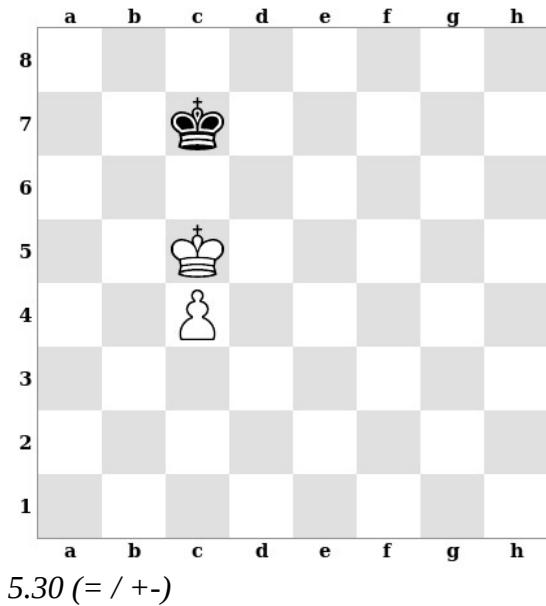
5.29 (= / =)

This position is drawn no matter who moves first because Black can reach the defensive key square c6: 1...Kc6 2.c5 Kc7 3.Kd5 Kd7 4.c6+ Kc7 5.Kc5 Kc8 6.Kd6 Kd8 7.c7+ Kc8 8.Kc6 =

Note that the position was drawn even though White initially had the opposition. Having the opposition generally does not benefit the stronger side when his king is beside or behind his pawn.

## The Fight for Key Squares

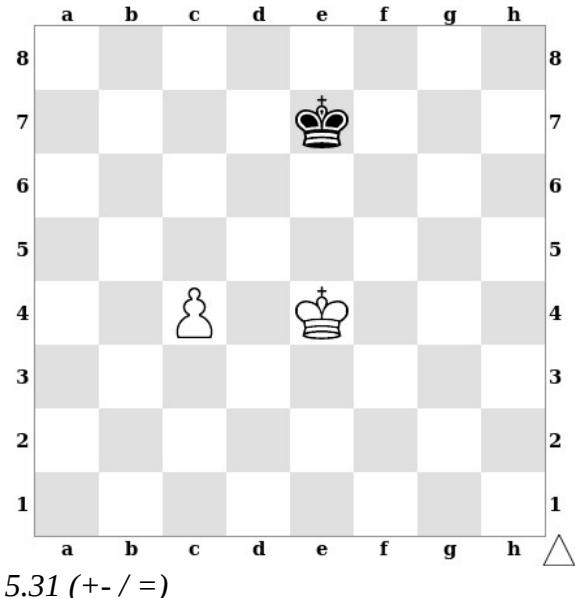
In many positions the two kings are still fighting to reach their respective key squares. We will look at a few representative examples.

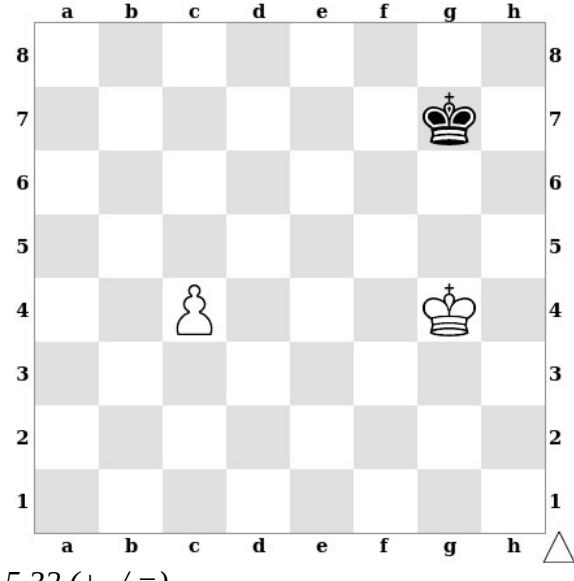


- White's key squares are on b6, c6, and d6, while Black's are on c5 and c6. Neither king has reached a key square, but both kings are on the verge of doing so. The result here depends on who has the opposition.
- With Black to move, White has the opposition and wins: 1...Kd7 2.Kb6 (White reaches a key square) 2...Kc8 3.Kc6 Kb8 4.Kd7 Kb7 5.c5 Kb8 6.c6 +-.
- This demonstrates **rule 5.2:** if the stronger side's king is ahead of his (non-rook) pawn and he has the opposition, then he wins.
- With White to move, Black has the opposition and draws: 1.Kd5 Kd7 2.Kc5 Kc7 3.Kb5 Kb7 4.c5 Kc7 (Black reaches a defensive key square) 5.c6 Kc8 6.Kb6 Kb8 7.c7+ Kc8 =.
- This demonstrates **rule 5.3:** if the stronger side's king is behind his key squares and the weaker side has direct opposition, then the position is drawn.

In this position, White needs to take the opposition to win: 1.Ke5 Kd7 2.Kd5 Kc7 3.Kc5 +-, reaching the previous diagram with Black to move.

Instead 1.Kd5? Kd7 2.Kc5 Kc7 = is drawn, as in the previous diagram with White to move.





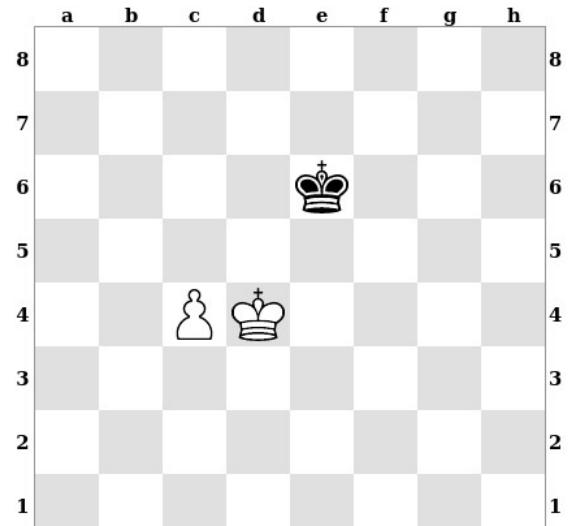
5.32 (+- / =)

It is sometimes necessary to begin using opposition quite some distance from the pawn. White can only win here with **1.Kg5 Kf7 2.Kf5 +-**, eventually reaching 5.30 with the opposition. Instead, **1.Kf5? Kf7 =** would lead to 5.30 with Black having the opposition.

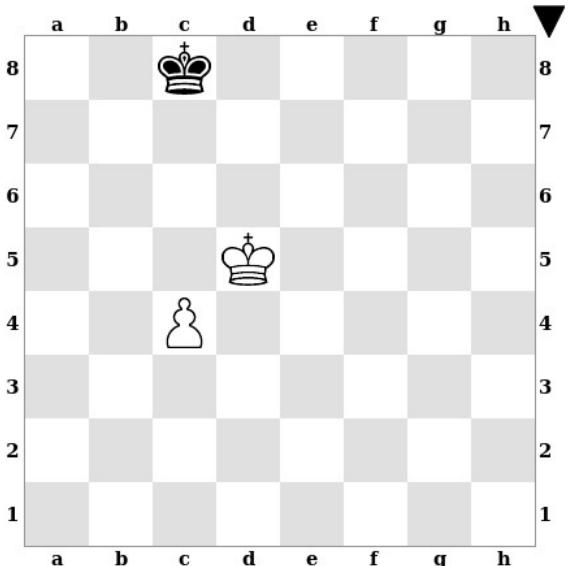
However, taking the opposition is not always the right choice. Remember that the goal is to reach key squares; opposition is just one tool for doing this.

Here **1.Ke4?** **Kd6 2.Kd4 Kc6** = lets Black reach a defensive key square, as in 5.29. As we have seen before, having the opposition is not generally useful for the stronger side when his king is not in front of the pawn.

Instead, White wins with **1.Kc5 Kd7 2.Kb6 +-** reaching a key square of his own and winning as in 5.23.



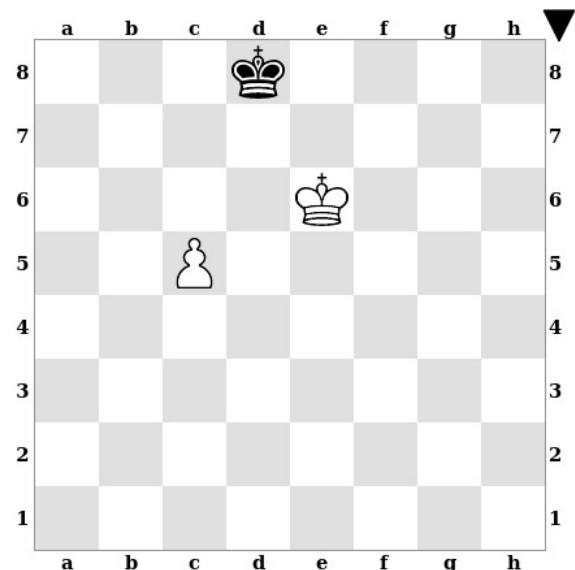
5.33 (+- / =)



5.34 (+- / =)

Here Black needs to take the opposition to draw:  
1...Kd7 2.Kc5 Kc7 =, as in 5.30.

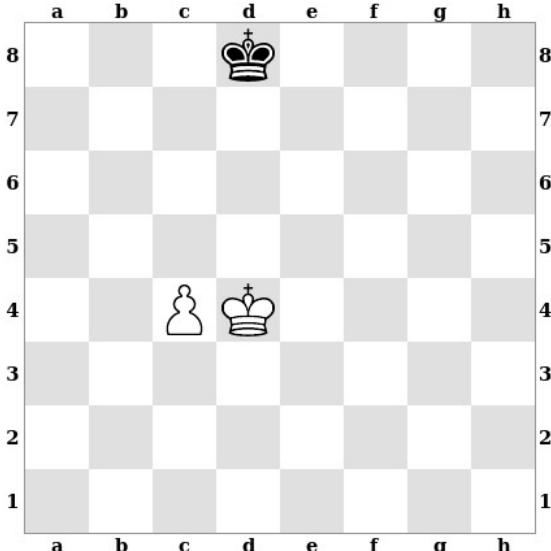
Instead, 1...Kc7? Kc5 +- lets White have the opposition with his king in front of the pawn. Also, 1...Kb7? 2.Kd6 +- lets White reach a key square directly.



5.35 (+- / =)

Here, however, Black would only lose if he were to be distracted by the opposition. After 1...Ke8?

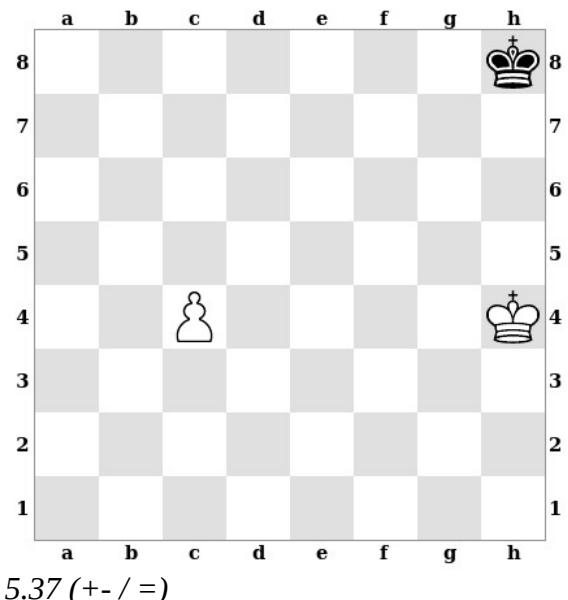
2.Kd6 +- White has reached a key square and wins as in 5.22. Instead, Black draws with 1...Kc7 =, reaching a key square of his own.



5.36 (= / =)

Surprisingly, this position is drawn no matter who moves first. With White to move, Black uses the opposition: **1.Kc5 Kc7 2.Kb5 Kb7 3.c5 Kc7 =**.

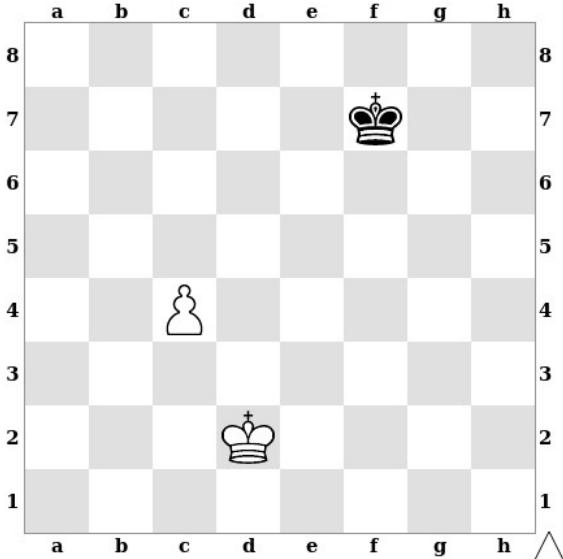
If Black moves first, then White has distant opposition. However, we again see that having the opposition is not useful for the stronger side when his king is not yet ahead of the pawn. Black can draw with **1...Kc8** when White has nothing better than **2.Kc5 Kc7 =**, as in the variation above.



5.37 (+- / =)

If Black moves first here, he draws with **1...Kg8**, which leads to an extended version of the previous position: **2.Kg4** (**2.Kg5 Kg7 =**) **2...Kf8 3.Kf4 Ke8 4.Ke4 Kd8 5.Kd4 Kc8 =**.

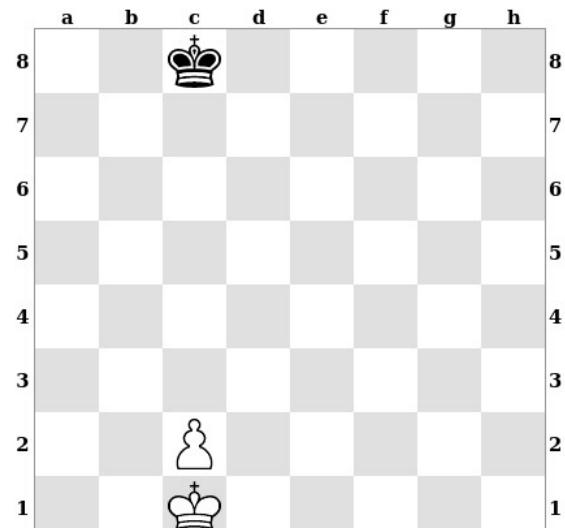
White to move wins because Black's king is outside the pawn's square: **1.c5 Kg7 2.c6 Kf7 3.c7 +-.**



5.38 (+- / =)

Here White wins by heading for the key square b6 using a route that Black can't easily interfere with:  
**1.Kc3 Ke6 2.Kb4 Kd6 3.Kb5 Kc7** (the only move that guards b6) **4.Kc5 Kb7 5.Kd6** +- (White reaches a key square).

The alternative **1.Kd3?** = is an instructive mistake:  
**1...Ke6 2.Ke4 Kd6 3.Kd4 Kc6** =. White takes the opposition in this variation, but his king is not in front of the pawn when does so and he can't reach the key squares.



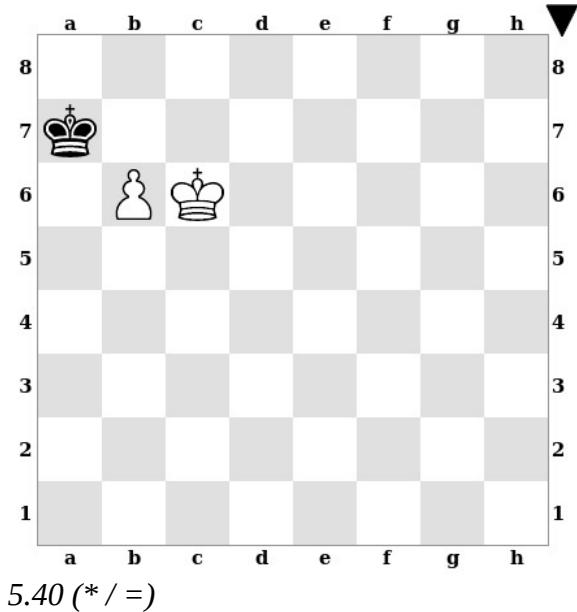
5.39 (+- / =)

If White moves first here, he reaches a key square easily: **1.Kd2 Kc7 2.Kd3 Kc6 3.Kc4** +- (or even **3.Kd4** +-). Of course it would be a huge mistake to play **1.c4?** =, when the key squares have all moved forward two ranks and White has no chance of reaching his squares first.

Black to move can draw by moving closer while taking the opposition when possible: **1...Kc7 2.Kd2 Kd6 3.Kd3 Kd5 4.c3 Kc5** =.

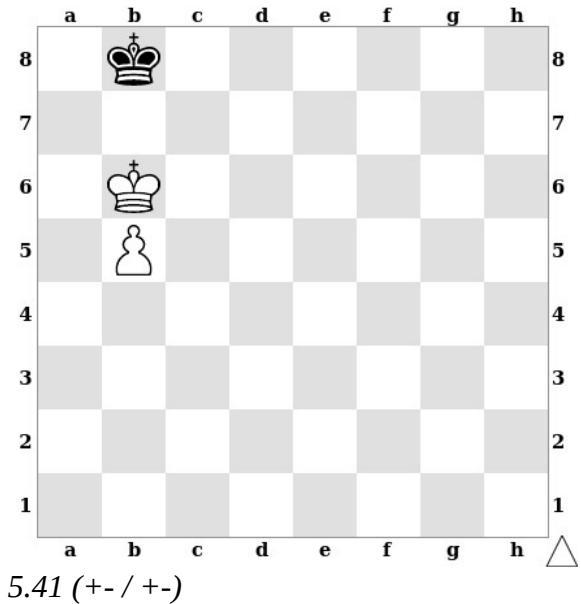
## Knight Pawns

Knight pawns mostly follow the same rules as bishop pawns and center pawns. However, there are a couple of important positions that are unique to knight pawns.



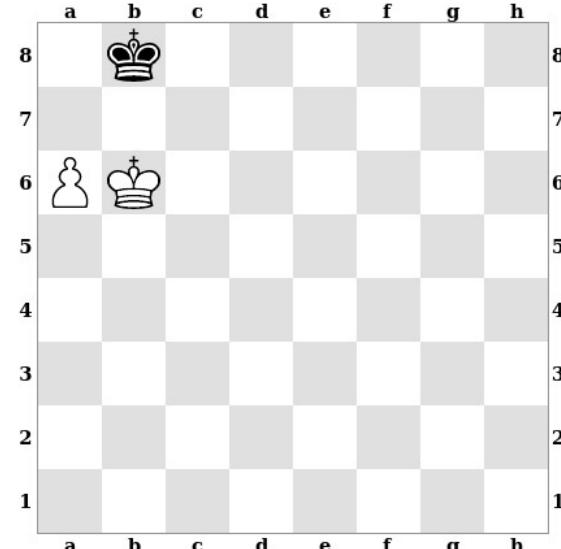
In this position Black would be losing against many pawns, but with a knight pawn he can draw with a clever move: 1...Ka8 2.b7+ Kb8 =. The problem for White is that 2.Kc7 = would be stalemate, which would not be true in the equivalent position with e.g. a bishop pawn.

White wins here after 1.Ka6 Ka8 2.b6 Kb8 3.b7 Kc7 4.Ka7 +- . With another kind of pawn it would work just as well to play 1.Kc6, but with a knight pawn this doesn't work. After 1...Ka7, White can only win by admitting his mistake and playing 2.Kc7 Ka8 3.Kb6 Kb8 4.Ka6 +- , transposing to 1.Ka6. Instead, White would give away the win with either 2.b6? = (leading to 5.40) or 3.b6? =, which is stalemate.



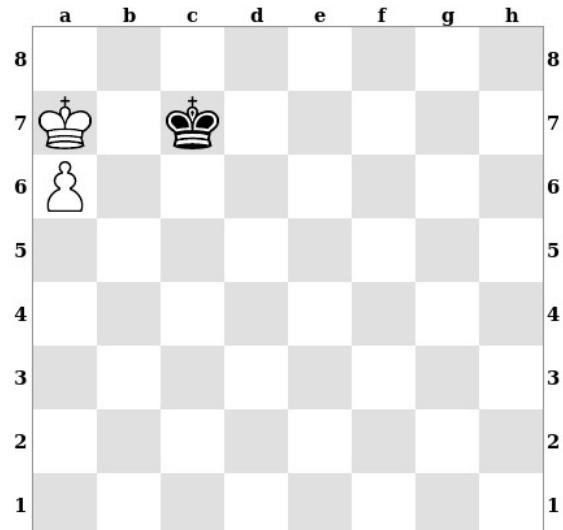
## Rook Pawns

Positions featuring rook pawns follow quite different rules from the positions covered so far. Luckily, these rules can still be understood in terms of key squares, it is just that the key squares are determined in a totally different way. First, let's see why rook pawns are special at all.



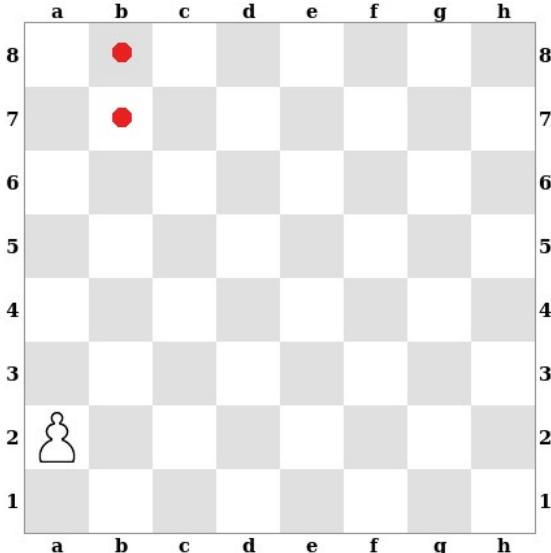
5.42 (= / =)

Here is the first sign that rook pawns have their own rules. With any other pawn, the result here would depend on who moved first. With a rook pawn, however, the position is simply drawn in any case. With Black to move, 1...Ka8 2.a7 = is stalemate, even though this would normally be a winning sequence for White. The problem is that Black's king has no way to exit to the left of the pawn, so it simply ends up getting stalemated. Once the weaker side's king gets into the corner against a rook pawn there is no way to drive it out again.



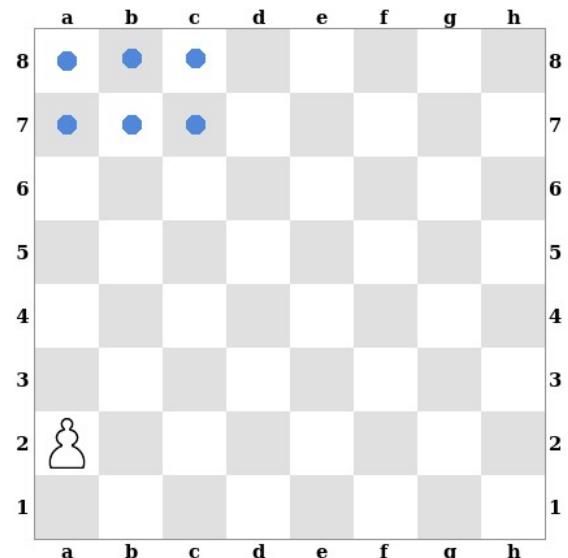
5.43 (= / =)

This position shows us the second difference. With another pawn White would win easily here, simply by moving his king out to the left and escorting the pawn through. However, with no file to the left of the a-file, White has no way out: **1.Ka8 Kc8 2.a7 Kc7 = stalemate.**



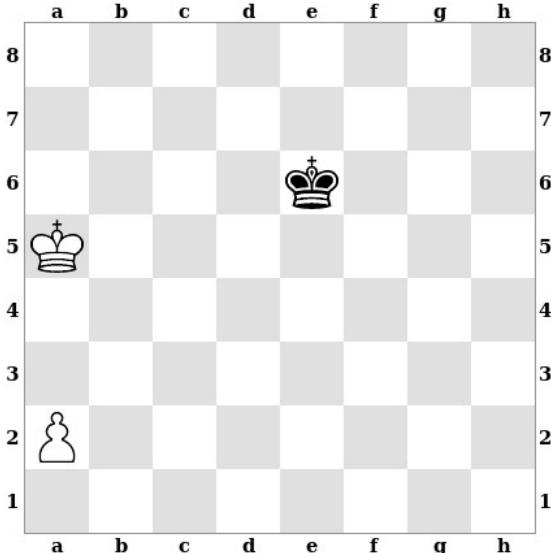
5.44

The rook pawn's key squares for the stronger side consist of the seventh and eighth rank squares on the adjacent knight file. Importantly, these are the key squares no matter what rank the pawn is on. The squares marked in the diagram would be the key squares whether this pawn was on a2 or a6, for example.



5.45

Similarly, the defensive key squares are the same no matter what rank the pawn is on. We could also mark all of the squares on the a-file that are still in front of the pawn (a6, a5, etc.). However, if the defender has reached these, the result is usually obvious in any case.



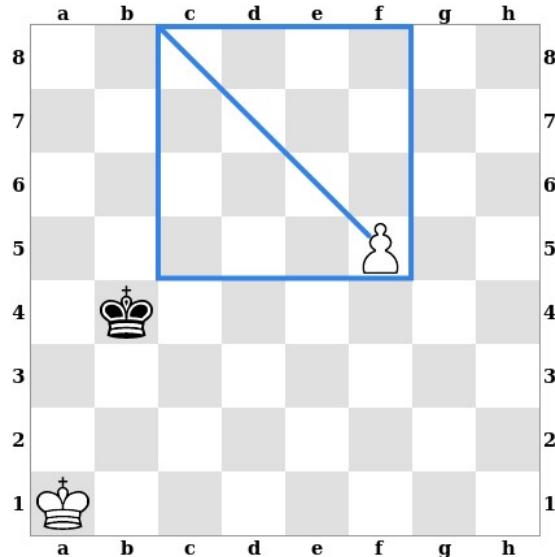
5.46 (+- / =)

Here the result depends on who moves first. If Black moves first, he draws by heading straight for the defensive key square c8: 1...Kd7 2.Kb6 Kc8 3.Ka7 Kc7 4.a4 Kc8 5.a5 Kc7 6.a6 Kc8 7.Ka8 Kc7 8.a7 Kc8 =. The game would also be drawn after 7.Kb6 Kb8 8.a7+ Ka8 =.

White can win if he moves first, but only by heading straight for the key square b7: 1.Kb6 Kd7 2.Kb7 Kd6 3.a4 Kc5 4.a5 Kb5 5.a6 +-.. Any delay would allow Black to draw, e.g. 2.a4? Kc8 =.

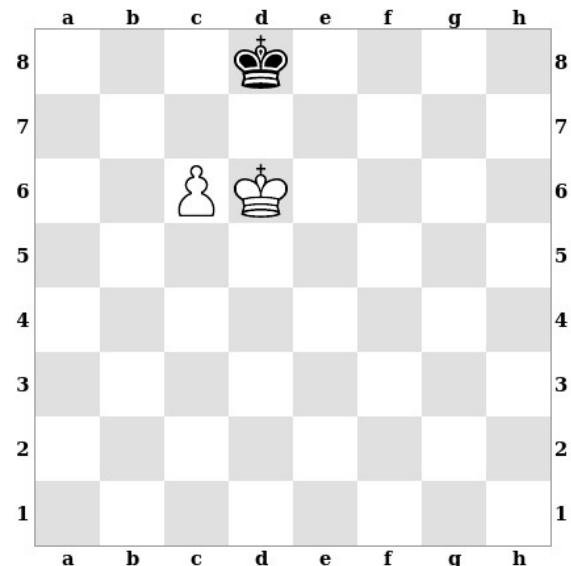
## Conclusion

Single pawn endings have a surprising amount of theory, and this is one endgame in which it is really worth taking the time to commit the most important results to memory. Many other endgames reduce to this one, and you will sometimes be judging these positions some ways in advance. For these reasons, it is important to be able to evaluate single pawn positions quickly and accurately. Here is a summary of the most important points that should be memorized.

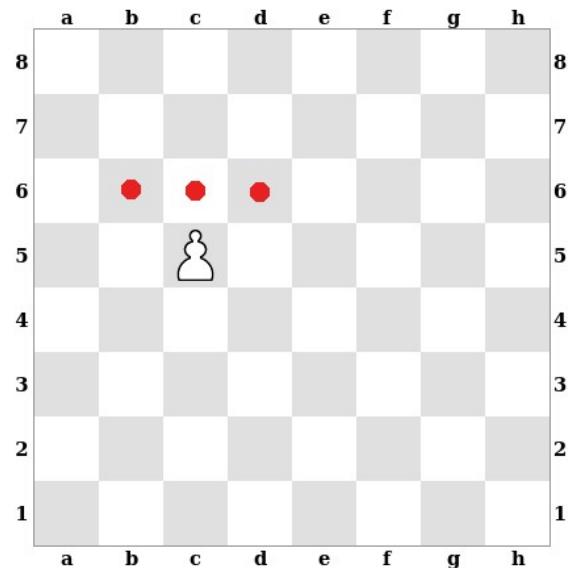
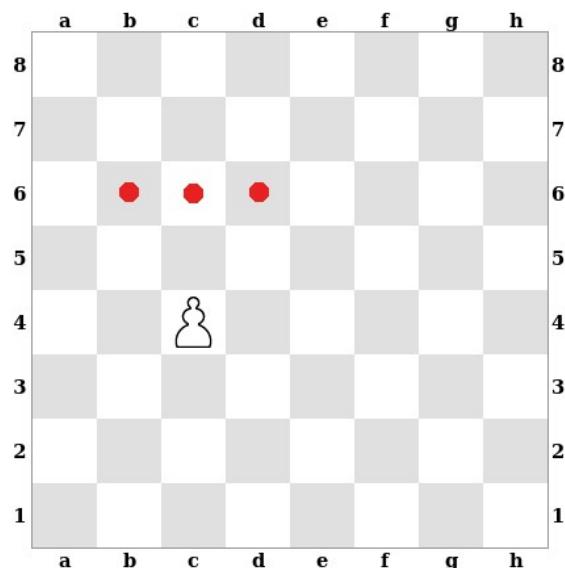
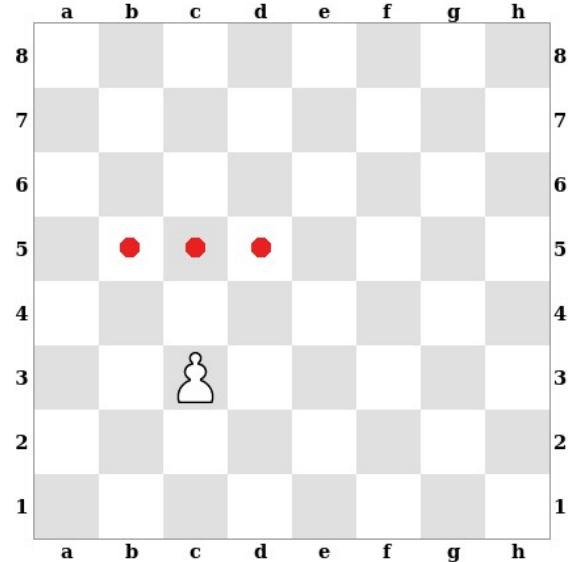
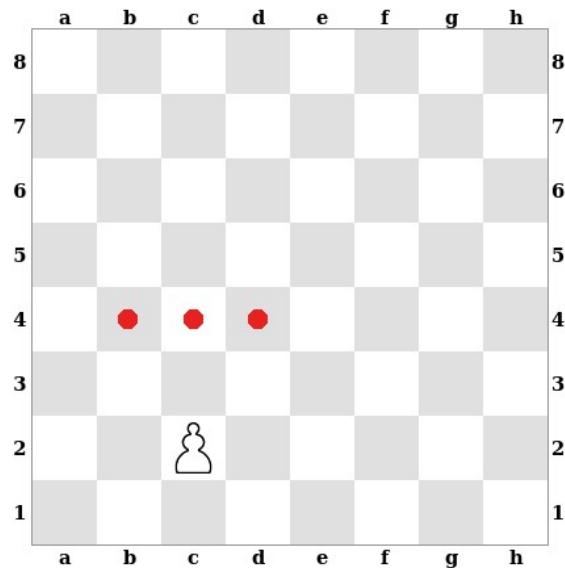


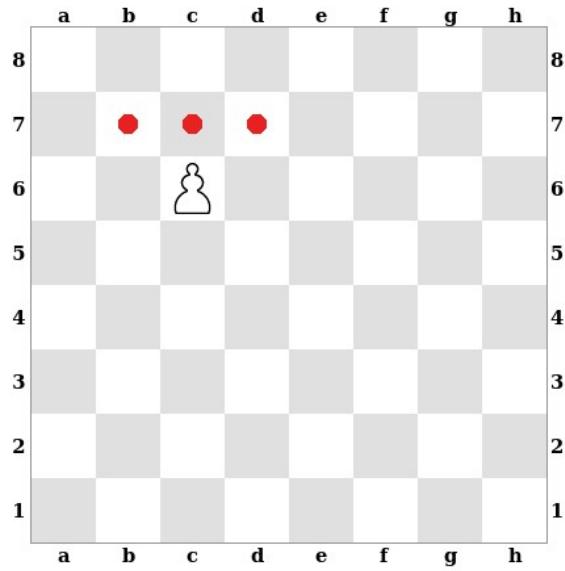
- 6 **Rule 5.1 (Rule of the Square):** draw a square from the pawn to the promotion rank. If the weaker side's king can enter the square, then it will catch the pawn in time. If the pawn can move with the king already outside the square, then the pawn will safely promote.

**Position 5.14**, which is drawn with White to move (the pawn advances with check) and winning with Black to move (the pawn advances without giving check).

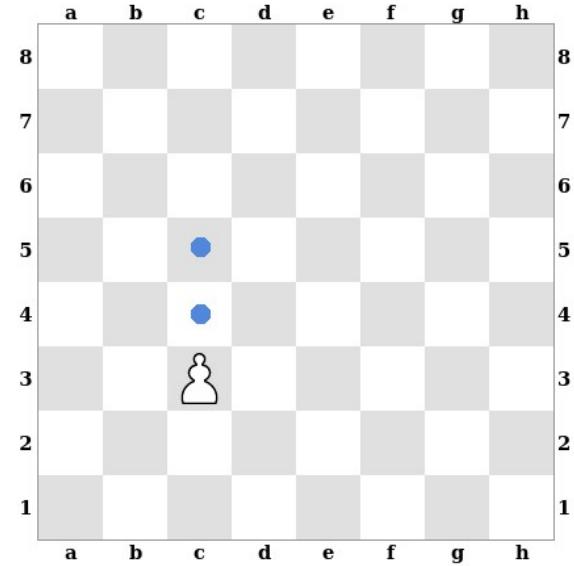
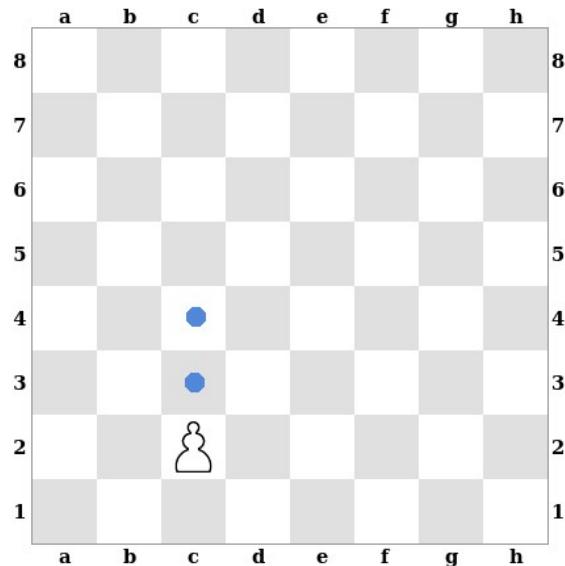


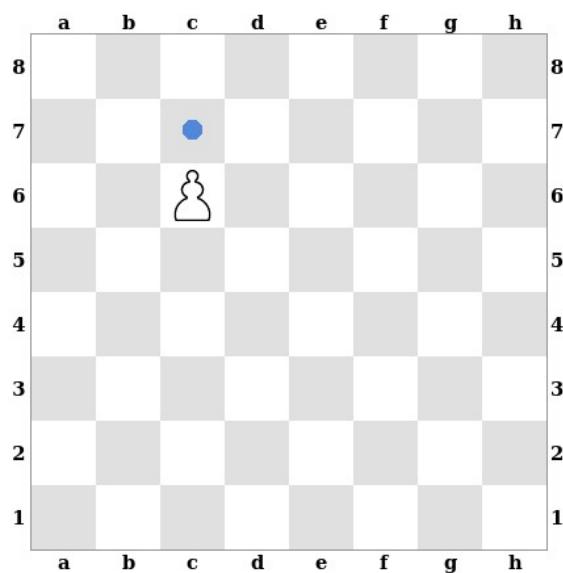
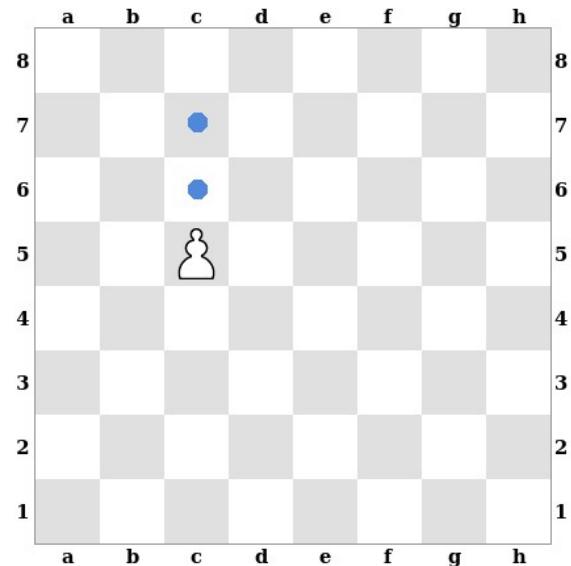
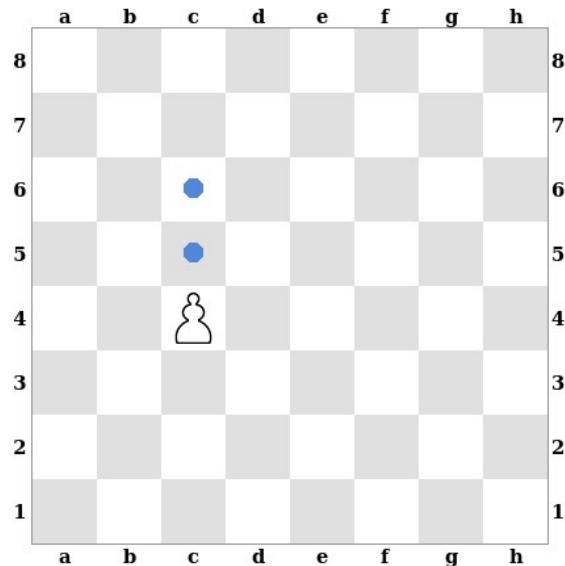
The **key squares** (for the attacker) of a non-rook pawn:



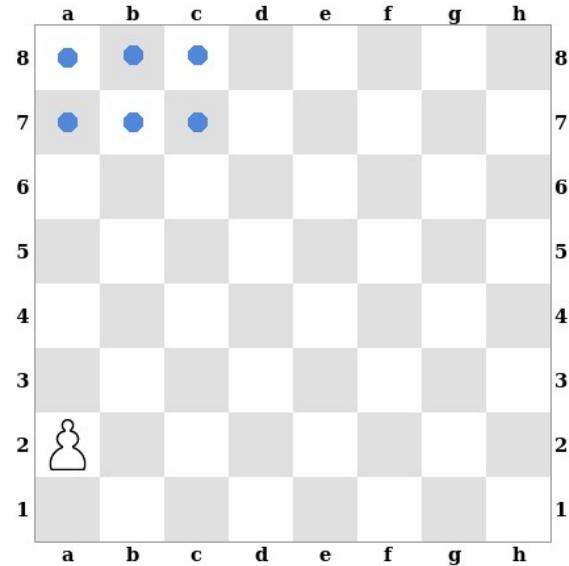
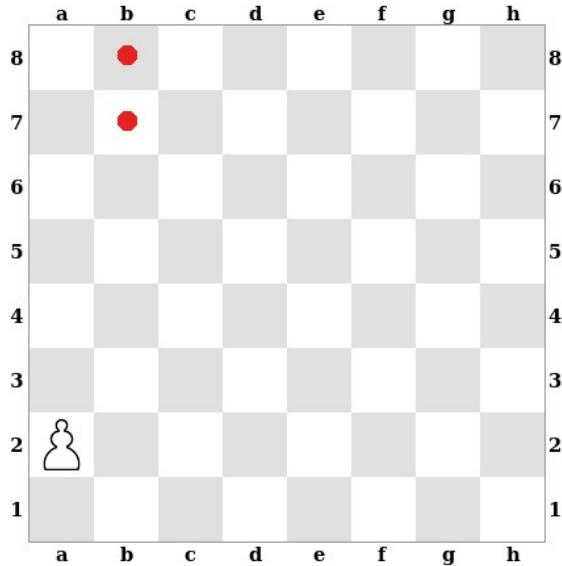


The **key squares** (for the defender) of a non-rook pawn:

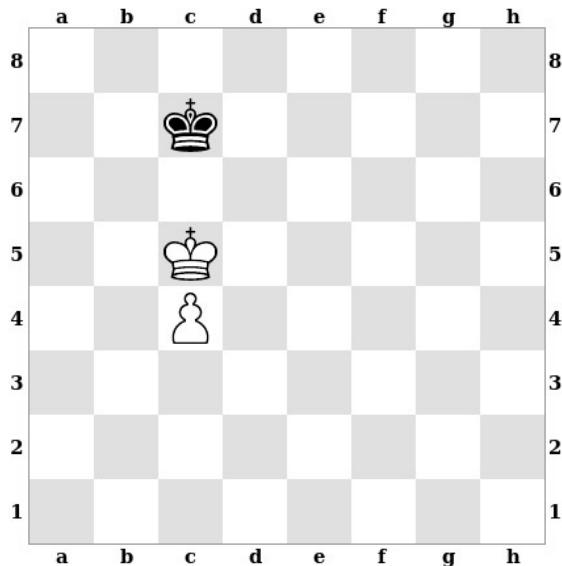




The **key squares** of a rook pawn, for the attacker and defender:



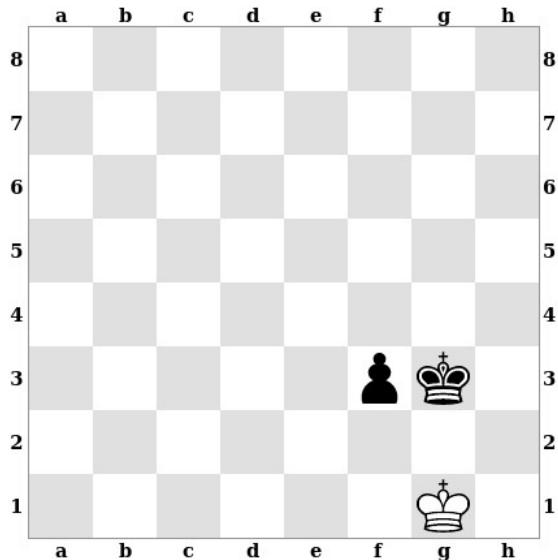
Finally, **rule 5.2** and **rule 5.3**:



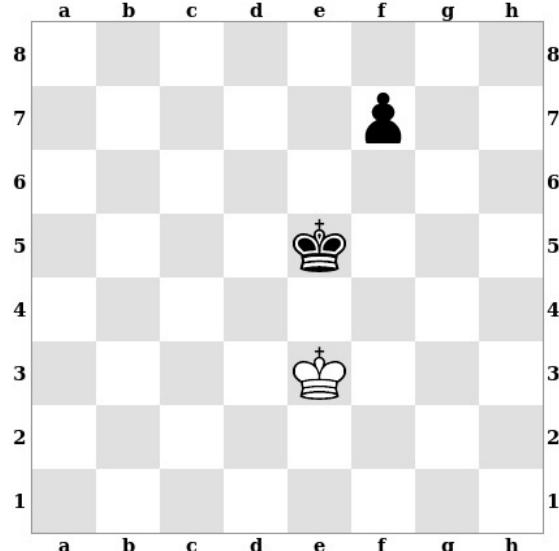
**rule 5.2:** if the stronger side's king is ahead of his (non-rook) pawn and he has the opposition, then he wins.

**rule 5.3:** if the stronger side's king is behind his key squares and the weaker side has direct opposition, then the position is drawn.

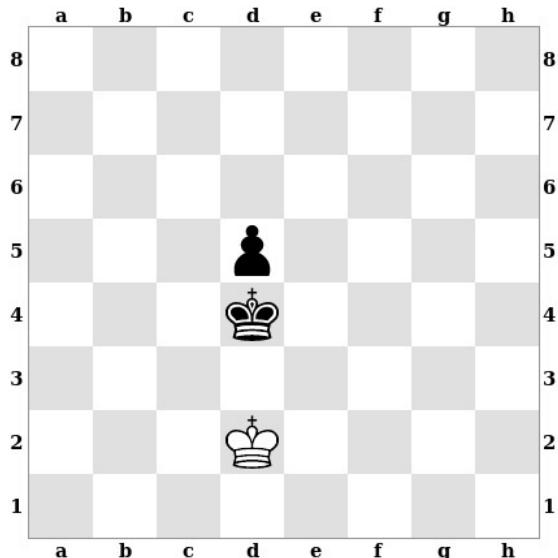
## Exercises



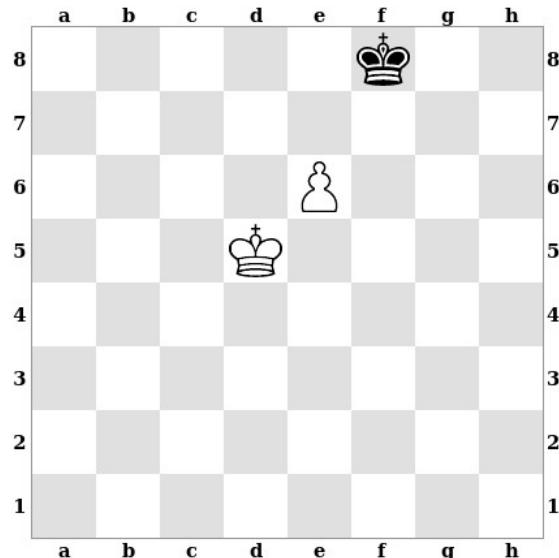
E5.01 (W/B)



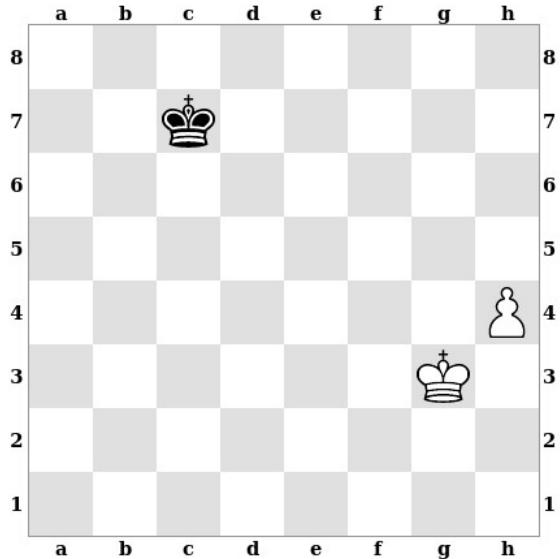
E5.02 (W/B)



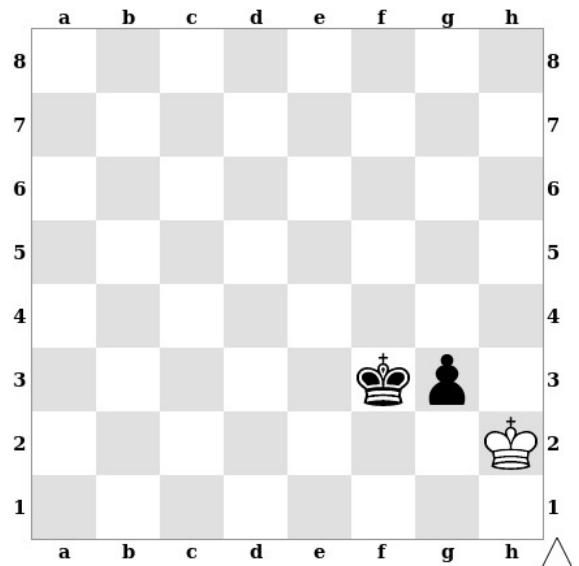
E5.03 (W/B)



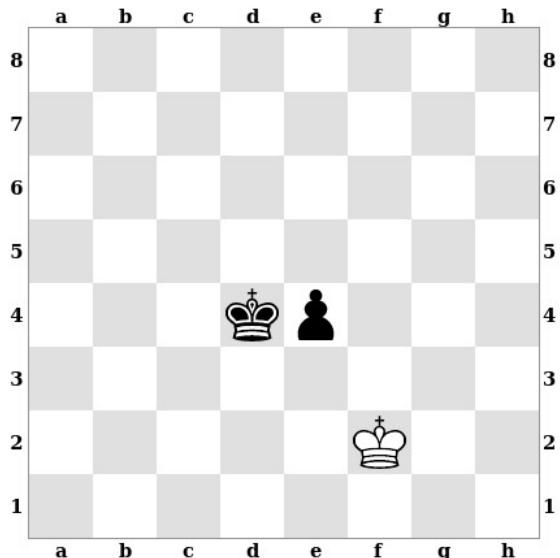
E5.04 (W/B)



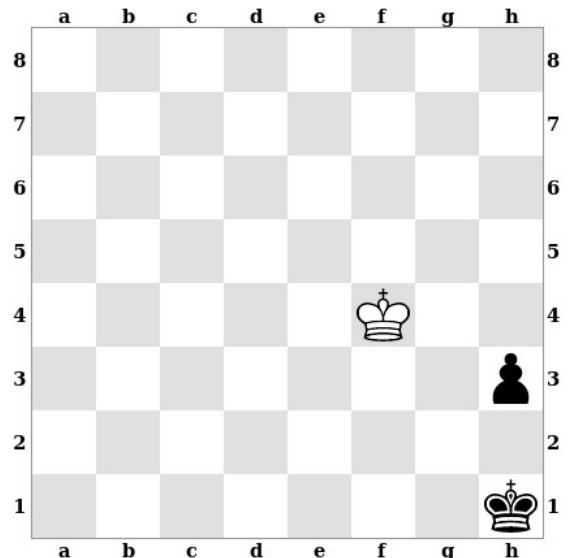
E5.05 (W/B)



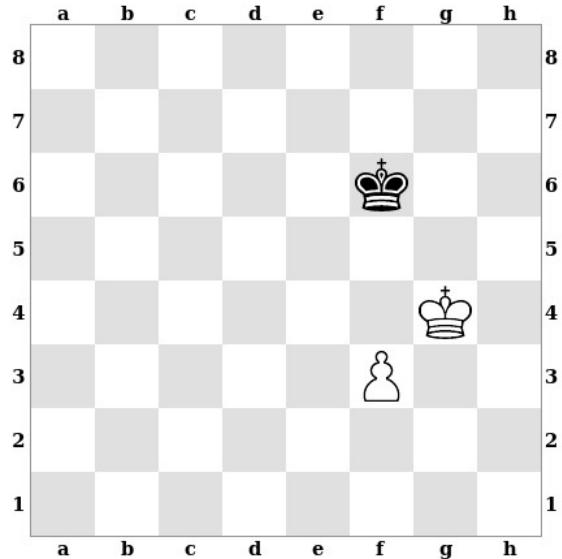
E5.06 (W)



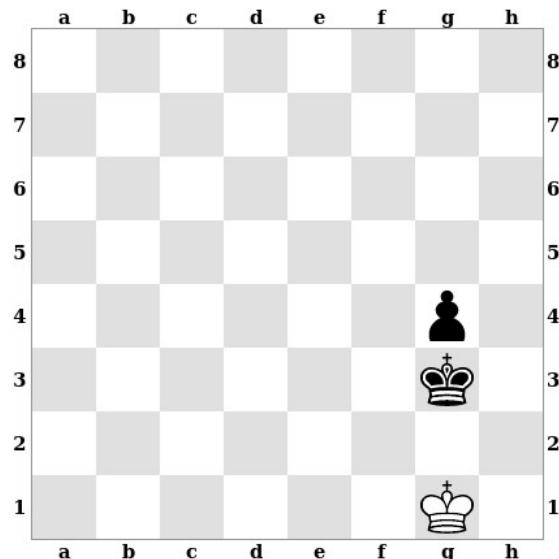
E5.07 (W/B)



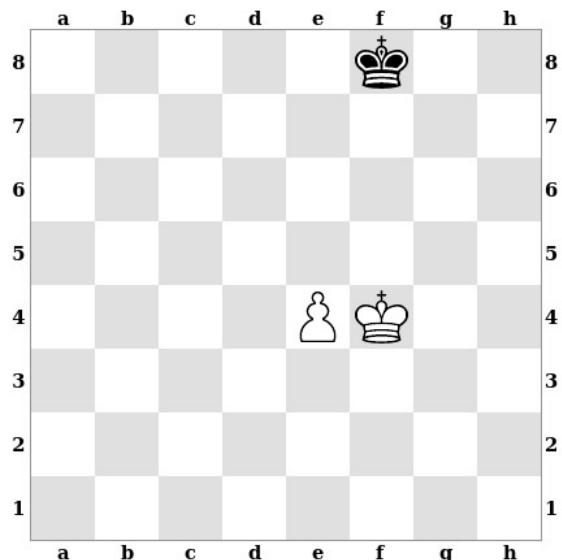
E5.08 (W/B)



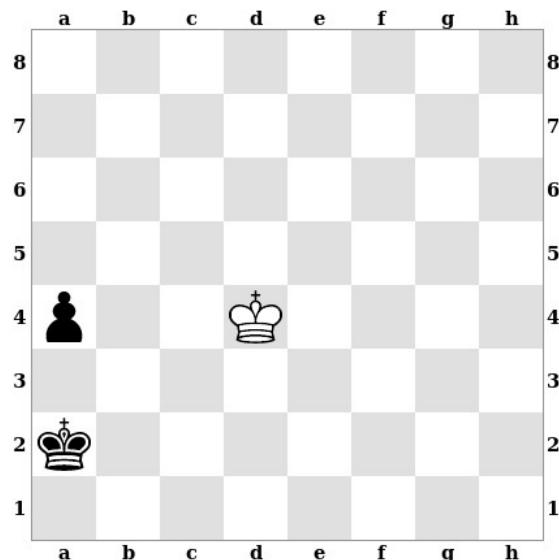
E5.09 (W/B)



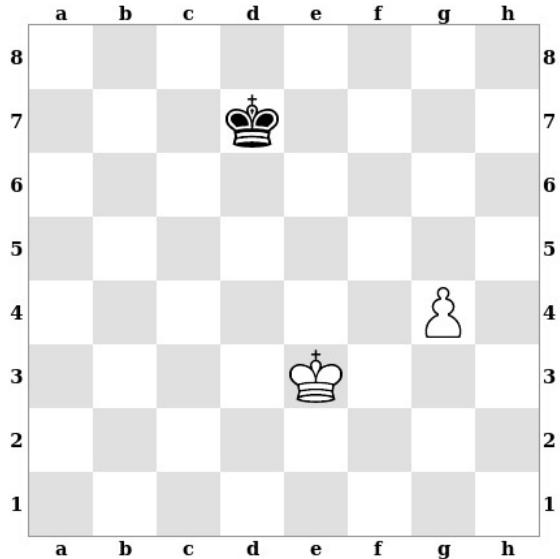
E5.10 (W/B)



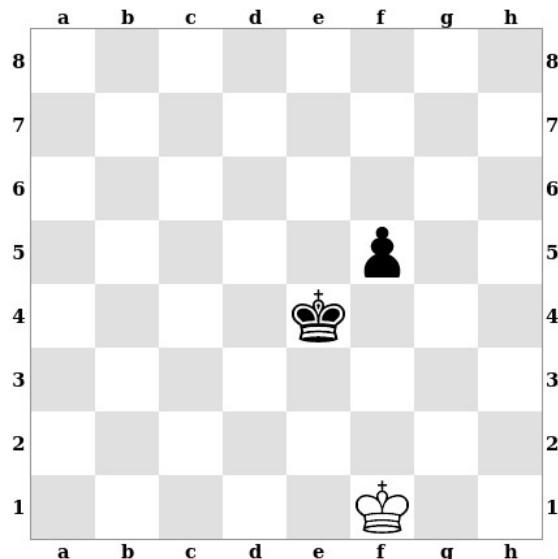
E5.11 (W/B)



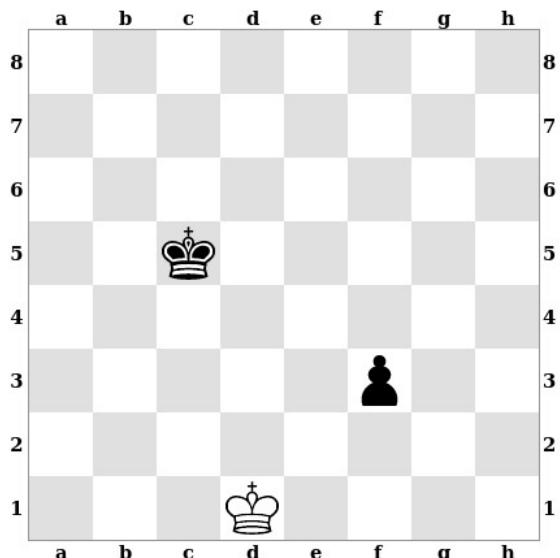
E5.12 (W/B)



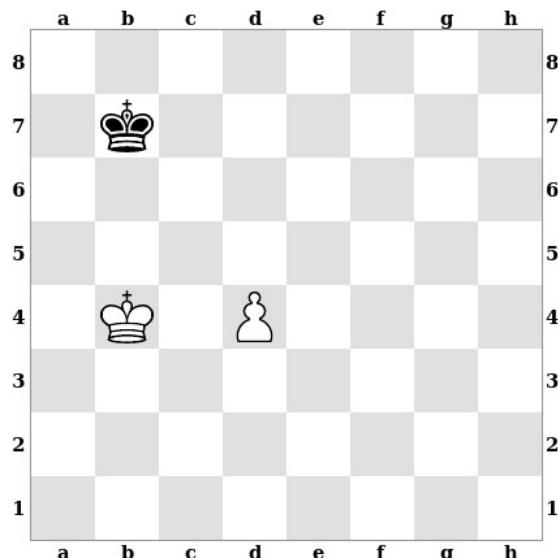
E5.13 (W/B)



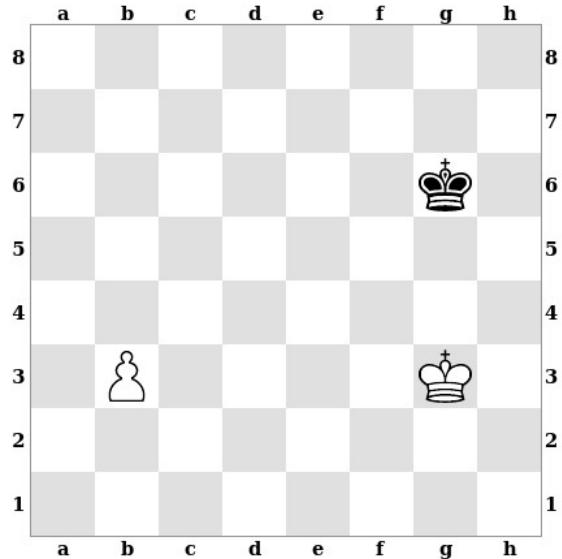
E5.14 (W/B)



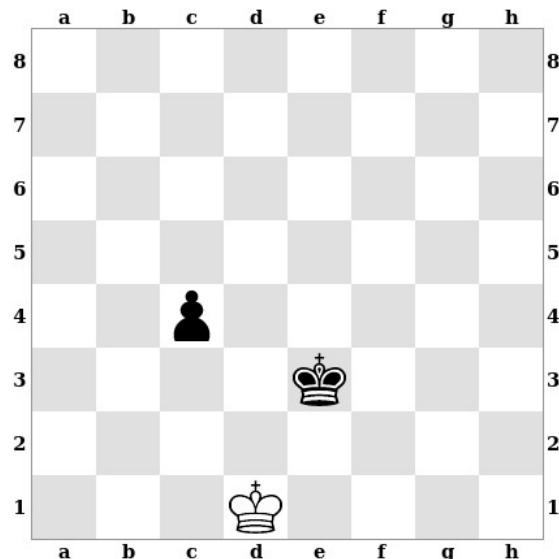
E5.15 (W/B)



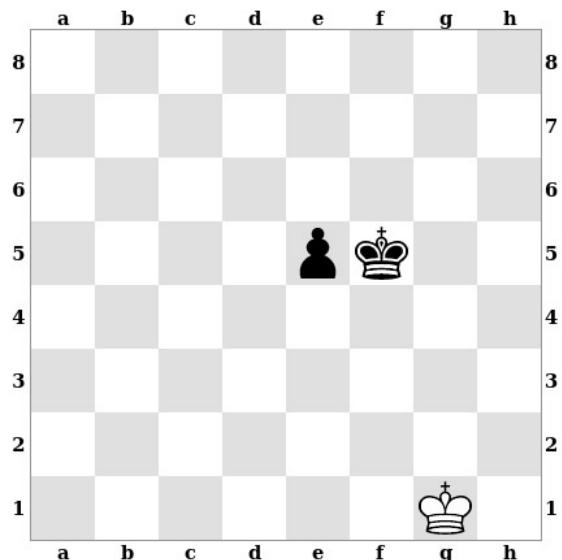
E5.16 (W/B)



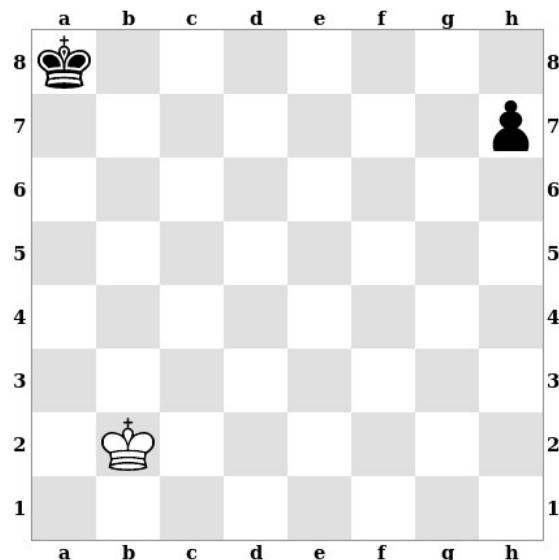
E5.17 (W/B)



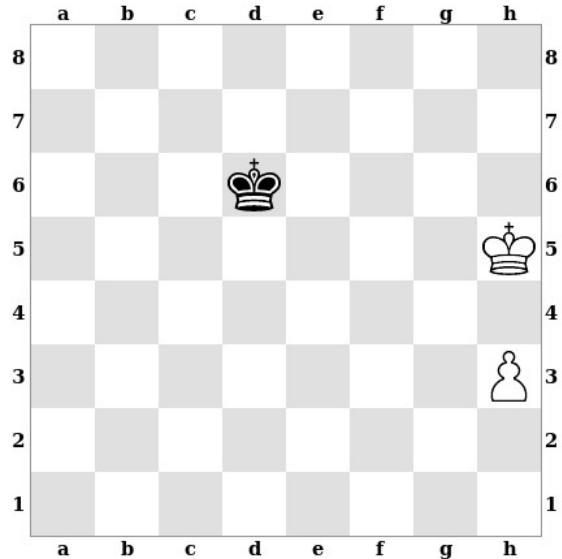
E5.18 (W/B)



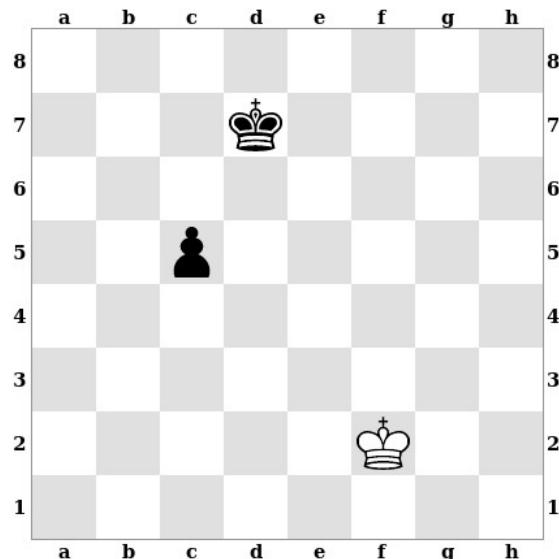
E5.19 (W/B)



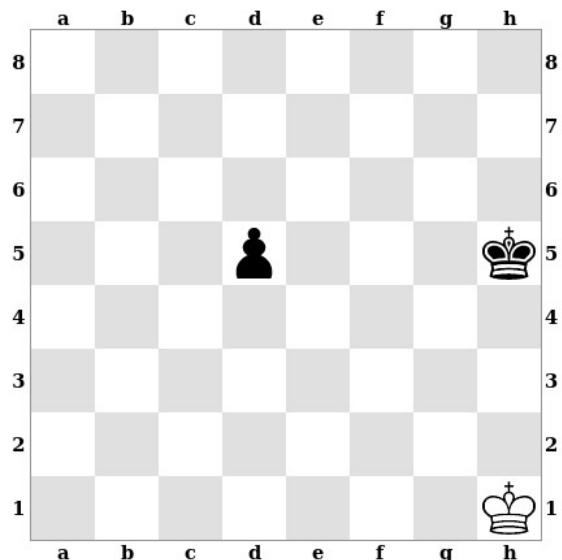
E5.20 (W/B)



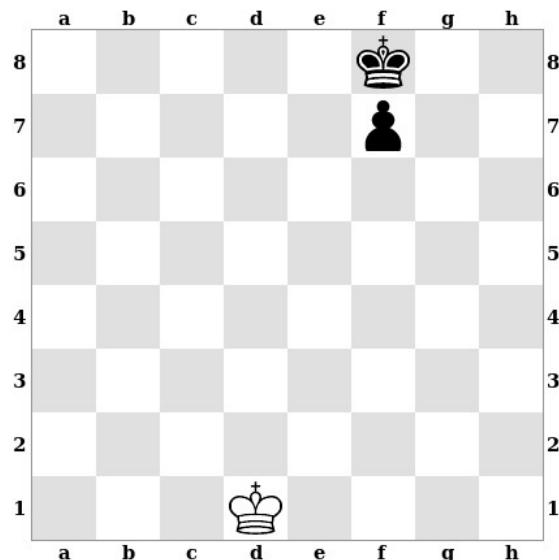
E5.21 (W/B)



E5.22 (W/B)



E5.23 (W/B)



E5.24 (W/B)

## Solutions

### E5.01

This is the important, nameless position from diagram 5.14.

**W:** (-+) 1.Kf1 f2 2.Ke2 Kg2 -+

**B:** (=) 1...Kf4 (1...f2+ 2.Kf1 =) 2.Kf2 Ke4 3.Kf1 (3.Ke1? Ke3 4.Kf1 f2 5.Kg2 Ke2 -+) 3...Ke3 4.Ke1 f2+ 5.Kf1 =

### E5.02

(-+) Black's king is on a key square of his pawn.

**W:** 1.Kf3 Kf5 2.Kg3 Ke4 3.Kf2 (3.Kg4 f5+ 4.Kg3 Ke3 5.Kg2 f4 6.Kf1 Kf3 -+) 3...Kf4 4.Ke2 Kg3 5.Kf1 (5.Ke3 f5 6.Ke2 f4 7.Kf1 Kf3 8.Kg1 Ke2 -+) 5...Kf3 6.Kg1 f5 7.Kf1 f4 8.Kg1 Ke2 9.Kg2 f3+ -+

**B:** 1...Kf5 2.Kf3 f6 3.Kg3 Ke4 4.Kf2 Kf4 5.Ke2 Kg3 6.Kf1 Kf3 7.Kg1 f5 8.Kf1 f4 9.Kg1 Ke2 10.Kg2 f3+ -+

### E5.03

**W:** (-+) Black is winning because of rule 5.2: he has the opposition with his king ahead of the pawn.

1.Ke2 Kc3 2.Kd1 Kd3 3.Kc1 Ke2 4.Kc2 d4 5.Kc1 d3 -+

**B:** (=) This is drawn because of rule 5.3: White has the direct opposition and Black's king is behind his key squares. 1...Ke4 2.Ke2 d4 3.Kd2 d3 4.Kd1 Ke3 5.Ke1 d2+ 6.Kd1 =

### E5.04

**W:** (+-) 1.Kd6 Ke8 2.e7 Kf7 3.Kd7 +-

**B:** (=) 1...Ke7 reaching a defensive key square. 2.Ke5 Ke8 3.Kd6 Kd8 4.e7+ Ke8 =

### E5.05

**W:** (-+) Black's king is outside the square of the pawn. 1.h5 (not 1.Kg4? Kd7 2.Kg5 Ke7 3.Kg6 Kf8 [key square] =) 1...Kd7 2.h6 Ke7 3.h7 +-

**B:** (=) 1...Kd7 2.Kf4 Ke7 3.Kg5 Kf7 [key square] =

### E5.06

(=) 1.Kh1 (1.Kg1? g2 2.Kh2 Kf2 -+) 1...g2+ 2.Kg1 =

### E5.07

**W:** (=) 1.Ke2 [key square] 1...e3 2.Ke1 Kd3 3.Kd1 e2+ 4.Ke1 Ke3 =

**B:** (-+) 1...Kd3 [key square] 2.Ke1 Ke3 3.Kd1 Kf2 4.Kd2 e3+ -+

### E5.08

W: (=) 1.Kg3 (1.Kf3? Kg1 2.Kg3 h2 -+) 1...h2 2.Kf2 =

B: (-+) 1...Kg2 [key square] 2.Ke3 h2 -+

### E5.09

W: (+-) 1.Kf4 [rule 5.2] 1...Ke6 2.Kg5 Kf7 3.Kf5 Kg7 4.Ke6 Kf8 5.Kf6 Kg8 6.f4 Kf8 7.f5 Kg8  
8.Ke7 +-

B: (=) 1...Kg6 [rule 5.3] 2.f4 Kf6 3.f5 Kf7 =

### E5.10

(-) Black's king is on a key square of the pawn, so he is winning no matter who moves first. However, he needs to be careful since the pawn is a knight pawn.

W: 1.Kh1 Kf2 2.Kh2 g3+ 3.Kh1 g2+ 4.Kh2 g1=Q+ -+

B: 1...Kh3 (After 1...Kf3 2.Kh2, Black can only win by admitting his mistake with 2...Kf2 [*but not 2...g3+? 3.Kh1 =*] 3.Kh1 Kg3 [*and not 3...g3? =*] 4.Kg1 Kh3 -+, transposing to 1...Kh3) 2.Kh1 g3 3.Kg1 g2 4.Kf2 Kh2 -+

### E5.11

(=)

W: 1.Kf5 Kf7 2.e5 Ke7 3.e6 Ke8 =

B: 1...Ke8 (not 1...Ke7? 2.Ke5 [rule 5.2] 2...Kf7 3.Kd6 Ke8 4.Ke6 Kd8 5.Kf7 Kd7 6.e5 +-) 2.Kf5 Kf7  
3.e5 Ke7 4.e6 Ke8 =

### E5.12

W: (=) 1.Kc3 a3 (1...Kb1 2.Kb4 =) 2.Kc2 [key square] 2...Ka1 3.Kc1 a2 4.Kc2 =

B: (-+) 1...Kb2 [key square] 2.Kd3 a3 -+

### E5.13

W: (+-) 1.Kf4 (1.Kd3? Ke6 =) 1...Ke6 2.Kg5 Kf7 3.Kh6 [key square] 3...Kg8 4.g5 Kh8 5.g6 Kg8  
6.g7 +-

B: (=) 1...Ke6 2.Kf4 Kf6 [rule 5.3] 3.g5+ Kg6 4.Kg4 Kg7 =

### E5.14

W: (=) 1.Ke2 [rule 5.3] 1...f4 2.Kf2 f3 3.Kf1 =

B: (-+) 1...Kf3 [key square] 2.Ke1 Kg2 3.Ke2 f4 -+

### E5.15

W: (=) 1.Ke1 Kd4 2.Kf2 [key square] =

B: (-+) 1...Kd4 2.Kd2 (2.Ke1 Ke3 3.Kf1 f2 -+) 2...Ke4 3.Kd1 Kd3 (3...Ke3? Ke1 =) 4.Ke1 Ke3 5.Kf1  
f2 -+

### E5.16

W: (+-) 1.Kb5 [rule 5.2] 1...Kc7 2.Kc5 Kd7 3.Kd5 Kc7 4.Ke6 Kd8 5.Kd6 Ke8 6.Kc7 Ke7 7.d5 +-  
B: (=) 1...Kc6 (or 1...Kb6 [rule 5.3] =) 2.Kc4 Kd6 [key square] 3.d5 Kd7 =

### E5.17

W: (+-) 1.Kg4 [rule 5.2] (not 1.Kf4? Kf6 [rule 5.3] =) 1...Kf6 2.Kf4 Ke6 3.Ke4 Kd6 4.Kd4 Kc6  
5.Kc4 Kb6 6.Kb4 Ka6 7.Kc5 Kb7 8.Kb5 +-

B: (=) 1...Kf5 (or 1...Kg5 [rule 5.3] =) 2.Kf3 Ke5 3.Ke3 Kd5 4.Kd3 Kc5 5.Kc3  
Kb5 [key square] =

### E5.18

W: (=) 1.Kc2 [key square] (not 1.Ke1? Kd3 [key square] 2.Kd1 c3 3.Kc1 c2 4.Kb2 Kd2 -+) 1...Kd4  
2.Kd2 c3+ 3.Kc2 Kc4 4.Kc1 =

B: (-+) 1...Kd3 [key square] 2.Kc1 Kc3 3.Kd1 Kb2 4.Kd2 c3+ +-

### E5.19

W: (=) 1.Kf1 (not 1.Kf2? Kf4 [rule 5.2] -+) 1...Ke4 2.Ke2 [rule 5.3] 2...Kd4 3.Kd2 e4 4.Ke2 e3  
5.Ke1 =

B: (-+) 1...Ke4 (not 1...Kf4? 2.Kf2 [rule 5.3] =; also, not 1...Kg5? 2.Kf1 Kf5 3.Ke1 =) 2.Kf2 Kd3  
[key square] 3.Ke1 Ke3 +-

### E5.20

Remember that the square of a pawn needs to be drawn differently if the pawn can still move two spaces in one move. In this position, the square of Black's pawn should be drawn as though the pawn were already on h6.

W: (=) 1.Kc2 entering the pawn's square. 1...Kb7 2.Kd2 Kc6 3.Ke2 Kd5 4.Kf2 [key square] =

B: (-+) White's king is outside of the pawn's square. 1...h5 2.Kc2 h4 3.Kd2 h3 +-

### E5.21

W: (+-) 1.Kg6 Ke7 2.Kg7 [key square] 2...Ke6 3.h4 Kf5 4.h5 Kg5 5.h6 +-

B: (=) 1...Ke7 (not 1...Ke6? 2.Kg6 Ke7 3.Kg7 [key square] +-) 2.Kg6 Kf8 [key square] 3.h4 Kg8 =

### E5.22

W: (=) 1.Ke3 Kc6 (1...Kd6 2.Kd3 Kd5 3.Kc3 [key square] =) 2.Kd3 Kb5 3.Kc3 [key square] =

B: (-+) 1...Kc6 2.Ke3 Kb5 3.Kd3 Kb4 threatening Kb3, which would reach a key square. 4.Kc2 Kc4  
[rule 5.2] 5.Kb2 Kd3 6.Kc1 Kc3 7.Kb1 Kd2 8.Kb2 c4 +-

**5.23**

(=)

**W:** 1.Kg1 (1.Kg2? Kg4 [rule 5.2] -+) 1...Kg5 2.Kf1 Kf5 3.Ke1 Ke5 4.Kd1 Ke4

5.Ke2 [rule 5.3] =

**B:** 1...Kg5 (1...Kg4 2.Kg2 [rule 5.3] =) 2.Kg1 Kf5 3.Kf1 Ke5 4.Ke1 Ke4 5.Ke2 [rule 5.3] =

**E5.24**

**W:** (=) 1.Ke2 Kg7 2.Kf3 (2.Ke3? Kg6 3.Kf4 Kf6 [rule 5.2] -+) 2...Kh7 3.Kf4 Kg6 4.Kg4 [rule 5.3]

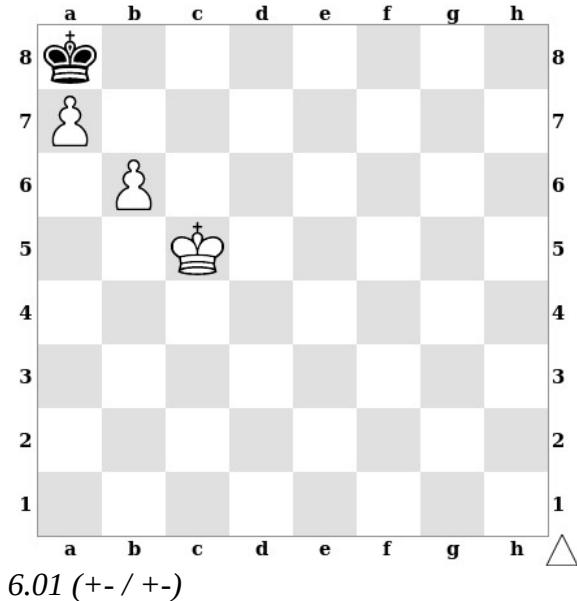
4...f6 5.Kf4 f5 6.Kf3 =

**B:** (-+) 1...Ke7 2.Ke1 Kf6 3.Kf2 Kf5 [key square] 4.Kf3 f6 5.Kg3 Ke4 6.Kf2 Kf4 -+

## 6 – Basic Multi-Pawn Endings

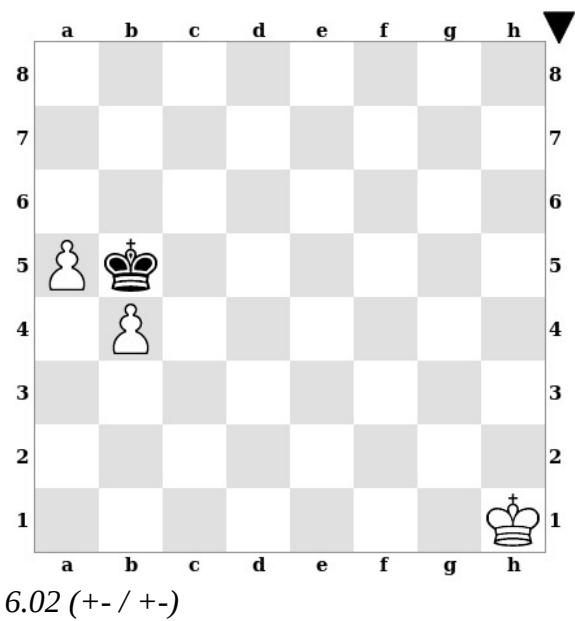
We will now begin looking at king and pawn endgames with more than one pawn on the board. In this chapter we are focused on the most basic positions. A later chapter deals with more complicated cases.

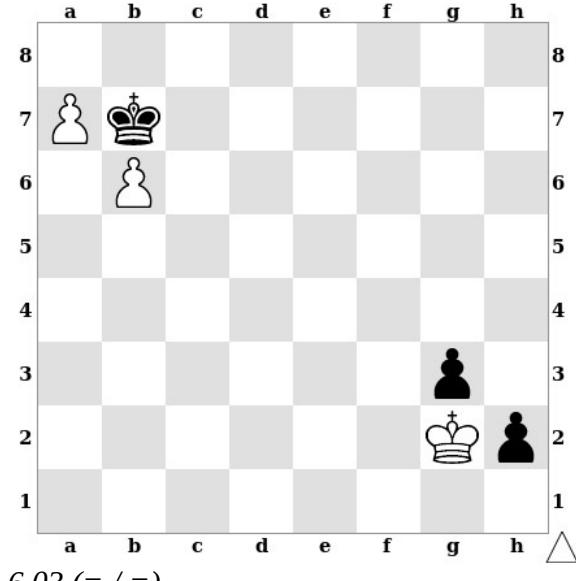
### Connected Pawns



Two extra connected pawns usually win with the king's support. An important possibility for the stronger side is to sacrifice one of the pawns to reach a winning single pawn ending. Here this is necessary, since 1.Kc6? = is stalemate. Instead, White wins with 1.Kb5 Kb7 2.a8=Q+ Kxa8 3.Ka6 Kb8 4.b7 Kc7 5.Ka7 +-.

Two connected pawns can normally defend themselves without their king's help, as long as they aren't forced to move. Here for example, if Black plays 1...Kxb4 2.a6 +-, then White's a-pawn promotes. If Black just waits, then White brings his king in and wins in a way similar to the previous example.



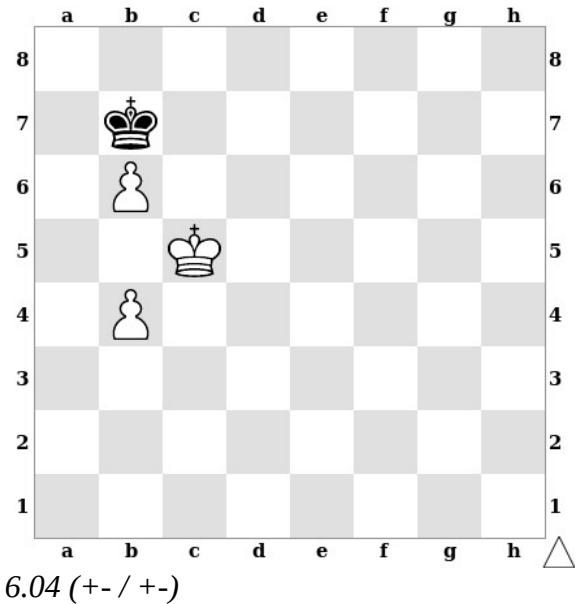


However, two connected pawns cannot win on their own. This position is drawn since neither king can come help his pawns: 1.Kh1 Ka8 2.Kg2 Kb7 =.

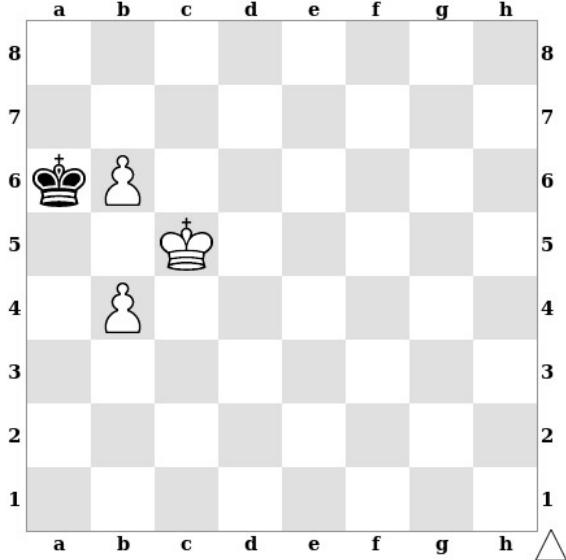
6.03 (= / =)

## Doubled Pawns

Doubled pawns also normally win with the support of the king, though there are more potential problems. In some cases the rear pawn can be used to break zugzwang. In this position, for instance, after 1.Kb5 Kb8 2.Ka6 Ka8 3.b7+ Kb8, a single pawn position would end in stalemate after Kb6, but here White wins with 4.b5 Kc7 5.Ka7 +-.

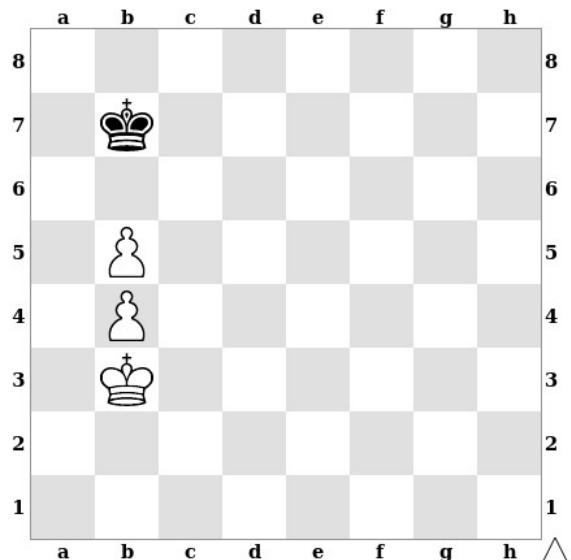


6.04 (+- / +-)



6.05 (+- / +-)

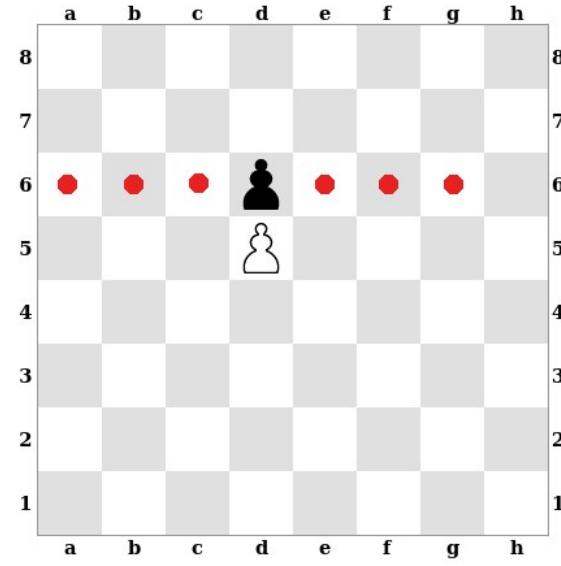
As with connected pawns, it is sometimes necessary to sacrifice one pawn to win. Here White would only draw after 1.Kc6? = (stalemate) or 1.b5+? Kb7 = when White can't help but to allow Kxb6 next move, which puts Black's king on a defensive key square of the b5 pawn. Instead, White wins with **1.b7 Kxb7 2.Kb5 +-**, taking the opposition in front of his pawn.



6.06 (+- / +-)

To win here White must first improve his king position, but he has to be careful how he does this. The immediate 1.Kc4? Kb6 = is mutual zugzwang and Black will play Kxb5 next. White should instead wait to play Kc4 when Black can't respond with Kb6: **1.Kc3 Kc7** (1...Kb6 2.Kc4 Kb7 3.Kc5 Kc7 4.b6+ Kb7 5.Kb5 +-, as in 6.04) **2.Kd4 Kd6 3.Kc4 Kc7 4.Kc5 Kb7 5.b6 Ka6 6.b7 +-**, as in 6.05.

## Blocked Pawns

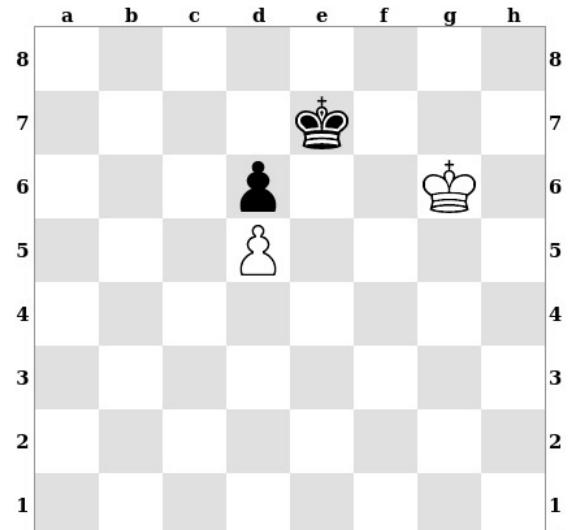


6.07

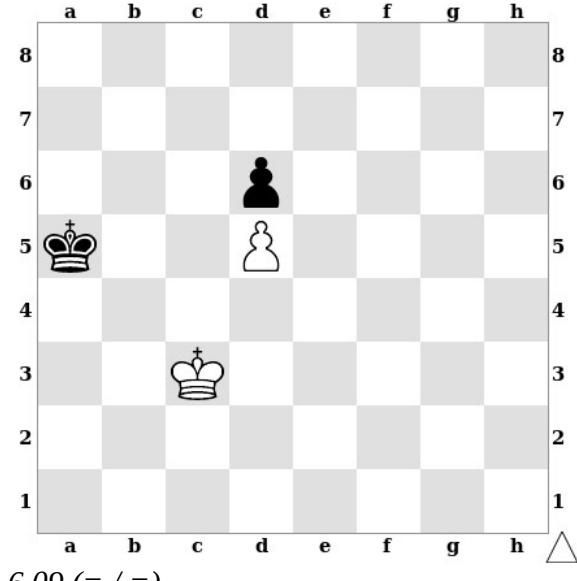
We now turn to positions in which both sides have one pawn. When the pawns block each other, each pawn has a set of (up to) six *critical squares* extending out from its sides in both directions. If the opponent's king can reach one of these squares then it will capture the pawn by force. In this diagram the critical squares of the black pawn are marked. If White's king occupies one of these squares then he will win Black's pawn. For the time being we ignore the possibility of a counterattack against White's pawn by Black's king.

Here White's king occupies one of the critical squares, so he can win Black's pawn no matter who moves first. With White to move, the game might end **1.Kg7 Ke8 2.Kf6 Kd7 3.Kf7 Kd8 4.Ke6 Kc7 5.Ke7 Kc8 6.Kxd6 +-**. If Black goes first, the game might end with **1...Ke8 2.Kf6 Kd7 3.Kf7 +-**, transposing to the previous variation.

Winning Black's pawn is enough for White to have a winning position in this example. This is because Black's pawn is on what will be a key square of White's pawn in the resulting single pawn ending.

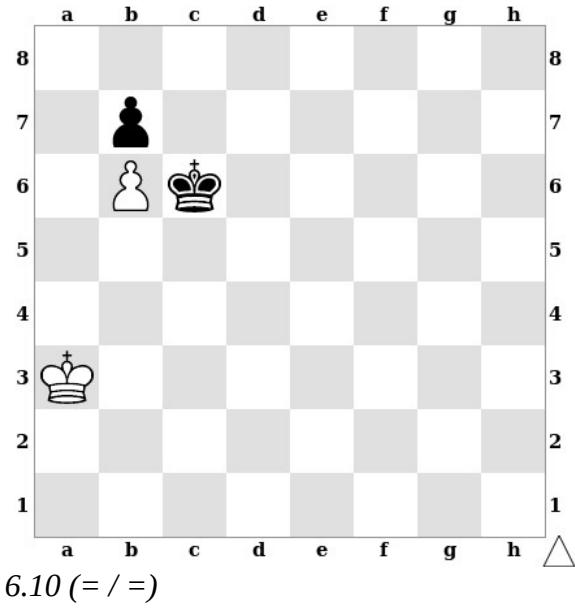


6.08 (+- / +-)



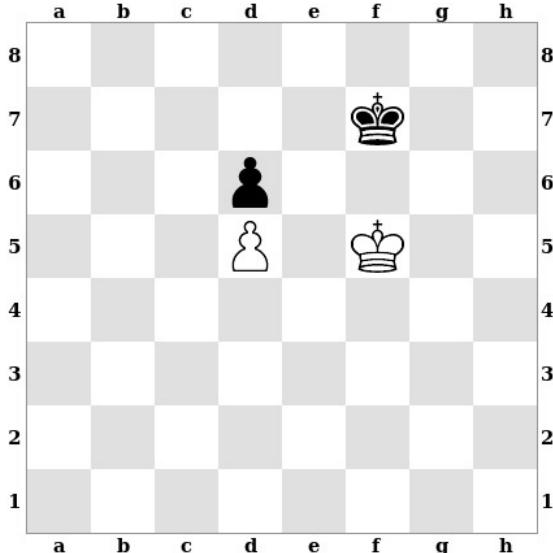
6.09 (= / =)

Winning such a pawn isn't always enough to win the game, as this position shows. Black's king has reached a critical square of White's pawn, so White cannot avoid losing the pawn: **1.Kc4 Ka4 2.Kc3 Kb5 3.Kd4 Kb4 4.Kd3 Kc5 5.Ke4 Kc4 6.Ke3 Kxd5.** However, the d5 square is not a key square of Black's d6 pawn; White draws the game with **7.Kd3** = because he has the direct opposition with Black's king behind his key squares.



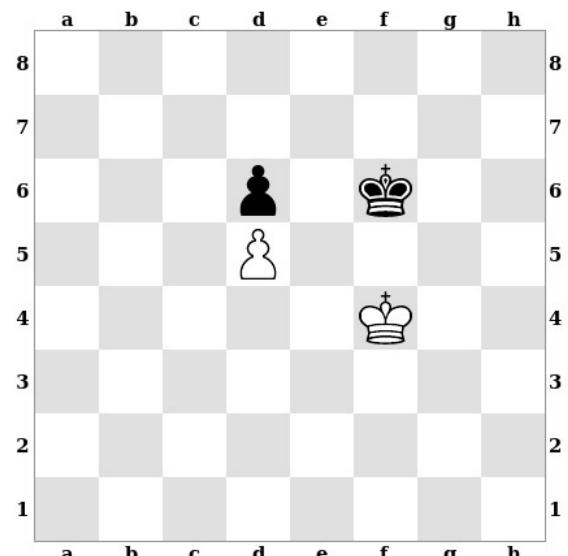
6.10 (= / =)

Here also, White cannot save his pawn but can draw the game. He needs to meet ...Kxb6 with Kb4, so he can draw with **1.Kb3** (or 1.Ka4 =) **1...Kxb6 2.Kb4** =. However he would lose after 1.Kb4? Kxb6 -+, when Black has the opposition in front of his pawn.



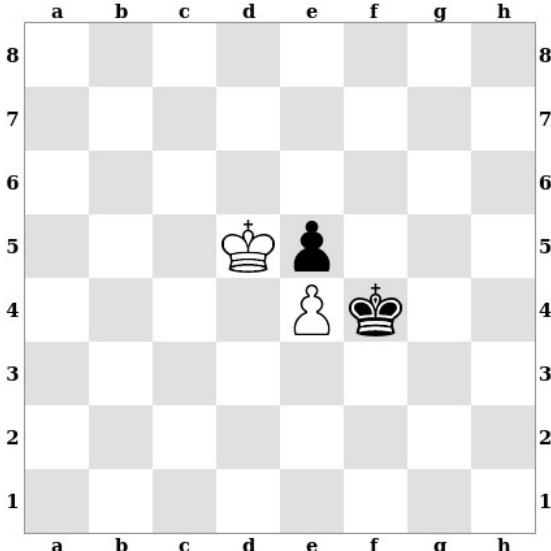
6.11 (= / +-)

Here the result depends on who has the opposition.  
If Black has the opposition he can keep White off of the pawn's critical squares: **1.Kg5 Kg7 2.Kf5 Kf7 =**. If White has the opposition then he reaches one of the critical squares: **1...Ke7 2.Kg6 +-.**



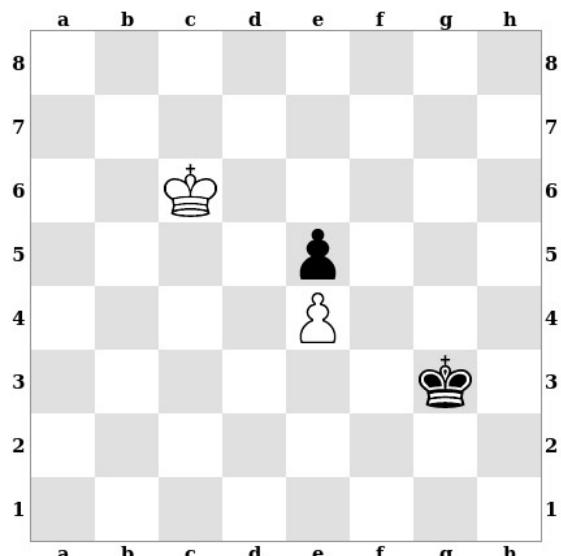
6.12 (= / =)

With the kings back a rank, Black can defend the critical squares even without the opposition. After **1...Kg6**, White would need to be able to play **2.Ke5** to make progress. Instead, **2.Kg4 Kf6 3.Kf4 Kg6 =** doesn't get White anywhere.



6.13 (-+ / +-)

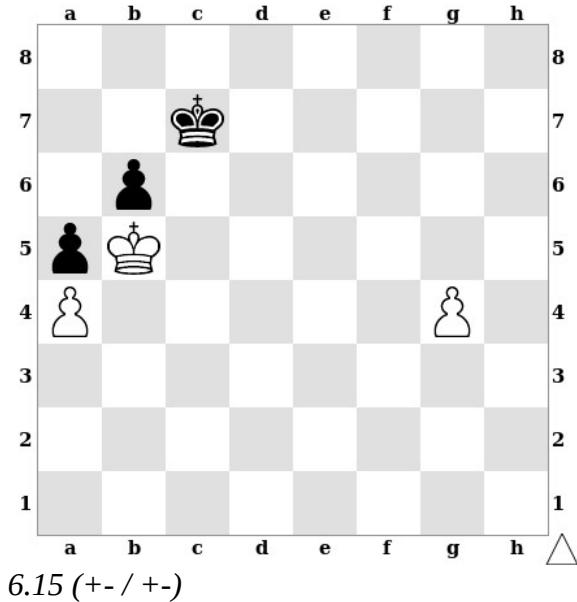
In some cases, both kings attack each other's pawns at the same time. This position is an important case of mutual zugzwang; whoever moves first loses his pawn. Furthermore, because the kings are not in position to defend, giving up this pawn loses the game. If White moves he loses after **1.Kc4 Kxe4 2.Kc3 Ke3 -+**, when Black has reached a key square of his pawn. Similarly, if Black moves first he loses after **1...Kg5 2.Kxe5 Kg6 3.Ke6 +-.**



6.14 (+- / -+)

In this position, by contrast, whoever moves first wins by arranging to move second when the previous position is reached. If White moves first, he wins with **1.Kd6** (not **1.Kd5?? Kf4 -+**) **1...Kf4 2.Kd5 +-.** If instead Black moves first, then he wins after **1...Kf3 2.Kd5 Kf4 -+.**

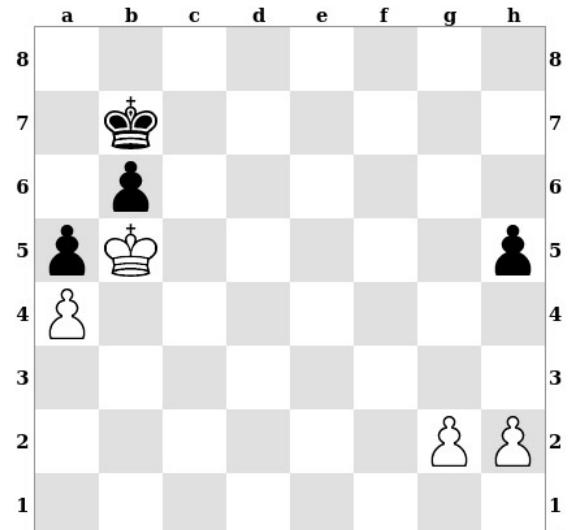
## Outside Passed Pawns



6.15 (+- / +-)

An *outside passed pawn* is a passed pawn that is far away from the other pawns in a position. Such a pawn is often a winning advantage because it forces the opponent's king to move away from the other pawns. In this position material is equal, but White's outside passed pawn on g4 gives him a win after 1.g5 Kd7 (Black's king must stay inside the g-pawn's square) 2.Kxb6 Ke6 3.Kxa5 Kf5 4.Kb6 +-.

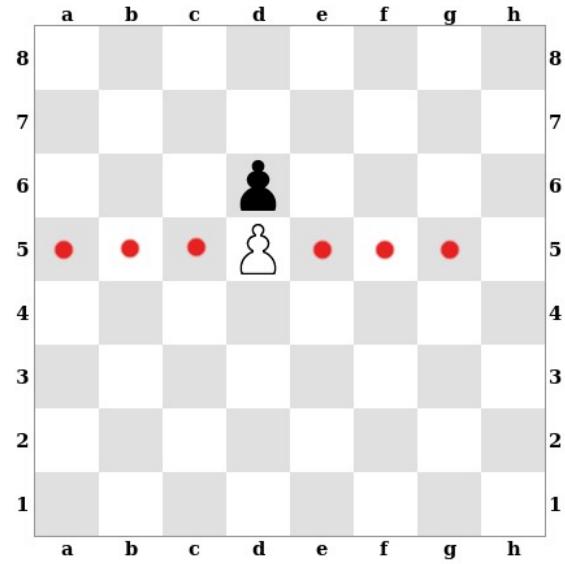
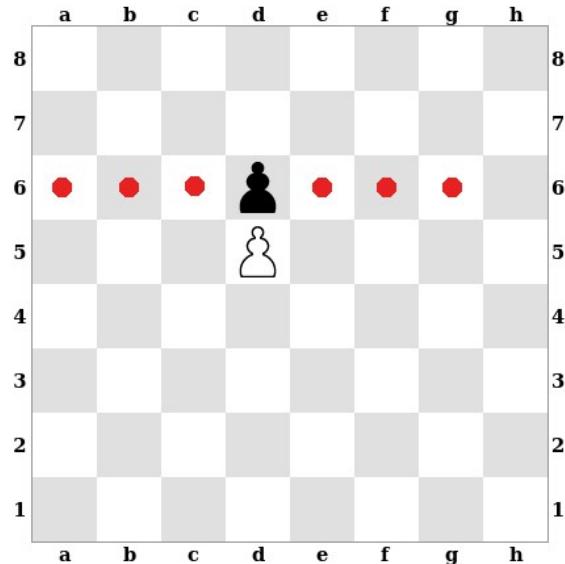
In this position White doesn't yet have an outside passed pawn, but he can create one using his kingside pawn majority. The best way to advance such a majority is usually to advance the unopposed pawn first (in this case, the g-pawn). White could only draw after 1.h4? Kc7 =, especially since he can't play 2.g4? hxg4 +-, when Black's g-pawn promotes first. Similarly, White loses after 1.h3?? h4 2.Kc4 Kc6 3.Kd3 b5 4.axb5+ Kxb5 +-, when Black is the one with an outside passed pawn. Instead, White wins with 1.g3 Kc7 2.h3 Kb7 3.g4 hxg4 4.hxg4 Kc7 5.g5 +-, continuing as in the previous example.



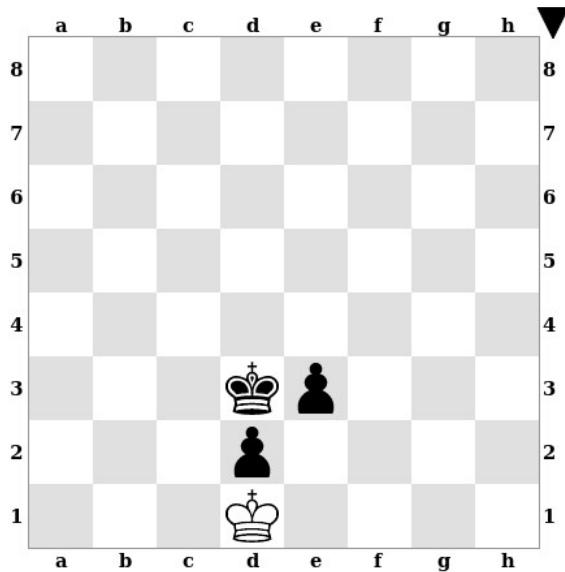
6.16 (+- / +-)

## Conclusion

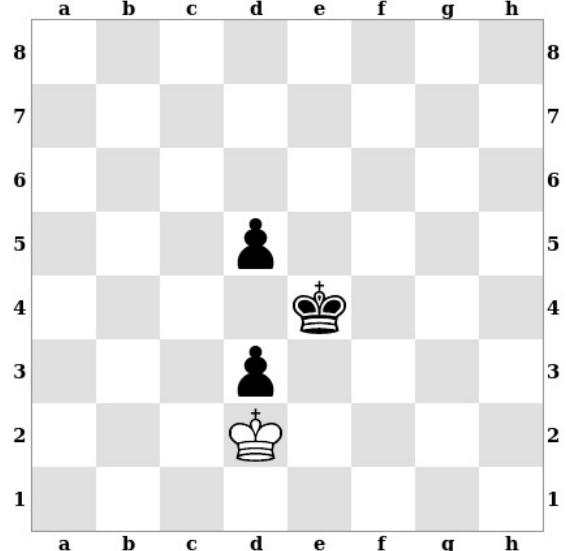
The only result from this chapter that you need to explicitly remember is the idea of critical squares. When two pawns are blocked, they each have a set of (up to) six critical squares extending away from them, three on each side. If the enemy king can reach one of these squares, then it is guaranteed to be able to capture the pawn (as long as there is no counterattack from the other king).



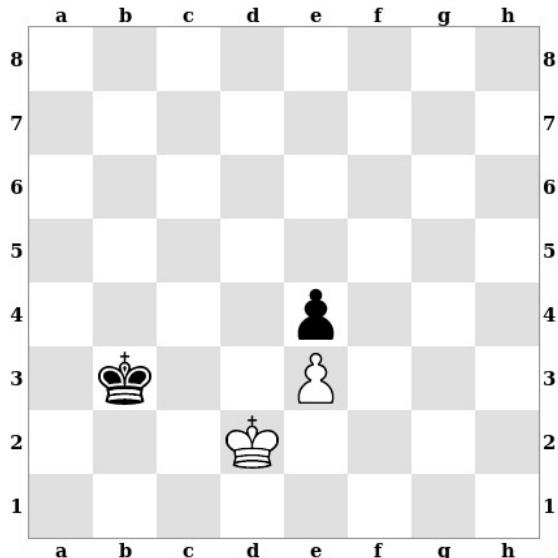
## Exercises



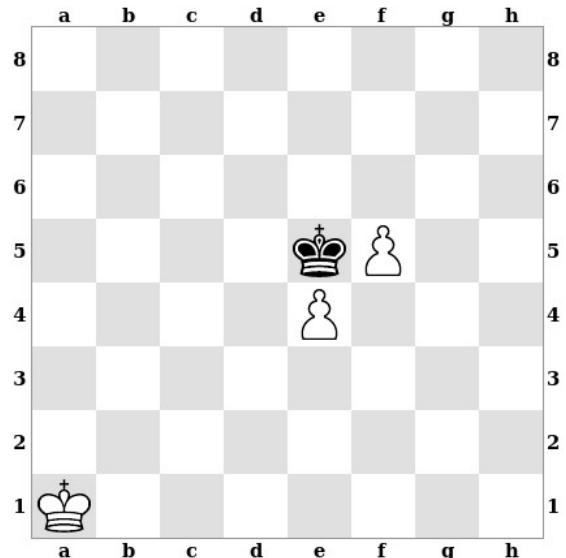
E6.01 (B)



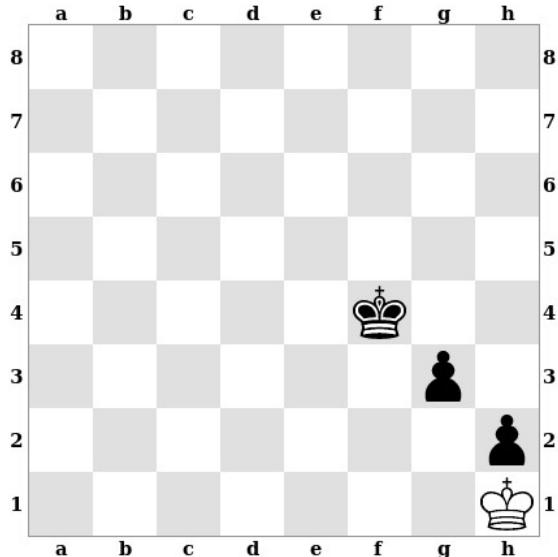
E6.02 (W/B)



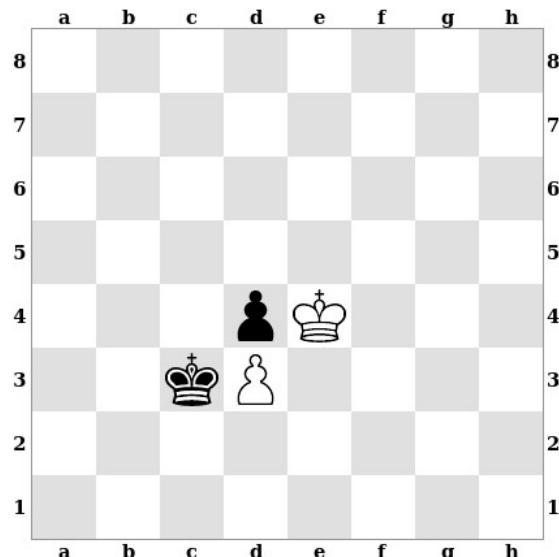
E6.03 (W/B)



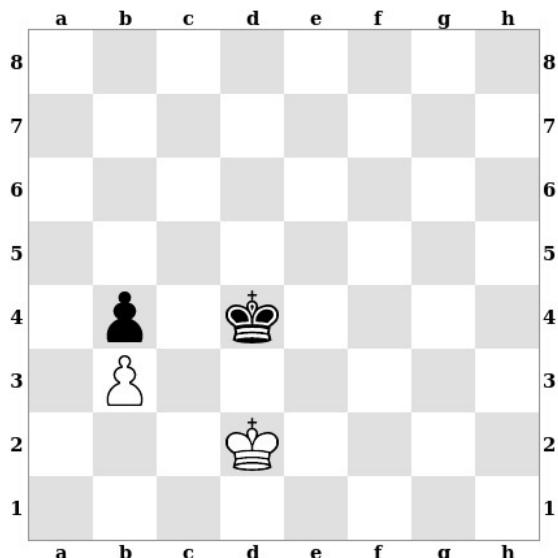
E6.04 (W/B)



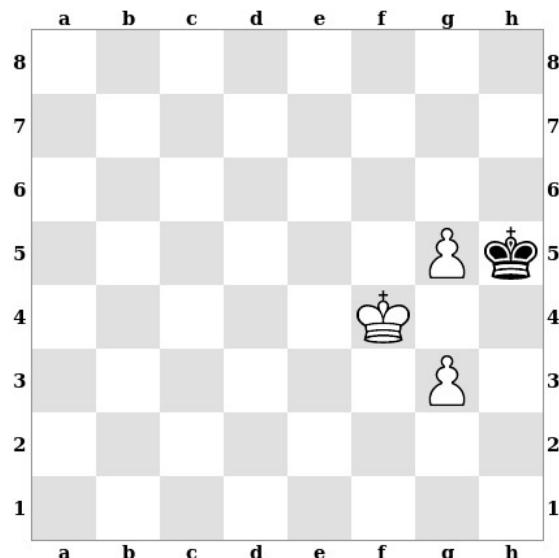
E6.05 (W/B)



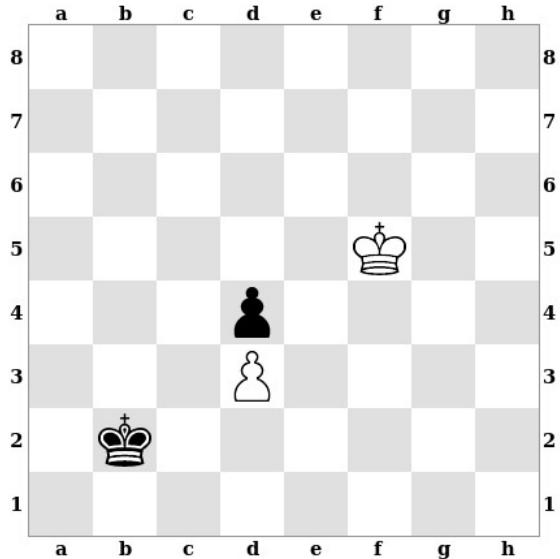
E6.06 (W/B)



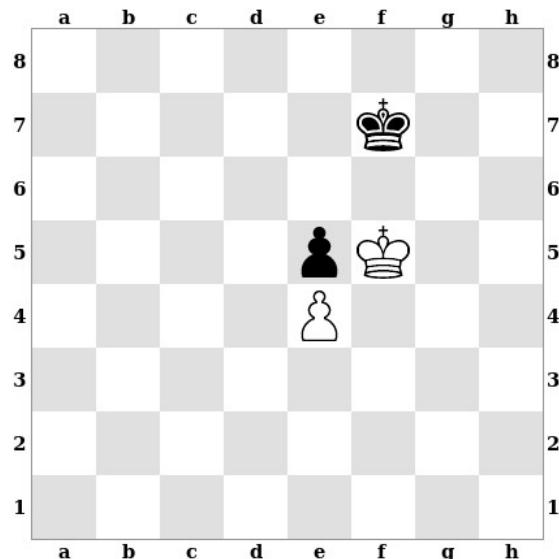
E6.07 (W/B)



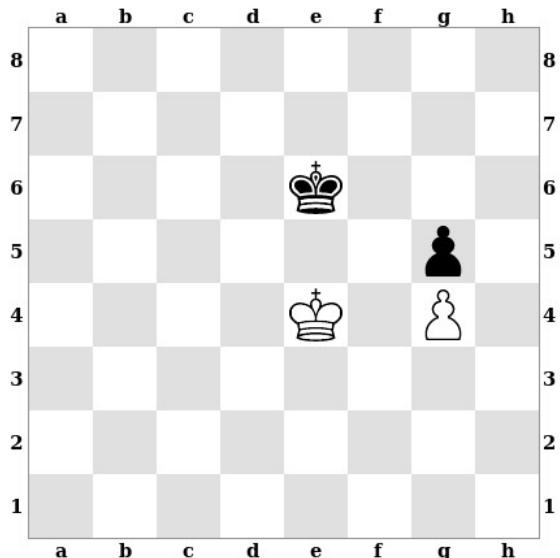
E6.08 (W/B)



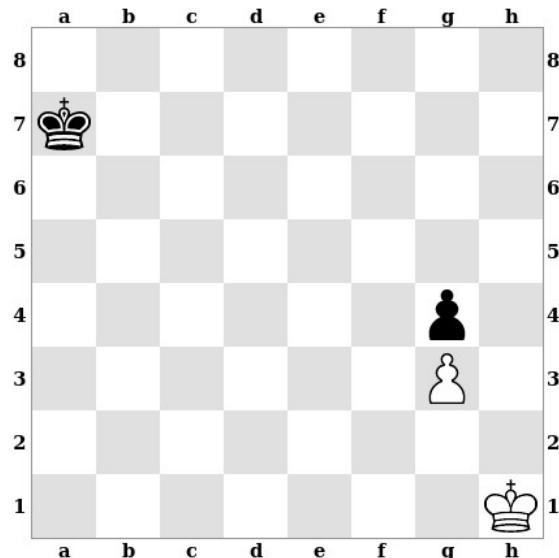
E6.09 (W/B)



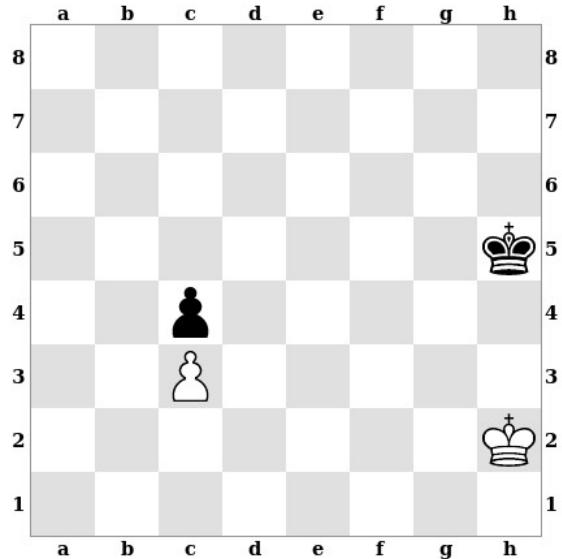
E6.10 (W/B)



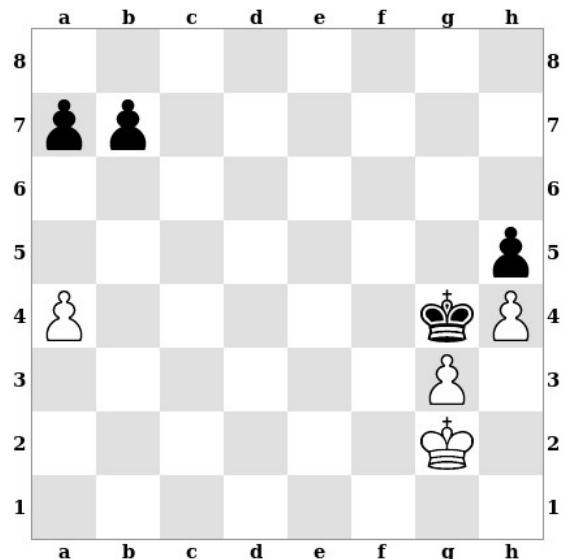
E6.11 (W/B)



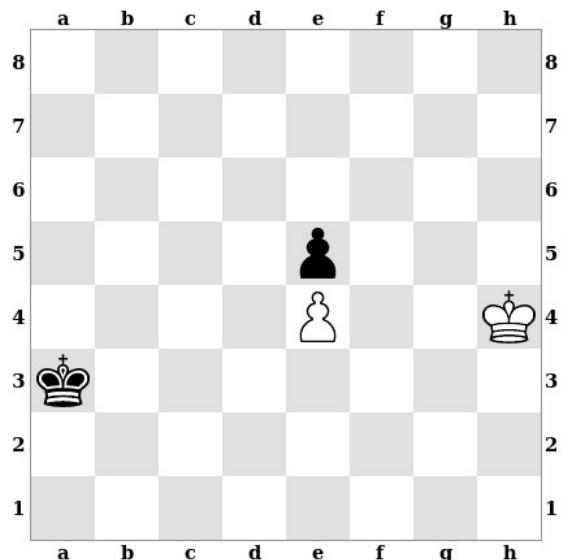
E6.12 (W/B)



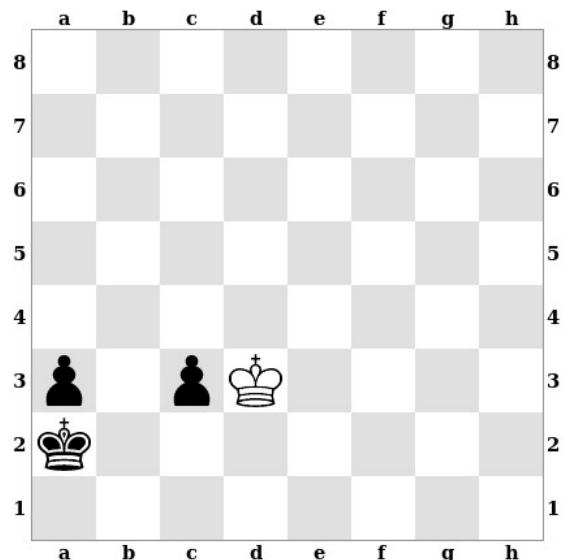
E6.13 (W/B)



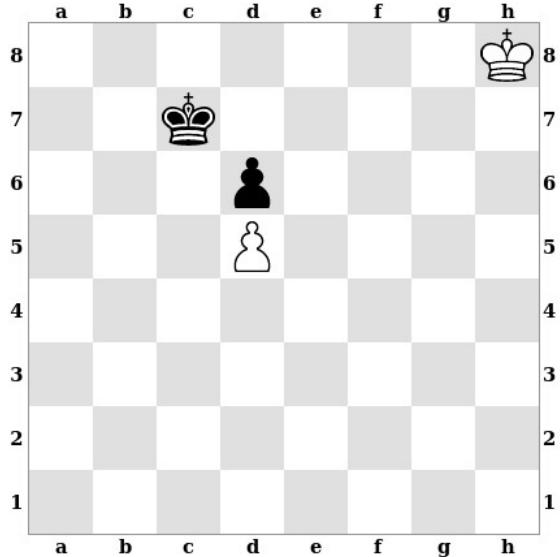
E6.14 (W/B)



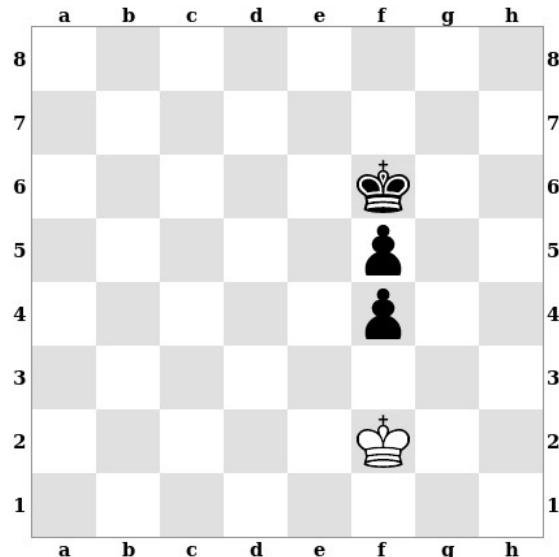
E6.15 (W/B)



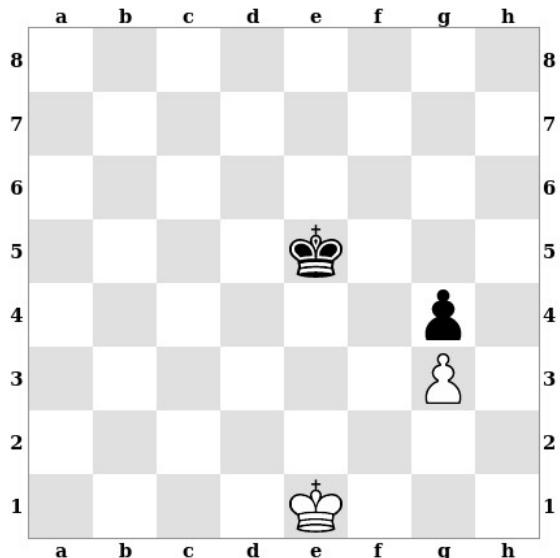
E6.16 (W/B)



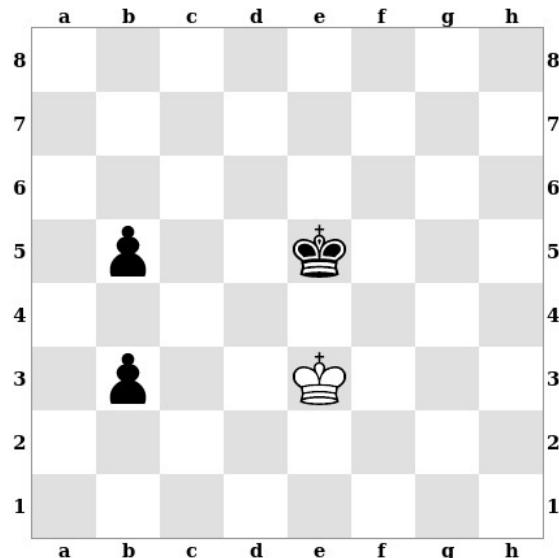
E6.17 (W/B)



E6.18 (W/B)



E6.19 (W/B)



E6.20 (W/B)

## Solutions

### E6.01

(-+) 1...e2#

### E6.02

(-+)

W: 1.Kd1 Ke3 -+ transposes to the line below

B: 1...Kd4 (1...d4? 2.Kd1 Ke3 3.Ke1 d2+ 4.Kd1 =) 2.Kd1 Ke3 3.Ke1 d2+ 4.Kd1 d4 5.Kc2 Ke2 -+

### E6.03

(-+) Black's king is on a critical square of White's pawn and White's pawn is on what will be a key square of Black's pawn.

W: 1.Kd1 Kc3 2.Ke2 Kc2 3.Ke1 Kd3 4.Kf2 Kd2 5.Kf1 Kxe3 6.Ke1 Kd3 7.Kd1 e3 8.Ke1 e2 9.Kf2 Kd2 -+

B: 1...Kb2 2.Kd1 Kc3 -+ transposes to the previous line.

### E6.04

(+-)

W: 1.Kb2 Kf6 (1...Kxe4 2.f6 +-) 2.Kc3 Ke5 3.Kd3 Kf6 4.Kd4 Ke7 5.Ke5 Kf7 6.f6 Kf8 7.Ke6 Ke8 8.f7+ Kf8 9.e5 Kg7 10.Ke7 +-

B: It doesn't change anything for Black to move first. For example, 1...Kf6 2.Kb2 Ke5 3.Kc3 Kf6 4.Kd4 +- transposes to the previous line.

### E6.05

(-+)

W: 1.Kg2 h1=Q+ (1...Kg4 2.Kh1 Kf3? =) 2.Kxh1 Kf3 3.Kg1 g2 4.Kh2 Kf2 -+

B: 1...Kg4 (not 1...Kf3? =) 2.Kg2 h1=Q+ 3.Kxh1 Kf3 -+ transposes to the previous line.

### E6.06

W: (-+) 1.Kf3 Kxd3 2.Kf2 Kc2 3.Ke2 d3+ -+

B: (+-) 1...Kb4 2.Kxd4 Kb5 3.Kd5 +-

### E6.07

W: (-+) 1.Kc2 Ke3 reaching a critical square of White's pawn. 2.Kc1 Kd3 3.Kb2 Kd2 4.Kb1 Kc3 5.Ka2 Kc2 6.Ka1 Kxb3 7.Kb1 Ka3 (7...Kc3 8.Ka2 Kc2 [8...b3+? 9.Ka1 =] 9.Ka1 Kb3 10.Kb1 Ka3 -+) 8.Ka1 b3 9.Kb1 b2 10.Kc2 Ka2 -+

B: (=) 1...Ke4 2.Ke2 Kd4 3.Kd2 =

## E6.08

(+-)

W: 1.g6 (1.Kf5? =; 1.g4+? Kg6 2.Kf3 Kxg5 =) 1...Kxg6 2.Kg4 +-

B: 1...Kg6 2.Kg4 Kg7 3.Kh5 Kh7 4.g6+ Kg7 5.Kg5 Kg8 6.Kh6 Kh8 7.g7+ Kg8 8.g4 Kf7 9.Kh7 +-

## E6.09

W: (+-) 1.Ke5 (1.Ke4?? Kc3 2.Kf3 Kxd3 -+) 1...Kc3 2.Ke4 Kb4 3.Kxd4 Kb5 4.Kd5 +-

B: (-+) 1...Kc2 (1...Kc3?? 2.Ke4 Kb4 3.Kxd4 Kb5 4.Kd5 +-) 2.Ke4 Kc3 3.Kf3 Kxd3 -+

## E6.10

(=)

W: 1.Kxe5 Ke7 =

B: 1...Kf8 (1...Ke7? 2.Kxe5 +-) 2.Kxe5 Ke7 =

## E6.11

(=)

W: 1.Kd4 Kd6 2.Ke4 Ke6

B: 1...Kf7 2.Kf5 Kg8 (2...Kg7? 3.Kxg5 +-) 3.Kxg5 Kg7 =

## E6.12

W: (+-) 1.Kg2 Kb6 2.Kf2 Kc5 3.Ke3 Kd5 4.Kf4 Ke6 5.Kxg4 Kf6 6.Kh5 +-

B: (-+) 1...Kb6 2.Kg2 Kc5 3.Kf2 Kd4 (threatening Kd3) 4.Ke2 Ke4 5.Kf2 Kd3 [critical square] 6.Kf1  
Ke3 7.Kg2 Ke2 8.Kg1 Kf3 9.Kh2 Kf2 10.Kh1 Kxg3 11.Kg1 Kh3 12.Kh1 g3 13.Kg1 g2  
14.Kf2 Kh2 -+

## E6.13

W: (=) 1.Kh3 Kg5 2.Kg3 Kf5 3.Kf3 Ke5 4.Ke3 Kd5 5.Kf4 White reaches a critical square, so he wins  
Black's pawn. However, this isn't enough to win the game. 5...Kd6 6.Ke4 Kd7 7.Kd4 Kc7  
8.Kxc4 Kc6 =

B: (-+) 1...Kh4 (1...Kg4? 2.Kg2 Kf4 3.Kf2 Ke4 4.Ke2 Kf4 5.Kf2 =) 2.Kg2 Kg4 3.Kf2 Kf4 4.Ke2 Ke4  
5.Kd2 Kf3 [critical square] 6.Kd1 Ke3 7.Kc2 Ke2 8.Kc1 Kd3 9.Kb2 Kd2 10.Kb1 Kxc3 -+

## E6.14

(-) Black is winning because he can use his queenside majority to create an outside passed pawn.

W: 1.a5 (other moves are similar to the variation below in which Black moves first) 1...b6 2.a6 b5  
3.Kf2 b4 4.Ke3 Kxg3 5.Kd3 Kxh4 6.Kc4 Kg3 7.Kxb4 h4 8.Kb5 h3 9.Kc6 h2 10.Kb7 h1=Q+  
11.Kxa7 Qc6 12.Kb8 Qxa6 -+

B: 1...b6 (not 1...a5? = or 1...a6?? 2.a5 +-) 2.Kf2 a6 3.Ke3 (or 3.Kg2 b5 4.axb5 axb5 5.Kf2 b4 6.Ke2  
Kxg3 7.Kd3 Kxh4 -+) 3...Kxg3 4.Kd4 Kxh4 5.Kd5 b5 6.a5 Kg3 7.Kc5 h4 8.Kb6 b4 9.Kxa6  
b3 10.Kb7 b2 11.a6 b1=Q+ 12.Kc7 Qa2 13.Kb7 Qxa6+ --

### E6.15

W: (+-) 1.Kg5 Kb4 2.Kf6 Kc5 3.Kxe5 Kc6 4.Ke6 +-

B: (=) 1...Kb4 2.Kg5 Kc5 (2...Kc3? 3.Kf6 Kd4 4.Kf5 Kc5 5.Kxe5 Kc6 6.Ke6 +-) 3.Kf6 Kd6 (3...Kd4? 4.Kf5 +-) 4.Kf5 Kd7 5.Kxe5 Ke7 =

### E6.16

W: (=) 1.Kc2 (1.Kxc3? Kb1 2.Kb3 a2 -+) 1...Ka1 2.Kc1 (2.Kb3? Kb1 +-) 2...c2 3.Kxc2 Ka2 4.Kc1

Kb3 5.Kb1 =

B: (-+) 1...Kb2 +-

### E6.17

W: (+-) 1.Kg7 Kb6 2.Kf6 Kb5 (2...Kc5 3.Ke6 +-) 3.Ke7 Kc5 4.Ke6 Kb6 5.Kxd6 +-

B: (-+) 1...Kb6 2.Kg7 Kc5 3.Kf6 Kxd5 4.Kf5 Kd4 +-

### E6.18

(-+)

W: 1.Ke2 Ke6 2.Kd3 (2.Kf2 Kd5 3.Kf3 Ke5 4.Kf2 Ke4 5.Ke2 f3+ 6.Kf2 Kf4 7.Kf1 Ke3 8.Ke1 f2+ 9.Kf1 f4 10.Kg2 Ke2 -+) 2...Ke5 3.Ke2 Ke4 4.Kf2 f3 5.Kf1 Ke3 6.Ke1 f2+ 7.Kf1 f4 8.Kg2 Ke2 +-

B: 1...Ke6 (1...Ke5? 2.Kf3 Ke6 3.Kxf4 =) 2.Ke2 (2.Kf3 Ke5 3.Kf2 Ke4 4.Ke2 f3+ 5.Kf2 Kf4 6.Kf1 Ke3 7.Ke1 f2+ 8.Kf1 f4 9.Kg2 Ke2 -+) 2...Kd5 3.Kd3 Ke5 4.Ke2 Ke4 5.Kf2 f3 6.Kf1 Ke3 7.Ke1 f2+ 8.Kf1 f4 9.Kg2 Ke2 +-

### E6.19

(=)

W: 1.Kd1 The only move to draw. (1.Kf1? Kd4 2.Ke2 Ke4 3.Kf2 Kd3 -+; 1.Kd2? Kd4 2.Ke2 Ke4 -+; 1.Kf2? Kd4 2.Ke2 Ke4 -+) 1...Kd5 2.Ke1 Kc4 3.Ke2 Kd4 4.Kd2 Ke4 5.Ke2 Kd4 6.Kd2 =

B: 1...Kd5 2.Kd1 Kd4 3.Kd2 Ke4 4.Ke2 =

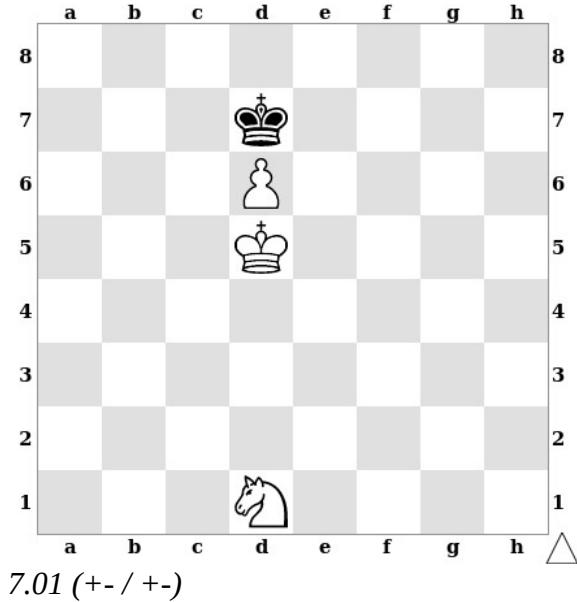
### E6.20

W: (-+) 1.Kd3 b4 (1...Kd5? 2.Kc3 Kc5 3.Kxb3 =) 2.Kd2 (2.Kc4 b2 -+) 2...Kd4 3.Kc1 Kc3 4.Kb1 b2 5.Ka2 b1=Q+ (5...b3+? 6.Kb1 Kc4 7.Kxb2 Kb4 8.Kb1 =) 6.Kxb1 Kb3 7.Ka1 Kc2 8.Ka2 b3+ -+

B: (-+) 1...b2 +-

## 7 – Minor Piece and Pawn Versus King

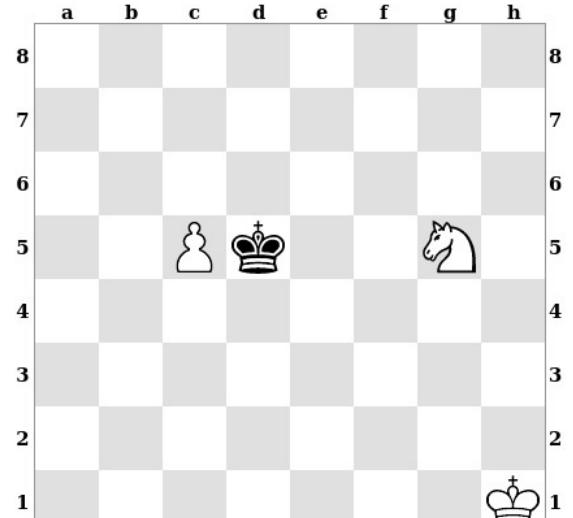
When the stronger side has a minor piece along with his pawn, the position is normally an easy win. However, there are a couple of tricks and surprising positions that are important to know about.



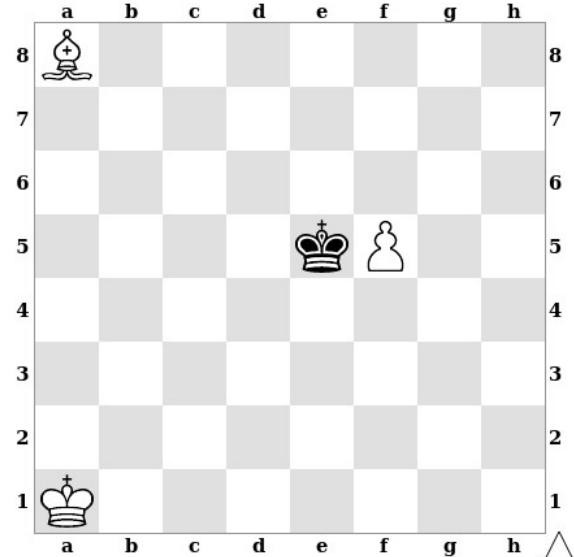
7.01 (+- / +-)

There are many ways for White to win here and you could use whichever seems easiest to you in a game. One instructive way to win, however, is to play out the position as though it were just a pawn endgame, and then use the knight for a single move to break zugzwang. White might play: **1.Ke5 Kd8 2.Ke6 Ke8 3.d7+ Kd8**. This would now be a draw without the knight, since **4.Kd6?** = is stalemate. But White's knight allows him to “skip a turn:” **4.Nf2 Kc7 5.Ke7 +-**. Of course, this trick works with any piece that can make an idle move, not just a knight.

In this position, the pawn is in danger of being lost. White's knight will need to guard it long enough for the king to get into the game, but it apparently doesn't have a safe way to do so. After **1.Ne6?** **Kxe6** = the knight is lost and the pawn can't be saved either. However, White can win with **1.Ne4** when **1...Kxe4** removes Black's king from the pawn's square, allowing White to win with **2.c6 +-**. If Black doesn't capture the knight, then White's king has time to come up and win in the same way as in the previous diagram.

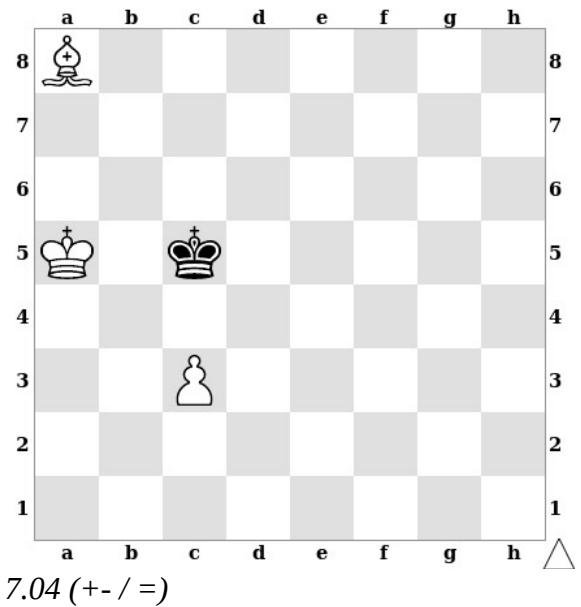


7.02 (+- / =)



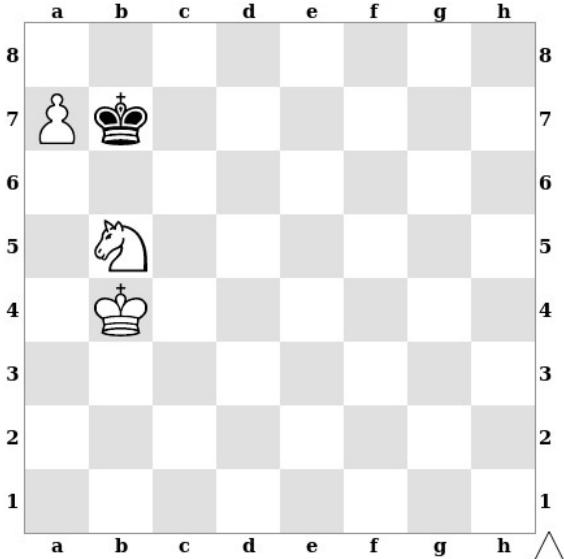
7.03 (+- / =)

A bishop can use the same trick. White can win here, but only with 1.Be4 +-, exploiting the fact that the king can't capture without leaving the pawn's square. Both this and the previous example show that, when guarding a pawn with a piece in this sort of endgame, it is normally better to guard the pawn from behind.



7.04 (+- / =)

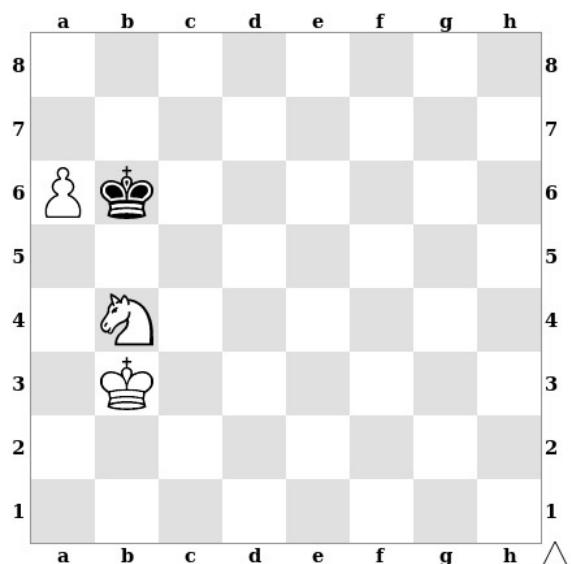
Here Black threatens 1...Kc4 and 2...Kxc3. This is surprisingly difficult to stop because of the awkward placement of White's pieces. However 1.Bd5 is enough to win, since 1...Kxd5 2.Kb5 +- puts White's king on a key square of the pawn.



7.05 (= / =)

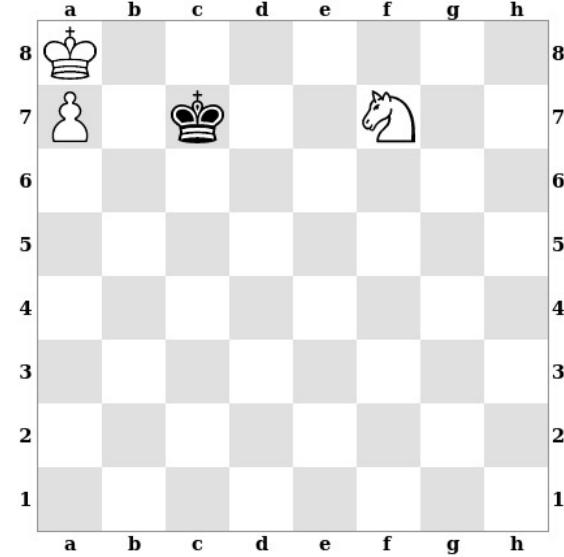
As is true in many endgames, rook pawns increase the chances of a draw. This position is drawn no matter who moves first. Black simply has to keep his king on b7 and a8 and there is nothing White can do. White would love to use the knight to control a8, but this would lose the pawn. Also, if White's king tries to guard the pawn in order to free the knight from its defense, this immediately creates stalemate: **1.Kc5 Ka8 2.Kb6 =.**

This position demonstrates **rule 7.1:** when a king faces a knight and a rook pawn, if the pawn is on the seventh rank and the king has reached the corner, then the position is drawn.



7.06 (+- / +-)

A knight and rook pawn can win, however, as long as the pawn hasn't been pushed all the way to the seventh rank. With the pawn on the sixth rank, White's king can guard the pawn without creating stalemate, at which point White's pieces can drive Black's king away from the corner. For example: **1.Kc4 Ka7 2.Kb5 Ka8 3.Kb6 Kb8 4.Nd5 Ka8 5.Nc7+ Kb8 6.a7+ Kc8 7.a8=Q+ +-.**



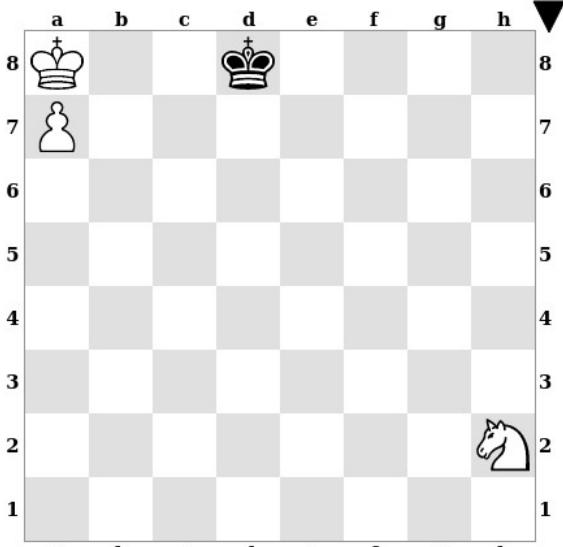
7.07 (+- / =)

This position shows an interesting draw that can occur when the stronger side's king is trapped in front of a seventh rank rook pawn. The outcome is determined by whether or not Black can keep his king on c7 and c8 in order to keep White's king trapped in front of the pawn. If White moves first he wins with **1.Nd6**, when Black's king is forced away from c7 and c8, e.g. after **1...Kxd6 2.Kb8 +-**. If instead Black moves first, he draws by simply shuffling his king between c7 and c8, e.g. **1...Kc8 2.Nd6+ Kc7 3.Nb5+ Kc8 4.Nd4 Kc7 5.Nc6 Kc8** (not **5...Kxc6? 6.Kb8 +-**) **6.Ne7+ Kc7 7.Nd5+ Kc8 =.**

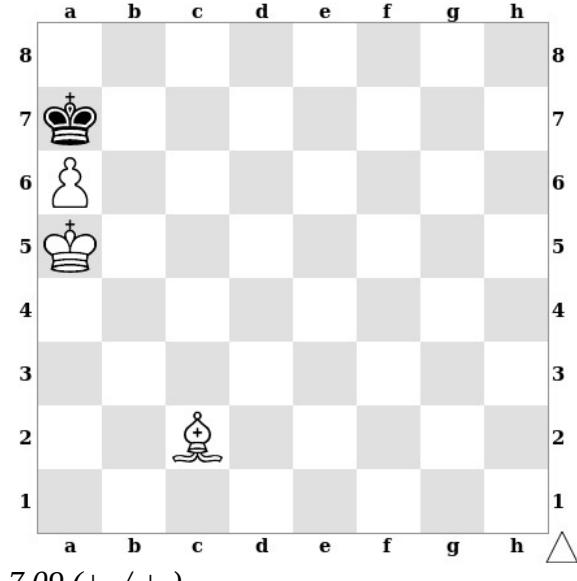
With the knight further away, it becomes difficult to determine through calculation which square Black's king should go to. Luckily, there is a rule ([rule 7.2](#)): when a king faces a knight and a rook pawn, if he has a chance to trap the stronger side's king in front its pawn, he should move to the square of the same color as that occupied by the knight.

Since the knight is on a dark square here, Black's king should go to c7: **1...Kc7 2.Ng4 Kc8 3.Ne5 Kc7 4.Nc6 Kc8 5.Ne7+ Kc7 =.** Instead, Black would lose with the other choice: **1...Kc8? 2.Ng4 Kc7 3.Nf6 Kc8 4.Nd5 +-**.

Two observations about knights are useful in justifying this rule. First, a knight never attacks any squares of the same color as the one it currently stands on. Second, any path that takes a knight from one square to another of the same color will involve an even number of moves.

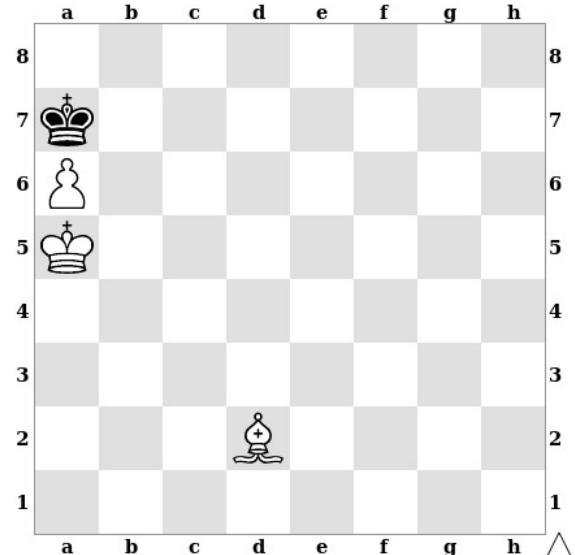


7.08 (+- / =)



7.09 (+- / +-)

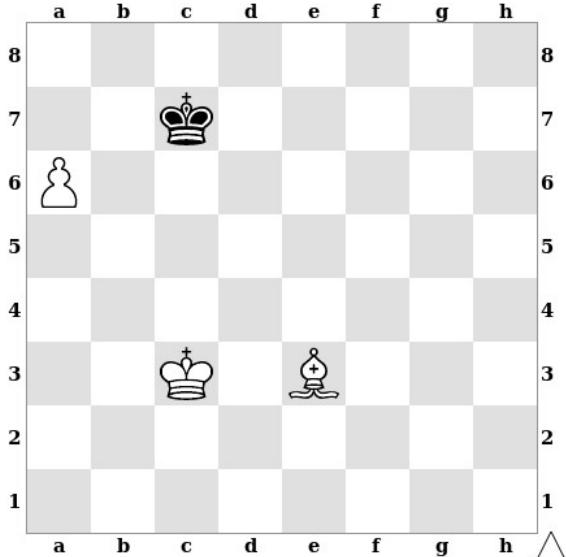
With a bishop and a rook pawn, the result is often determined by what color bishop the stronger side has. When the bishop can control the corner square, the win is usually easy. In this position, White's light-square bishop can control the a8 square so White wins: **1.Be4 Kb8 2.Kb6 Kc8 3.a7 +-.**



7.10 (= / =)

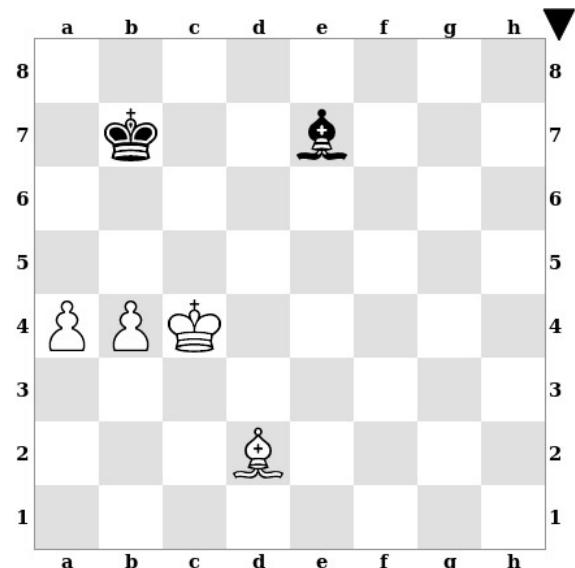
When the bishop is the wrong color to control the corner square, things are different. **Rule 7.3 (rook pawn and wrong-color bishop draw):** when a king faces a bishop and a rook pawn, if the bishop is the wrong color to control the corner square and the defending king has reached the corner, then the position is draw.

That is the case here; White simply has no way to try to drive Black's king from the corner without creating a stalemate in the process. Black just needs to keep his king in the corner: **1.Be3+ Ka8 2.Kb6 Kb8 3.Bf4+ Ka8 4.Be3 Kb8 5.a7+ Ka8 6.Kb5 Kb7 =.**



7.11 (+- / =)

However, even the wrong bishop can win if the defending king can be kept out of the corner. White wins here with 1.Ba7 Kc6 2.Kc4 Kc7 3.Kc5 Kc8 4.Kc6 Kd8 5.Kb7 +-.

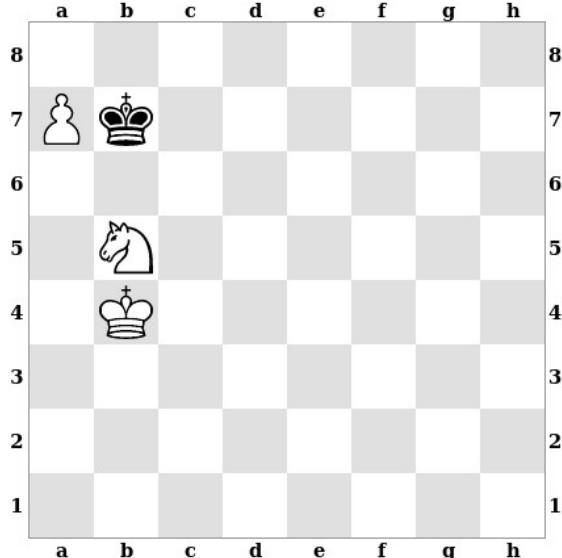


7.12 (+- / =)

The rook pawn and wrong color bishop draw is the basis for many clever ways of saving otherwise losing positions. Here Black would be losing if not for 1...Bxb4 =, when the game is drawn no matter how White responds.

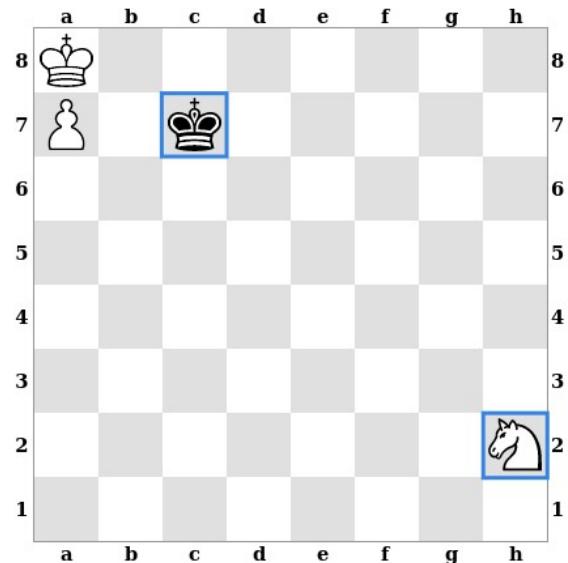
## Conclusion

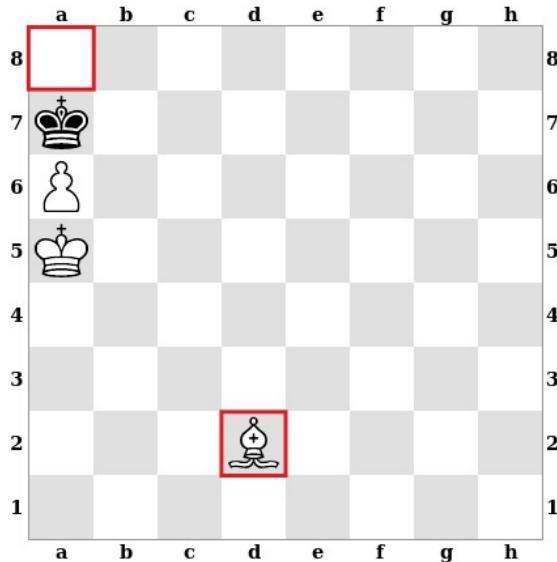
There are three rules from this chapter that you should remember, though the third one is the most important.



**Rule 7.1:** when a king faces a knight and a rook pawn, if the pawn is on the seventh rank and the defending king has reached the corner, then the position is drawn.

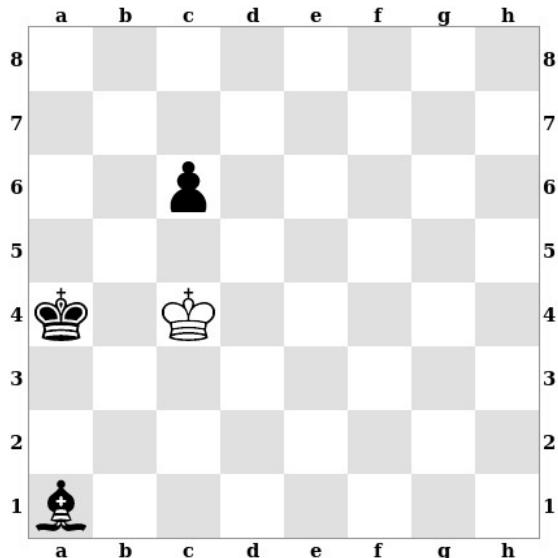
**Rule 7.2:** when a king faces a knight and a rook pawn, if he has a chance to trap the stronger side's king in front its pawn, he should move to the square of the same color as that occupied by the knight.



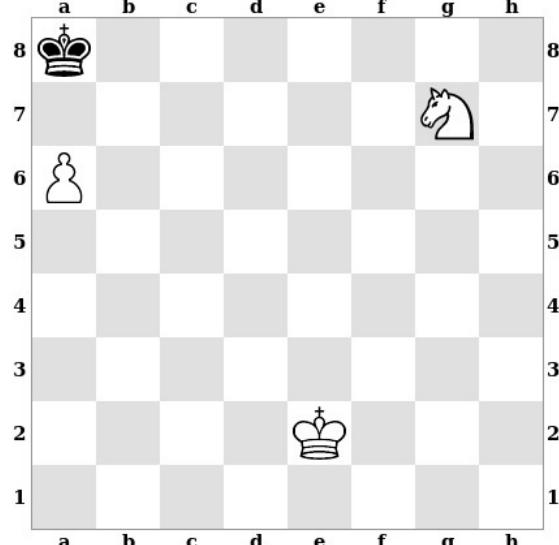


**Rule 7.3 (rook pawn and wrong-color bishop draw):** when a king faces a bishop and a rook pawn, if the bishop is the wrong color to control the corner square and the defending king has reached the corner, then the position is drawn.

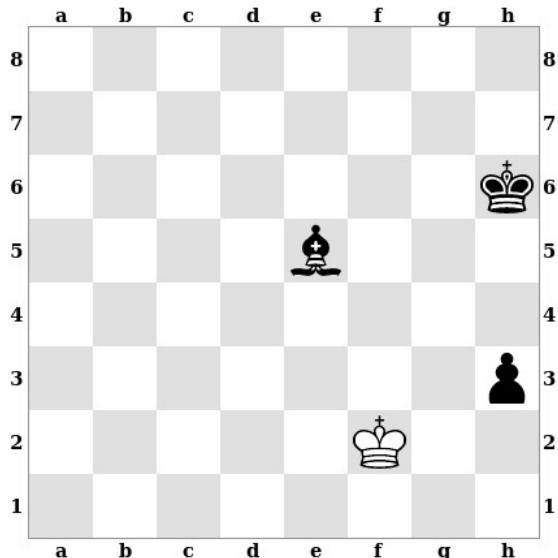
## Exercises



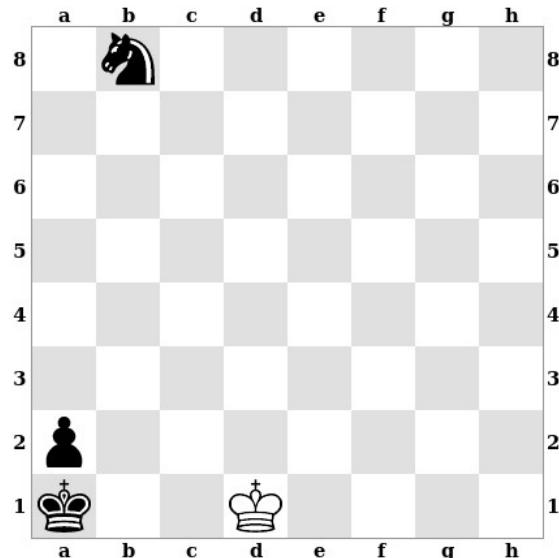
E7.01 (W/B)



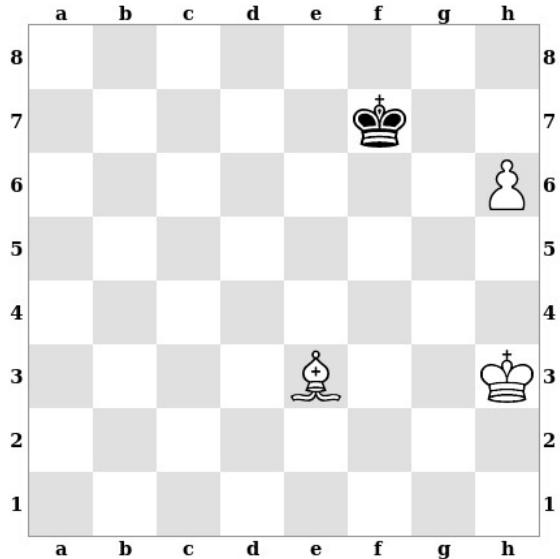
E7.02 (W/B)



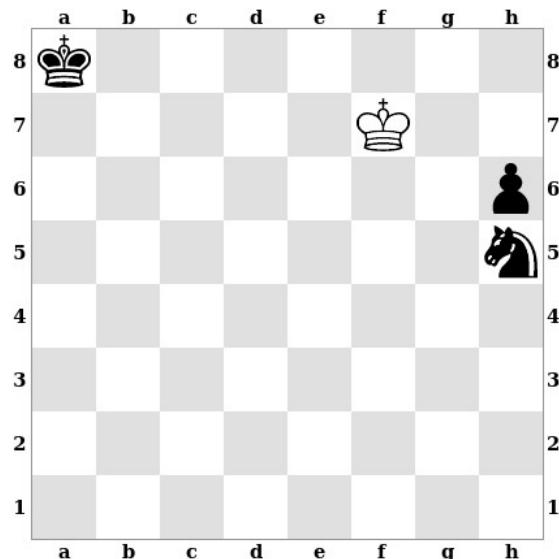
E7.03 (W/B)



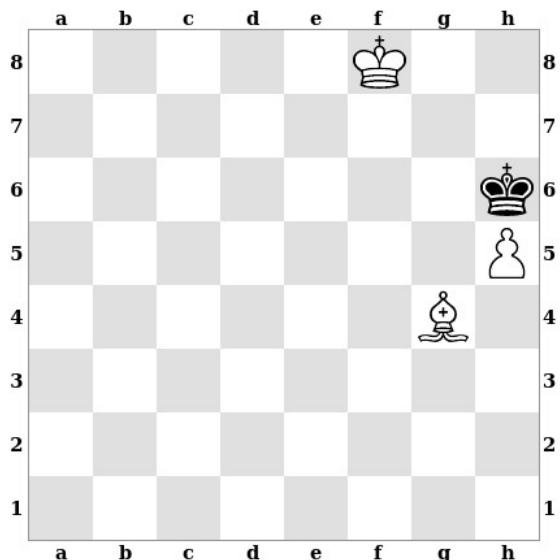
E7.04 (W/B)



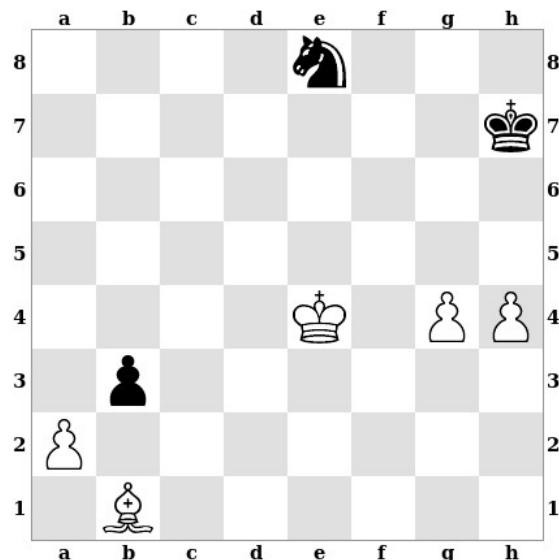
E7.05 (W/B)



E7.06 (W/B)



E7.07 (W/B)



E7.08 (W/B)

## Solutions

### E7.01

W: (=) 1.Kc5, followed by Kxc6

B: (-+) 1...Bd4 2.Kxd4 Kb4 3.Kd3 c5 4.Kc2 Kc4 -+

### E7.02

W: (+-) 1.Ne6 Ka7 2.Nc5 (2.Nc7? Kb6 =) 2...Kb6 3.Kd3 Ka7 4.Kc4 Kb6 5.Kb4 Ka7 6.Kb5 Ka8  
7.Kb6 Kb8 8.Ne6 Ka8 9.Nc7+ Kb8 10.a7+ Kc8 11.a8=Q+ +-

B: (=) 1...Ka7, followed by Kxa6

### E7.03

W: (=) 1.Kg1 is drawn by rule 7.3.

B: (-+) 1...Bh2 2.Kf3 Kg5 3.Kf2 Kg4 4.Kf1 Kg3 5.Ke2 Kg2 -+

### E7.04

W: (=) White draws by applying rule 7.2: 1.Kc1 (1.Kc2? Nc6 2.Kc1 Nd4 3.Kd2 Kb2 -+) 1...Nd7 2.Kc2  
Nb6 3.Kc1 Nd5 4.Kc2 Nc3 5.Kc1 Ne2+ 6.Kc2 Nd4+ 7.Kc1 =

B: (-+) 1...Kb2

### E7.05

(+-) White is winning no matter who moves first since he has the right bishop to control the corner. For example, with Black to move first, play might go 1...Kg6 2.Kg4 Kh7 3.Kg5 Kh8 4.Kg6 Kg8  
5.Bd4 Kf8 6.h7 +-.

### E7.06

W: (=) 1.Kg6

B: (-+) 1...Ng7 (1...Nf6? 2.Kxf6 =) 2.Kg6 h5 3.Kg5 Kb7 -+

### E7.07

W: (+-) 1.Kg8 Kg5 2.Kg7 Kxg4 3.h6 +-

B: (=) 1...Kh7 2.Kf7 Kh8 =, drawn by rule 7.3.

### E7.08

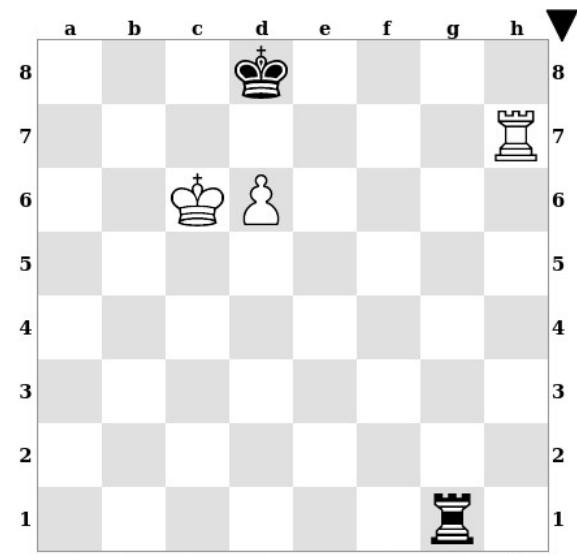
W: (+-) 1.axb3

B: (=) 1...bxa2 (1...Nf6+? 2.Kf4+ Kg7 3.axb3 +-) 2.Bxa2 (2.Kf4+? axb1=Q -+) 2...Nf6+ 3.Kf4 Nxg4  
4.Kxg4 =, drawn by rule 7.3

## 8 – Basic Rook Endings

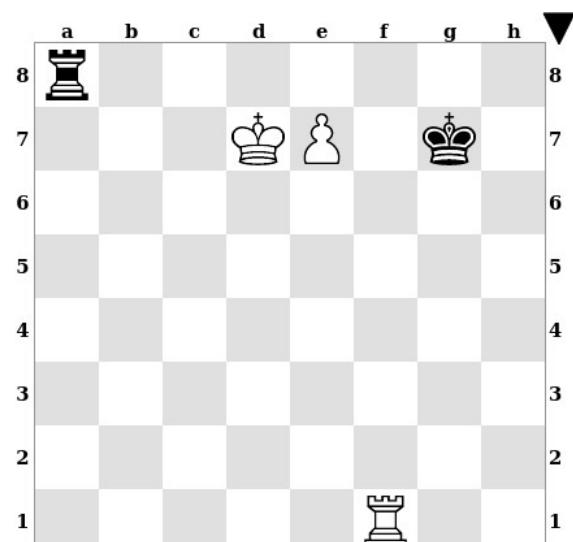
Rook and pawn endings are the most common type of endings and so there is a lot of practical value in learning about them. On the other hand, these endings have quite a lot of theory and trying to learn them in any real depth would quickly become a very ambitious project. For now we will learn about a few of the most important ideas and positions.

### Repeated Checks



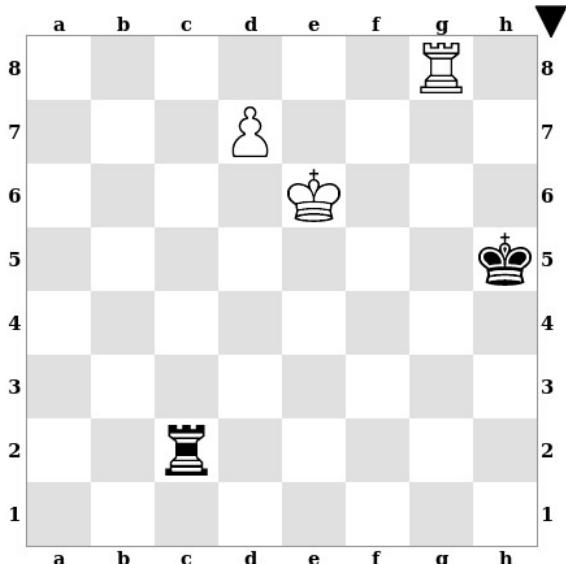
8.01 (+- / =)

The defender can sometimes use his rook to repeatedly check the stronger side's king. In this position White's king has no shelter, so repeated checks work well: 1...Rc1+ 2.Kd5 Rd1+ 3.Ke5 Re1+ 4.Kf5 Rf1+ 5.Kg5. Now that the king is far away, Black can even win the pawn: 5...Rd1 6.Rh6 Kd7 =.



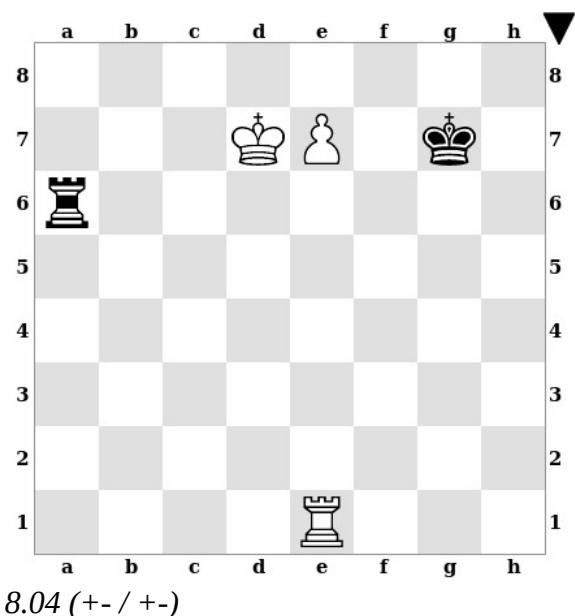
8.02 (+- / =)

Such a series of checks can also come from the side. For example, Black can hold a draw here with 1...Ra7+ 2.Kd6 Ra6+ 3.Kc7 (3.Kc5 Re6 =) 3...Ra7+ 4.Kd8 Ra8+ =.



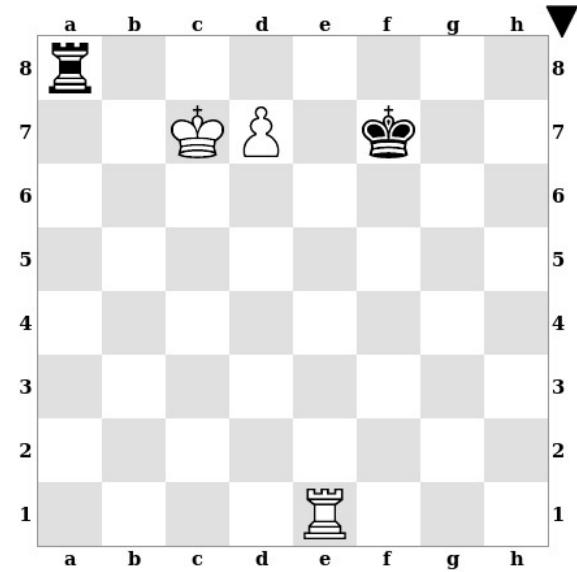
8.03 (+- / +-)

Repeated checks don't usually work if the stronger side's king can safely approach the rook. Here White's rook already guards the promotion square, so White's king is no longer needed near the pawn. When the rook checks start, White's king just "walks down the staircase" and the rook can't keep checking: 1...Re2+ 2.Kf5 Rf2+ 3.Ke4 Re2+ 4.Kf3 Rd2 5.d8=Q +-.



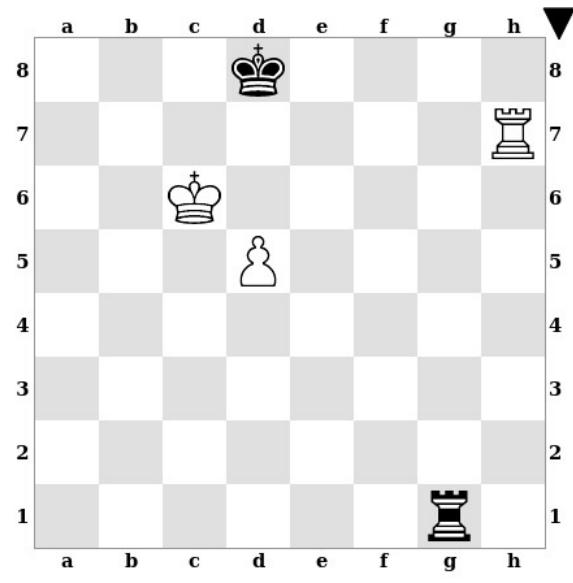
8.04 (+- / +-)

When the stronger side's rook is behind the pawn, it is prepared to control the promotion square if the pawn promotes. White can again approach the rook safely: 1...Ra7+ 2.Kc6 Ra6+ 3.Kb5 Ra8 4.e8=Q +-.



8.05 (+- / +-)

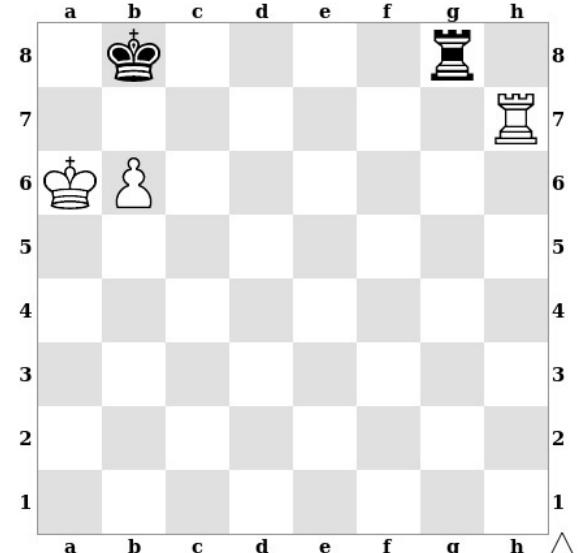
Repeated checks sometimes don't work because the rook doesn't have enough *checking distance*. In this position, Black's rook is too close to White's king for repeated checks to accomplish anything; White wins after 1...Ra7+ 2.Kc8 Ra8+ 3.Kb7 Rd8 4.Kc7 +-.



8.06 (+- / =)

Finally, in some cases repeated checks aren't effective because the king has *shelter* from the checks. Here, after either 1...Rc1+? 2.Kd6 +- or 1...Rg6+? 2.d6 +-, the checks can't continue. We won't look at the full theory of this position yet, but checking with the rook is not the correct approach for Black.

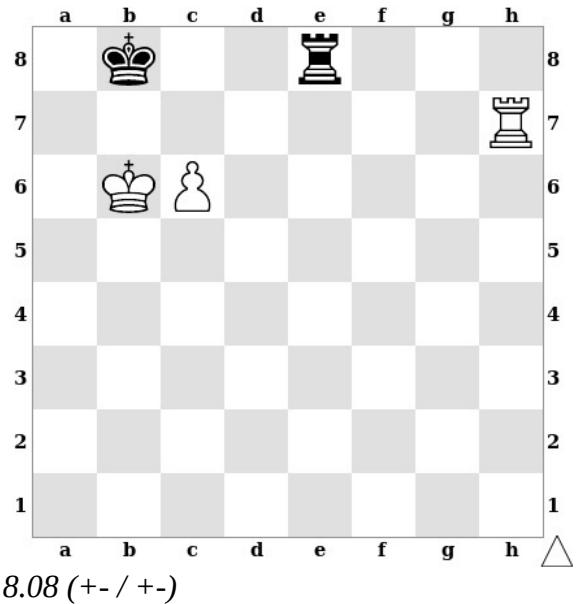
## Passive Defense



8.07 (= / =)

One of the most straightforward defensive strategies is *passive defense*: the weaker side's pieces both stay on the eighth rank, with the king standing roughly in front of the pawn. Keeping the rook on the eighth rank is important; without Black's rook on the eighth rank here, White would win immediately with Rh8+(#).

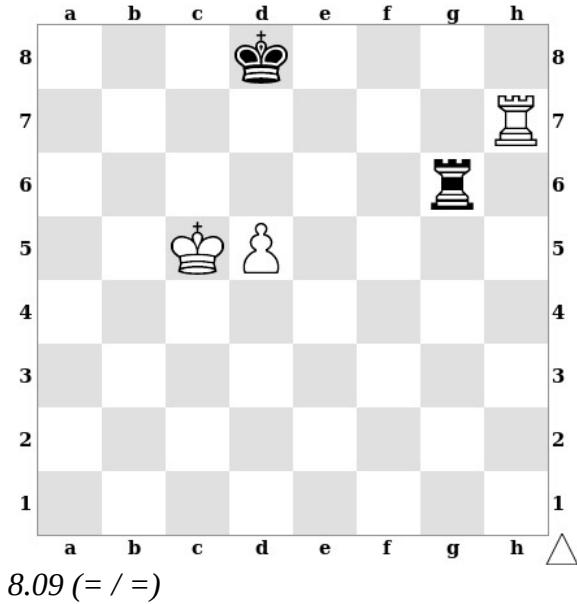
Against a knight pawn or rook pawn, passive defense is an easy and effective way of defending. In this position Black can draw effortlessly, as long as he keeps his king as close to the corner as possible and never removes his rook from the back rank. After **1.Rb7+ Ka8** (but not 1...Kc8? 2.Ka7 +-) **2.Ra7+ Kb8** **3.Rb7+ (3.b7? Rg6+ +-)** **3...Ka8 =**, White isn't getting anywhere.



8.08 (+- / +-)

Unfortunately, passive defense doesn't work against a bishop pawn or center pawn. White wins here by arranging a check on a8 to trade the rooks: **1.Rb7+ Kc8** (1...Ka8 2.Ra7+ Kb8 3.c7+ Kc8 4.Ra8+ +-) **2.Ra7 Kb8** **3.c7+ Kc8** **4.Ra8+ Kd7** **5.Rxe8 Kxe8** **6.Kb7 +-.**

## Philidor's Position

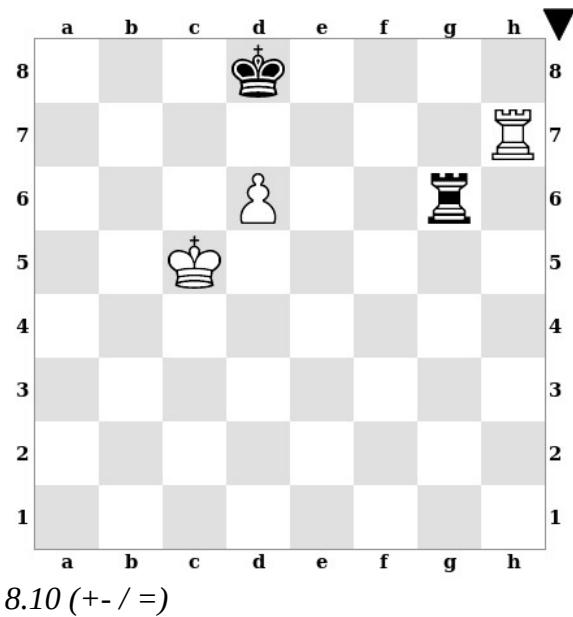


Philidor's position is the most important drawn position in rook endgames. Unlike passive defense, Philidor's position is a reliable draw with any kind of pawn.

The defining characteristics of the position are these: the stronger side's king and pawn are not yet on the sixth rank, the defending king stands in the pawn's path, and his rook occupies the sixth rank, preventing the stronger side's king from moving forward.

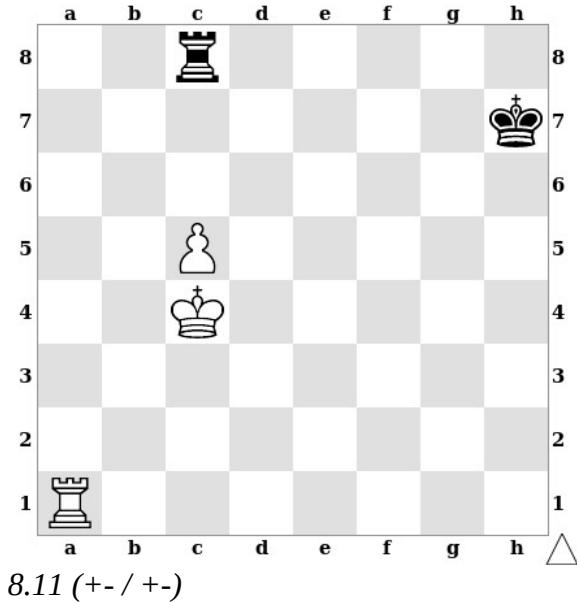
If White does nothing, then Black's rook waits on the sixth rank: 1.Rf7 Rh6 2.Rg7 Rf6 =. Checks also don't accomplish anything: 1.Rh8+ Kd7 2.Rh7+ Kd8 =. Finally, rook trades lead to a drawn pawn ending: 1.Rb7 Rh6 2.Rb6 Rxb6 3.Kxb6 Kd7 =.

White's only other try is to move his pawn: **1.d6**, leading to the next diagram.



Now White's king lacks shelter from rear checks, so Black brings his rook to the first rank: **1...Rg1 =**. There is nothing White can do to escape an endless series of checks, other than to trade into a drawn pawn ending: 2.Rh5 Rc1+ 3.Kd5 Rd1+ 4.Ke6 Re1+ 5.Re5 Rxе5+ 6.Kxе5 Kd7 =.

## Cutting Off the King



One important idea for the stronger side is to *cut off* the weaker side's king away from the action. The value of doing this depends on a number of factors, such as how many files the king is cut off by. Here White wins easily with **1.Rg1**, when Black's king is trapped a long ways away from the pawn. Black can only free the king by offering a rook trade, but **1...Rg8 2.Rxg8 Kxg8 +-** is a hopeless pawn ending.

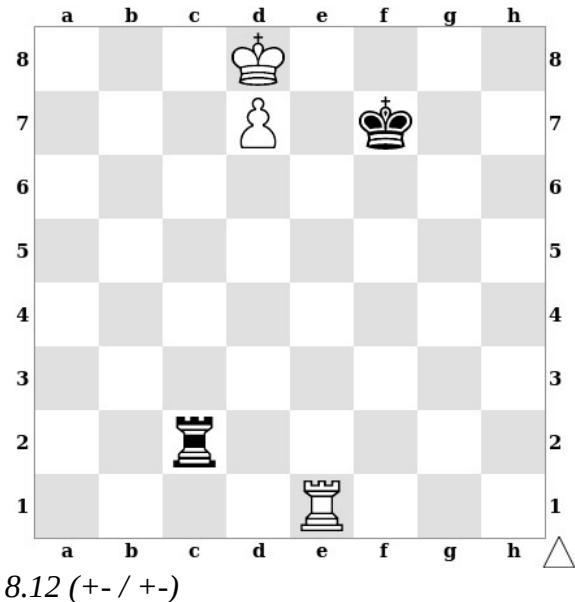
With Black's king unable to participate, White's king easily makes progress against Black's rook: **1...Kh6 2.Kd5 Rd8+ 3.Kc6 Rc8+ 4.Kd6 Rd8+ 5.Kc7 Rd2 6.c6 Rc2 7.Kd7 Rd2+ 8.Kc8 Rc2 9.c7 Rb2**. Now White brings his rook back to create shelter on the d-file: **10.Rd1 Kg7 11.Kd7 +-**.

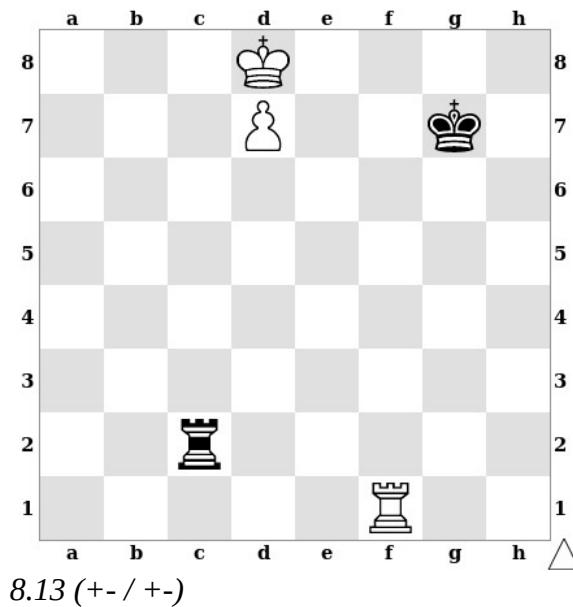
When the king cannot be cut off as far, things become more complicated.

## The Lucena Position

This is the Lucena position, the main winning position for the stronger side in rook endgames. The defining characteristics of the Lucena position are these: the pawn is any non-rook pawn on the seventh rank, the stronger side's king is in front of it, and the weaker side's king is cut off by one file.

To promote his pawn White first needs to get his king out, so he starts with **1.Rf1+**, driving Black's king away. Now **1...Ke6** creates shelter for White's king, allowing White to play **2.Ke8 +-** and promote the pawn. Black's more challenging responses are **1...Kg7** and **1...Kg6**, which we will examine in turn.

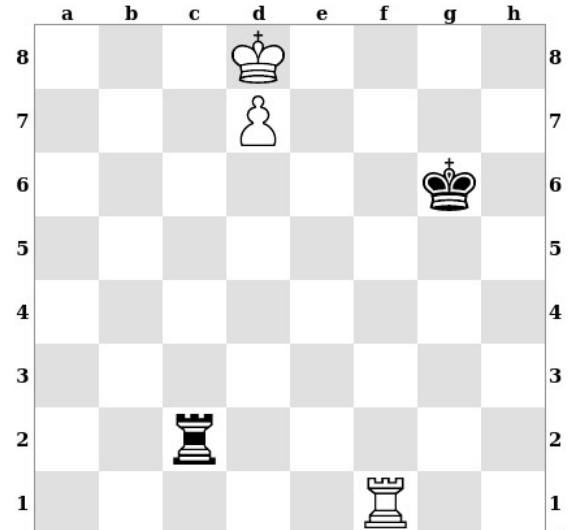




First we will look at 1...Kg7 (D). Now White would like to move his king out, but then Black can start checking. Walking down the stairs doesn't work here because White's rook doesn't control the promotion square: 2.Ke7 Re2+ 3.Kd6 Rd2+ 4.Kc6 Rc2+ 5.Kb5 Rd2 6.Kc6 Rc1+ 7.Kb7 Rd2 etc.

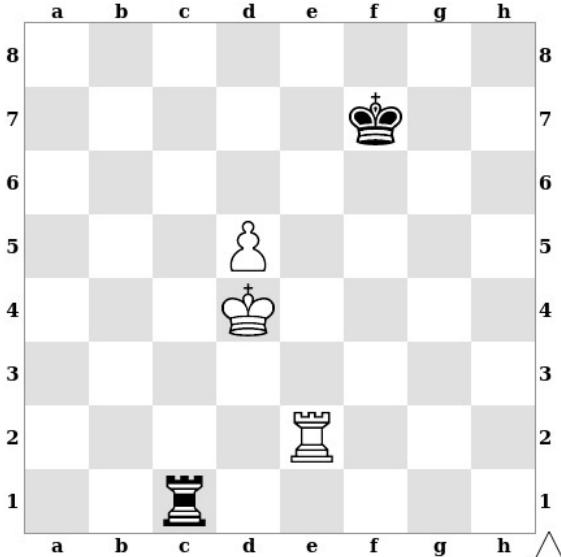
Instead, White wins by "building a bridge:" 2.Rf4 Rc1 (1...Kg6 2.Ke8 Re2+ 3.Kf8 Rd2 4.Rf7 +- transposes to the analysis of 1...Kg6 below) 3.Ke7 Re1+ 4.Kd6 Rd1+ 5.Ke6 Re1+ 6.Kd5 Rd1+ 7.Rd4 +-, when the pawn promotes.

Black could also play 5...Rd2, but then White has 6.Rf5 +- followed by Rd5. Another try for Black is 5...Kg6, when 6.Rf5? Rxd7 = doesn't work. However, White has 6.Rf8, guarding the promotion square, followed by walking down the staircase after 5...Re1+ 6.Kd6 Rd1+ 7.Kc6 Rc1+ 8.Kb5 Rb1+ 9.Ka4 +-.



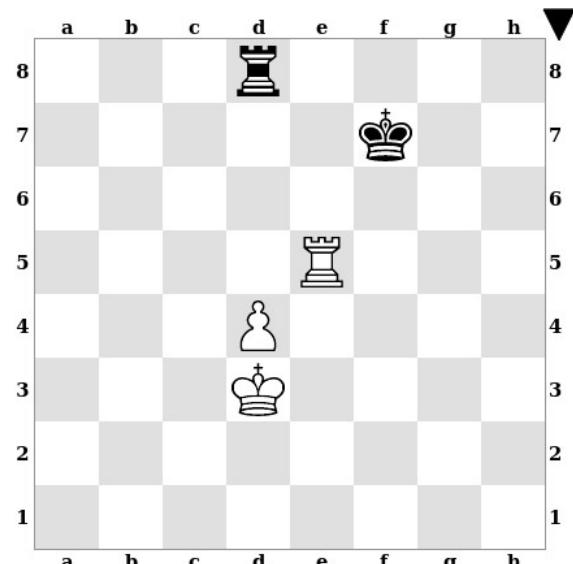
Black can instead respond to 1.Rg7+ with 1...Kg6 (D), with the idea of harassing White's rook when it tries to build a bridge. White can still play 2.Rf4, but this is more complicated than after 1...Kg7.

An easier way to win here is to play 2.Ke8 Re2+ 3.Kf8 Rd2 4.Rf7 +-, with the idea of 5.Ke8, 6.Re7 and 7.d8=Q.



8.15 (+- / +-)

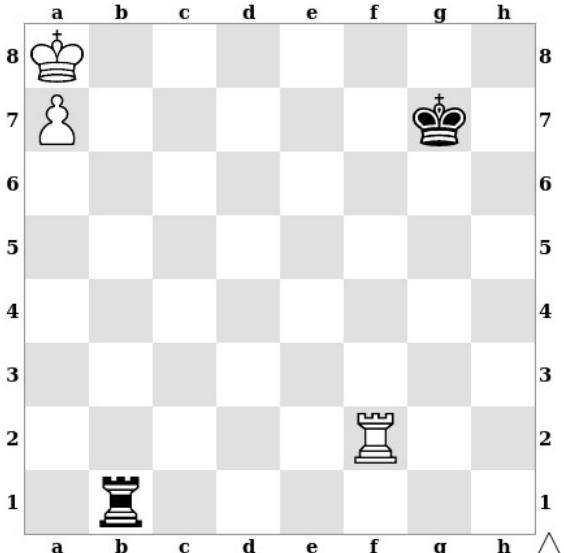
Reaching a Lucena position is the standard way for the stronger side to win. In this position, there is nothing Black can do to stop this: **1.d6 Rd1+ (1...Rc8 2.Kd5 Re8 3.Rxe8 Kxe8 4.Ke6 +-)** **2.Kc5 Rc1+ 3.Kb6 Rd1 4.Kc7 Rc1+ 5.Kd8 Rd1** (after 5...Ra1 6.d7 Ra8+ 7.Kc7 +-, Black's rook lacks checking distance) **6.d7 Rc1 7.Rf2+ +-**, with a Lucena position.



8.16 (= / =)

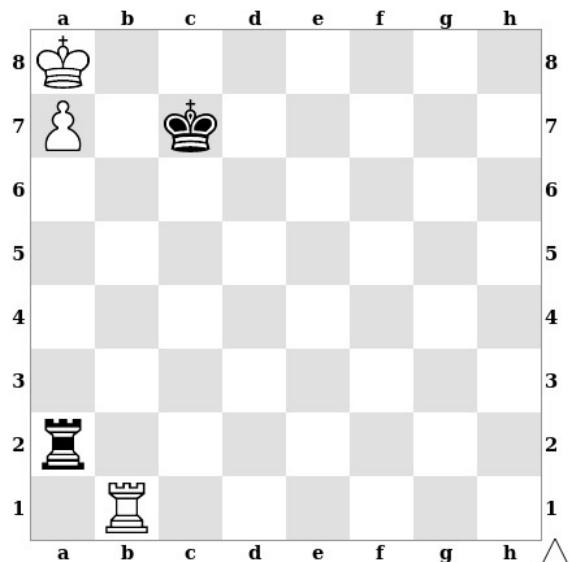
In other cases, the defender can draw by reaching a Philidor position. Here Black draws with **1...Re8 2.Rh5 (2.Rxe8 Kxe8 3.Kc4 Kd8 =)** **2...Ke7 3.d5 Kd7 4.Kd4 Rg8 5.Kc5** (5.Rh6 tries to stop Black's rook from occupying the sixth rank, but White's king already lacks shelter from checks, so Black can play 5...Rg1, and meet 6.Rh7+ with 6...Kd6 =) **5...Rg6 6.Rh7+ Kd8 =**, with a Philidor.

## Rook Pawn Positions



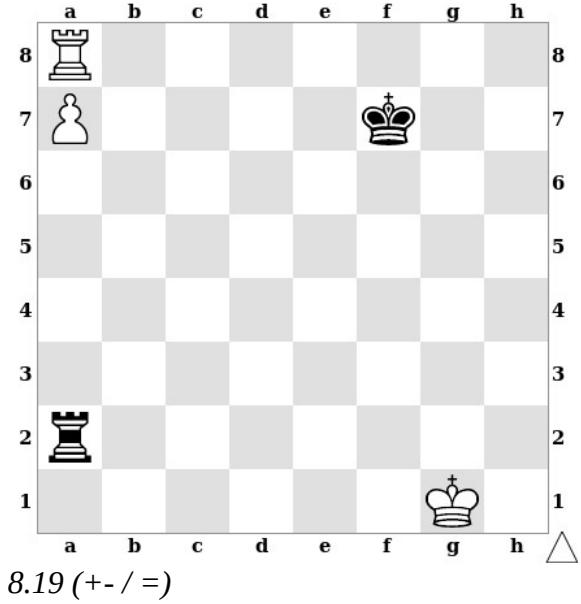
8.17 (+- / +-)

With a rook pawn, the winning chances go down since the king only has one file that it can move to. However, a win is sometimes possible if the defender's king is cut off a long way from the pawn. Here White wins by bringing his rook over to challenge Black's rook and release his king: **1.Rc2 Kf7 2.Rc8 Ke7 3.Rb8 Ra1 4.Kb7 Rb1+ 5.Kc6 +-**, followed by walking down the staircase.



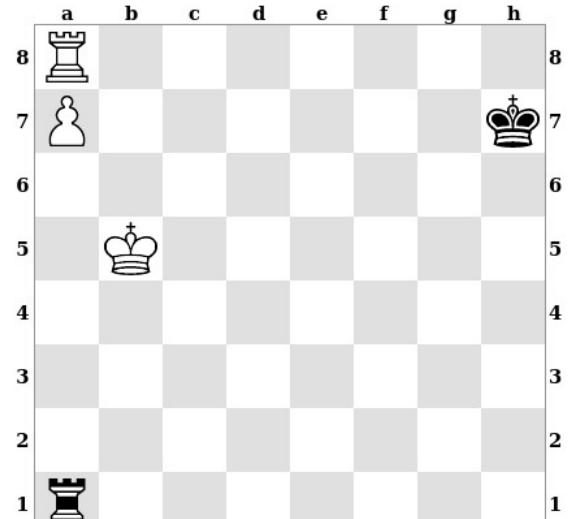
8.18 (= / =)

Positions with the defending king nearby are typically drawn. White's best try here begins **1.Rc1+ Kd7**. Now bringing White's king out leads nowhere: **2.Kb7 Rb2+ 3.Ka6 Ra2+ 4.Kb6 Rb2+ 5.Kc5 Ra2=**. Instead, White might try to build a bridge with **2.Rc4**, but then Black's rook takes control of the b-file with **2...Rb2**. White's last try is to bring his rook to the b-file to drive Black's rook away. However, **3.Rh4** allows Black's king back with **3...Kc7**. Now after **4.Rh7+ Kc8 5.Rh8+ Kc7 6.Rb8**, Black's rook must abandon the b-file, but this hardly matters with Black's king so well-placed. Black can really stop any further trouble with **6...Rc1=**, when White will never again drive the Black king off of the c-file. As long as Black keeps his king on c7 and c8, White can do nothing more.



Another important position with a rook pawn features the stronger side's rook standing in front of the pawn, instead of his king. With the pawn on the seventh rank, the position is drawn if Black's king occupies g7 or h7. If White moves first here then he wins, since Black's king hasn't reached g7 or h7 yet: **1.Rh8 Rxa7** (or 1...Kg7 2.a8=Q +-) **2.Rh7+ +-**, winning Black's rook with a skewer.

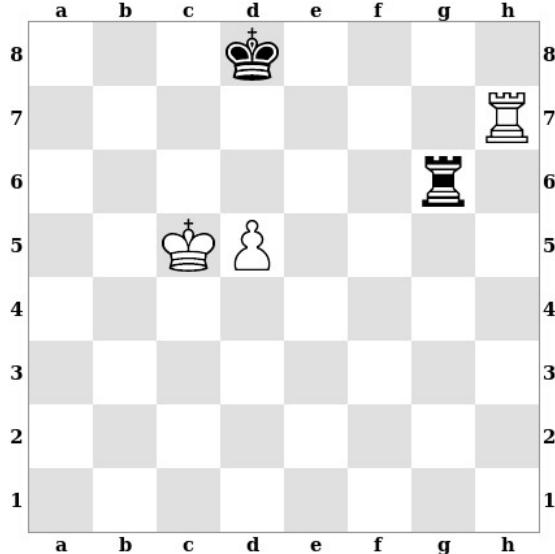
If Black moves first, he can draw with 1...Kg7. Now the skewer trick is no longer available to White and Black will simply move his king back and forth between g7 and h7. White's only winning try is to bring his king up to support the pawn, as in the next diagram (which he might reach by **2.Kf1 Kh7 3.Ke1 Kg7 4.Kd1 Kh7 5.Kc1 Kg7 6.Kb1 Ra4 7.Kb2 Kh7 8.Kb3 Ra1 9.Kb4 Kg7 10.Kb5 Kh7**).



So far Black has simply been moving his king back and forth between g7 and h7, but once White's king is guarding the pawn Black must start checking: **11.Kb6 Rb1+ 12.Ka6 Ra1+ 13.Kb7 Rb1+ 14.Kc7**. Now that White's king isn't guarding the pawn, the rook can simply return to its position behind the pawn: **14...Ra1 =**.

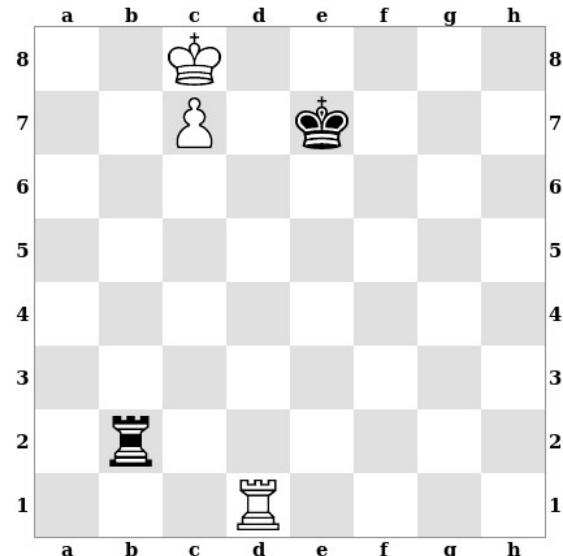
## Conclusion

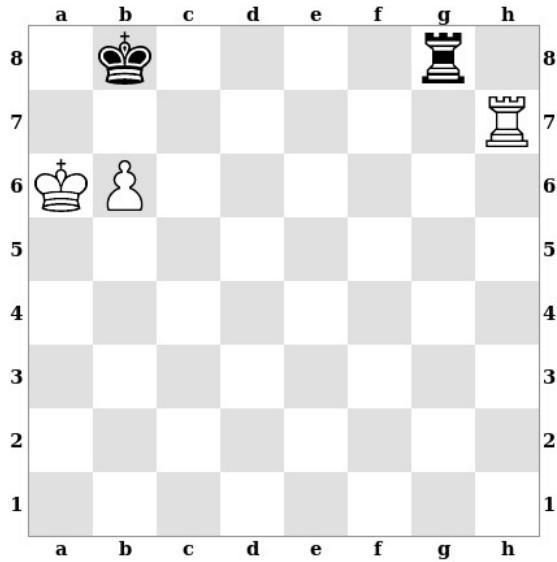
There are a few important things that you should remember from this chapter. The two most important, by far, are Philidor's position and the Lucena position.



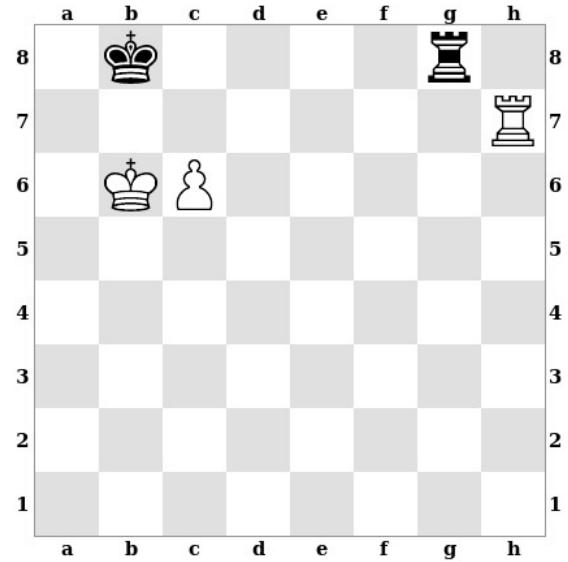
Philidor's position is a draw. It is characterized by the defending king standing in the pawn's path, the stronger side's king and pawn not having reached the sixth rank, and the defender's rook occupying the sixth rank. The pawn can be on any file.

The Lucena position is winning for the stronger side. Its characteristic features are as follows: the pawn is any non-rook pawn on the seventh rank, the stronger side's king is in front of it, and the weaker side's king is cut off by one file.

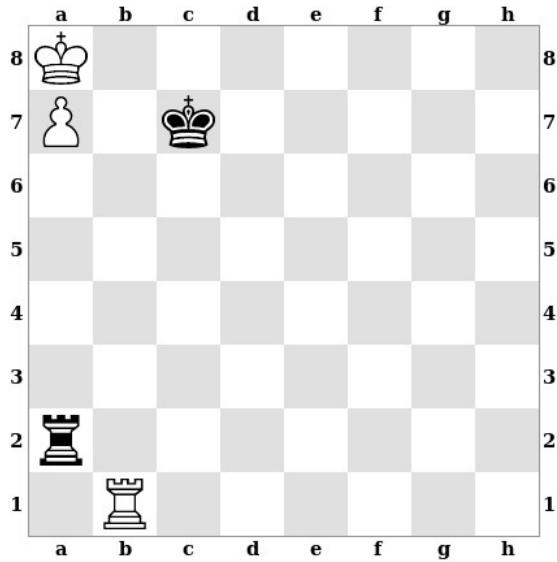




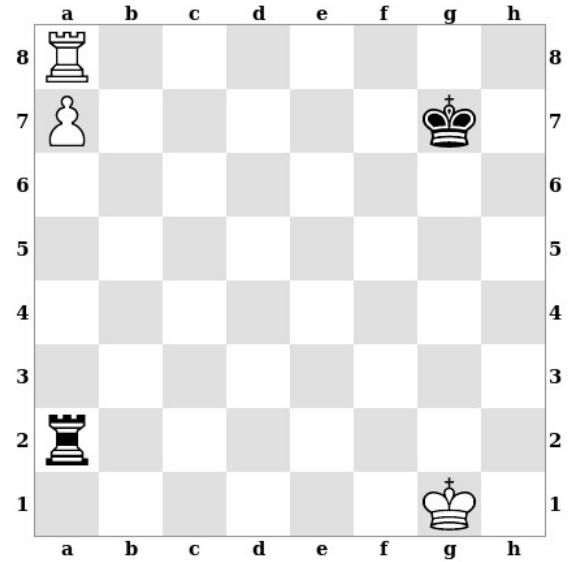
Passive defense (keeping the king and rook on the back rank) draws against a rook pawn or knight pawn.  
This is an easy and reliable defense.



Passive defense loses against a bishop pawn or center pawn.

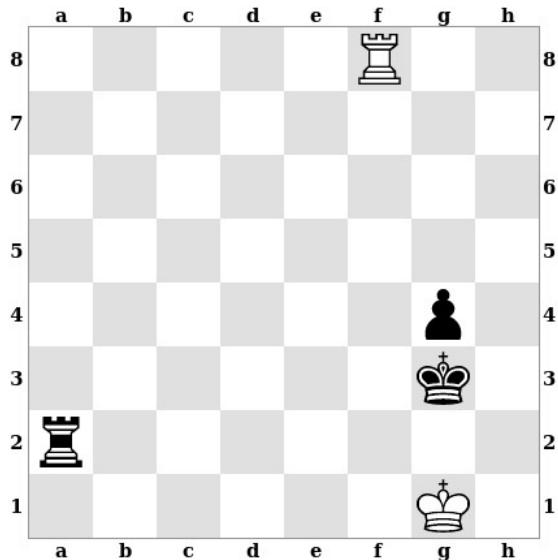


With a rook pawn, a Lucena-style position is only drawn. The weaker side's king needs to be cut off by a number of files in order for a win to be possible.

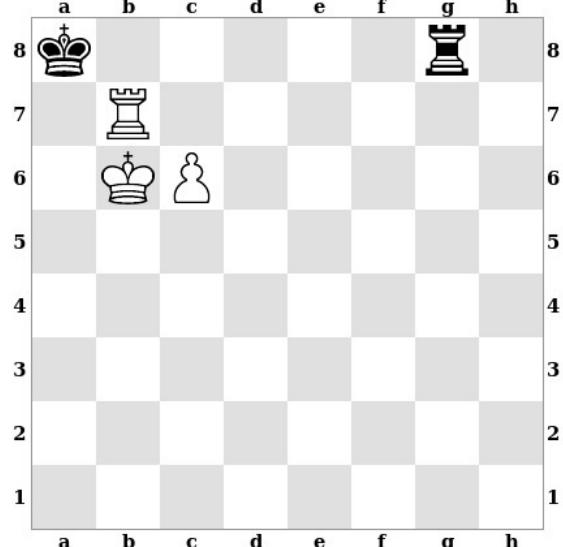


With the stronger side's rook in front of a rook pawn on the seventh rank, the position is drawn if the defending king is on g7/h7 (for a white pawn on a7).

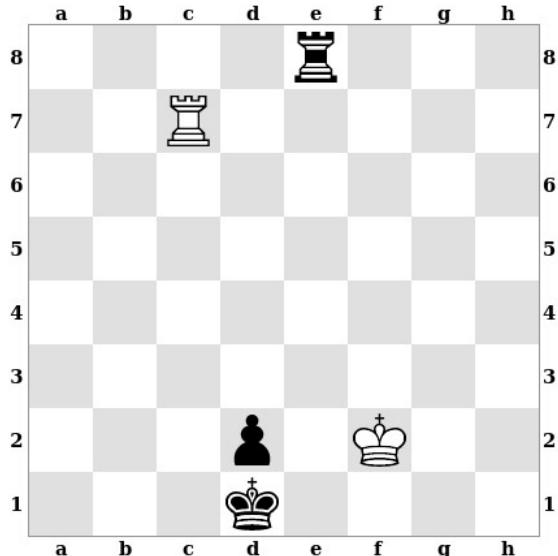
## Exercises



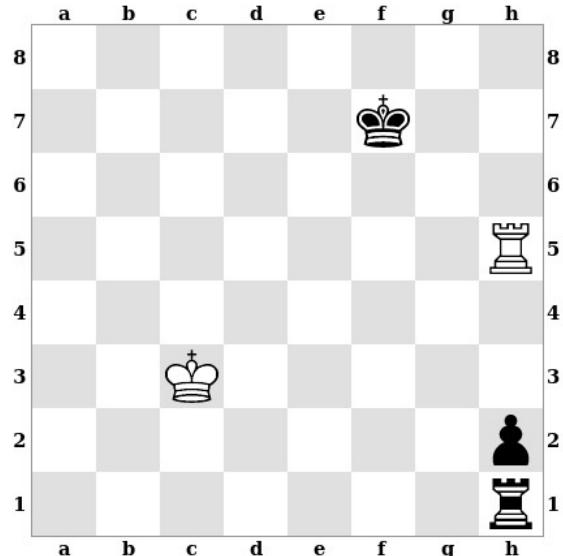
E8.01 (W/B)



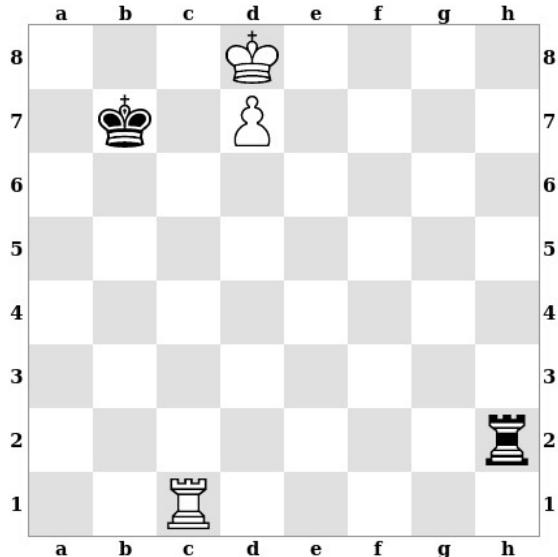
E8.02 (W/B)



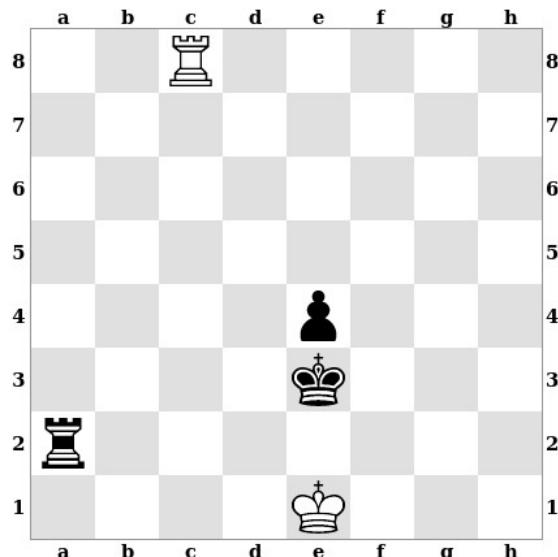
E8.03 (W/B)



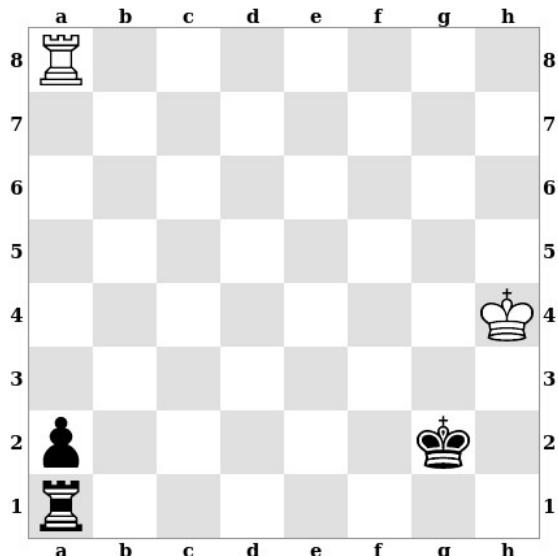
E8.04 (W/B)



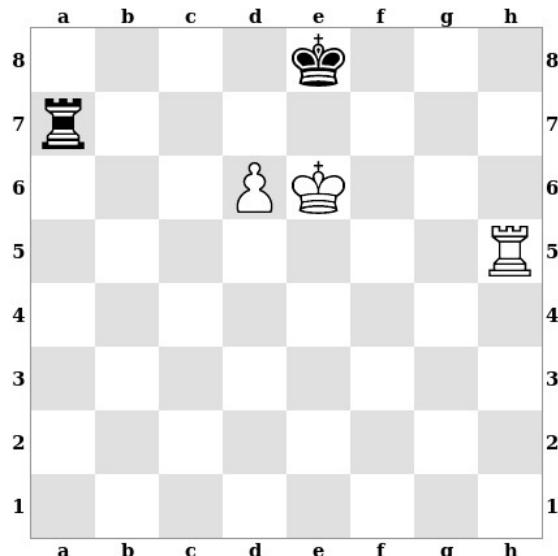
E8.05 (W/B)



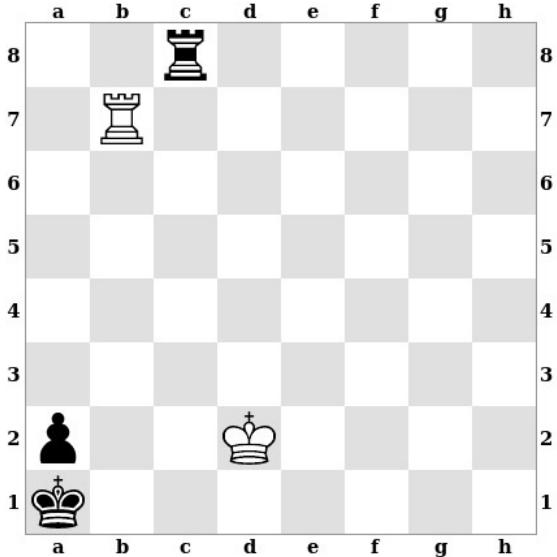
E8.06 (W/B)



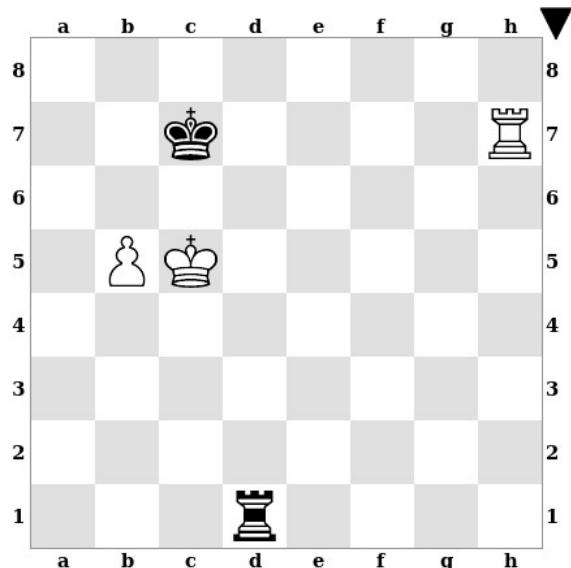
E8.07 (W/B)



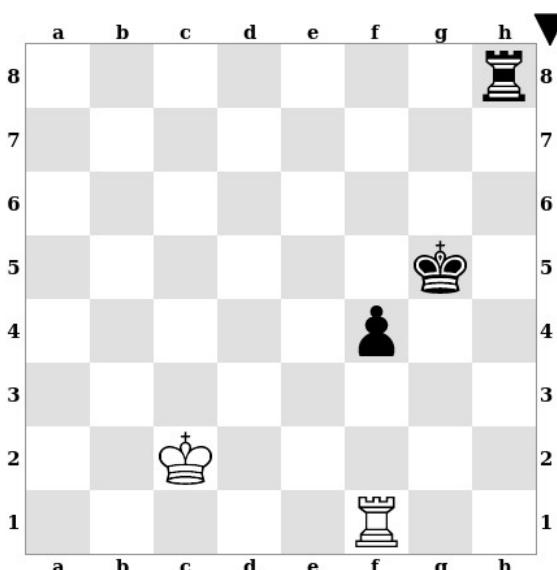
E8.08 (W/B)



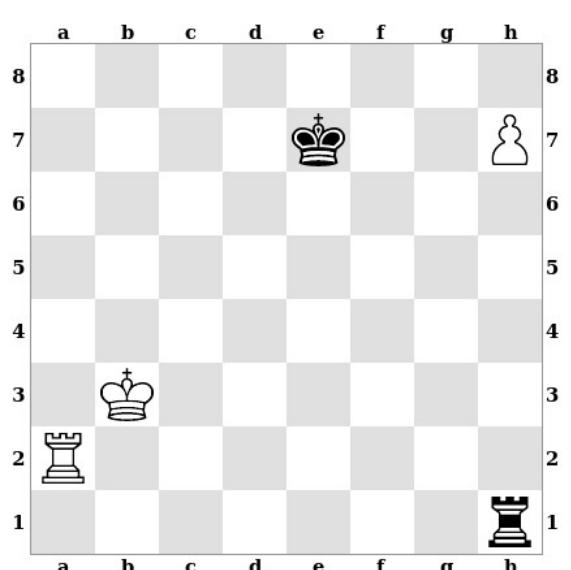
E8.09 (W/B)



E8.10 (B)



E8.11 (B)



E8.12 (W/B)

## Solutions

### E8.01

**W:** (=) White is too late to get a Philidor, but because the pawn is a knight pawn he can draw with passive defense. **1.Rf1 Kh3 2.Rb1 g3 3.Rc1 Rg2+ 4.Kh1 Rh2+ 5.Kg1 =**

**B:** (-+) **1...Ra1+ 2.Rf1 Rxf1+ 3.Kxf1 Kh2 -+**

### E8.02

**W:** (-+) Black's passive defense doesn't work against White's bishop pawn. **1.Ra7+ Kb8 2.c7+ Kc8 3.Ra8+ Kd7 4.Rxg8**

**B:** (=) **1...Rb8 2.Kc7 (2.Rxb8+ Kxb8 3.c7+ Kc8 = ; 2.Ka6 Rxb7 3.cxb7+ Kb8 =) 2...Rc8+ 3.Kd7 Rxc6 =**

### E8.03

(-+) (Lucena position)

**W:** Against most moves, Black wins the same way as in the variation below. After 1.Rf7, Black has **1...Rc8 -+, followed by ...Kc2.**

**B:** **1...Rf8+ 2.Kg2 Rf5 3.Rc8 (3.Kg3 Ke2 4.Re7+ Kf1 5.Rd7 Rf2 6.Rd8 Ke1 7.Re8+ Re2 -+) 3...Ke2 4.Re8+ Kd3 5.Rd8+ Ke3 6.Kg3 (6.Re8+ Kd4 7.Rd8+ Rd5 -+ ; 6.Rd7 Rf4 7.Kg3 Rd4 -+) 6...Rf1 7.Re8+ Kd3 8.Rd8+ Kc3 9.Rc8+ Kb4 10.Rb8+ Ka5 -+**

### E8.04

**W:** (=) **1.Kb2 (1.Kd2? Ra1 2.Rxh2 Ra2+ -+; 1.Rh7+? Kg6 2.Rh8 Rc1+ 3.Kd2 h1=Q -+) 1...Kg6 2.Rh8 Kg5 3.Ka2 Kg4 4.Kb2 Kg3 5.Rg8+ Kh3 6.Rh8+ Kg2 7.Rg8+ Kf2 8.Rh8 =**

**B:** (-+) **1...Rc1+ 2.Kb2 h1=Q -+**

### E8.05

This is nearly a Lucena, but Black's rook is ready to start a strong series of checks against White's king.

**W:** (-+) **1.Rb1+ Ka7 2.Rb4 Rc2 (2...Rh8+ 3.Kc7 +-) 3.Ke7 Re2+ 4.Kd6 Rd2+ 5.Ke6 Ka6 6.Rb8 Re2+ 7.Kf5 Rf2+ 8.Kg4 +-**

**B:** (=) **1...Rh8+ 2.Ke7 Rh7+ 3.Ke6 Rh6+ 4.Kf5 Rd6 =**

### E8.06

**W:** (=) White can draw by getting a Philidor after **1.Rc3+ (but he loses after 1.Rc1?, since passive defense doesn't work against a center pawn: 1...Kf3 2.Rb1 e3 3.Rc1 Rh2 4.Kd1 Rh1+ 5.Kc2 Rxc1+ 6.Kxc1 Kf2 -+) 1...Kf4 2.Rb3 e3 3.Rb8 Kf3 4.Rf8+ Ke4 5.Re8+ Kd3 6.Rd8+ =**

**B:** (-+) **1...Ra1+ 2.Rc1 Rxc1#**

### E8.07

W: (=) White can draw by keeping his own king sheltered by Black's king. 1.Kg4 Kf2 2.Kf4 Ke2 3.Ke4 Kd2 4.Kd4 Kc2 5.Kc4 Kb2 6.Rb8+ Ka3 7.Ra8+ Kb2 8.Rb8+ Kc1 9.Ra8 Kc2 10.Ra7 Kd1 11.Kd3 Kc1 12.Kc3 =  
B: (-+) 1...Rh1+ 2.Kg4 a1=Q -+

### E8.08

W: (+-) 1.Rh8#  
B: (=) 1...Re7+ 2.Kd5 Re1 3.Kc6 Rc1+ 4.Kb6 Kd7 =

### E8.09

(=)  
W: 1.Rg6 = is not meaningfully different than the line with Black to move first.  
B: 1...Rh8 2.Kc2 Rh2+ 3.Kc1 Rh1+ 4.Kc2 Rb1 5.Rc7 Rb2+ 6.Kc1 Rb1+ 7.Kc2 =

### E8.10

(=) Black can only draw with passive defense. 1...Kb8 2.Rh8+ Kb7 3.Rh7+ Kb8 4.Kc6 Rd8 =

### E8.11

(-+) Black can win in a number of ways, but the easiest is to cut off White's king. 1...Rd8 2.Rg1+ Kh4 3.Rf1 Kg3 4.Rg1+ Kf2 5.Rg7 f3 6.Rf7 Kg2 7.Rg7+ Kf1 8.Rf7 f2. Black now has a Lucena position. 9.Rg7 Rd5 10.Rg8 Ke2 11.Re8+ Kf3 12.Rf8+ Ke3 13.Re8+ Kf4 14.Rf8+ Rf5 -+

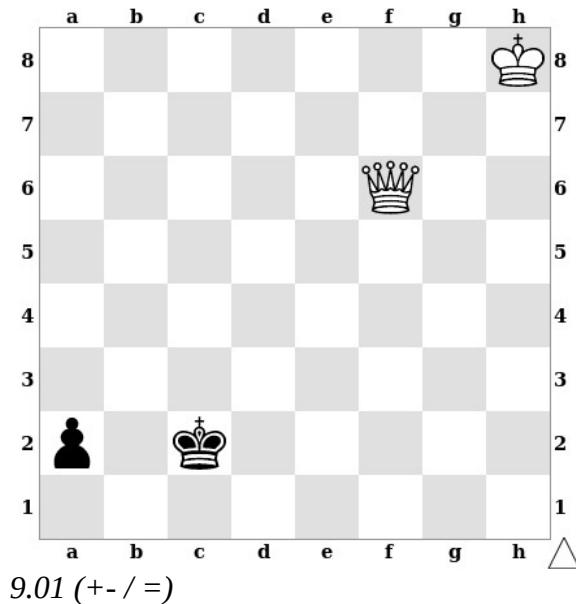
### E8.12

W: (+-) 1.Ra8 Rxh7 (1...Kf7 2.h8=Q +-) 2.Ra7+ Kf8 3.Rxh7 +-  
B: (=) 1...Kf7 2.Ra7+ Kg6 =

## 9 – Piece Versus Pawn(s)

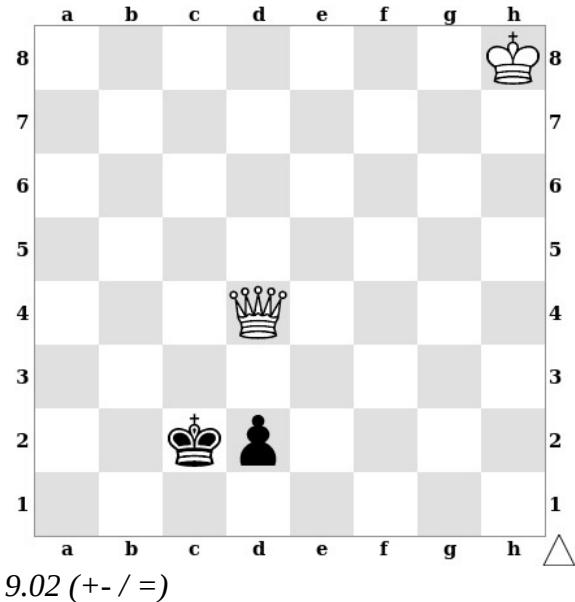
In this chapter, we will look at different struggles between a piece and one or more pawns. In some cases the side with the piece is trying to win, while in others he is only hoping to draw.

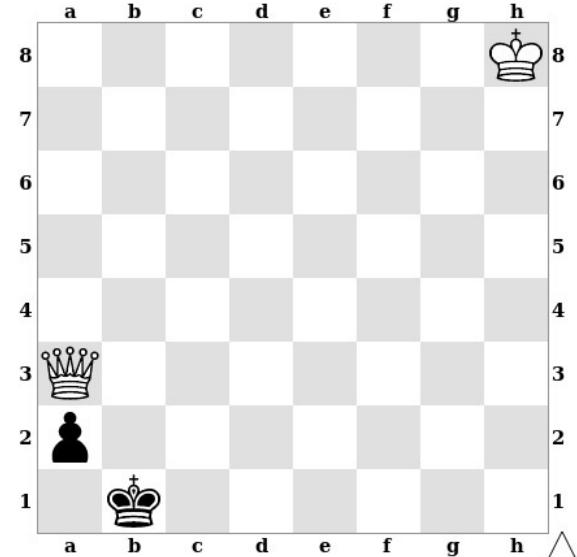
### Queen Versus Pawn(s)



A queen very often wins against a pawn. This is easy if the queen can occupy a square in the pawn's path, as White can here after **1.Qa1 Kb3 2.Kg7 +-**. White simply brings his king in and uses both his pieces to drive Black's king away and win the pawn.

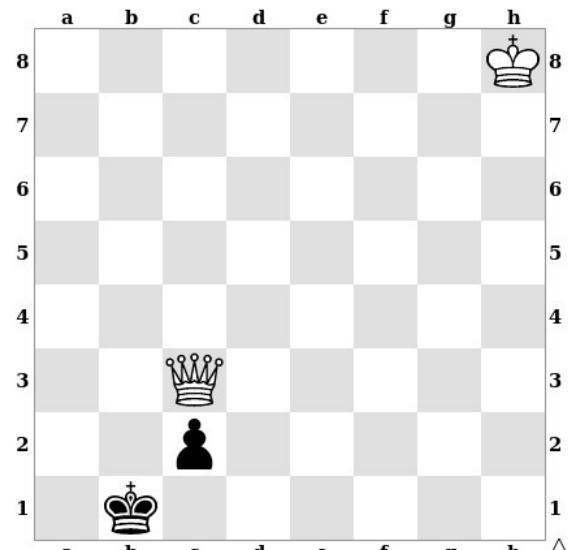
With the pawn on the seventh rank and the queen unable to stand in its path, things become more complicated. However, if the pawn is a center pawn or knight pawn, then the stronger side should win. To win here, White must first force Black's king to block his own pawn: **1.Qc4+ Kb2 2.Qd3 Kc1 3.Qc3+ Kd1**. Now White's king has a free move to come closer with **4.Kg7**. White now repeats this process until his king is close enough to help win the pawn: **4...Ke2 5.Qc2 Ke1 6.Qe4+ Kf2 7.Qd3 Ke1 8.Qe3+ Kd1 9.Kf6 Kc2 10.Qe2 Kc1 11.Qc4+ Kb2 12.Qd3 Kc1 13.Qc3+ Kd1 14.Ke5 Ke2 15.Qc2 Ke1 16.Qe4+ Kf2 17.Qd3 Ke1 18.Qe3+ Kd1 19.Kd4 Kc2 20.Qc3+ Kd1 21.Ke3 Ke1 22.Qxd2+ +-.**





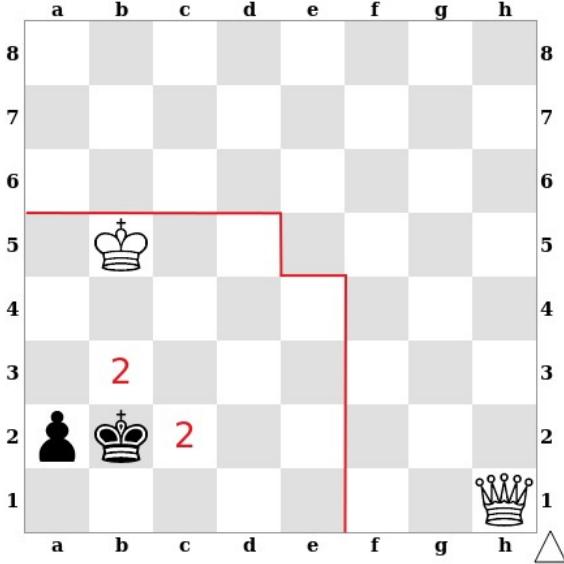
9.03 (= / =)

Positions with a rook pawn are often drawn, as this example shows after **1.Qb3+ Ka1 =.** Now 2.Kg7 is just stalemate and more checks don't accomplish anything.  
White can't make any progress.



9.04 (= / =)

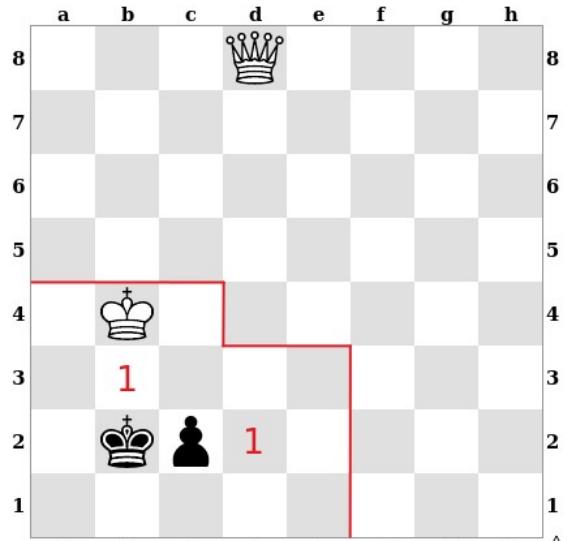
More surprisingly, positions with a bishop pawn are also frequently drawn, once again because of stalemate issues. Here, after **1.Qb3+ Ka1 =,** 2.Qxc2 is stalemate and 2.Kg7 allows 2...c1=Q. As a result, White has to keep checking which never gets him anywhere.



9.05 (+- / =)

However, positions with rook and bishop pawns are not always drawn. If the stronger side's king is close enough to begin with, he can win.

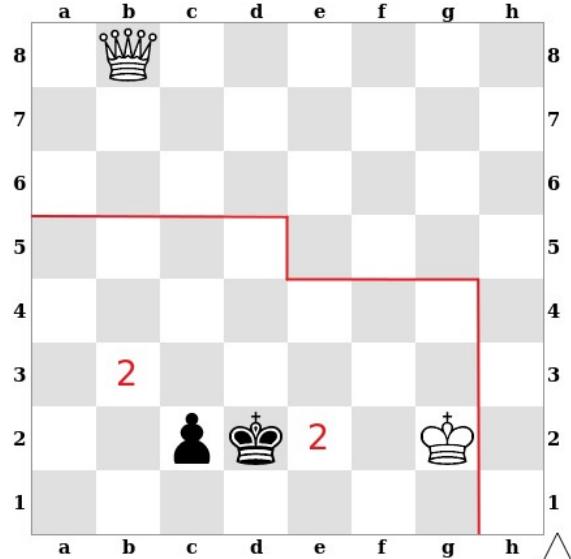
With a rook pawn there is a single winning zone, shown here, that the stronger side's king must be inside of to win. The zone consists of all of the squares that let White's king reach b3 or c2 in no more than two moves. White's king is just inside the zone, so he wins: **1.Qg2+ Kb1** (or 1...Kb3 2.Qg7 Kc2 3.Qa1 +-) **2.Kb4 a1=Q 3.Kb3** +-, when checkmate can't be stopped.



9.06 (+- / =)

With a bishop pawn there are two different winning zones, depending on which side of the pawn the weaker side's king is on. The weaker side is better off with his king closer to the corner since this is where it needs to be to arrange stalemate. As a result, this winning zone is the smaller of the two.

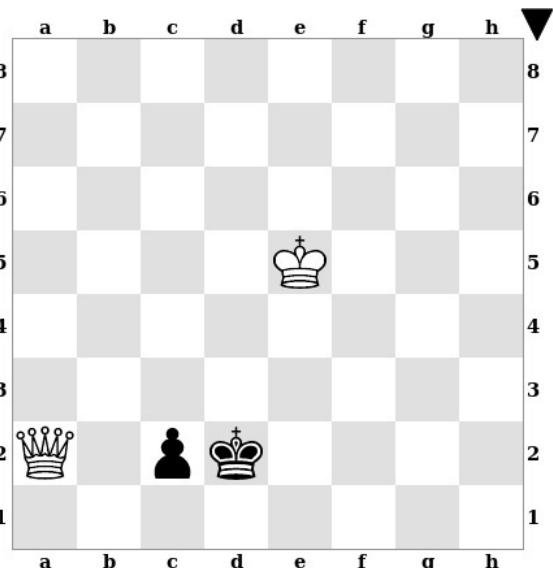
White's king is close enough to be inside the winning zone here, and he wins after **1.Qd2 Kb1** **2.Kb3 c1=N+ 3.Ka3** +-, when mate is coming soon.



9.07 (+- / =)

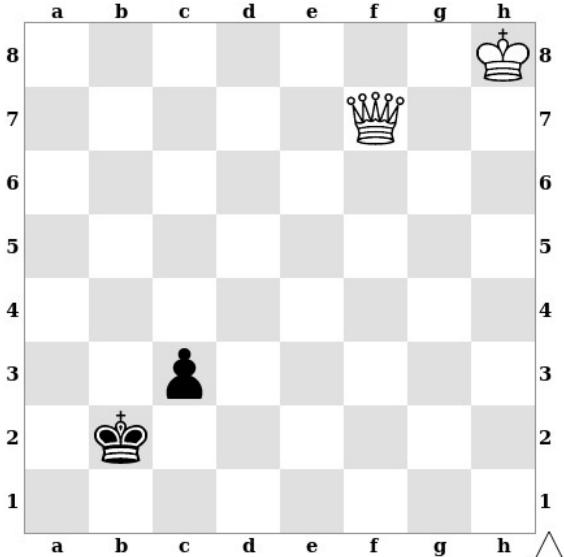
It is worse for the defender to have his king on the other side of a bishop pawn since it will take him longer to set up a stalemate. As a result, the winning zone is larger in this case.

White's king is close enough to win here and the game might end **1.Qb2 Kd1 2.Kf2 Kd2 (2...c1=Q 3.Qe2#) 3.Qd4+ Kc1 4.Qb4 Kd1 5.Qe1#.**



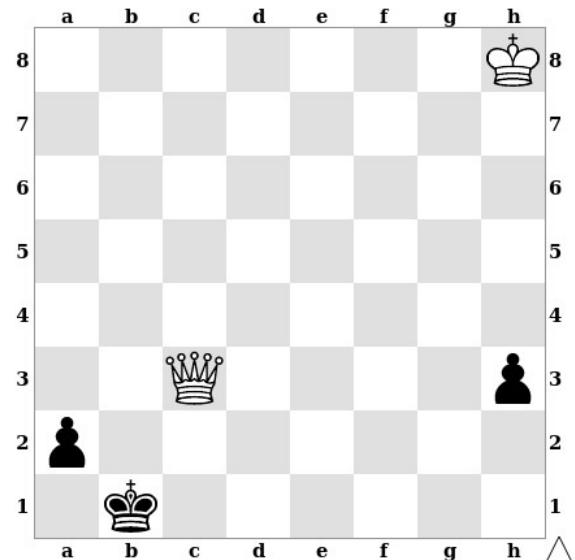
9.08 (+- / =)

It is sometimes necessary for the defender to play a counterintuitive move for concrete reasons. Black can draw if he moves first here because White's king is not yet in the winning zone. However, the obvious move **1...Kd1?** loses to **2.Kd4 c1=Q 3.Kd3 +-** when Black can't stop checkmate. Instead, Black draws with the surprising **1...Kc3.** Importantly, White can't get his queen to c1 in response, which would be the obvious punishment for Black's unusual move. After **2.Qa1+ Kd2 3.Qb2,** Black should now play the more standard **3...Kd1 =.** The difference is that now White doesn't have time for **4.Kd4 c1=Q 5.Kd3? -+** because his queen is hanging. Also, after **3...Kd3?** White would have **4.Qc1 +-,** which wasn't available after **1...Kc3.**



9.09 (+-/-=)

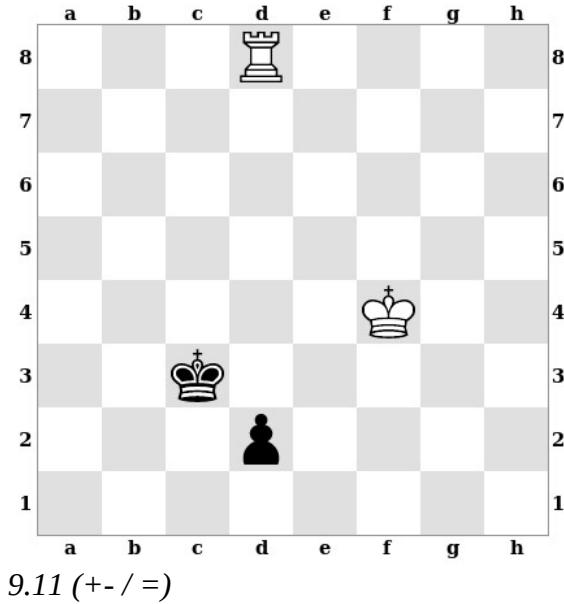
Against a bishop pawn on the sixth rank there is an important technique for the stronger side. If Black can play ...c2 here the position will be drawn since White's king is many moves away from the winning zone. This is exactly what happens after 1.Qb7+? Kc1 2.Qh1+ Kb2 =, when ...c2 can't be stopped. Instead, White wins by putting his queen on the long diagonal with 1.Qf6 (or 1.Qg7). Now Black can't move his pawn with his king on b2 (since it is pinned) or with his king on c2 (since it would be blocked). If Black plays ...Kb3 and ...c2 then White wins with Qa1 and Qc1. Black's only chance is to get his king to d2, but after 1...Kc2 2.Qd4 White stops this forever. Black now has nothing better than to wait for White's king to arrive and win the pawn, e.g. 2...Kb3 3.Kg7 Kb2 4.Kf6 Kc2 5.Ke5 +-.



9.10 (+-/-+)

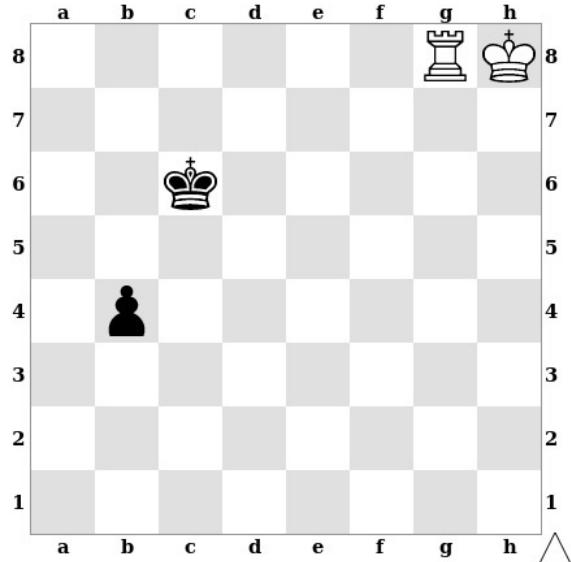
Strangely, it often just hurts the weaker side to have an additional pawn on the board. The extra pawn interferes with stalemates and gives the stronger side added possibilities. This position would be drawn if Black didn't have the pawn on h3, but with this pawn he quickly loses: 1.Qb3+ Ka1 (1...Kc1 2.Qxa2 Kd1 3.Qh2 +-) 2.Qc2 h2 3.Qc1#.

## Rook Versus Pawn(s)



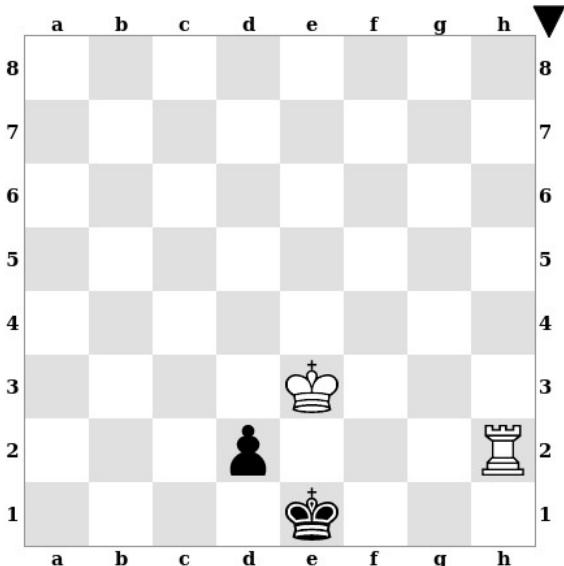
9.11 (+- / =)

A rook can also frequently win against a lone pawn. This position is a borderline case in which the result depends on who moves first. If Black moves first, then he draws after 1...Kc2 2.Rc8+ Kb2 3.Rd8 Kc2 4.Ke3 d1=Q =. If White moves first, then his king is just in time to help win the pawn: 1.Ke3 Kc2 2.Rxd2+ +-.



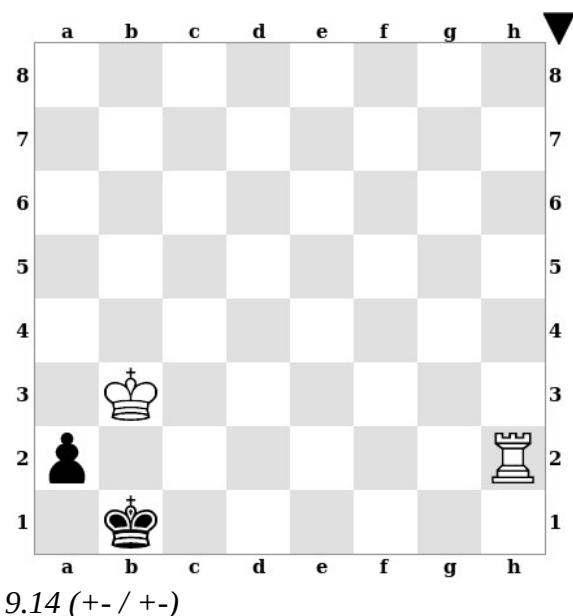
9.12 (+- / =)

In this position White's rook can cut off Black's king with 1.Rg5. If Black's king now tries to approach the white rook, he loses his pawn: 1...Kd6 2.Rb5 +- . If Black instead just plays waiting moves, then White can bring his king in, for example by coming down the h file to h1 and then across the first rank to b1. Black's only other try is to push his pawn, but this just results in the pawn being captured after 1...b3 2.Rg3 b2 3.Rb3 +- . This sort of cutoff is usually only effective if the weaker side's king can be restricted to the three back ranks, furthest from the promotion rank.



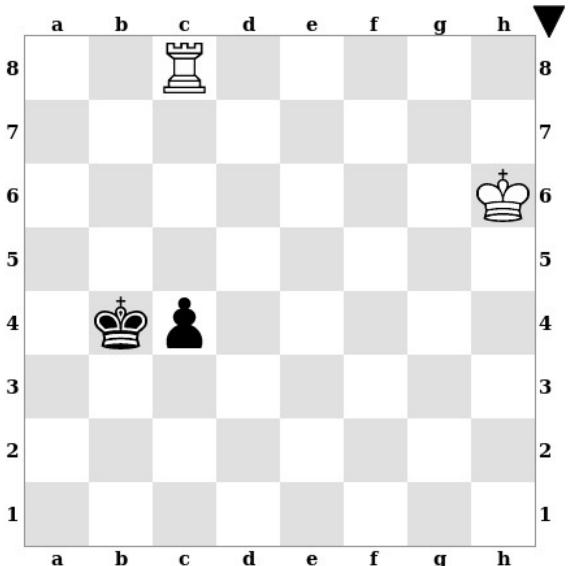
9.13 (+- / =)

One important resource for the weaker side is underpromotion. In this position, White is about to win Black's pawn and 1...d1=Q? runs into 2.Rh1#. Instead, Black can draw with 1...d1=N+, taking advantage of the fact that the endgame rook versus knight is normally a draw. Black now holds by keeping his king and knight together: 2.Kf3 Nc3 3.Rc2 Nd1 4.Re2+ Kf1 5.Rh2 Ke1=.



9.14 (+- / +-)

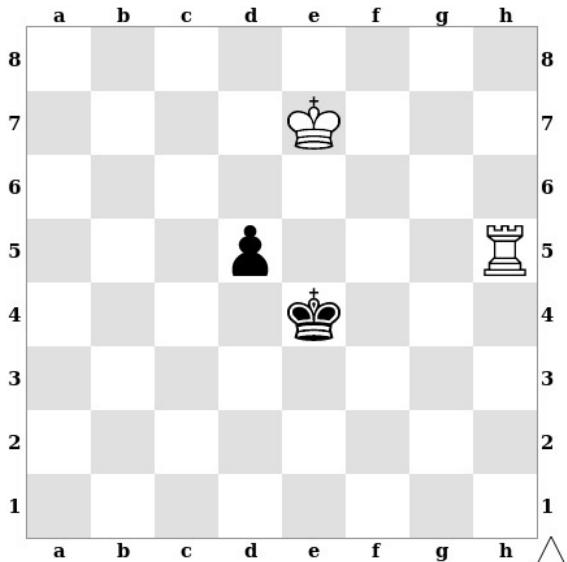
As this position shows, underpromotion doesn't work with a rook pawn because the knight ends up trapped in the corner. White wins after 1...a1=N+ 2.Kc3 +-.



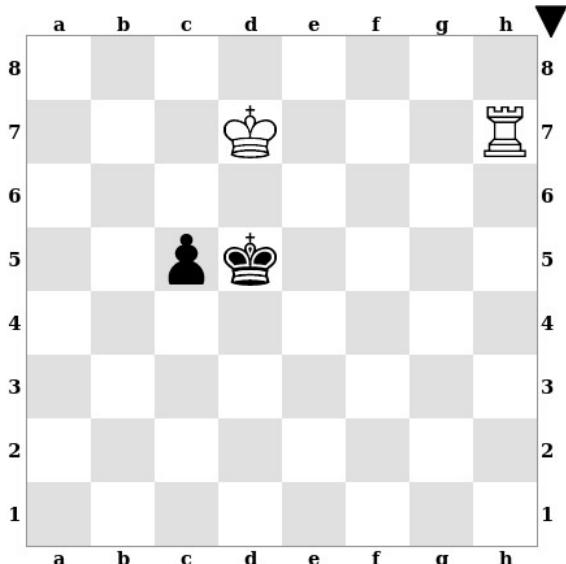
9.15 (+- / =)

One important technique for both sides is *shoulder-charging*: using a king to interfere with the other king's movement. Black loses here after the straightforward 1...c3? 2.Kg5 Kb3 3.Kf4 c2 4.Ke3 Kb2 5.Kd2 +-. Instead, he can draw with the surprising move 1...Kc3. Blocking the pawn looks wrong, but Black's king will need to move forward anyway and it is worth putting the king on a square where it stops White's king from rushing in. Black draws after the further 2.Kg5 Kd3 3.Kf4 c3 4.Kf3 c2 5.Kf2 (5.Rd8+ Kc3 6.Rc8+ Kd2 =) 5...Kd2 6.Rd8+ Kc3 7.Rc8+ Kd2 =.

White only gets a draw here after 1.Ke6? d4 2.Rh4+ Ke3 3.Ke5 d3 4.Rh3+ Ke2 5.Ke4 d2 6.Rh2+ Ke1 7.Ke3, when Black is saved by the underpromotion 7...d1=N+ =. The problem for White in this variation is that he needs a rook check at each step before his king can move forward. Black's king gets in the way and prevents White's king from moving forward, essentially a form of rear shoulder-charging. Instead, White can win by *outflanking* with his king: 1.Kd6 d4 2.Kc5 d3 3.Kc4 d2 4.Rd5 Ke3 5.Kc3 Ke2 6.Rxd2+ +-. In this variation White's king goes around to the side, avoiding Black's king and reducing the number of rook moves he needs to get his pieces in position to stop the pawn.



9.16 (+- / =)

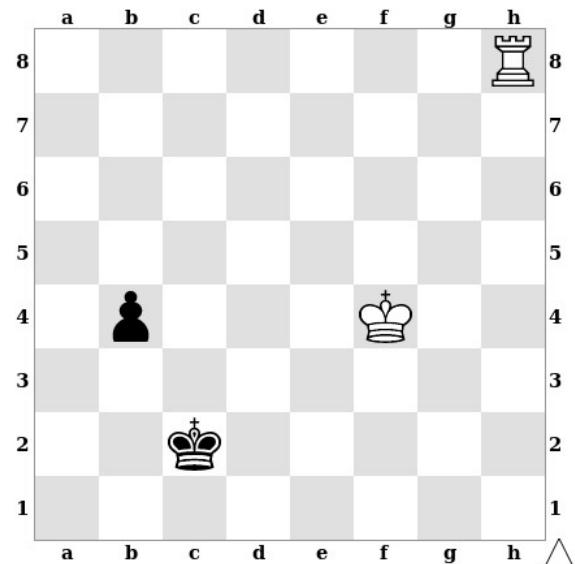


9.17 (+- / =)

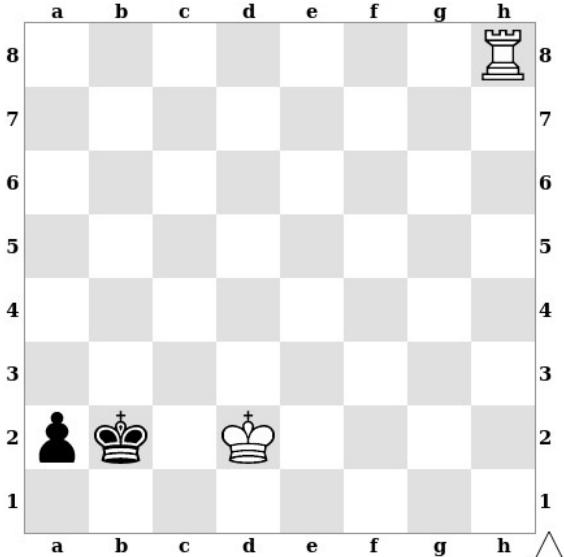
Here Black loses if he allows White to start outflanking right away: 1...Kd4? 2.Kc6 c4 3.Kb5 c3 4.Kb4 c2 5.Rc7 Kd3 6.Kb3 Kd2 7.Rxc2+ +- . Instead, Black should move his pawn first in order to slow down White's outflanking by one turn: 1...c4 2.Rh5+ **Kd4** 3.Kc6 c3 4.Kb5 c2 5.Rc5 Kd3 6.Kb4 Kd2 = . The move 1...c4 is effectively a way of shoulder-charging with the king by leaving it in its current position for an extra move.

If the weaker side's king gets too far ahead of his pawn, the stronger side can sometimes reposition his rook with a time-gaining check. In this position White can play 1.Rc8+ Kd2 (1...Kb2 2.Ke3 b3 3.Kd2 +- transposes) 2.Rb8 Kc3 3.Ke3. White has succeeded in putting his rook behind the pawn and moving his king forward while Black hasn't made any progress. White wins after the further 3...b3 4.Rc8+ Kb2 5.Kd2 Ka2 6.Kc3 b2 7.Ra8+ Kb1 8.Rb8 Ka1 9.Kc2 +-.

White's checks were helpful here because Black's king was already ahead of his pawn. Such a series of checks can be counterproductive if they instead help the defending king make useful moves.

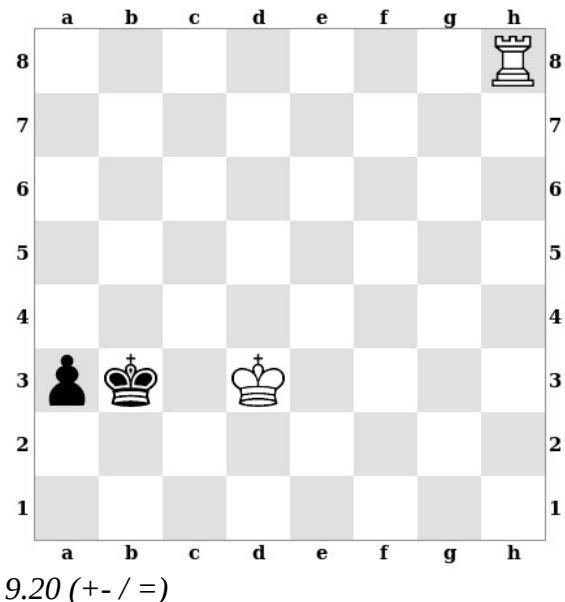


9.18 (+- / =)



9.19 (= / =)

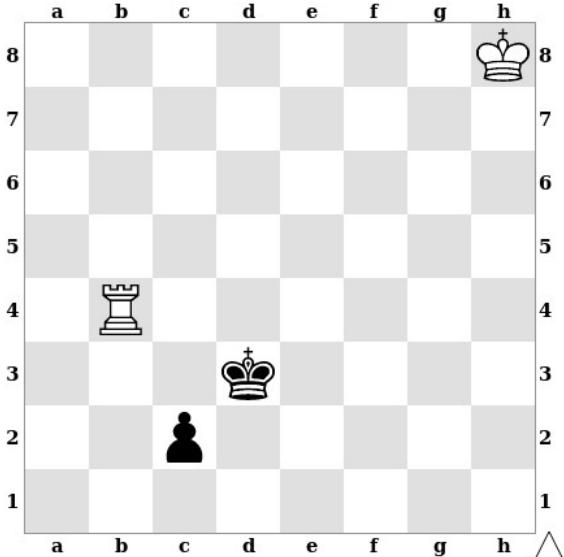
This is an important position with a rook pawn. Even with White to move first, Black can draw after **1.Rb8+ Ka1 2.Ra8** (or 2.Kc2 =, stalemate) **2...Kb2** =, when White can't make progress. However, he would lose if he instead responded to 1.Rb8+ with **1...Ka3? 2.Kc2** (threatening Ra8+) **2...a1=N+ 3.Kc3 Ka2 4.Rb7** +-, with zugzwang.



9.20 (+- / =)

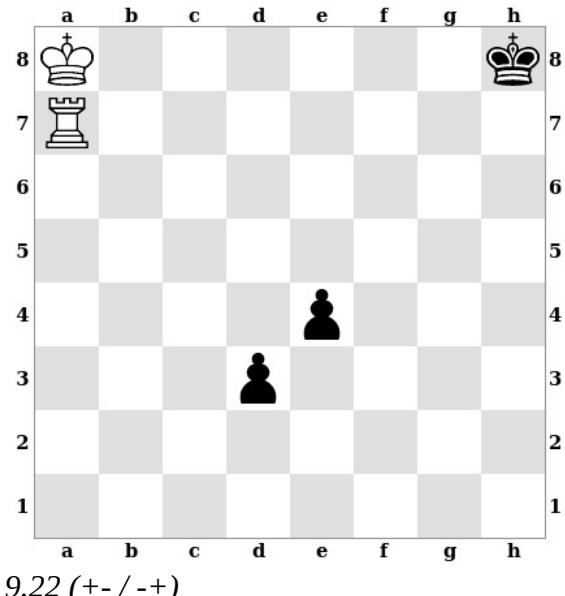
This is a second important rook pawn position. If White moves first, he wins with **1.Rb8+ Ka2 2.Kc2 Ka1 3.Kb3 a2 4.Rh8 Kb1 5.Rh1#**.

If Black moves first, he can draw with **1...Kb2 2.Rb8+ (2.Kd2 a2 = reaches the previous diagram) 2...Kc1 3.Kc3 a2 4.Ra8 Kb1 =**. Instead, Black would lose after **1...a2? 2.Rb8+ Ka3 (2...Ka4 3.Kc2 a1=N+ 4.Kb2 +-)** **3.Kc2 a1=N+ 4.Kc3 Ka2 5.Rb7** +-, with zugzwang.



9.21 (-+ / -+)

The pawn can sometimes even win if it is far advanced and the rook is awkwardly placed. In this position White would be happy to simply trade his rook for Black's pawn, but with careful play Black can prevent this and successfully queen. After **1.Rb3+** Black must avoid both **1...Kd2? 2.Rb2 =** and **1...Kc4? 2.Rb8 =**. The latter variation sets up a rook skewer on c8 and could even be winning if Black played the further error **2...c1=Q? 3.Rc8+ +-.** Instead of all this, Black wins by bringing his king back along the d-file until it can prevent these sorts of tricks: **1...Kd4 2.Rb4+ Kd5 3.Rb5+ Kd6 4.Rb6+ Kc7 +-**, followed by promoting next turn.

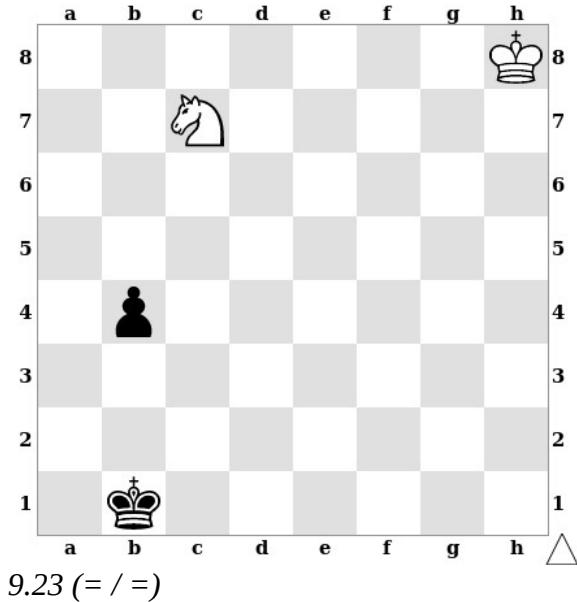


9.22 (+- / -+)

A rook can sometimes stop two connected pawns if the rook is well-placed. The key idea is to attack the more advanced pawn to stop either pawn from moving. If White moves first here, he wins with **1.Rd7 Kg8 (1...e3 2.Rxd3 e2 3.Re3 +-) 2.Rd4 Kf7 3.Rxe4 Kf6 4.Rd4 +-.**

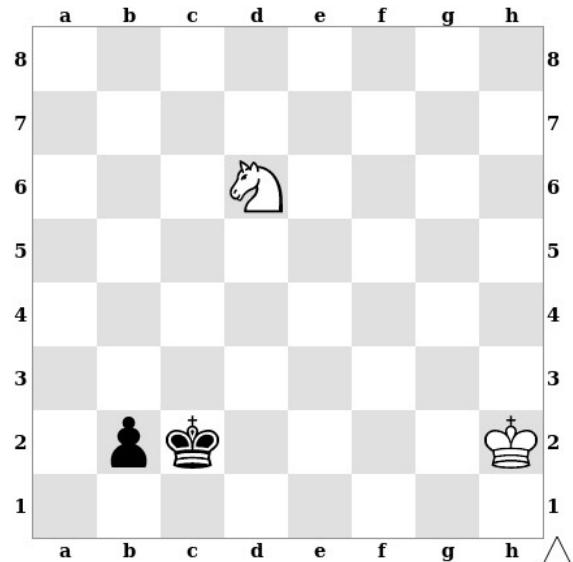
If instead Black moves first, then he can play **1...e3 2.Re7 d2 3.Rd7 e2 +-**, when White can't stop him from promoting. A rook generally can't stop two connected pawns once they have both reached the sixth rank.

## Knight Versus Pawn



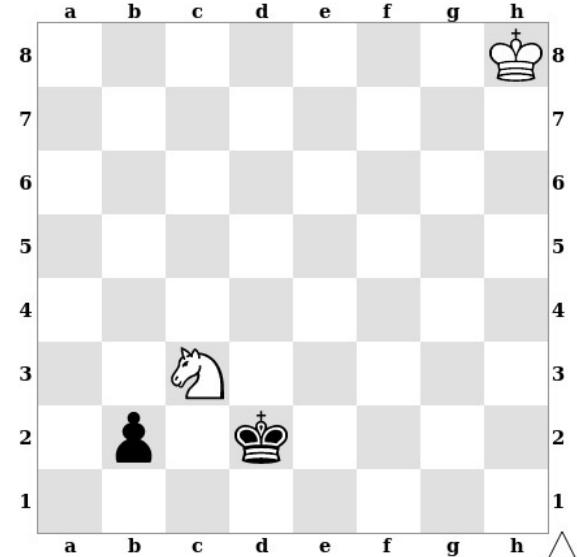
9.23 (= / =)

A knight can usually only hope to draw against a pawn. The most straightforward way to do this, when possible, is to sacrifice the knight for the pawn directly. Here White can hunt the pawn down with a forcing series of moves: 1.Na6 b3 2.Nc5 b2 3.Na4 Kc2 4.Nxb2 =.



9.24 (= / - +)

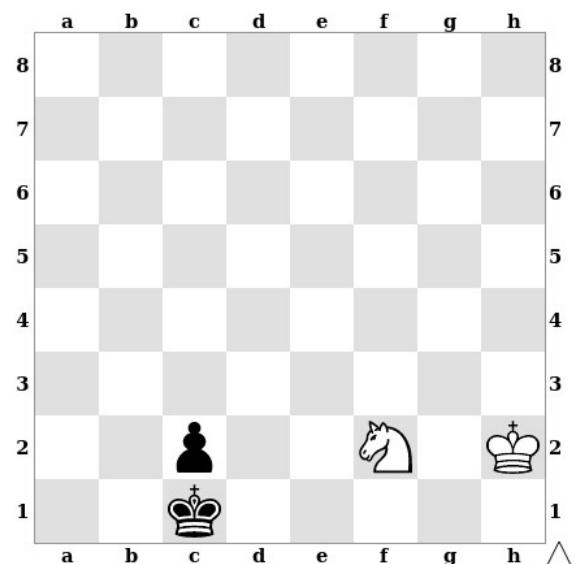
In this position the knight can use a fork to draw: 1.Nc4 b1=Q 2.Na3+ Kb2 3.Nxb1 =.



9.25 (= / -+)

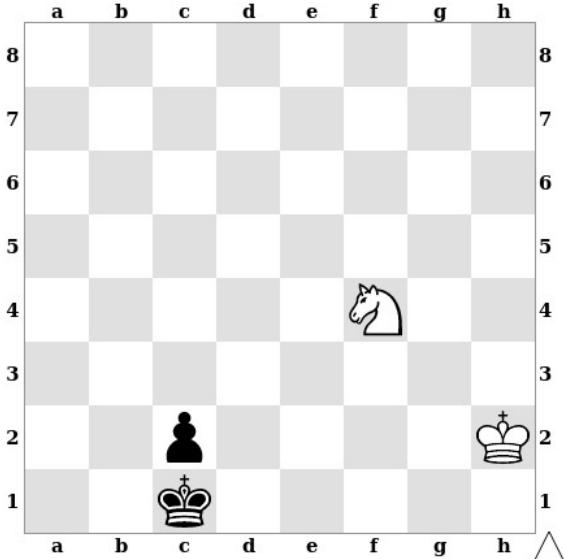
In many positions the knight can draw even without being able to capture the pawn. Here White draws with  
**1.Nb1+ Kc2 2.Na3+ Kb3 3.Nb1 Ka2 4.Nd2 =.** The knight simply jumps back and forth in front of the pawn and to different sides of it and Black can't do anything to stop it.

This position illustrates **rule 9.1 (Occupation Rule):** if the pawn is not a rook pawn and the knight can safely occupy the square directly in front of it, then the position is drawn.



9.26 (-+ / -+)

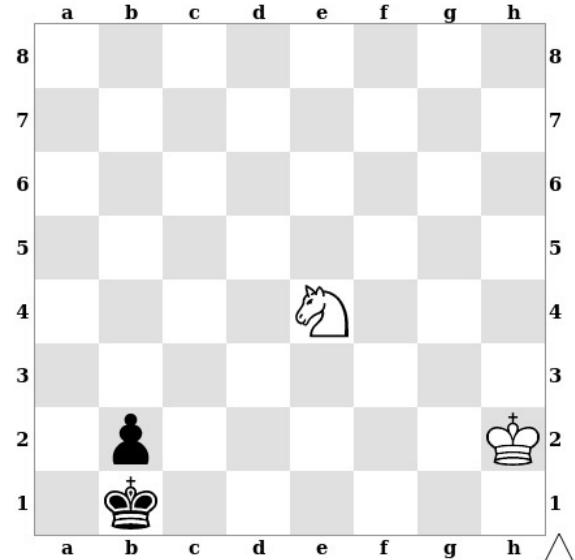
White's knight isn't in front of the pawn here, so he has to try to control the square in front of the pawn by giving check. In this position, this turns out not to work. Black wins after **1.Nd3+ (1.Ne4 Kb2 2.Nc5 Kc3 -+ transposes) 1...Kd2 2.Nc5 Kc3 (2...c1=Q? 3.Nb3+ Kc2 4.Nxc1 =) 3.Ne4+ Kd3 4.Nc5+ Kc4 -+.**



9.27 (= / -+)

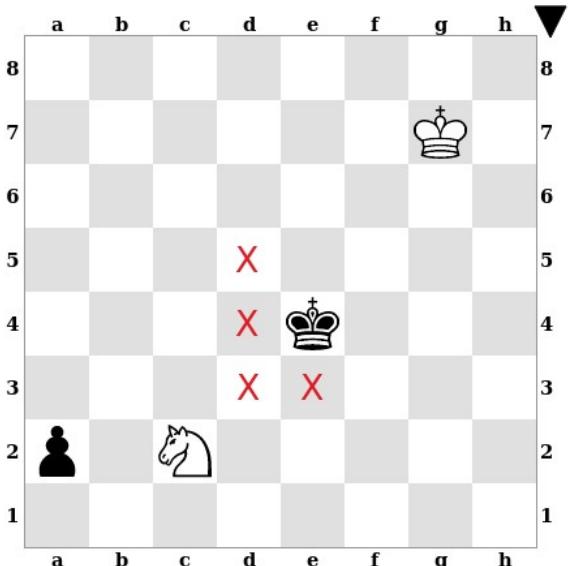
In this position, White has a choice of two knight checks. We know that he loses after 1.Nd3? -+, as in the previous example. Instead, he can draw by attacking c1 laterally (with the knight standing on the same rank as the pawn): 1.Ne2+ Kd1 (1...Kd2 2.Nd4 c1=Q 3.Nb3+ =) 2.Nc3+ Kd2 3.Na2 Kd3 4.Kg2 Kc4. In fact, White's knight doesn't even need help from his king: 5.Kh2 Kb3 6.Nc1+ Kb2 7.Ne2 =.

This example demonstrates **rule 9.2 (Lateral Control Rule)**: with a bishop pawn or center pawn, if the knight can safely attack the square in front of it laterally, then the position is drawn. There are sometimes exceptions if the pawn can promote with check.



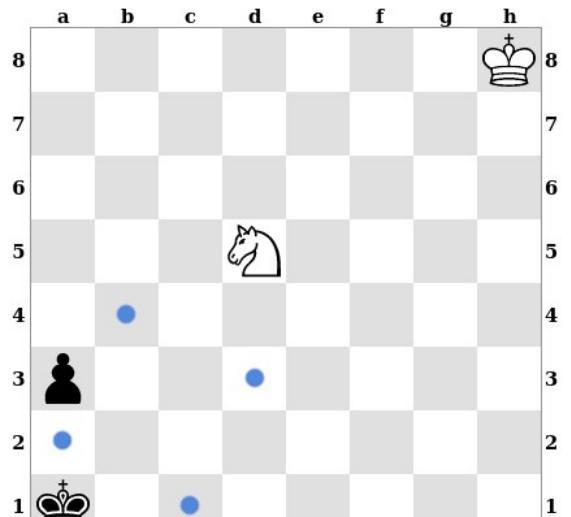
9.28 (-+/-+)

This position shows why rule 9.2 doesn't apply to knight pawns. After 1.Nd2+, White has controlled the square in front of the pawn laterally, but Black wins after 1...Kc1 2.Nb3+ Kd1 -+. If White could now put his knight on a square to the left of a2, then he would draw. Such a square would exist with a bishop pawn or center pawn, but does not exist with a knight pawn. As a result, the knight really needs to occupy the square in front of the pawn to guarantee a draw against a knight pawn.



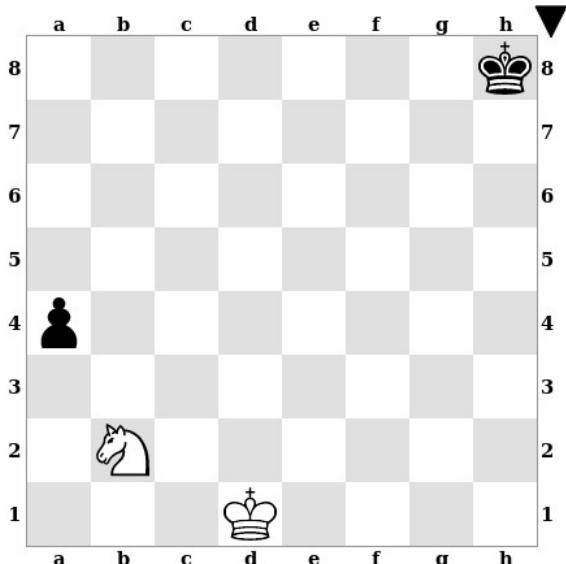
9.29 (= / =)

A knight generally has more trouble against a rook pawn. With the pawn on the seventh rank, the side with the knight is usually just trying to slow the opposing king down. A useful concept in this effort is the knight's ability to create a *barrier*, such as the one drawn here. White's knight controls e3 and d4 directly and controls d3 and d5 indirectly because of the possible forking move Nb4. As a result, Black's king needs to take an inconvenient route toward White's knight, giving White's king time to come in and arrange stalemate: 1...Kf3 2.Kf6 Ke2 3.Ke5 Kd2 4.Na1 Kc3 5.Ke4 Kb2 6.Kd3 Kxa1 7.Kc2 =.



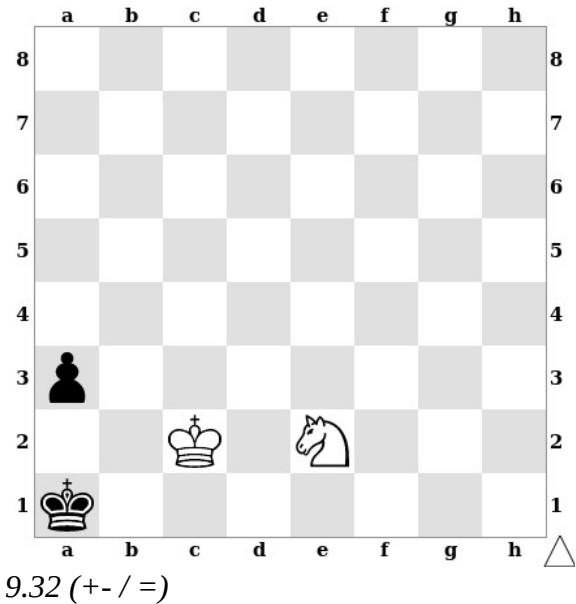
9.30 (= / =)

Against a rook pawn on the sixth rank, the knight can draw if it can get onto the right *circuit* of squares. White loses here after 1.Nc3? Kb2 2.Nd1+ Kb3 -+, when the knight is poorly placed to stop the pawn. Instead, White draws with 1.Nb4, when Black can't get rid of the knight, e.g. 1...Kb2 2.Kg7 Kb3 3.Nd3 Kc3 (3...a2 4.Nc1+ =) 4.Nc1 Kc2 5.Na2 Kb2 6.Nb4 Kc3 7.Na2+ Kb3 8.Nc1+ Kb2 9.Nd3+ Kc2 10.Nb4+ Kc3 11.Na2+ =.



9.31 (= / -+)

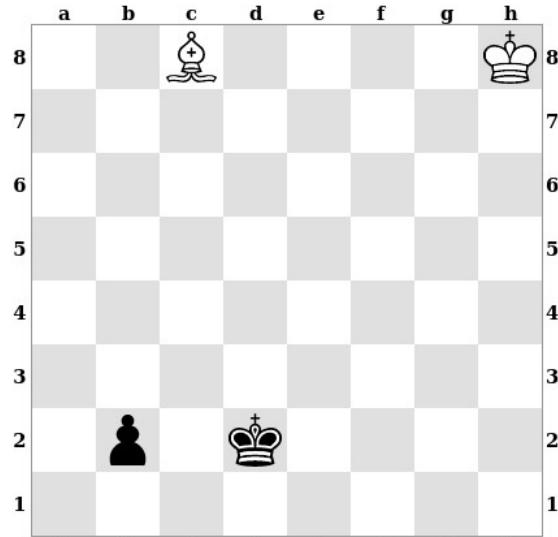
The knight's strange movement allows a surprising trick in which both the knight and king are powerless to stop a pawn despite standing quite near it. Black wins here after 1...a3 2.Kc2 a2 -+ when there is no way to stop 3...a1=Q.



9.32 (+- / =)

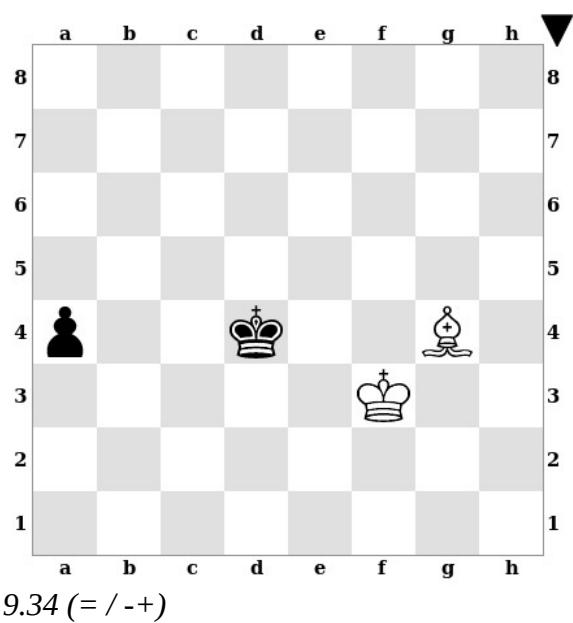
On the other hand, it is occasionally even possible for the knight to win. White can mate in two moves here with 1.Nc1 a2 2.Nb3#.

## Bishop Versus Pawn



9.33 (= / -+)

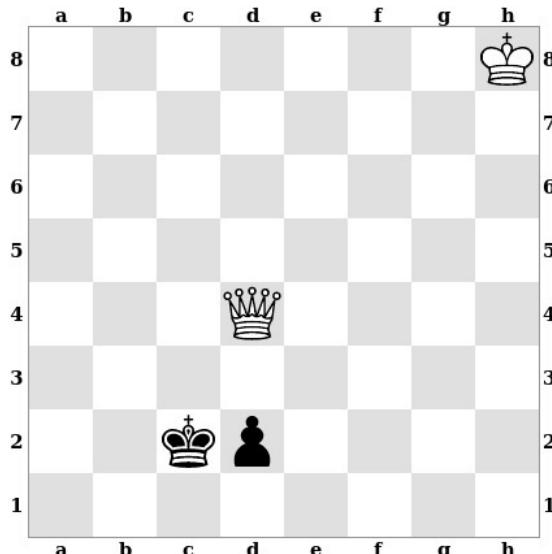
In contrast to the previous endgames in this chapter, positions in which a bishop faces a pawn are rarely interesting or difficult. If White moves first here, he draws with 1.Bf5 =, while if Black moves first, then he wins with 1...b1=Q -+.



9.34 (= / -+)

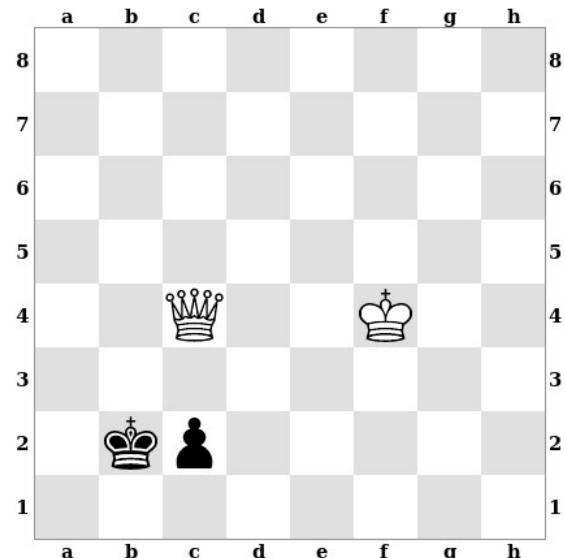
Here Black can win by keeping the bishop off of the a2-g8 diagonal: 1...Ke5 2.Bh5 Kf6 3.Ke3 a3 -+.

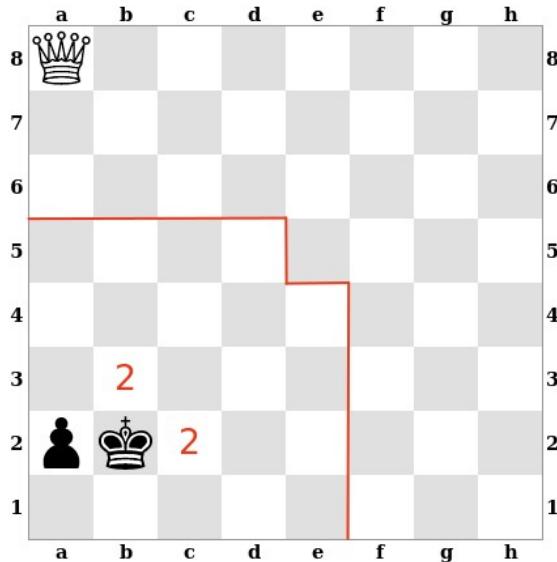
## Conclusion



There are a few important rules and results from this chapter that you should remember. You should know that a queen normally wins against a knight pawn or center pawn, no matter how far away the stronger side's king is. The winning technique is also something that is worth practicing a few times until you can do it easily.

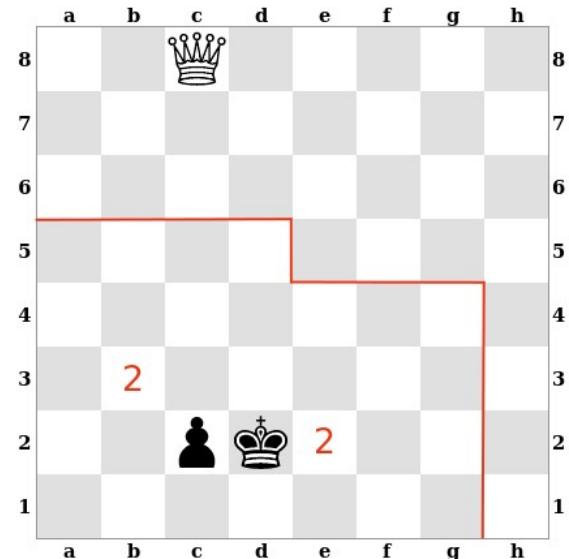
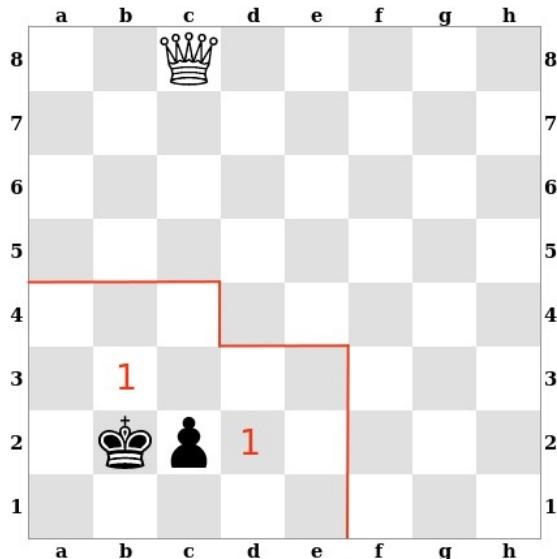
You should also remember that when a queen faces a rook pawn or bishop pawn the position is often drawn, though the exact result depends on how close the stronger side's king is.

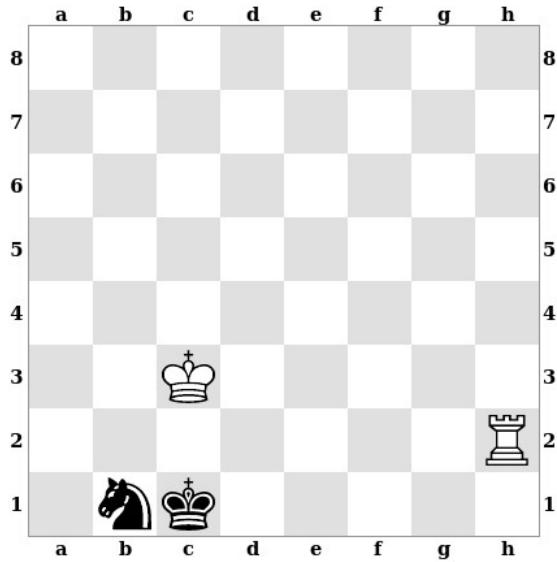




If you want to be very ambitious you could try to remember the winning zones for a rook pawn and a bishop pawn. The rook pawn has a single winning zone, regardless of the position of the weaker side's king.

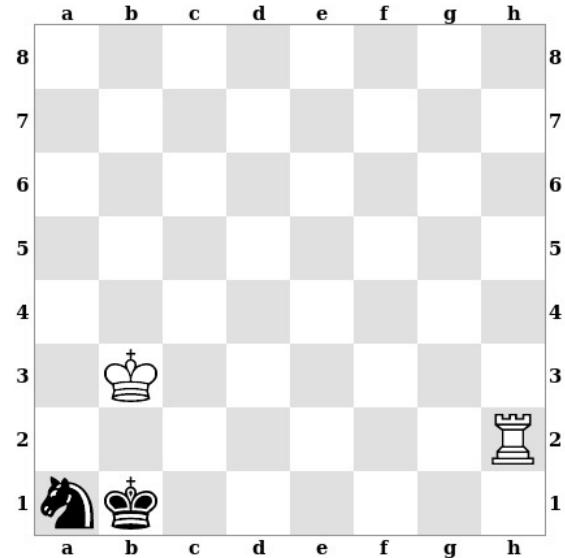
The bishop pawn has two winning zones, depending on whether the weaker side's king is on the better side of the pawn (closer to the corner) or the worse side.

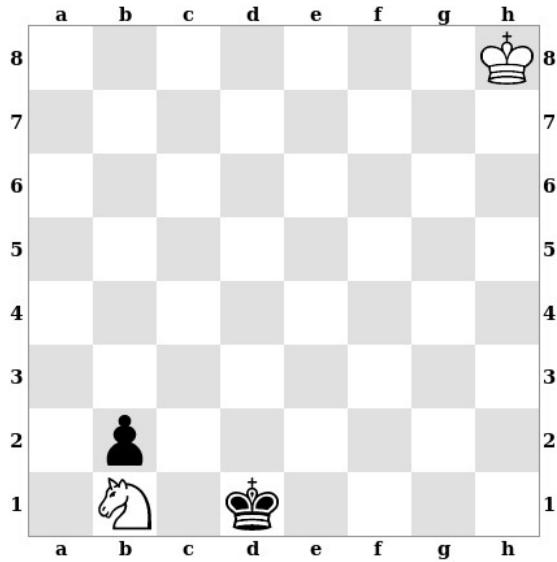




The ending rook versus pawn contains few rules, but it is worth knowing that underpromotion is a valid defense with any pawn other than rook pawn.

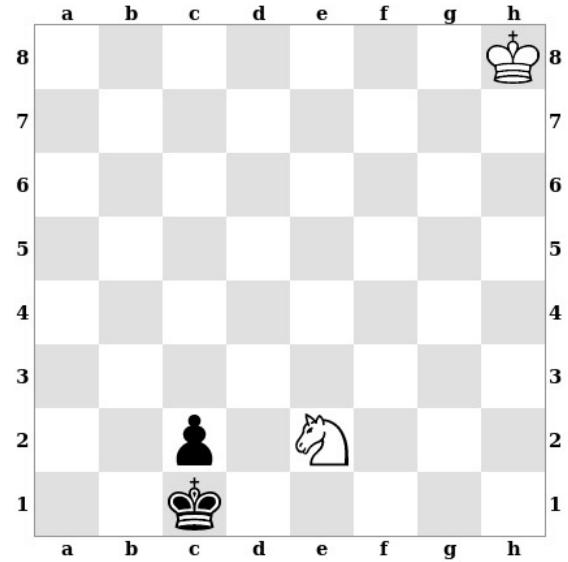
However, underpromotion loses with a rook pawn since the knight ends up trapped in the corner.





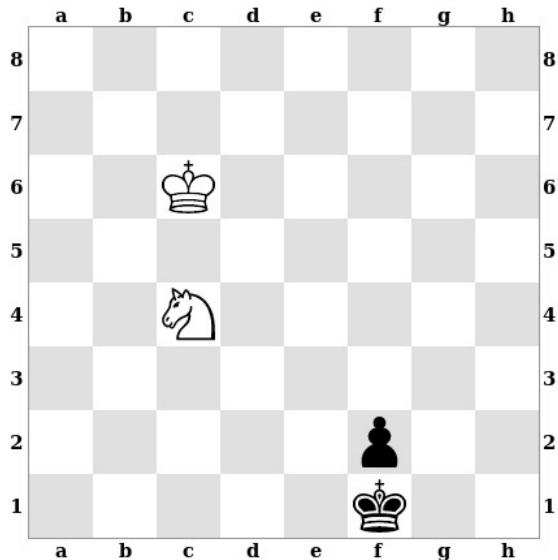
Finally, there are two important rules from the knight versus pawn ending that you should remember.

First, **rule 9.1 (Occupation Rule):** if the pawn is not a rook pawn and the knight can safely occupy the square directly in front of it, then the position is drawn.

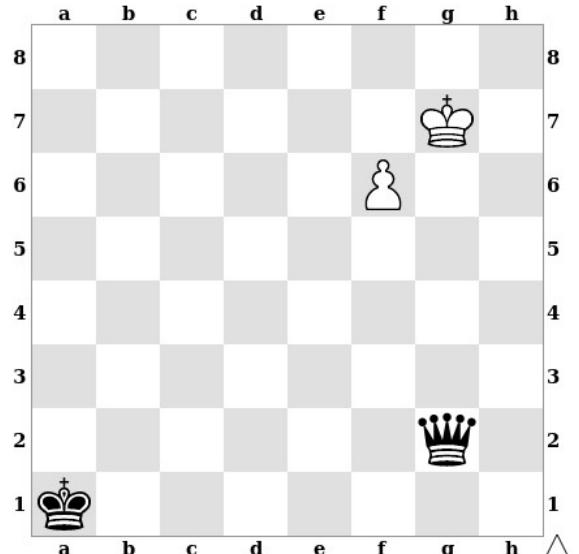


Second, **rule 9.2 (Lateral Control Rule):** with a bishop pawn or center pawn, if the knight can safely attack the square in front of it laterally, then the position is drawn.

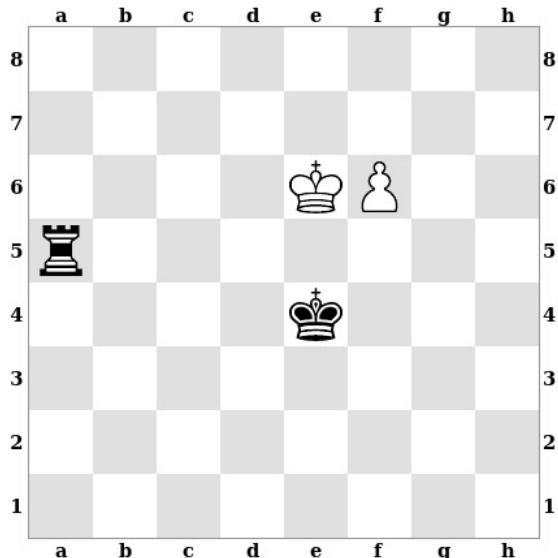
## Exercises



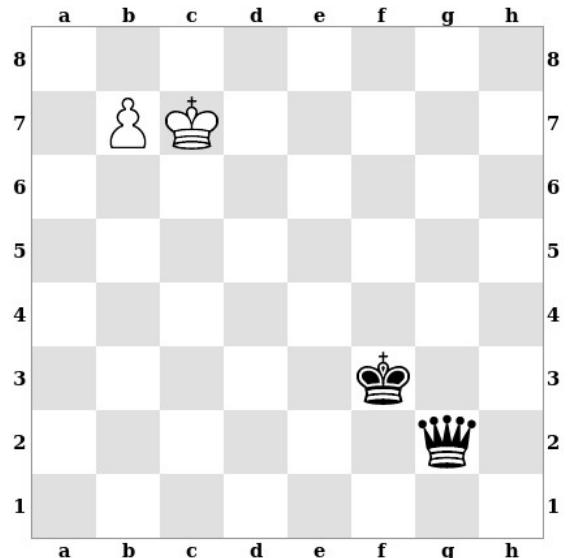
E9.01 (W/B)



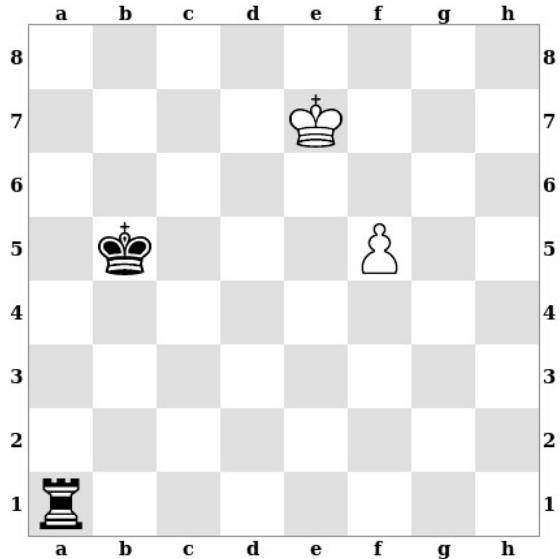
E9.02 (W)



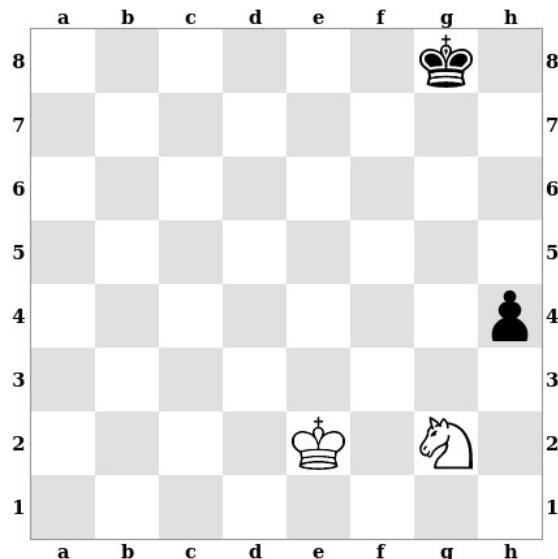
E9.03 (W/B)



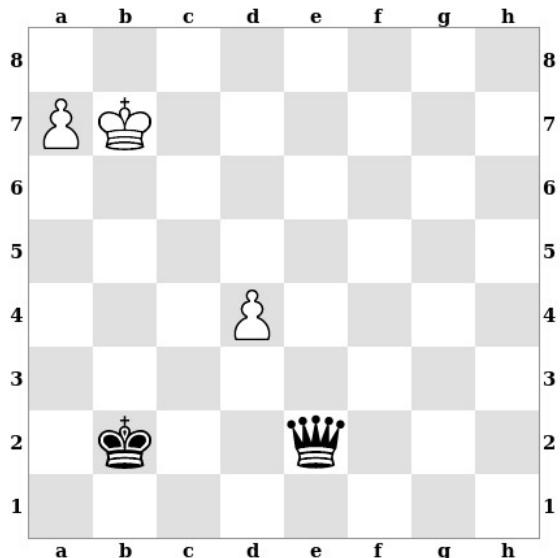
E9.04 (W/B)



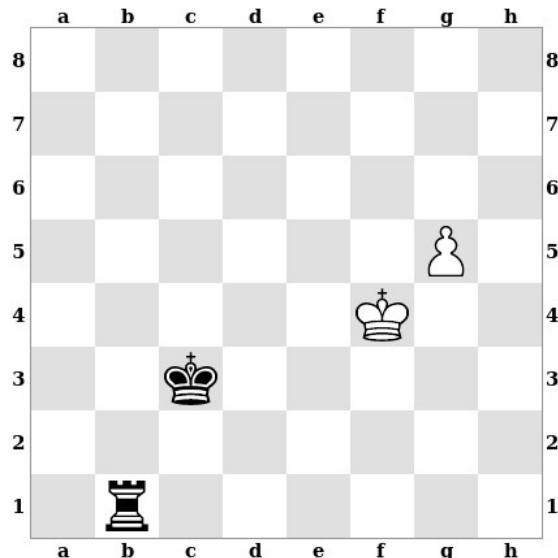
E9.05 (W/B)



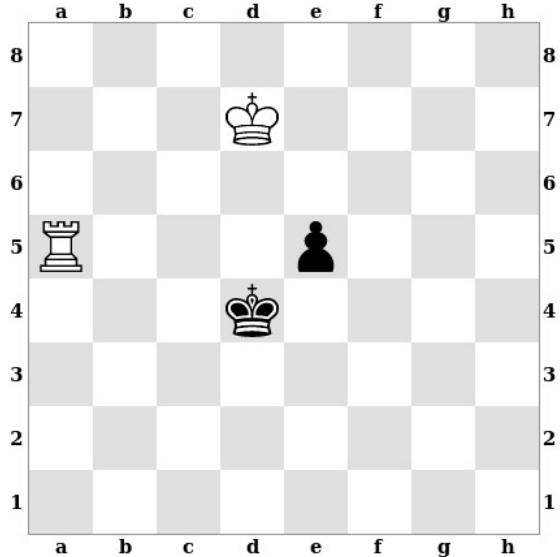
E9.06 (W/B)



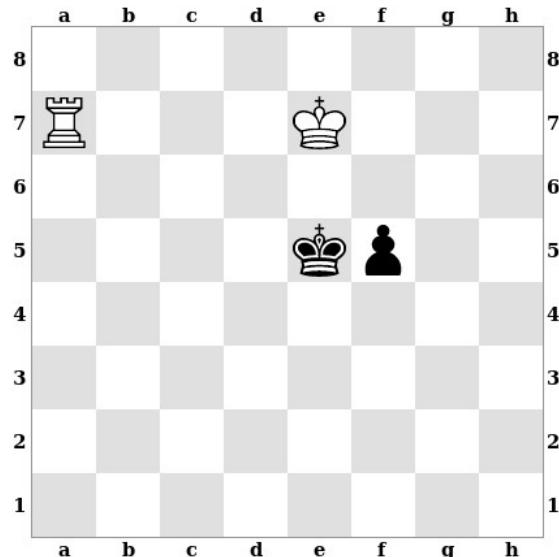
E9.07 (W/B)



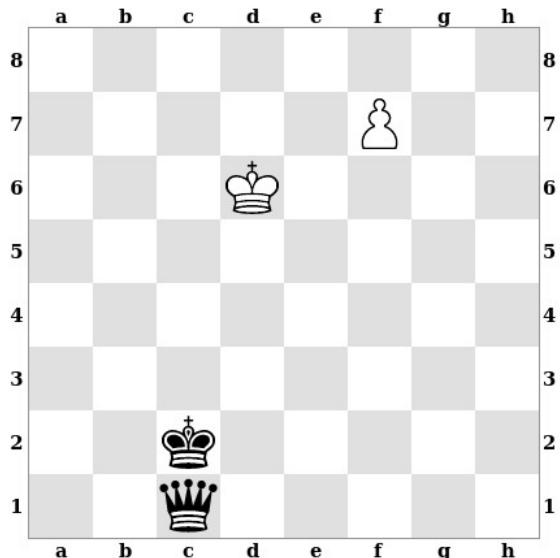
E9.08 (W/B)



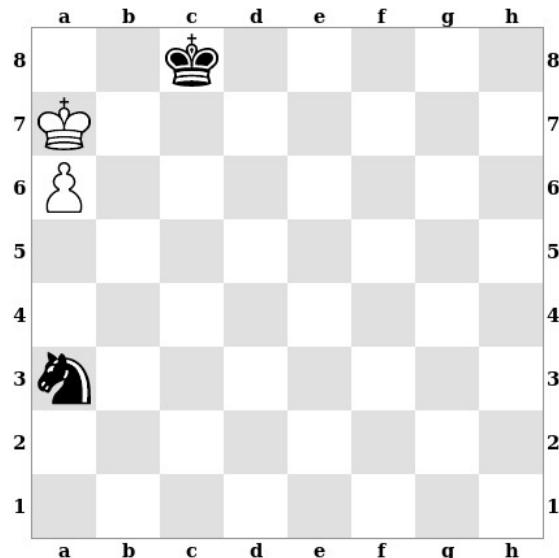
E9.09 (W/B)



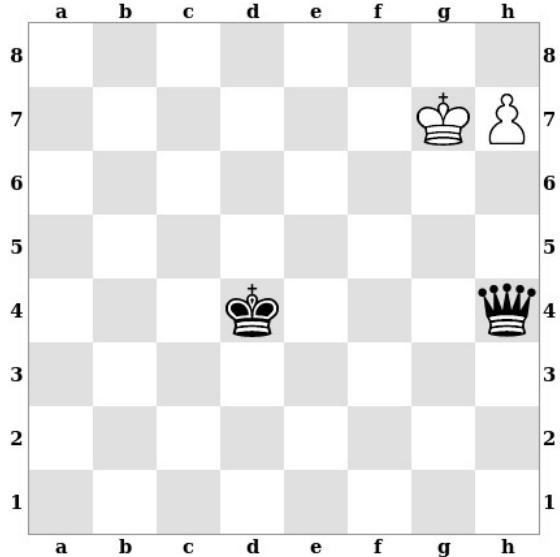
E9.10 (W/B)



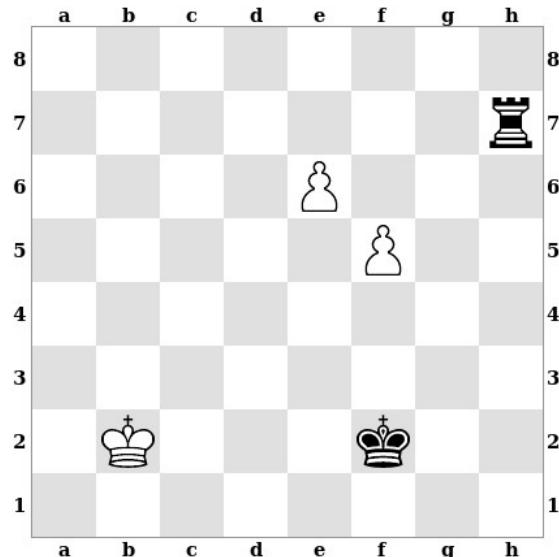
E9.11 (W/B)



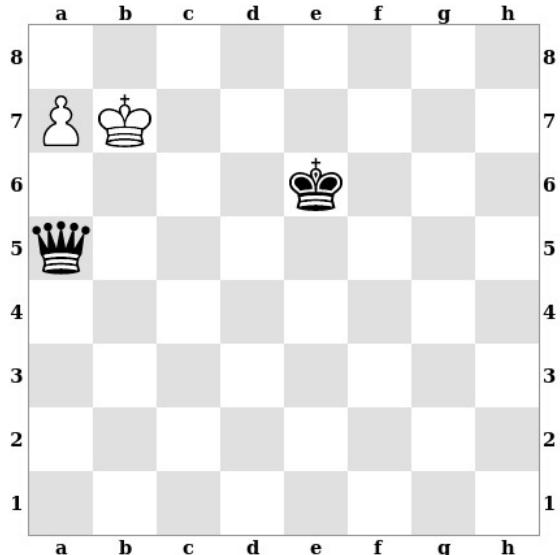
E9.12 (W/B)



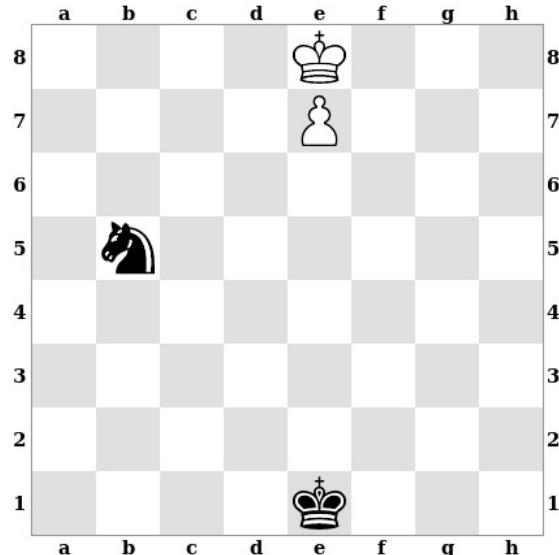
E9.13 (W/B)



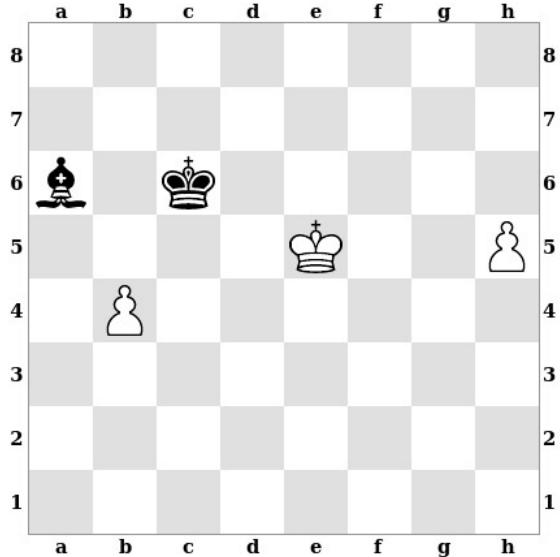
E9.14 (W/B)



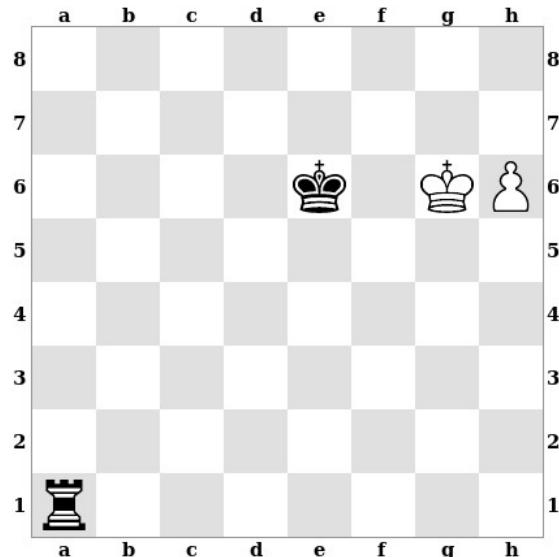
E9.15 (W/B)



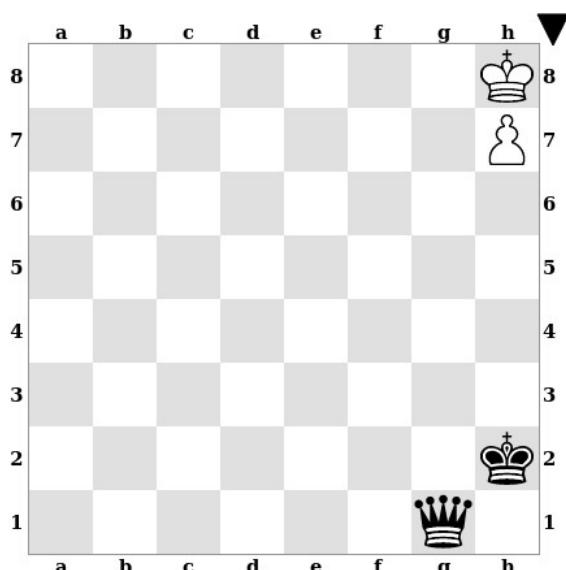
E9.16 (W/B)



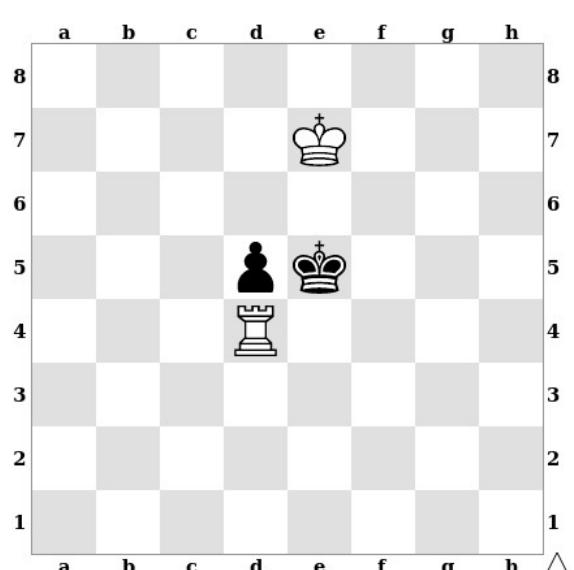
E9.17 (W/B)



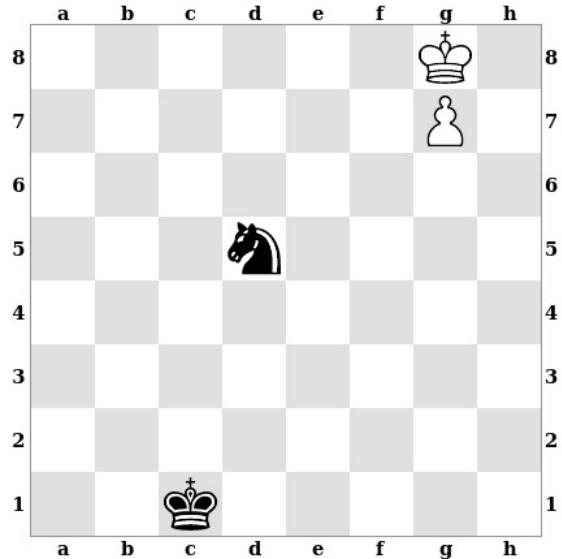
E9.18 (W/B)



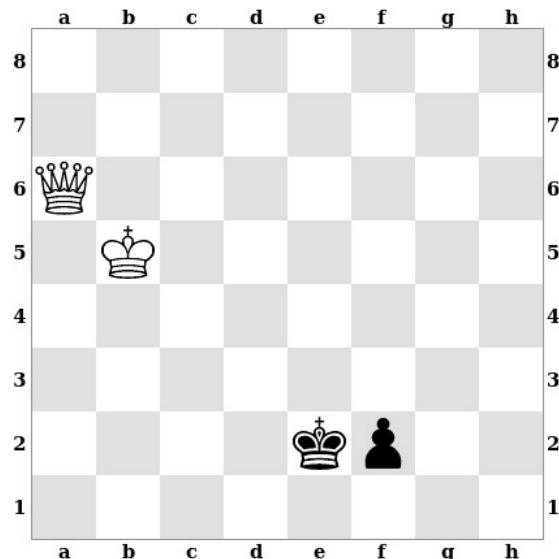
E9.19 (B)



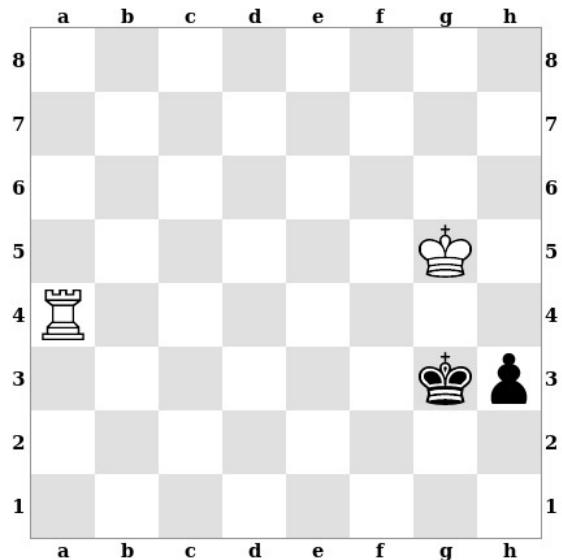
E9.20 (W)



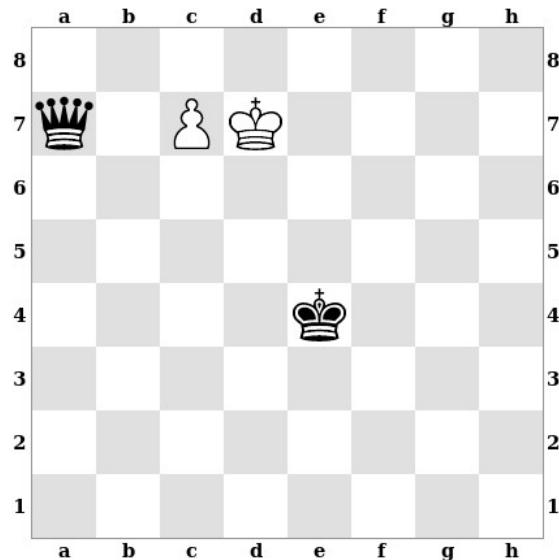
E9.21 (W/B)



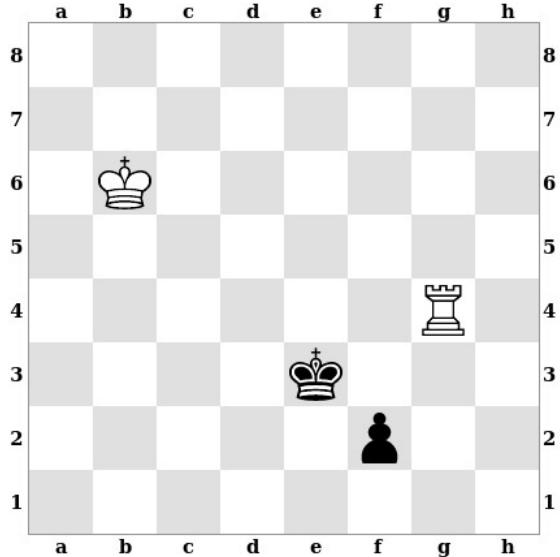
E9.22 (W/B)



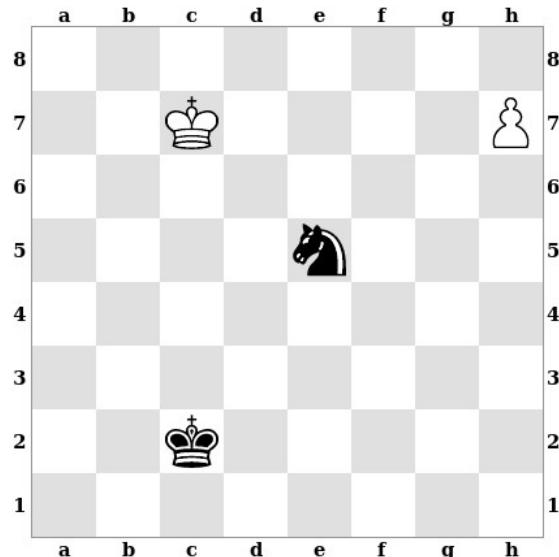
E9.23 (W/B)



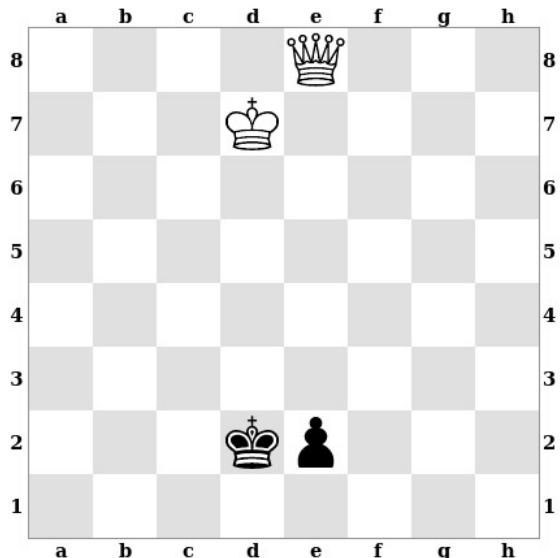
E9.24 (W/B)



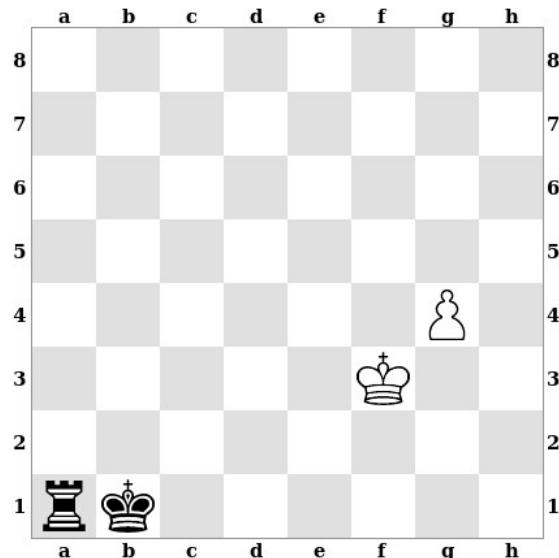
E9.25 (W/B)



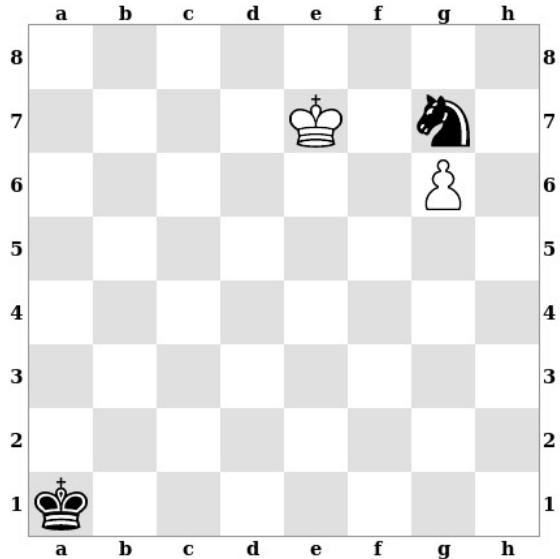
E9.26 (W/B)



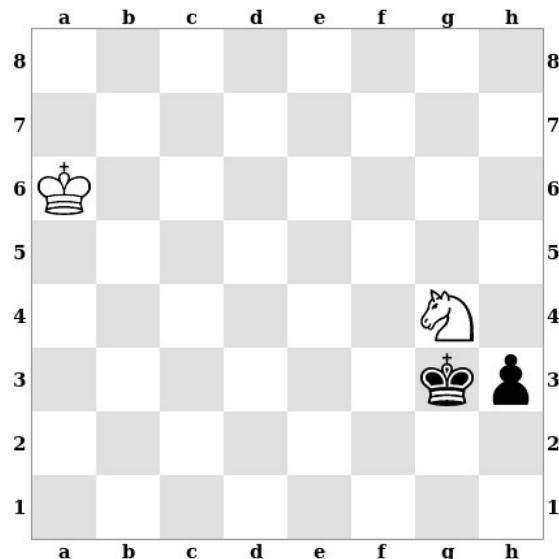
E9.27 (W/B)



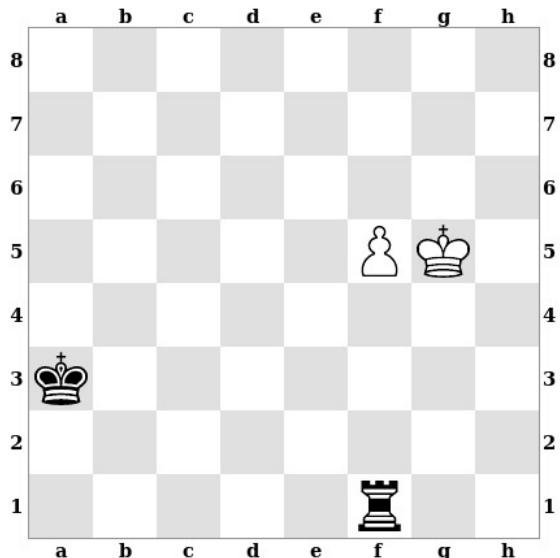
E9.28 (W/B)



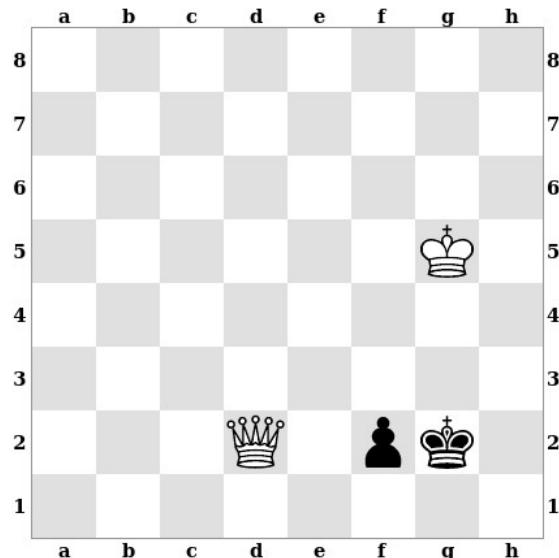
E9.29 (W/B)



E9.30 (W/B)



E9.31 (W/B)



E9.32 (W/B)

## Solutions

### E9.01

**W:** (=) 1.Nd2+ and the position is drawn by the lateral control rule (1.Ne3+? Ke2 2.Nf5 Kf3 3.Nd4+ Ke3 4.Nf5+ Kf4 -+) 1...Ke1 (1...Ke2 2.Ne4 f1=Q 3.Ng3+ =) 2.Nf3+ Ke2 3.Nh2 =

**B:** (-+) 1...Ke2 2.Ne5 f1=Q -+

### E9.02

(=) 1.Kf8 (1.Kh8? Qb2 2.Kg7 Qe5 3.Kg6 Kb2 4.Kf7 Kc3 5.Kg6 Kd4 6.f7 Qh8 7.Kg5 Qf8 -+; 1.Kh7? Qe4+ 2.Kg7 Qe5 -+) 1...Qa8+ 2.Kg7 Qg2+ 3.Kf8 Kb2 4.f7 Kc3 5.Ke7 Black's king is outside the winning zone. 5...Qg7 6.Ke8 Qe5+ 7.Kf8 Kd4 8.Kg8 Qg5+ 9.Kh7 Qf6 10.Kg8 Qg6+ 11.Kh8 =

### E9.03

(=)

**W:** 1.f7 Rf5 2.Ke7 Ke5 3.f8=Q Rxf8 4.Kxf8 =

**B:** Black doesn't have time for anything other than following behind with his king, but White draws with underpromotion: 1...Ra6+ 2.Ke7 Ke5 3.f7 Ra7+ 4.Ke8 Ke6 5.f8=N+ Kf6 6.Nd7+ Ke6 7.Nf8+ Kd6 8.Ng6 Rg7 9.Nf8 Re7+ 10.Kd8 Ra7 11.Ke8 =

### E9.04

**W:** (=) 1.b8=Q

**B:** (-+) The queen wins against a knight pawn: 1...Qg7+ 2.Kc8 Qc3+ 3.Kd7 Qb4 4.Kc7 Qc5+ 5.Kd7 Qb6 6.Kc8 Qc6+ 7.Kb8 Ke4 8.Ka7 Qc7 9.Ka8 Qa5+ 10.Kb8 Kd5 11.Kc8 Qc5+ 12.Kd7 Qb6 13.Kc8 Qc6+ 14.Kb8 Kd6 15.Ka7 Qa4+ 16.Kb8 Qa6 17.Kc8 Kc6 18.Kd8 Qxb7 -+

### E9.05

**W:** (=) 1.f6 Re1+ 2.Kd7 Rf1 3.Ke7 Kc6 4.f7 Kc7 5.f8=Q Rxf8 6.Kxf8 =

**B:** (-+) Black wins with a time-gaining check: 1...Re1+ (1...Rf1? 2.f6 Kc6 3.f7 Re1+ 4.Kd8 =; 1...Kc6? 2.f6 Re1+ 3.Kd8 Rf1 4.Ke7 Kc7 5.f7 Re1+ 6.Kf6 Kd7 7.f8=Q Rf1+ 8.Kg7 Rxf8 9.Kxf8 =) 2.Kd7 Rf1 3.Ke6 Kc6 4.f6 Re1+ 5.Kf7 Kd7 6.Kg7 Rf1 7.f7 Ke7 -+

### E9.06

**W:** (=) 1.Nxh4 =

**B:** (-+) 1...h3 2.Kf2 h2 -+

### E9.07

**W:** (=) 1.a8=Q Qe4+ 2.Kb8 Qxd4 =

**B:** (-+) 1...Qb5+ 2.Kc7 (2.Ka8 Qb6 3.d5 Qc7 4.d6 Qc8#) 2...Qa6 3.Kb8 Qb6+ 4.Ka8 Qc7 5.d5 Qc8#

### E9.08

**W:** (=) 1.Kf5 Kd4 2.g6 Kd5 3.Kf6 Kd6 4.g7 Rf1+ 5.Kg6 Ke7 6.g8=Q Rg1+ 7.Kh7 =

**B:** (-+) 1...Kd4 (but not 1...Rb5? because cutting off doesn't normally help if the king can't be trapped on the back three ranks. 2.g6 Kd4 3.g7 Rb1 4.Kf5 Rg1 5.Kf6 Kd5 6.Kf7 =) 2.Kf5 Kd5 3.Kf6 Kd6 4.g6 Rf1+ 5.Kg7 Ke7 6.Kh8 Rg1 7.g7 Kf7 -+

### E9.09

**W:** (+-) 1.Ke6 White wins by outflanking (but following behind with the king only draws: 1.Kd6? e4 2.Ra4+ Kd3 3.Kd5 e3 4.Ra3+ Kd2 5.Kd4 e2 6.Ra2+ Kd1 7.Kd3 e1=N+ =) 1...e4 2.Kf5 e3 3.Re5 Kd3 4.Kf4 e2 5.Kf3 +-

**B:** (=) 1...e4 2.Ke6 (2.Kd6 e3 3.Ra4+ Kd3 4.Kd5 e2 5.Ra1 =) 2...e3 3.Kf5 e2 4.Re5 Kd3 5.Kf4 Kd2 6.Kf3 e1=Q 7.Rxe1 Kxe1 =

### E9.10

**W:** (+-) 1.Ra5+ Ke4 2.Kf6 (2.Ke6? f4 =) 2...f4 3.Kg5 f3 4.Rf5 Ke3 5.Kg4 f2 6.Kg3 +-

**B:** (=) 1...f4 Holding White's king back one move longer. (Instead, Black loses after 1...Ke4? 2.Kf6 f4 3.Kg5 f3 4.Rf7 Ke3 5.Kg4 f2 6.Kg3 +-) 2.Ra8 f3 3.Rf8 Ke4 4.Kf6 f2 5.Kg5 Ke3 6.Kg4 Ke2 =

### E9.11

In fact, White is not threatening to promote yet, because of the skewer ...Qa3+. However, Black's king is two moves away from the winning zone.

**W:** (=) 1.Ke7 =

**B:** (-+) 1...Qa3+ (1...Kd3? 2.Ke7 =; 1...Qe3? 2.Kd7 [2.f8=Q? Qa3+ -+] 2...Qf4 3.Ke7 =) 2.Kd7 Qf8 +-

### E9.12

**W:** (=) 1.Kb6 = (1.Ka8? Nb5 2.a7 Nc7#)

**B:** (-+) 1...Nc4 (1...Kc7 2.Ka8 Nc4 -+ transposes) 2.Ka8 Kc7 3.Ka7 Nd6 4.Ka8 Nc8 5.a7 Nb6#

### E9.13

(=) Black's king is outside the winning zone.

**W:** 1.h8=Q =

**B:** 1...Qg5+ 2.Kf7 Qh6 3.Kg8 Qg6+ 4.Kh8 =

### E9.14

**W:** (+-) 1.f6 with two connected passed pawns on the sixth rank, White wins 1...Ke3 2.e7 Rh8 3.f7 +-

**B:** (-+) 1...Re7 Black draws by attacking the more advanced pawn (but not 1...Ke3? 2.f6 Ke4 3.e7 Rh8 4.f7 +-) 2.Kc3 Ke3 3.Kc4 Ke4 4.f6 Rxe6 5.f7 Rf6 -+

### E9.15

**W:** (=) 1.a8=Q

**B:** (-+) Black's king is inside the winning zone. 1...Kd6 2.a8=Q Qb5+ 3.Ka7 (3.Kc8 Qd7+ 4.Kb8 Qc7#) 3...Kc7 -+

### E9.16

(+-)

**W:** 1.Kd7 +-

**B:** This position is an exception to the lateral control rule since the pawn queens with check. 1...Nc7+ (1...Nd6+ 2.Kd7 Ne4 3.e8=Q +-) 2.Kd7 Nd5 3.e8=Q+ +-

### E9.17

**W:** (+-) 1.b5+ Bxb5 (1...Kxb5 2.h6 +-) 2.Kd4 Ba4 3.Kc3 Kd6 4.h6 Be8 5.h7 +-

**B:** (=) 1...Bd3

### E9.18

**W:** (=) 1.Kg7 (1.h7? Rg1+ 2.Kh6 Kf7 3.h8=N+ Kg6 4.Kh7 Rg2 -+) 1...Rg1+ (1...Ra7+ 2.Kg6 [2.Kg8? Kf6 -+] 2...Ke7 3.h7 Ra8 4.Kg7 =) 2.Kf8 Rh1 3.Kg7 Ke7 4.h7 Rg1+ 5.Kh8 Rh1 6.Kg7 =  
**B:** (-+) 1...Rg1+ 2.Kh7 (2.Kh5 Kf7 3.h7 Kg7 -+) 2...Kf7 3.Kh8 Ra1 4.Kh7 Rh1 5.Kh8 Rxh6#

### E9.19

(-+) Black's king is outside the winning zone, but he has a trick to move the king forward without losing any time: 1...Kg3 (1...Qd4+? = gives away the opportunity) 2.Kg7 Kf4+ 3.Kf7 Qa7+ 4.Kg8 Kg5 5.h8=Q Kg6 -+

### E9.20

(-+) In this famous study from Réti, White needs to choose the right square for his rook. The correct choice is the surprising 1.Rd2 (or 1.Rd3 +-; but not 1.Rd1? d4 2.Kd7 Kd5 3.Kc7 Kc5 when White can't make progress without a further rook move, but any rook move is counterproductive. 4.Rh1 d3 5.Rg1 Kg4 6.Kd6 Ke3 7.Ke5 d2 8.Ra1 Ke2 9.Ke4 d1=Q =; Also wrong is 1.Rh4? d4 2.Rh5+ Ke4 3.Kd6 d3 4.Rd5 Ke3 5.Kc5 d2 6.Kc4 Ke2 =) 1...d4 2.Rd1 Kd5 (2...Ke4 3.Kd6 Ke3 4.Kc5 d3 5.Kc4 d2 6.Kc3 +-) 3.Kd7 Black is now in zugzwang and must allow White's king forward. 3...Ke4 (3...Kc5 4.Ke6 +-) 4.Kd6 d3 (4...Ke3 5.Kd5 d3 6.Kc4 d2 7.Kc3 +-) 5.Kc5 Ke3 6.Kc4 d2 7.Kc3 +-

### E9.21

(-+)

**W:** 1.Kf7 +-

**B:** Lateral control doesn't guarantee a draw against a knight pawn. 1...Ne7+ (1...Nf6+ 2.Kf7 Ng4 3.Kg6 Ne5+ 4.Kf6 Ng4+ 5.Kg5 +-) 2.Kf8 Ng6+ 3.Ke8 +-

### E9.22

**W:** (-+) White can make a free king move with a discovered check, but he must choose the right one to get into the winning zone. 1.Kb4+ This puts White's king in the winning zone. (1.Kc5+? leaves the king outside the winning zone. 1...Ke1 2.Qa1+ Ke2 3.Qe5+ Kd1 4.Qd4+ Ke2 5.Qe4+ Kd2 6.Qf3 Ke1 7.Qe3+ Kf1 8.Kd4 Kg2 =; of course 1.Qa2+? just wastes the potential discovery altogether 1...Kf1 2.Kc4 Kg1 3.Qa7 Kg2 =) 1...Ke1 2.Qe6+ Kf1 (2...Kd2 3.Qf5 Ke2 4.Qe4+ Kd2 5.Qg2 Ke1 6.Kc3 Ke2 7.Kd4 Ke1 8.Ke3 f1=N+ 9.Kd3 +-) 3.Qg4 Ke1 4.Qg3 Ke2 5.Qg2 Ke1 6.Kc3 Ke2 7.Kd4 Ke1 8.Ke3 f1=N+ 9.Kd3 +-

**B:** (=) 1...f1=Q

### E9.23

**W:** (-+) Following behind with the king is enough to win against a rook pawn. 1.Ra3+ Kg2 2.Kg4 h2 3.Ra2+ Kg1 4.Kg3 h1=N+ 5.Kf3 +-

**B:** (=) 1...h2 With Black to move first, he can hold White's king back for one crucial move. 2.Ra3+ Kg2 3.Kg4 h1=N 4.Ra2+ Kg1 5.Ra1+ (5.Kg3? Qh8 -+) 5...Kg2 6.Rxh1 Kxh1 =

### E9.24

**W:** (=) Black's king is outside the winning zone, but White needs to be accurate to draw. **1.Kc6**  
(1.Kd8? Ke5 2.c8=Q Kd6 -+) **1...Qa6+ 2.Kd7 Qb7 3.Kd8** (3.Kd6? Qc8 -+) **3...Qb6 4.Kd7 Qa7 5.Kc6 =**

**B:** (-+) White's pawn is currently pinned, so Black's king has a free move to step into the winning zone. **1...Kd5 2.Kd8 Ke6** (2...Kd6? 3.c8=N+ =) **3.c8=Q+ Kd6 -+**

### E9.25

(-+)

**W:** **1.Rg3+ Ke4** (1...Ke2? 2.Rg2 =; 1...Kf4? 2.Rg8 Ke3 3.Rf8 =) **2.Rg4+ Ke5 3.Rg5+ Ke6 4.Rg6+ Kf7 -+**

**B:** **1...f1=Q -+**

### E9.26

**W:** (+-) **1.h8=Q**

**B:** (=) Black needs to set up a barrier with his knight in order to slow down White's king. **1...Ng6**  
(1...Nf7? 2.Kd7 Kd3 3.Ke7 Nh8 [3...Ne5 4.Kf6 +-] 4.Kf6 Ke4 5.Kg7 Kf5 6.Kxh8 Kf6 7.Kg8 -+) **2.Kd8** (2.Kd7 Nf8+ =; 2.Kd6 Kd3 3.Ke6 Nf8+ =) **2...Kd3 3.Ke8 Ke4 4.Kf7 Kf5 =**

### E9.27

(=) A queen normally wins against a center pawn, but this position is exceptional; due to the awkward placement of White's king and queen, the queen can't find a check or pin to get the winning procedure started.

**W:** **1.Qe4 e1=Q =**

**B:** **1...e1=Q =**

### E9.28

**W:** (=) **1.Kf4 Kc2 2.g5 Kd3 3.Kf5 Kd4 4.g6 Kd5 5.g7 Rg1 6.Kf6 Kd6 7.Kf7 Rf1+ 8.Ke8 Rg1 9.Kf7 =**

**B:** (-+) **1...Ra4** This cutoff doesn't permanently stop White's king from moving forward, but it slows it down enough for Black to win. **2.Kg3** (2.g5 Kc2 3.g6 Ra6 4.g7 Rg6 -+) **2...Kc2 3.Kh4 Kd3 4.Kg5 Ke4 5.Kh6 Kf4 6.g5 Kf5 7.g6 Ra6 8.Kh7 Rxg6 -+**

### E9.29

(=) The position is drawn no matter who moves first by the occupation rule.

**W:** **1.Kf6 Nh5+ 2.Kg5 Ng7 3.Kh6 Ne6 =**

**B:** Black can make any king move and draw as in the previous variation.

### E9.30

**W:** (=) White's knight is already on the right circuit of squares to draw against Black's sixth-rank rook pawn. **1.Ne3 Kf3** (1...h2 2.Nf1+ =) **2.Nf1 Kf2 3.Nh2 Kg2 4.Ng4 Kf3 5.Nh2+ Kg3 6.Nf1+ Kg2 7.Ne3+ Kf2 8.Ng4+ Kf3 9.Nh2+ =**

**B:** (+-) **1...Kxg4**

### E9.31

**W:** (=) White must start shoulder-charging right away to draw. **1.Kf6** (1.Kg6? Kb4 2.f6 Kc5 3.f7 Kd6 4.Kg7 Ke7 -+) 1...Kb4 2.Ke6 Kc5 3.f6 Kc6 4.f7 Kc7 5.Ke7 Re1+ 6.Kf6 Kd7 7.f8=Q Rf1+ 8.Kg7 Rx<sub>f</sub>8 9.Kxf8 =

**B:** (-+) 1...Kb4 2.Kf6 Kc5 3.Ke6 Kc6 4.f6 Re1+ 5.Kf7 Kd7 6.Kg7 Ke6 7.f7 Rg1+ 8.Kf8 Rf1 9.Kg8 Rx<sub>f</sub>7 -+

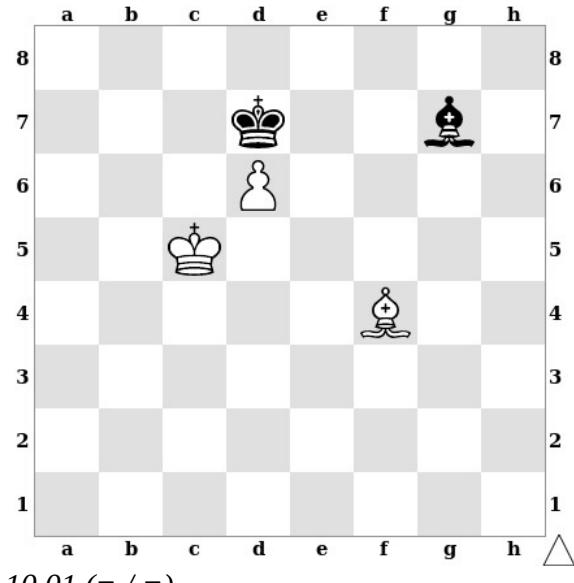
### E9.32

**W:** (+-) Black's pawn is pinned so White gets a free move to get his king into the winning zone. **1.Kg4 Kg1** (1...Kh1 2.Qe2 Kg1 3.Kg3 f1=N+ 4.Kh3 -+) 2.Kf3 f1=Q+ 3.Kg3 +-

**B:** (=) White's king is outside the winning zone, but Black must be accurate to draw. **1...Kh1** (1...Kg1? 2.Kg4 f1=Q 3.Kg3 +-) 2.Qf4 Kg2 3.Qg4+ Kh2 4.Qe2 Kg1 5.Qe3 Kg2 6.Qd2 Kh1 7.Qe2 Kg1 =

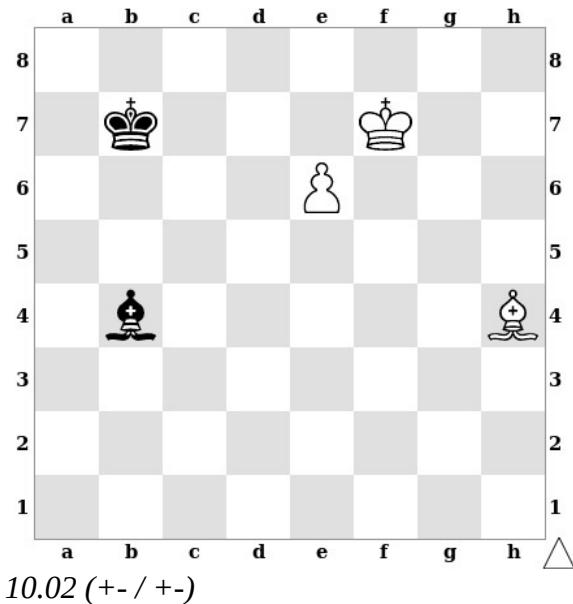
# 10 – Minor Piece Endings

## Bishop Versus Bishop (Same Color)



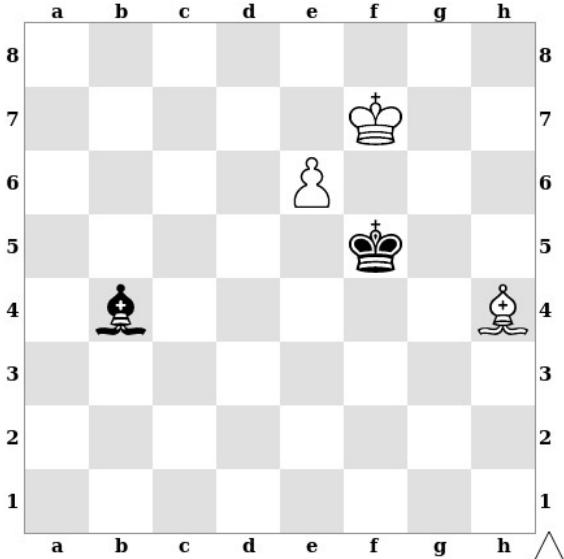
10.01 (= / =)

We begin with positions in which a bishop and pawn face a bishop of the same color. With the weaker side's king standing in front of the pawn on a square of the opposite color to the bishops, the position is an easy draw. Here Black could simply shuffle his bishop around and refuse to move his king, or he could sacrifice his bishop for the pawn. In either case the game is drawn.



10.02 (+- / +-)

In this position Black's king is not well-placed, but his bishop is currently stopping the pawn's advance. White wins with a standard technique. First, he forces Black's bishop to the other controlling diagonal (h4-d8), since this is the one White's king can help to block:  
**1.Be7 Be1 2.Bf8 Bh4.** Now White brings his bishop to f6, in order to block Black's bishop: **3.Bg7 Kc8** (3...Bd8 4.Bf6 +-)  
**4.Bf6 Bxf6 5.Kxf6 Kd8 6.Kf7 +-.**

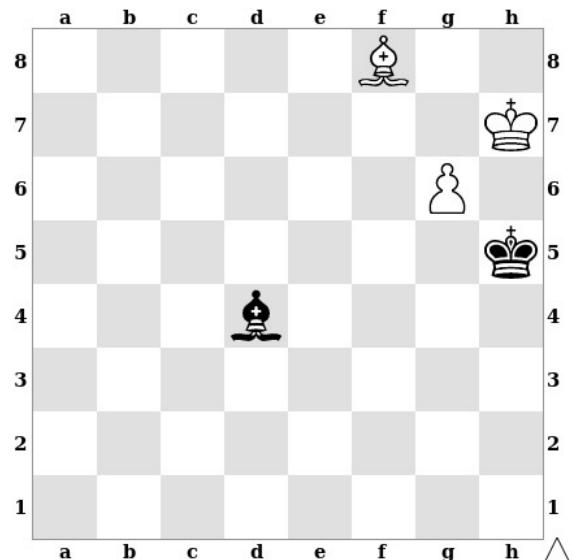


10.03 (= / =)

Here Black has adopted the standard defensive technique in these positions: his king stands directly behind White's king, in contact with the pawn. After **1.Be7 Be1 2.Bf8 Bh4**, Black's bishop has been forced to the other diagonal, just like before. However, after **3.Bg7 Bg5** the value of Black's king position becomes clear: White can't play **4.Bf6**. White can repeatedly challenge Black's bishop, but because this can only be done with White's bishop going to e7, where it blocks White's pawn, Black always has time to get to the other diagonal before the pawn can advance. After **4.Bd4 Bh4 5.Bc5 Bg5 6.Be7 Bd2 7.Bh4 Bb4 =**, it is obvious that White can't make progress.

The defender often runs into problems when one of the controlling diagonals is short. In this position, because the h6-f8 diagonal is only three squares long, Black loses in spite of his excellent king position. White begins with **1.Bg7**, forcing Black to bring his bishop to the short diagonal. Black can choose whether to put the bishop on f8 or h6, but he loses in either case:

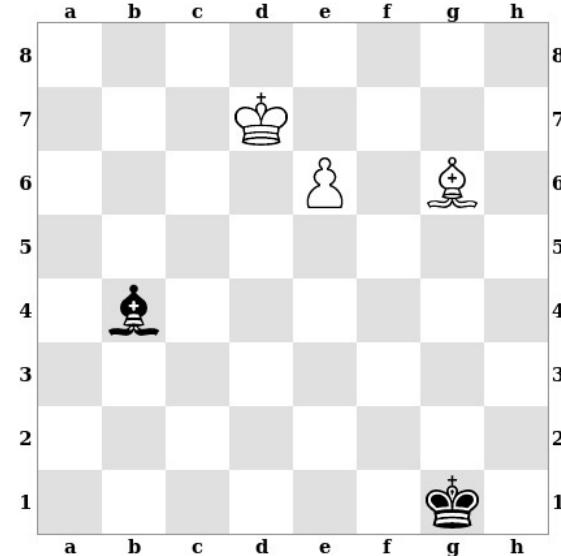
- a.) **1...Bc5 2.Bc3 Bf8 3.Bd2** (zugzwang) **Kg4 4.Bh6 +-**
- b.) **1...Be3 2.Bc3 Bh6 3.Bb4** (zugzwang) **Kg5 4.Bd2+ +-**



10.04 (+- / +-)

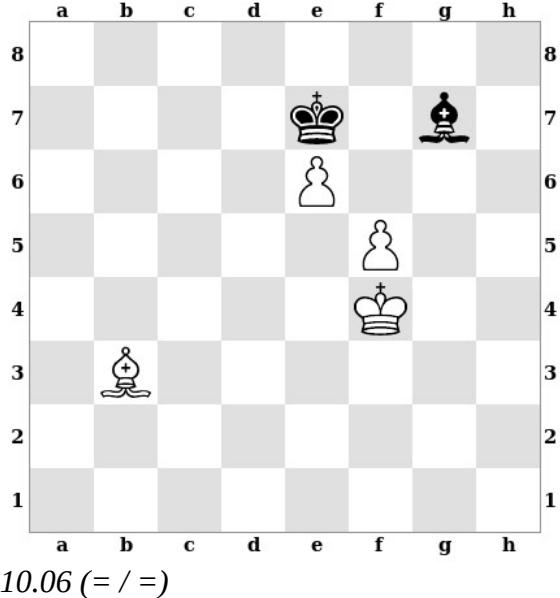
## Bishop Versus Bishop (Opposite Color)

We now look at endgames in which each player has a bishop but the bishops move on opposite color squares from one another. These endgames are very often drawn, even when one of the players is down multiple pawns. For this reason, heading into an opposite color bishop endgame is a common way of saving an otherwise losing position.



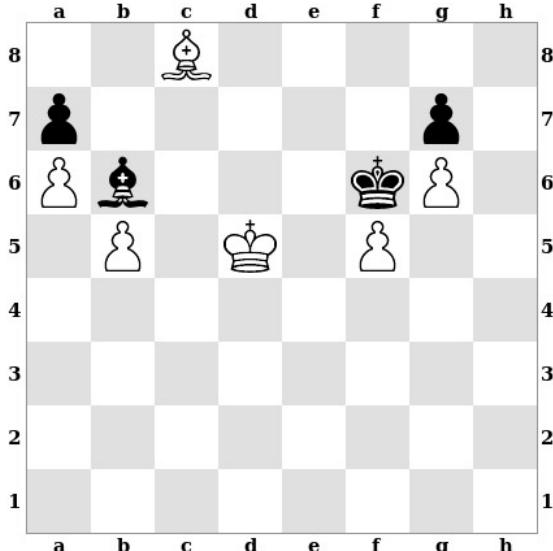
10.05 (= / =)

Positions with a single pawn are not usually very complicated. Here Black's bishop attacks a square in the pawn's path, so the position is completely drawn. All Black needs to do is to shuffle his bishop around on the a3-f8 diagonal and sacrifice it for the pawn if the pawn ever moves forward. Since White's bishop can't challenge Black's on its diagonal, there is really nothing that White can even try.



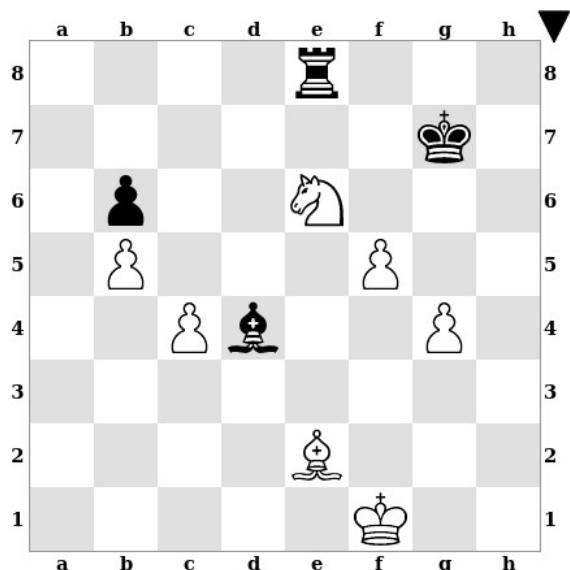
10.06 (= / =)

Even two extra pawns can't win in many cases. In this position the pawns have both been moved to light squares, allowing Black to set up a *blockade* on the dark squares. The position is completely drawn as long as Black keeps his bishop on the a1-h8 diagonal and doesn't move his king: 1.Kg5 Ba1 2.Kg6 Bb2 =.



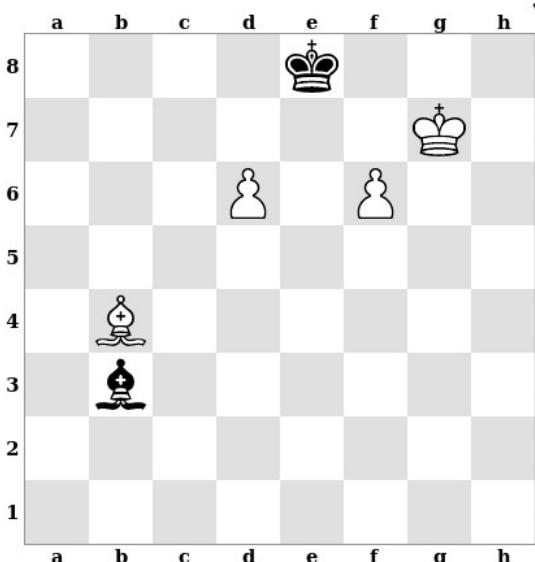
10.07 (= / =)

Positions with more pawns on the board may still be easily drawn if the defender can blockade the stronger side's pawns. In this position, Black can just move his bishop around on the g1-a7 diagonal, leaving his king where it is.



10.08 (\* / =)

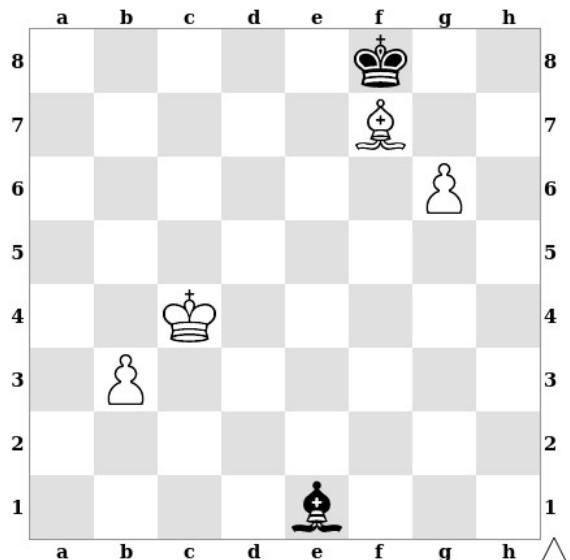
The drawish nature of opposite color bishop endings is the basis for many important defensive resources. In this position, for example, White has just played Ne6+, forking Black's king and bishop. If Black responds with something like 1...Kf6? 2.Nxd4 +-, then he will be losing. However, Black can save the game by sacrificing the exchange to reach a completely drawn opposite color bishop ending: **1...Rxe6 2.fxe6 Kf6 =**. Black's plan now is to play ...Kxe6, bring his king to g7, and to then shuffle his bishop around on the g1-a7 diagonal, never moving his king again. There is nothing White can do to stop this, so the game is completely drawn.



10.09 (= / =)

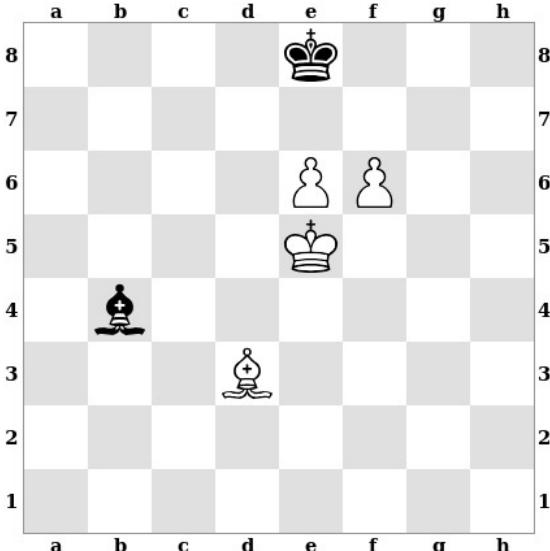
In this position, the pawns are separated by one file. Thanks to the good position of his king, Black can restrain both pawns. He simply needs to keep his bishop on the appropriate diagonal to help restrain whichever pawn White's king is currently supporting. With the white king on g7 Black's bishop needs to occupy the a2-g8 diagonal. For the time being Black can just wait: 1...Bc4 2.Kg6 Bb3 3.Kf5 Bc4 4.Ke5 Bb3 5.Kd4 Ba2 6.Kc5 Bb3. Once White's king threatens to advance the d-pawn, the bishop must switch diagonals: 7.Kc6 Be6 8.Kc7 Bh3 9.Ba3 Bg4 =.

It is worth noting that White can win Black's bishop in the initial position after 1...Bc4 2.f7+ Bxf7 3.d7+ Kxd7 4.Kxf7 =. This is just a draw here, but with other pawns on the board it might be winning.



10.10 (+- / +-)

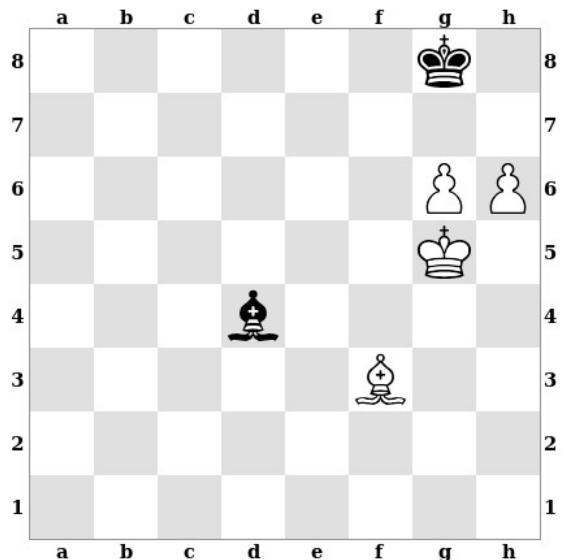
Of course, not all opposite color bishop endings are drawn. Things are harder for the defender when the pawns are separated by many files. White wins easily here by forcing Black to sacrifice his bishop for one of the pawns: 1.b4 Bf2 2.b5 Bg1 3.Kd5 Bf2 4.Kc6 Be3 5.b6 Bf4 6.Kd7 Be3 7.b7 Bf4 8.Kc8 Bg3 9.b8=Q Bxb8 10.Kxb8 +-.



10.11 (+- / +-)

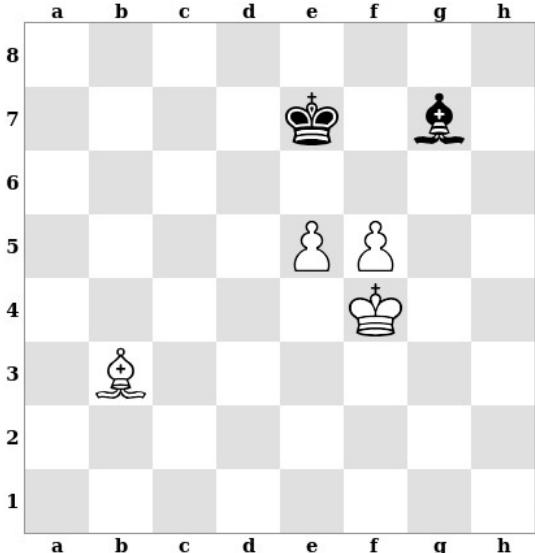
With two connected pawns on the sixth rank, the stronger side can normally win. Here White begins with **1.Bb5+**, forcing Black to commit his king to a particular side of the pawn. After **1...Kf8**, White brings his king around on the queenside: **2.Kd5 Ba3** (**2...Bc3 3.e7+ +-**) **3.Kc6 Ke8** **4.Kc7+ Kf8** **5.Kd7 Bb4**. Now, with his king on d7, White is ready to advance his pawns: **6.e7+ Bxe7** **7.fxe7+ Kf7** **8.e8=Q+ +-**.

If Black instead played **1...Kd8**, then White's king would approach on the kingside: **2.Kf5 Bc5** **3.Kg6 Bb4** **4.Kf7 Bc5** **5.e7+ +-**.



10.12 (= / =)

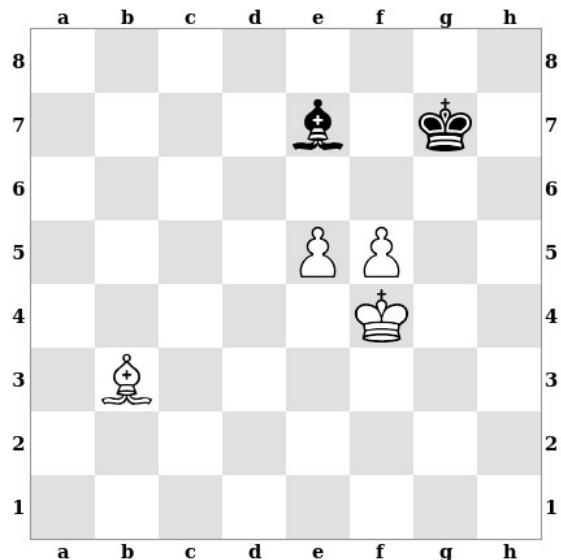
When one of the pawns is a rook pawn then the position may be drawn even with the pawns on the sixth rank. This is because the stronger side's king only has one side that it can use to advance. Here, after **1.Bd5+**, Black simply needs to play **1...Kf8** followed by moving his bishop around on the a1-h8 diagonal. White has no way to make progress. On the other hand, **1...Kh8?** would lose to **Kf5-e6** etc.



10.13 (= / =)

If the pawns are on the fifth rank (or farther back), then it is sometimes possible for the defender to draw. In this position, Black draws since he has achieved the correct defensive setup. His king and bishop prevent the advance of White's f-pawn (which Black would meet by sacrificing his bishop to capture both pawns). If White plays e6, then Black has the blockade shown in 10.06. Finally, Black's bishop attacks one of the white pawns, preventing White from playing Kg5-g6. Black can draw by simply shuffling his bishop between g7 and h8.

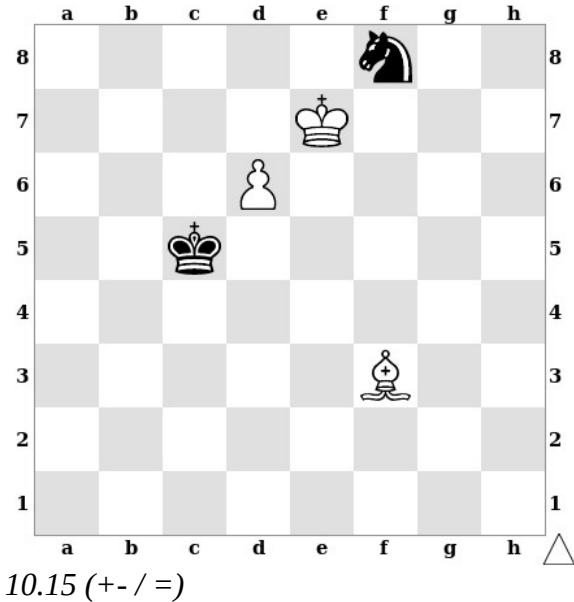
This defensive setup is not available once the pawns have reached the sixth rank because the bishop doesn't have room to shuffle around.



10.14 (+- / +-)

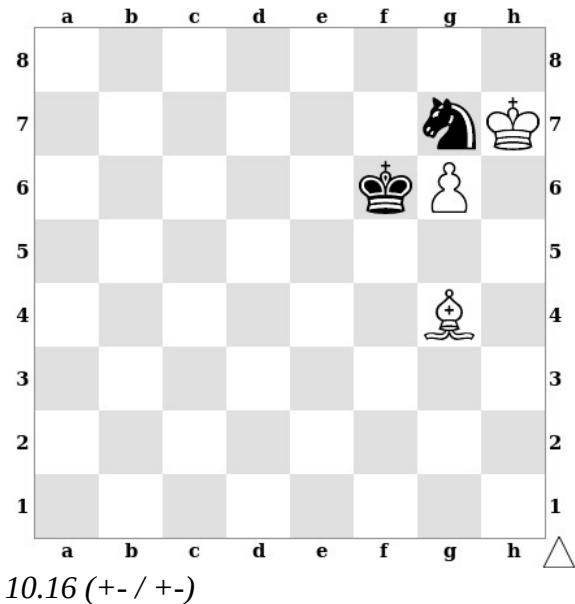
The placement of Black's pieces in the previous example was quite important. While this position looks similar, the king and bishop have swapped places, allowing White's king to get in front of the pawns:  
**1.Ke4 Bd8 2.Kd5 Kf7 3.Kd6+ Kg7 4.Ke6 Bh4 5.f6+ Kf8 6.Kf5 Be1 7.e6** +- . White now wins with his sixth rank pawns as shown in 10.11.

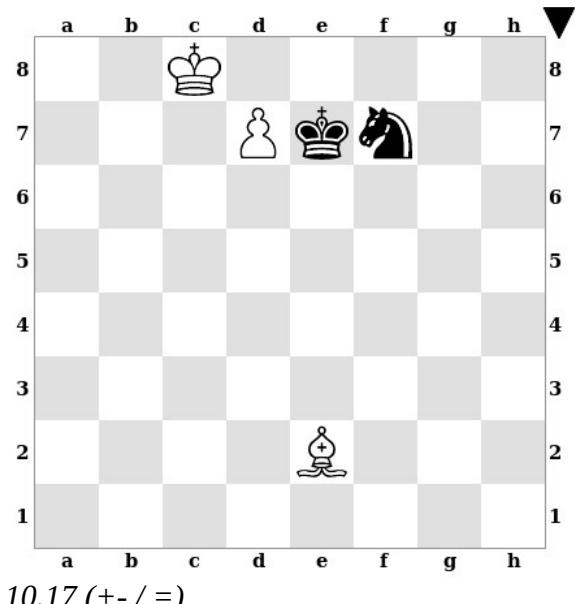
## Bishop and Pawn Versus Knight



A bishop can often dominate a knight on an open board due to the bishop being a long range piece. Here White wins with the simple **1.Be4**, corraling the knight (controlling all of its available squares). This leaves Black in zugzwang and he loses after **1...Kd4** **2.Kxf8 +-**.

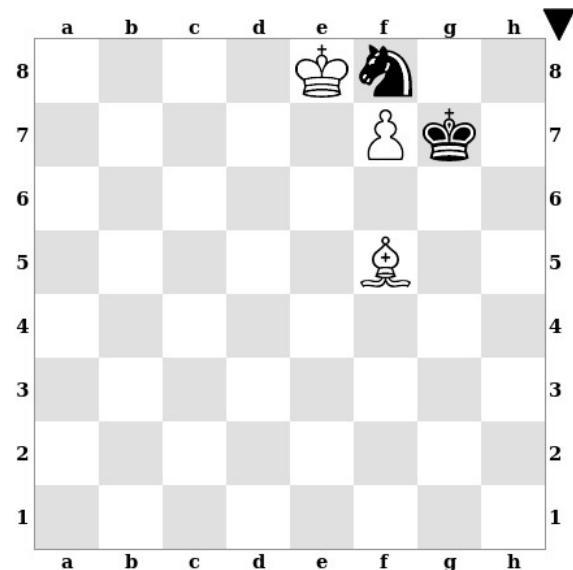
This position shows a slightly more complicated version of the same idea. White wins by arranging a zugzwang for Black that forces him to give up his knight: **1.Bh3 Nh5** (**1...Ne8 2.Bd7 Ng7 3.Kh6 +-**) **2.Kh6 Ng7 3.Bd7 +-**.





10.17 (+/-/=)

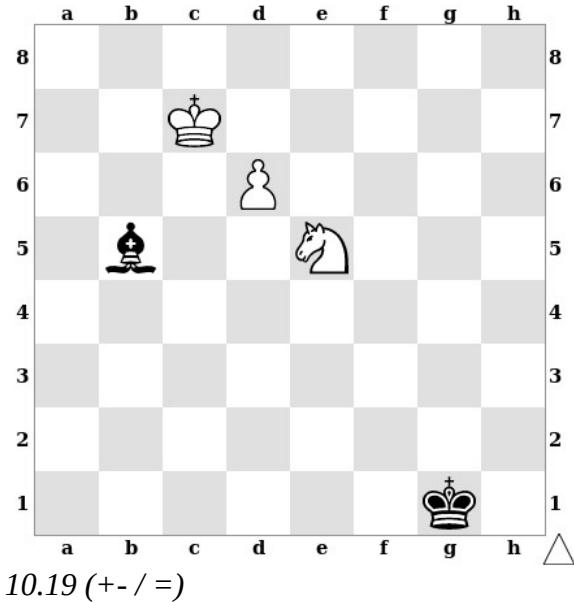
This position shows an important resource for the defender. Black draws with 1...Nd6+ 2.Kc7 Ne8+ 3.Kc6 (3.Kc8 Nd6+ =) 3...Nf6 4.Bg4 Nxd7 =.



10.18 (= / =)

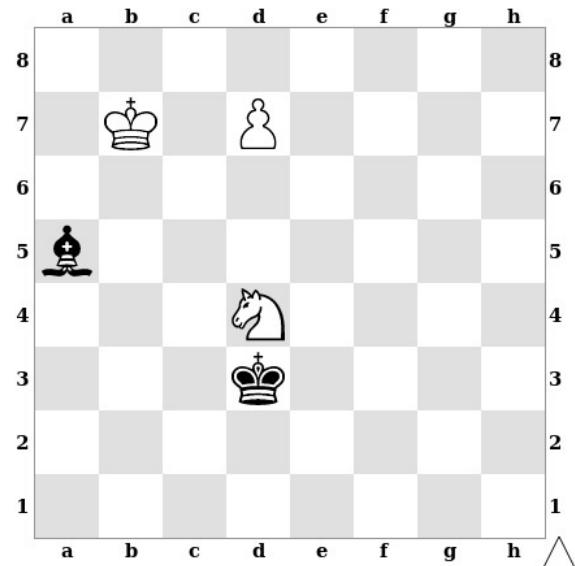
Black is apparently in zugzwang here, but he is saved by a surprising stalemate possibility: 1...Kh8 2.Ke7 (2.Kxf8 =, stalemate) 2...Kg7 3.Ke8 Kh8 4.Bh3 Kg7 5.Ke7 Nh7 6.Bg4 Nf8 7.Bf5 Kh8 =.

## Knight and Pawn Versus Bishop



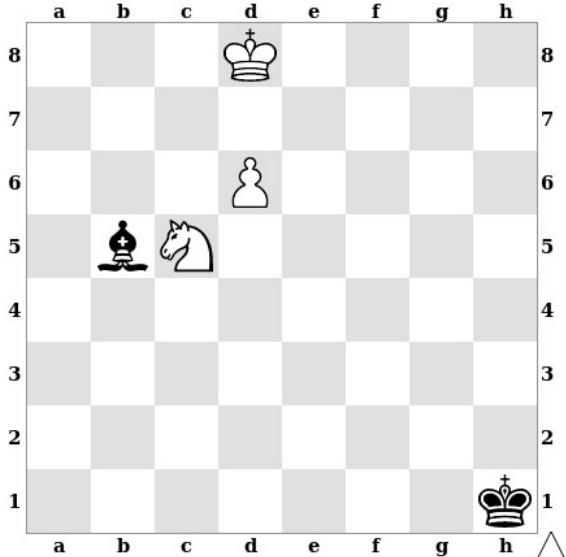
10.19 (+- / =)

When a knight and pawn face a bishop, the fight generally revolves around the stronger side trying to block the bishop or control all of the available squares on the diagonal. Here White wins by blocking the bishop with **1.Nc6 +-**, followed by pushing his pawn.



10.20 (+- / =)

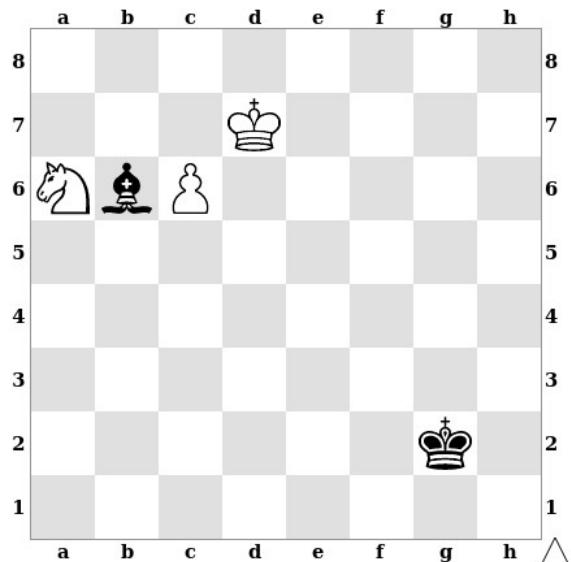
Here White wins by controlling all of the squares on the bishop's diagonal with **1.Nc6 +-**. It is important that the diagonal in question is only four squares long. White's king controls two squares and his knight controls two more, which is just enough to cover the entire diagonal.



10.21 (= / =)

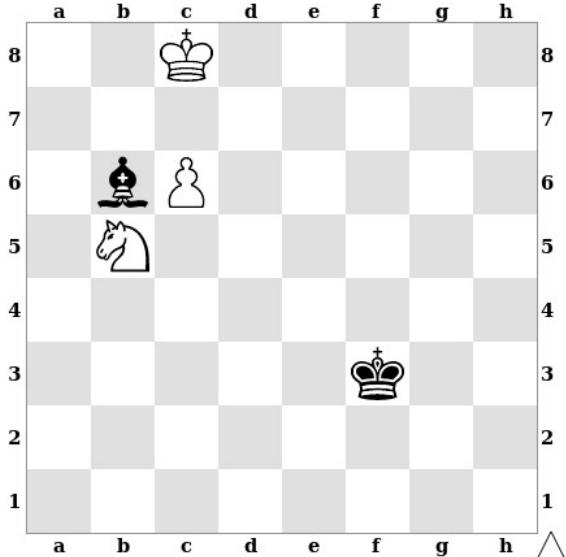
The bishop can often defend successfully when the two controlling diagonals available to it are both fairly long. Even the shorter of the two diagonals available to Black's bishop here is five squares long, which allows him to hold the draw. Black must simply notice when White's knight is threatening to block his bishop so that he can avoid this.

**1.Kc7 Kg1 2.Nd3 Kh1 3.Ne5** (threatening Nc6) **3...Be8 4.Nd7 Kg1 5.Kd8 Bg6 6.Ke7 Bf5 7.Nc5** (threatening Ne6) **7...Bc8 8.Nd7 Kh1 9.Kd8 Ba6 10.Kc7 Bb5 11.Ne5 Be8 =.**



10.22 (+- / =)

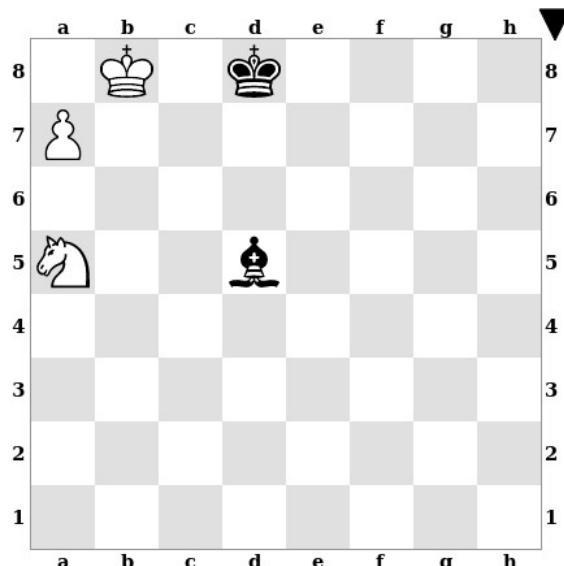
Here the bishop is on a four square diagonal and White's knight can soon take control of the last two safe squares: **1.Nc5 Kf3 2.Na4 Ba5 3.Nb2 Ke4 4.Nc4 +-**.



10.23 (+- / =)

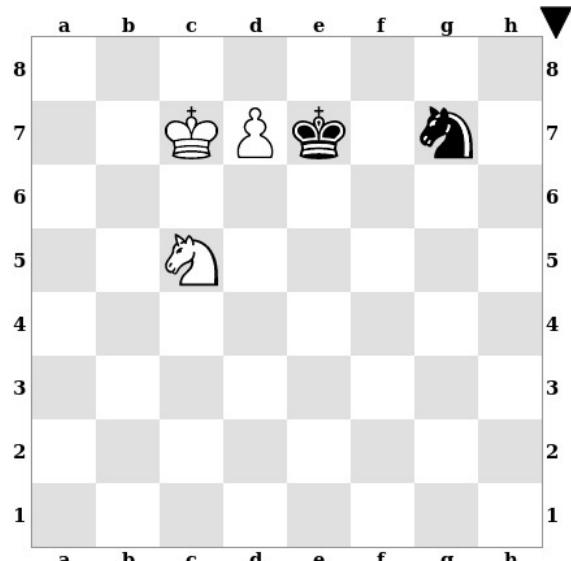
The stronger side often wins by combining threats to control a key diagonal and threats to block another. Here White wins after **1.Nd6 Ke2** (1...Kf4 2.Kd7 +- followed by Nc4) **2.Nc4 Bg1 3.Kd7 Bh2 4.Nd6 +-.**

Here White is threatening Nb7 but Black can draw with a surprising resource that is sometimes available against a rook pawn: **1...Ba8**. If White ever takes the bishop, then Black draws by moving his king to the same color square as that occupied by the knight, e.g. 2.Kxa8 Kc7 =. If White instead plays 2.Nc4, then he is no longer threatening Nb7 and the bishop can leave the corner: 2...Bg2 (but not 2...Kd7? 3.Nb6+ +-) 3.Nb6 Bf3 4.Nc8 Bg2 5.Nd6. Now White is threatening Nb7 again, so it's time for 5...Ba8 =. Finally, Black also draws after **2.Nb7+ Kd7 3.Nc5+ (3.Kxa8 Kc8 =) 3...Kd8 4.Ne6+ Kd7 5.Nf8+ Kd8 6.Kxa8 Kc7 =.**



10.24 (+- / =)

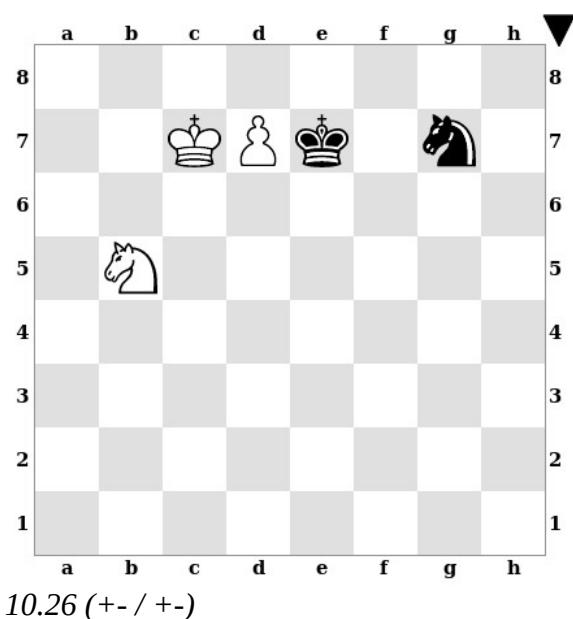
## Knight Versus Knight



10.25 (+- / =)

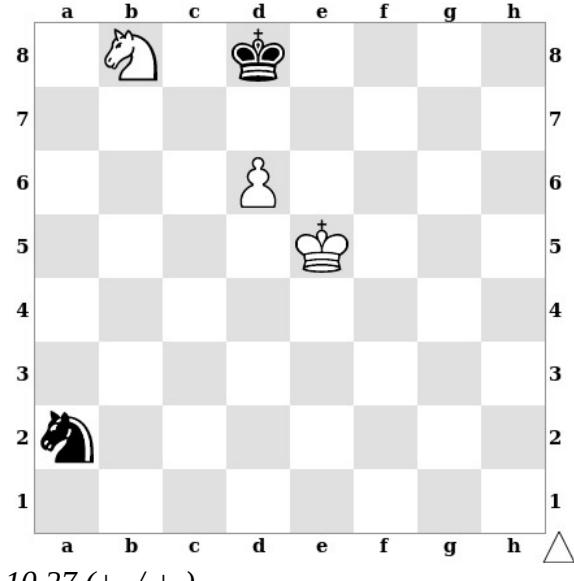
Positions in which both sides have a knight and pawns are similar in some ways to pawn endings: they are very concrete and difficult with many chances for zugzwang. There aren't many useful generalizations possible.

This position features a defensive resource that we already saw the knight use against a bishop and pawn in an earlier example. Black draws with 1...Ne8+ 2.Kc8 (2.Kc6 Nf6 = followed by ...Nxd7) 2...Nd6+ 3.Kb8 (3.Kc7 Ne8+ = just repeats the position) 3...Kd8 4.Ka7 Ne4 5.Kb6 Nxc5 =.

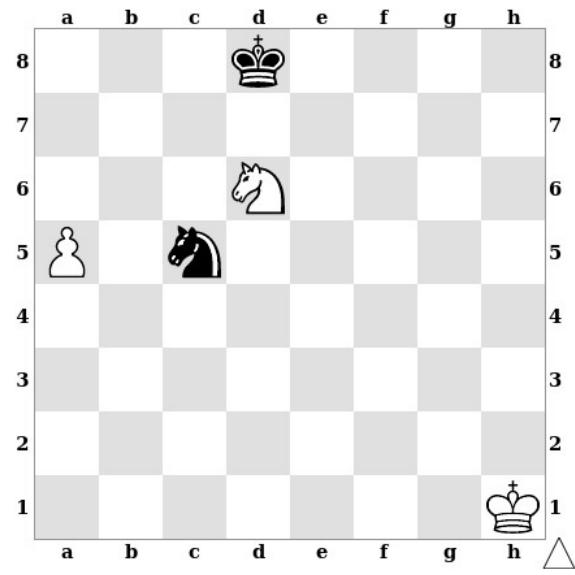


10.26 (+- / +-)

With White's knight attacking d6, the previous method isn't available and Black loses: 1...Ne6+ (1...Ne8+ 2.Kc8 +-) 2.Kc8 Nd8 (2...Kf7 3.Nc7 +-)  
3.Nd6 Ne6 (3...Nc6 4.Kc7 Nd8 5.Nf5+ +-) 4.Nf5+ Kf7  
5.Nd4 Ke7 (5...Nxd4 6.d8=Q +-) 6.Nxe6 +-.



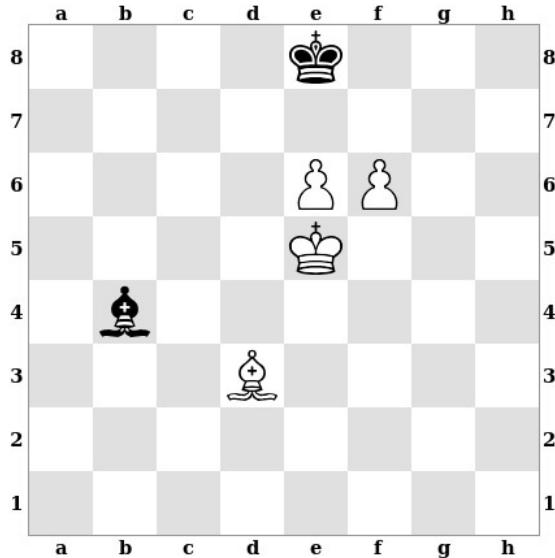
As the previous example already showed, the attacker often sacrifices his knight at an important moment to deflect the defending knight. White can use the same idea here to win after 1.Ke6 (threatening Nc6 and d7) 1...Nb4 2.d7 Kc7 3.Na6+ Nxa6 4.Ke7 +-.



Finally, the same surprising win that we saw in the fight between a knight and a rook shows up again here: 1.Nb7+ Nxb7 2.a6 Kc7 3.a7 +-.

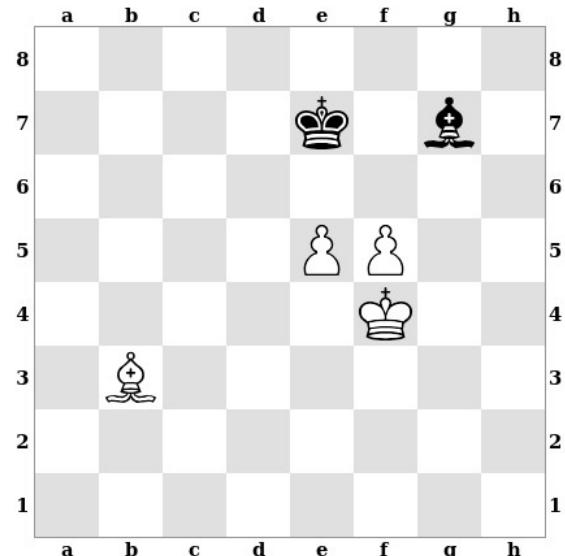
## Conclusion

This chapter has been more focused on typical ideas than on rules or specific results, so there really isn't much to memorize. The two most important results come from the section on opposite color bishops.

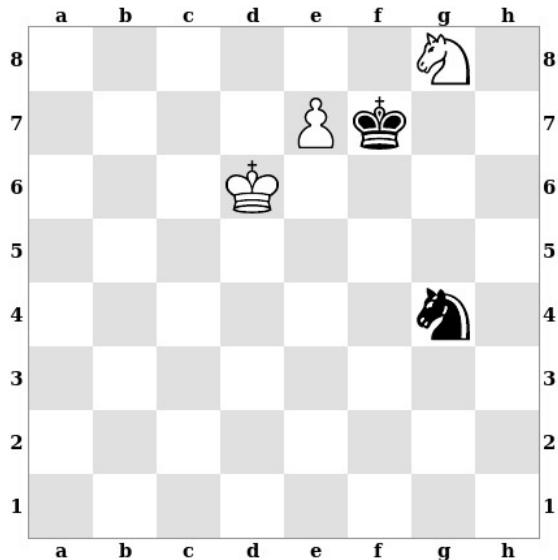


First, you should remember that two connected pawns normally win if they have reached the sixth rank. (Positions with a rook pawn are a notable exception.)

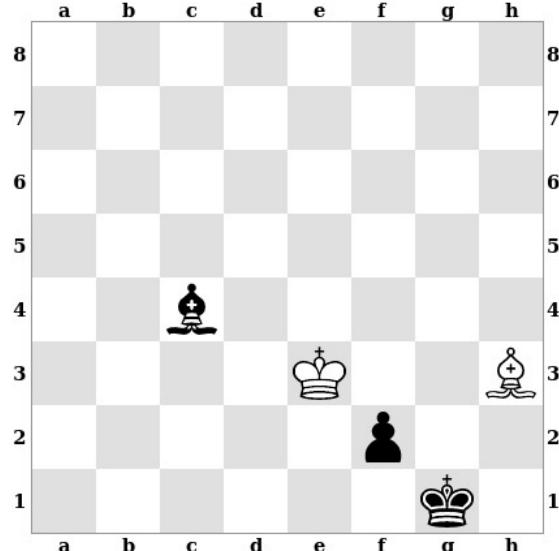
Second, you should remember that the defender can draw against pawns on the fifth rank (and farther back) if he can achieve the correct defensive setup. This setup involves having the bishop attack one of the pawns while the king and bishop work together to prevent the pawns from advancing.



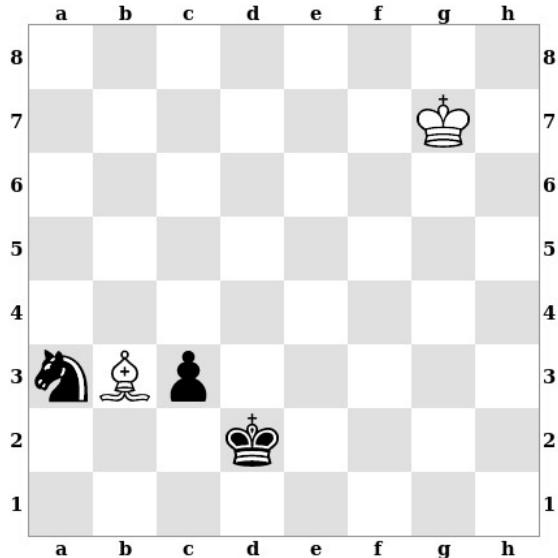
## Exercises



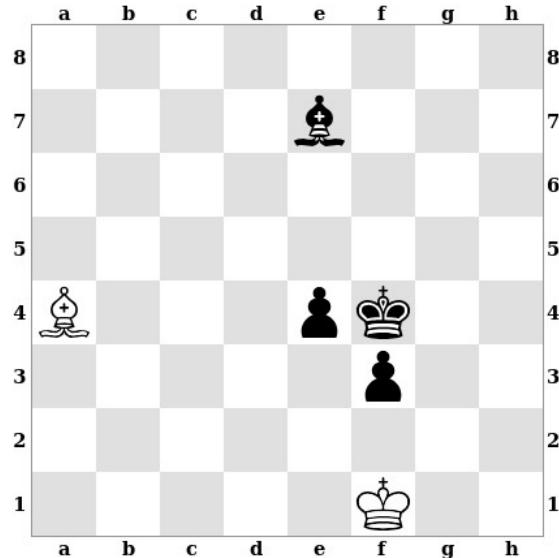
E10.01 (W/B)



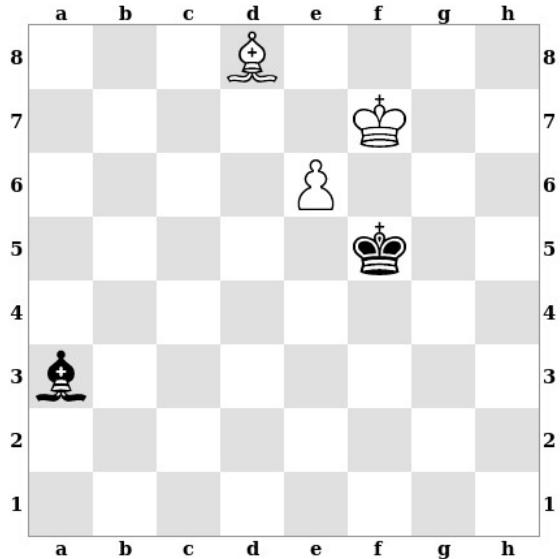
E10.02 (W/B)



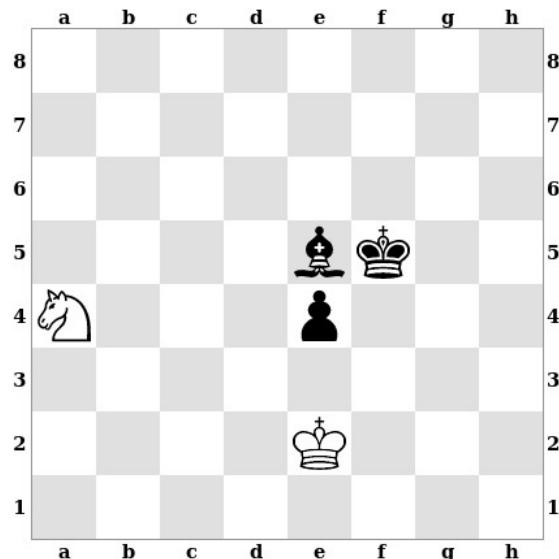
E10.03 (W/B)



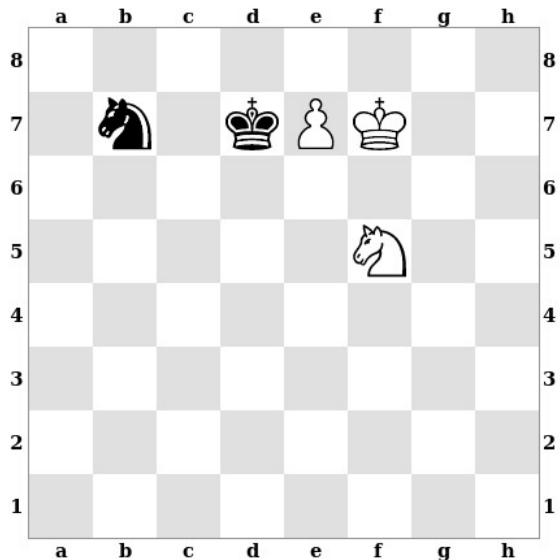
E10.04 (W/B)



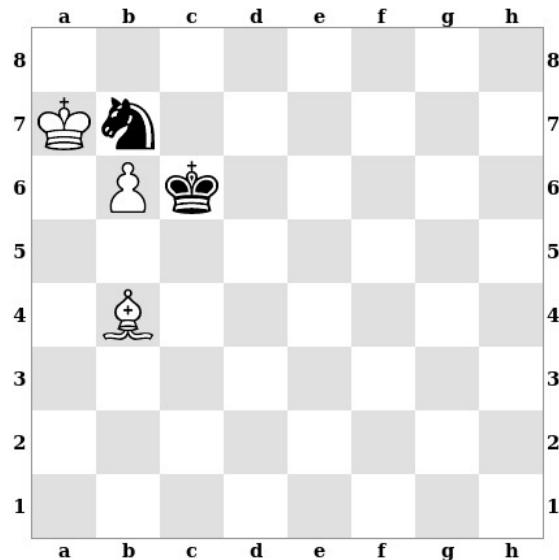
E10.05 (W/B)



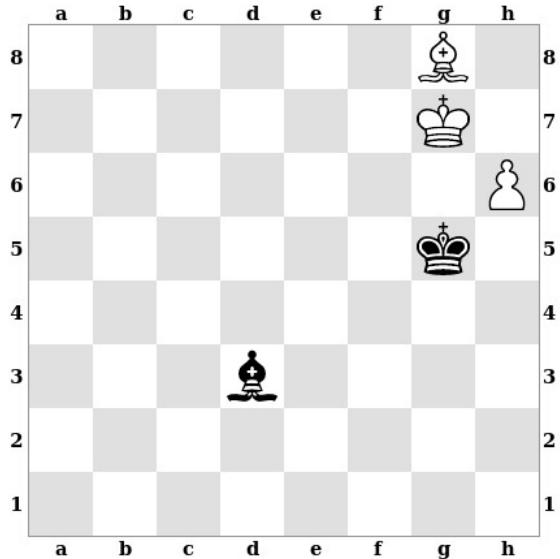
E10.06 (W/B)



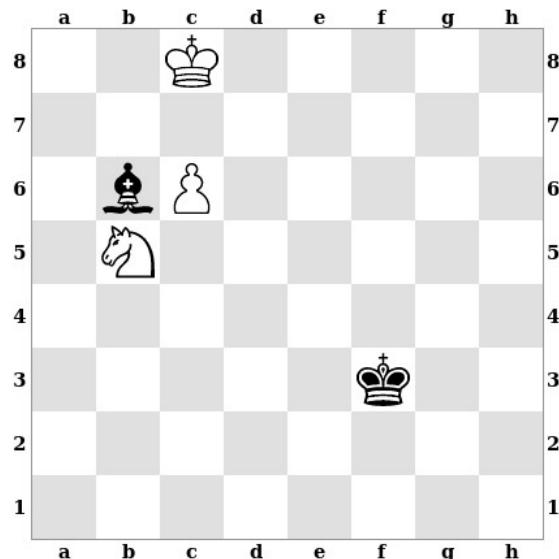
E10.07 (W/B)



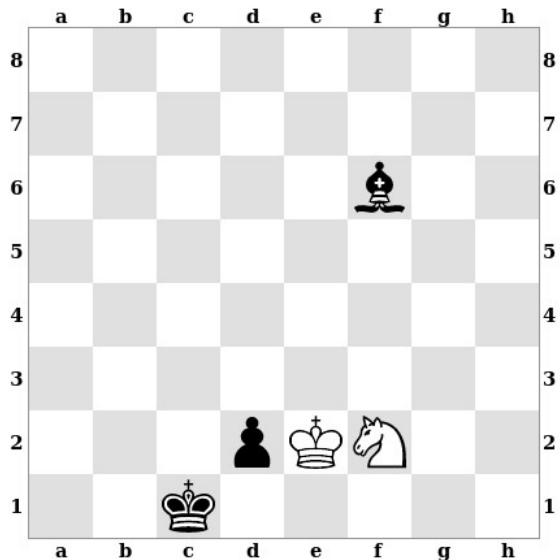
E10.08 (W/B)



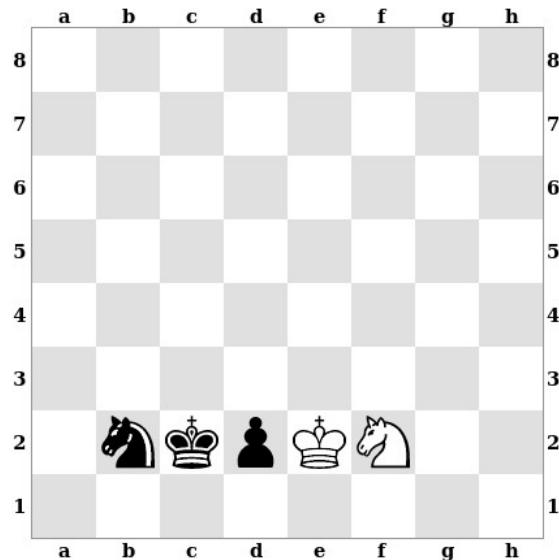
E10.09 (W/B)



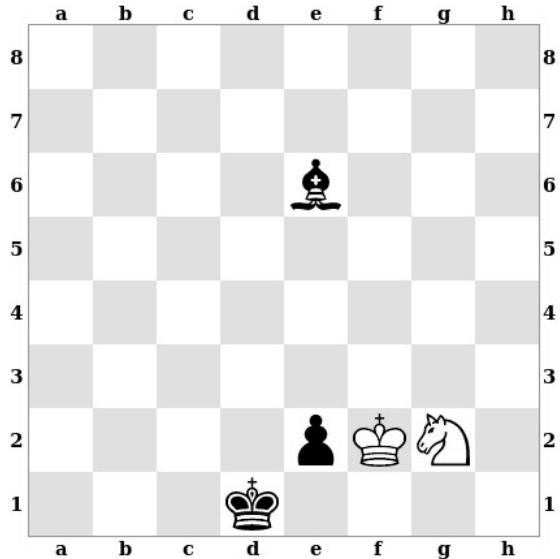
E10.10 (W/B)



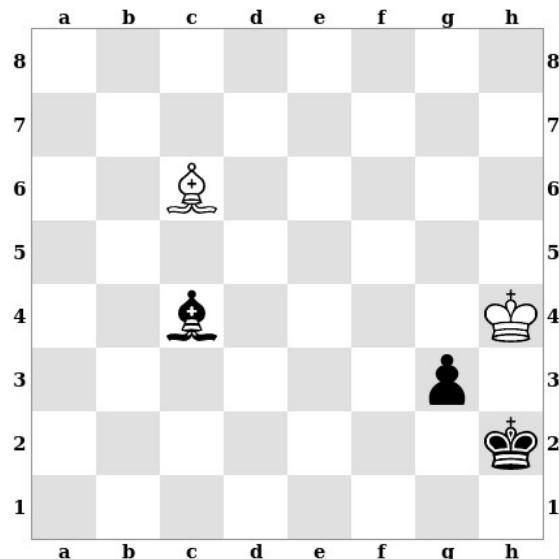
E10.11 (W/B)



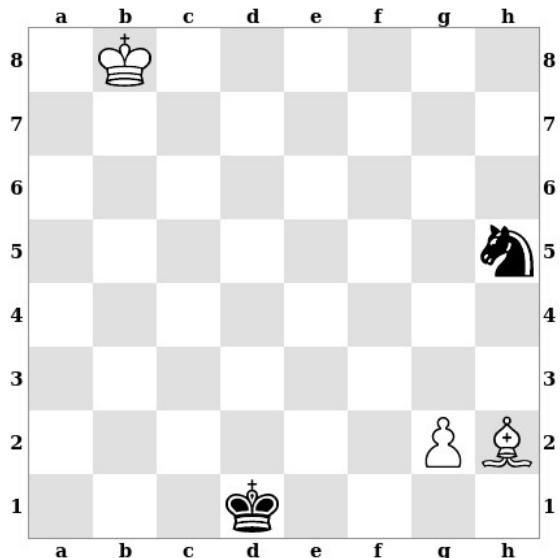
E10.12 (W/B)



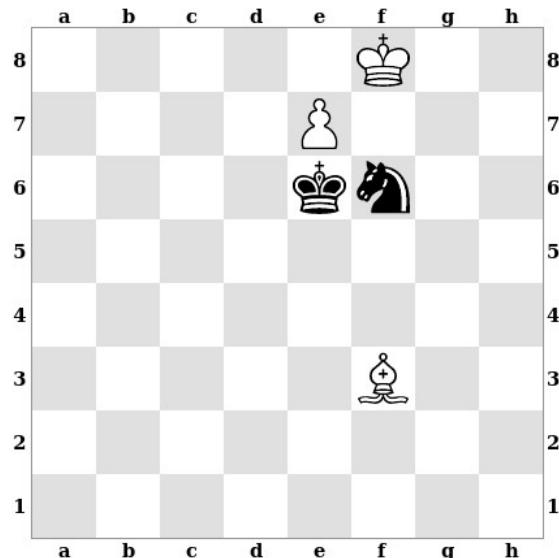
E10.13 (W/B)



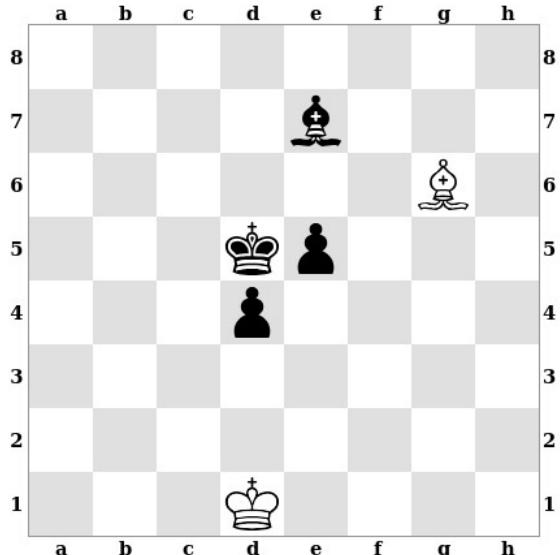
E10.14 (W/B)



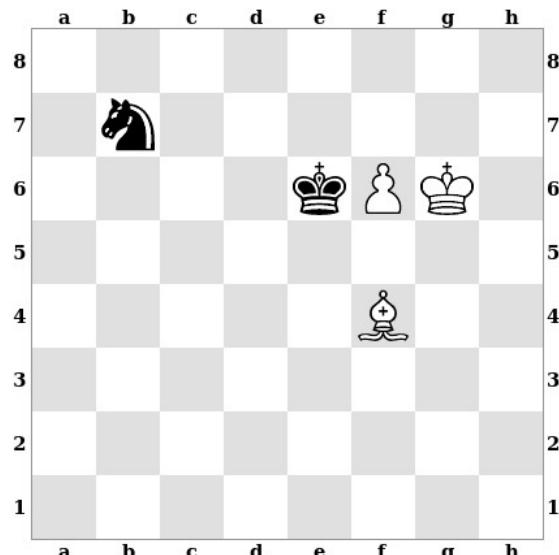
E10.15 (W/B)



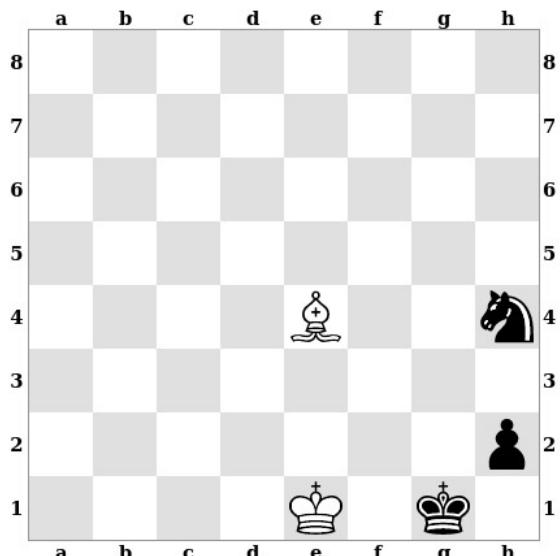
E10.16 (W/B)



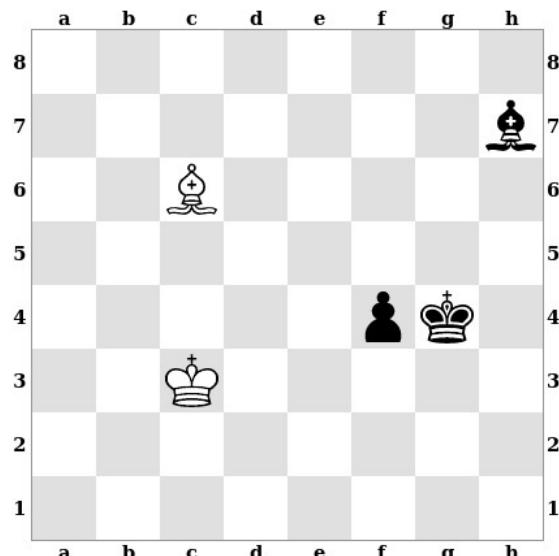
E10.17 (W/B)



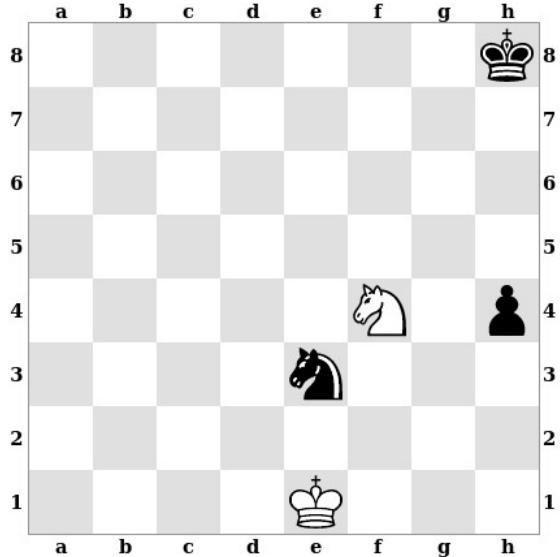
E10.18 (W/B)



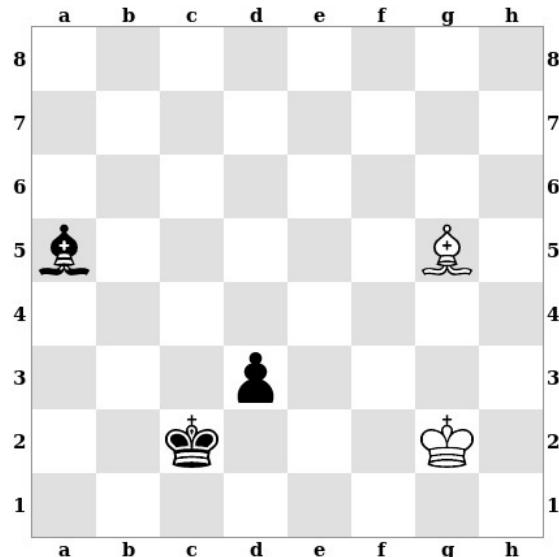
E10.19 (W/B)



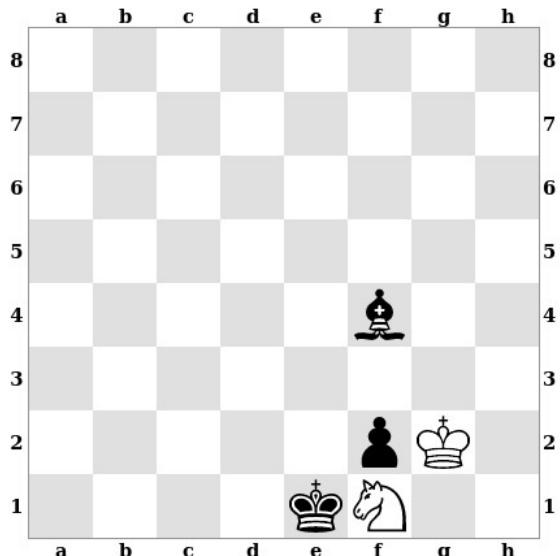
E10.20 (W/B)



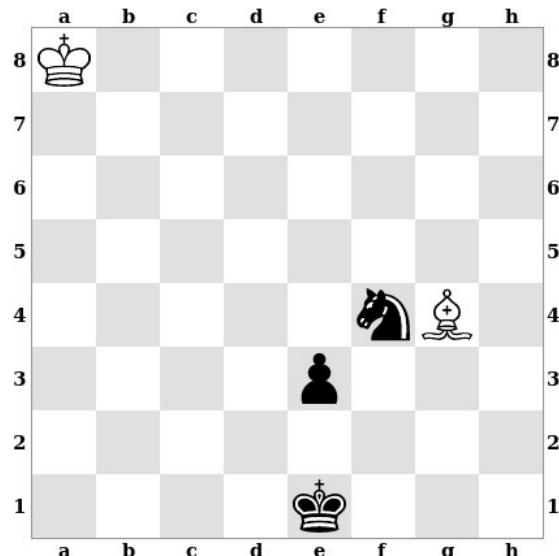
E10.21 (W/B)



E10.22 (W/B)



E10.23 (W/B)



E10.24 (W/B)

## Solutions

### E10.01

(+ -)

W: 1.Nh6+ Nxh6 2.Kd7 +-

B: 1...Ne3 (1...Ke8 2.Ke6 and 3.Nf6#) 2.Kd7 Nf5 3.e8=Q+ +-

### E10.02

(- +)

W: 1.Kf4 Be6 2.Kg3 Bxh3 3.Kxh3 f1=Q+ +-

B: 1...Be6 2.Bxe6 f1=Q +-

### E10.03

W: (=) White's king is just in time to help his bishop. 1.Kf6 Nc4 2.Ke6 Na5 3.Ba4 Nb7 4.Kd5 =

B: (-+) 1...Nc4 2.Kf6 Na5 3.Ba4 Nb7 4.Ke5 Nc5 +-

### E10.04

(- +) [opposite color bishops] White can't prevent e4-e3, giving Black connected pawns on the sixth rank.

W: 1.Bc2 e3 2.Bd3 (2.Bd1 Bh4 zugzwang -+) 2...Bh4 3.Ba6 Ke5 4.Bb5 Kd4 5.Ba6 Kc3 6.Bb5 Kd2 7.Ba6 e2+ +-

B: 1...e3 +- with similar play.

### E10.05

(=) Black draws due to his good king position and the controlling diagonals being long enough.

W: 1.Be7 Bc1 2.Bc5 Bg5 3.Bf8 Bh4 4.Bg7 Bg5 5.Bf8 Bh4 6.Be7 Be1 7.Bd8 Bb4 8.Bh4 Bc5 =

B: 1...Bb4 =, followed by playing as in the previous line.

### E10.06

W: (=) 1.Nc5 Kf4 2.Nxe4 =

B: (-+) 1...Bd4 corralling the knight 2.Kd2 Kf4 3.Ke2 e3 4.Kd3 Kf3 5.Kxd4 e2 +-

### E10.07

W: (+ -) 1.e8=Q+

B: (=) 1...Nd8+ 2.Kf8 Ne6+ 3.Kf7 Nd8+ 4.Kf6 Nc6 5.Kf7 Nxe7 =

### E10.08

(+ -)

W: 1.Ba3 Nd8 2.Be7 +- transposes to the line below

B: 1...Nd8 2.Be7 Nb7 3.Ka6 Nc5 4.Bxc5 +-

### E10.09

(+-)

**W:** 1.Bh7 Bc4 (1...Bxh7 2.Kxh7 Kf6 3.Kg8 Kg6 4.h7 +-)

**2.Bc2 Bg8 3.Bb3 Bxb3 4.h7 +-**

**B:** Black has no useful moves, e.g. 1...Bc2 2.Bh7 +- works out the same as the previous line.

### E10.10

**W:** (+-) 1.Nd6 Ke2 (1...Kf4 2.Kd7 Ba5 3.Nc4 +-)

**2.Nc4 Bg1 3.Kd7 Bh2 4.Nd6 +-**

**B:** (=) 1...Ke4 2.Nd6+ (2.Kb7 Bd8 3.Nc7 Ke5 4.Kc8 Kd6 =)

**2...Kd5 3.Kd7 Ba5 4.Nb7 Bb6 =**

### E10.11

**W:** (=) 1.Nd3+ Kc2 2.Ne1+ Kc1 3.Nd3+ Kc2 4.Ne1+ Kc3 5.Nf3 Kc2 6.Nxd2 =

**B:** (-+) 1...Kc2 2.Nd1 (2.Kf3 Bh4 3.Ke2 Bxf2 +-)

**2...Bd4 +-**

### E10.12

(-+)

**W:** 1.Kf1 Nd3 2.Ke2 Nxf2 3.Kxf2 d1=Q +-

**B:** 1...Nd3 2.Nd1 Nf4+ 3.Ke3 Kxd1 4.Kxf4 Ke2 +-

### E10.13

**W:** (=) 1.Ne3+ Kd2 2.Nf1+ Kd1 3.Ne3+ Kd2 4.Nf1+ Kd3 5.Ng3 Bg4 6.Nxe2 =

**B:** (-+) 1...Kd2 2.Kg3 (2.Ne1 Bd5 +-)

**2...Bd5 3.Kf2 Bxg2 4.Kxg2 e1=Q +-**

### E10.14

(-+)

**W:** White doesn't have a useful move, e.g. 1.Bb7 Bf1 +- works out the same way as the line below.

**B:** 1...Bf1 2.Bd5 Bg2 3.Be6 (3.Bc4 Bc6 4.Bf1 Bd7 5.Kg5 Bh3 6.Bxh3 Kxh3 +-)

**3...Bc6 4.Bh3 Bb5 5.Kg4 Bd7+ +-**

### E10.15

**W:** (+-) 1.Be5 corralling the knight 1...Ke2 2.g4 Ng7 3.Bxg7 Kf3 4.g5 Kf4 5.Bf6 +-

**B:** (=) 1...Nf6 2.Kc7 (2.Be5 Ng4 3.Bf4 Ke2 4.Kc7 Kf1 5.g3 Nf6 6.Kd8 Kf2 7.Ke7 Ne4 8.g4 Kf3 9.g5

Nxg5 =)

**2...Ke2 3.Be5 (3.Kd8 Kf1 4.g3 Ng4 =)**

**3...Kf1 4.g3 Kg2 5.Kd8 Kf3 6.Ke7 Ne4 7.Ke6**

Nxg3 =

### E10.16

**W:** (+-) 1.Bg4+ Kd6 2.Kf7 Ke5 3.Bh3 +-

**B:** (=) 1...Nd7+ 2.Ke8 Nf6+ 3.Kd8 Kf7 4.Bg4 Kg7 5.Kc7 Kf7 6.Kd8 Kg7 =

### E10.17

[opposite color bishops]

**W:** (=) White draws by preparing to adopt the standard defensive setup against Black's fifth rank pawns. **1.Bb1** (or 1...Bc2 =) **1...e4 2.Ke2 Bg5 3.Bc2 Ke5 4.Bb1** =

**B:** (-+) With Black to move, White doesn't have time to reach the defensive setup. **1...e4** Black is now threatening to play d3, Kd4 and e3, which White can't stop. **2.Bf7+ Ke5 3.Bc4 d3 4.Kd2 Kd4 5.Bf7 e3+** -+, winning because the pawns have reached the sixth rank.

### E10.18

**W:** (-+) **1.f7 Ke7 2.Kg7 Nd8 3.f8=Q** +-

**B:** (=) **1...Nd8 2.Bc7 Nf7 3.Bg3 Nd8 4.Bf4 Nf7 5.Bc7 Nh6** =

### E10.19

**W:** (=) **1.Bh1 Nf5 2.Bc6 Ne3 3.Bh1 Ng2+ 4.Ke2 Nh4 5.Ke1 Kxh1 6.Kf2 Nf3 7.Kf1 Nd2+ 8.Kf2 Ne4+ 9.Kf1** =

**B:** (-+) **1...Ng2+ 2.Ke2 h1=Q** -+

### E10.20

(=) White draws by quickly bringing his king to its ideal defensive position: directly behind Black's king and in contact with the pawn.

**W:** **1.Kd4** = is an easier version of the draw below.

**B:** **1...Kg3 2.Kd4 Bf5 3.Ke5 Bh3 4.Bd5** (White's king must stay in contact with the pawn when ...Bg2 is threatened. Black advances his pawn after 4.Kf6? Bg2 5.Bb5 f3 6.Kg5 f2. Now Black wins because one of the controlling diagonals is too short: 7.Bc4 Kh2 8.Kf4 Kg1 9.Kg3 Bf1 10.Bd5 Bb5 11.Bg2 Bd7 12.Kf3 Bc6+ -+) **4...Bg4 5.Kf6** (White's king needs to get to g5 without delay. Black advances his pawn after 5.Bc6? Bf3 6.Ba4 Bg2 7.Bd1 Bh3 8.Kf6 Bg4 9.Bb3 f3 10.Kg5 f2 11.Bc4 -+ and wins in the same way as in the previous note.) **5...Bf3 6.Bb3 Be2 7.Bd5 Bf1 8.Kg5 Be2 9.Bc6 Bf3 10.Ba4 Bd5 11.Bd1** =

### E10.21

**W:** (=) **1.Ng6+ Kg7 2.Nxh4** =

**B:** (-+) **1...Ng2+ 2.Nxg2 h3 3.Kf2 h2** -+

### E10.22

(-) White's king should be on c4 to defend, but he is not in time to get there.

**W:** **1.Kf1** (It is too late to go to c4: 1.Kf3 Bd2 2.Bd8 Bc1 3.Ba5 Bb2 4.Ke4 Bc3 -+) **1...Bd2 2.Bd8 Bc1 3.Ba5 Bb2 4.Ke1 Bc3+ 5.Bxc3 Kxc3 6.Kd1 d2** -+

**B:** Black has an even easier version of the above variations after **1...Bd2 2.Bd8 Bc1** -+

### **E10.23**

(=) White draws with a stalemate defense no matter who moves first: **1...Ke2 2.Kh1 Kf3 3.Ng3 Be5 4.Nf1 Ke2 5.Kg2** (but not 5.Ng3+? Bxg3 -+) **5...Bd4 6.Ng3+ Ke1 7.Nf1 =**

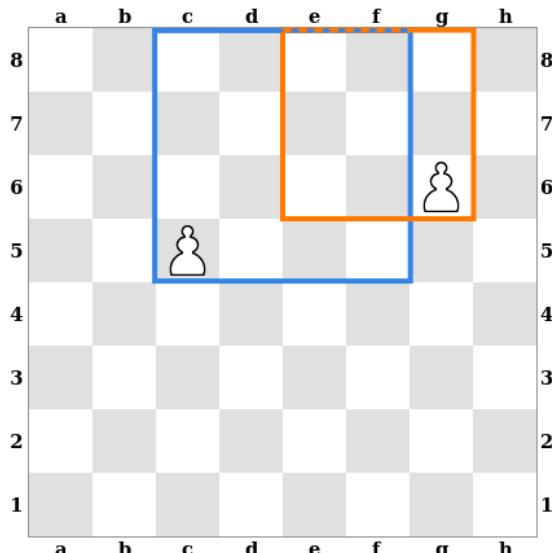
### **E10.24**

(=) White draws the same way no matter who moves first: he shuffles his king between a8 and b8 and relocates his bishop when the knight threatens to block it. For example: **1.Kb8 Kf2 2.Ka8 Ne6 3.Kb8 Nd4** (threatening ...Nf3) **4.Bd1 Ne2 5.Ka8 Ke1 6.Bb3 Kd2 7.Bc4 Nf4** (threatening ...Nd3) **8.Bf1 Ne2 9.Kb8 Ke1 10.Bh3 Kf2 11.Bg4 Nd4 12.Bd1 =**

# 11 – Further Pawn Endings

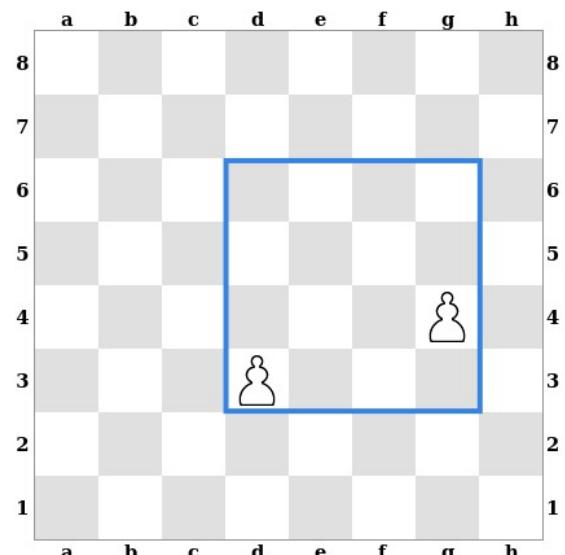
In an earlier chapter we began looking at king and pawn endgames with more than one pawn on the board. We now pick that topic back up and look at some more sophisticated ideas and positions.

## Separated Pawns – The Mutual Square



11.01

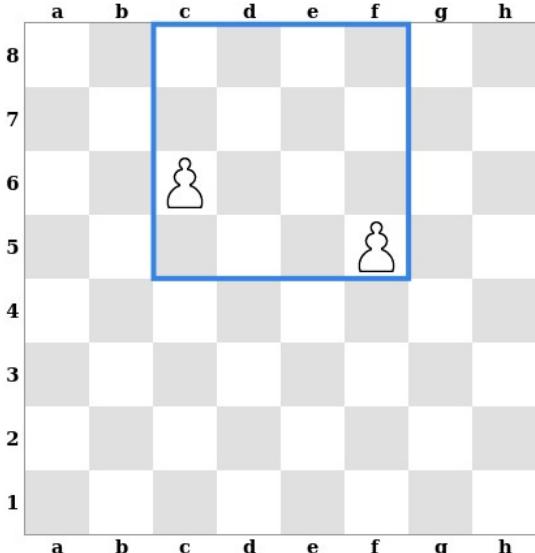
Two pawns separated by one or more files can sometimes promote by themselves. For example, when each pawn is outside the square of the other pawn, one of the pawns is guaranteed to promote. Even if the defending king is ready to capture one of the pawns immediately, he will be outside the square of the other pawn after doing so and will no longer be able to catch that pawn. For example, if Black's king were on g7 here he could capture the g6 pawn, but this would leave him unable to catch the c5 pawn.



11.02

This observation serves as the basis for an important rule. **Rule 11.1 (Rule of the Mutual Square):** when two pawns face a lone king, begin with the less advanced pawn and draw the smallest square that includes the more advanced pawn. If this square reaches the eighth rank with the pawns to move, then the defending king cannot stop both pawns.

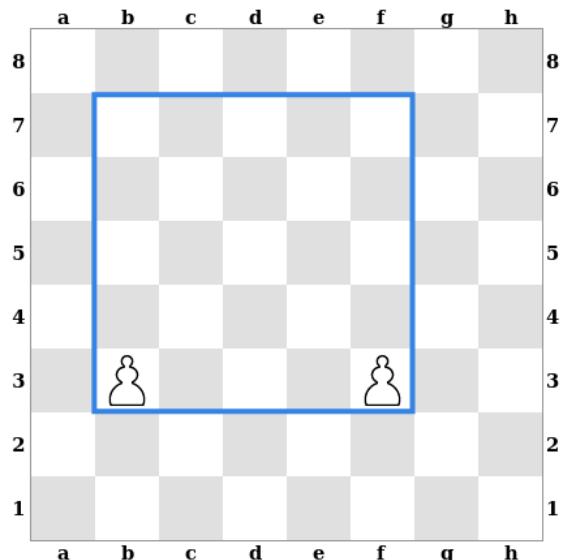
In this position the mutual square has not reached the eighth rank yet, so the pawns aren't guaranteed to promote on their own.



11.03

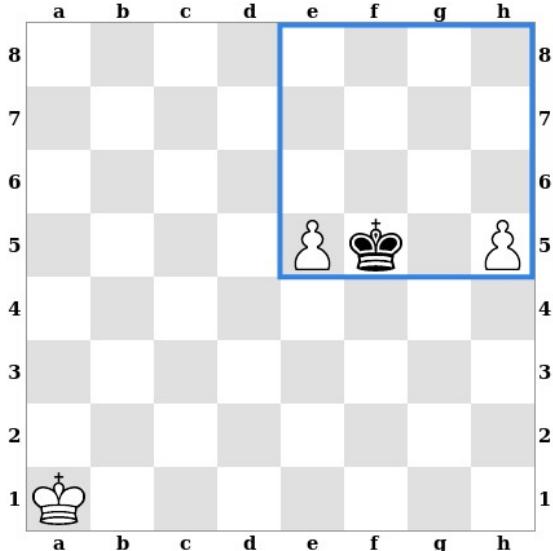
In this position the mutual square has reached the eighth rank. As long as White moves before either pawn is captured, one of the pawns is guaranteed to promote. This position helps reveal the logic behind the rule. If White moves first he will play 1.f6, putting the pawns outside one another's squares. If Black's king were itself on f6, then it would already be outside of the square of the c-pawn. In any case, the pawns are now unstoppable.

Of course, it is important that the pawns have a chance to move. If Black moved first here with his king on c7, then he would play 1...Kxc6 and capture both pawns.



11.04

The rule still applies when the pawns are on the same rank as one another. In this case, the square can be drawn beginning with either pawn chosen as the "less advanced" pawn.

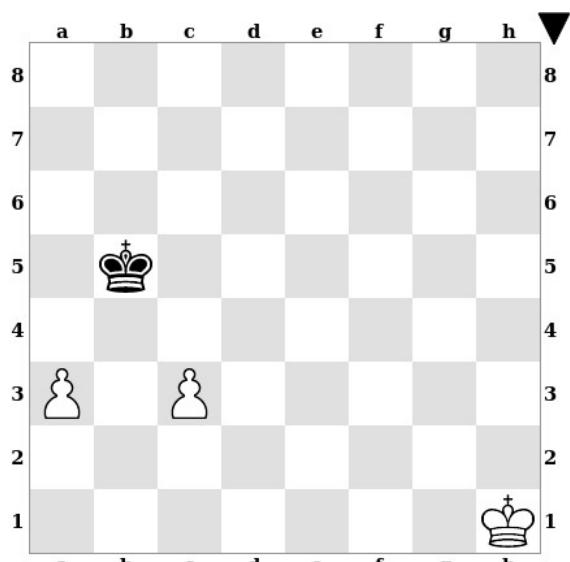


11.05 (+- / =)

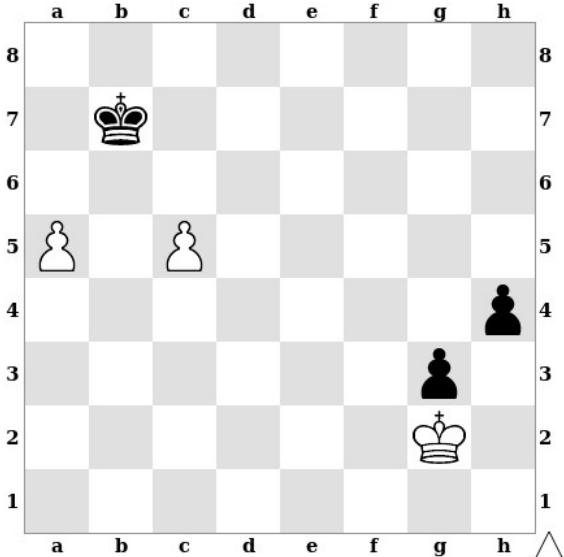
Here, the mutual square has reached the eighth rank. If White moves first, he wins: **1.h6 Kg6 2.e6 Kxh6 3.e7 +-**. If Black moves first, he draws with **1...Kxe5 =**, leaving him in the square of the h-pawn.

## Separated Pawns – Other Cases

Two pawns separated by one file can protect each other, as long as they can't be put in zugzwang. In this position, the mutual square hasn't reached the eighth rank and the pawns can't promote on their own. However, Black's king can't successfully attack them. For example, after **1...Kc4 2.a4**, Black can't play **2...Kxc3** because he would leave the square of the a-pawn, allowing **3.a5 +-**. After **2...Kc5 3.Kg2 Kb6 4.c4**, the pawns have safely moved forward a rank. Now **4...Ka5 5.c5 Ka6 6.Kf3 Kb7 7.a5** is the same idea, and it is clear that the pawns can defend themselves. White eventually brings his king in and promotes one of the pawns.

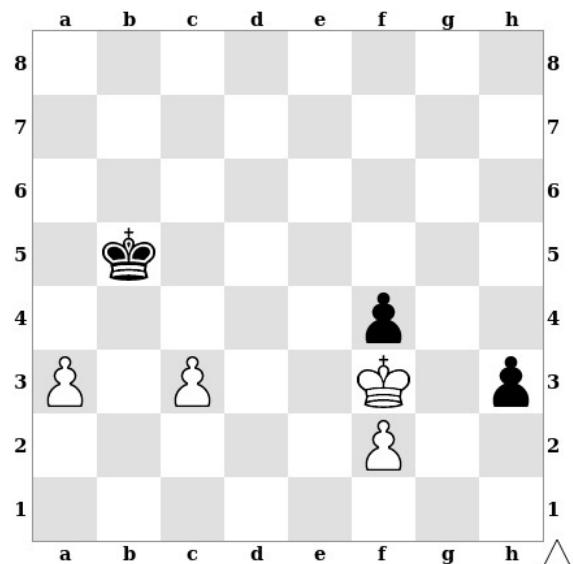


11.06 (+- / +-)



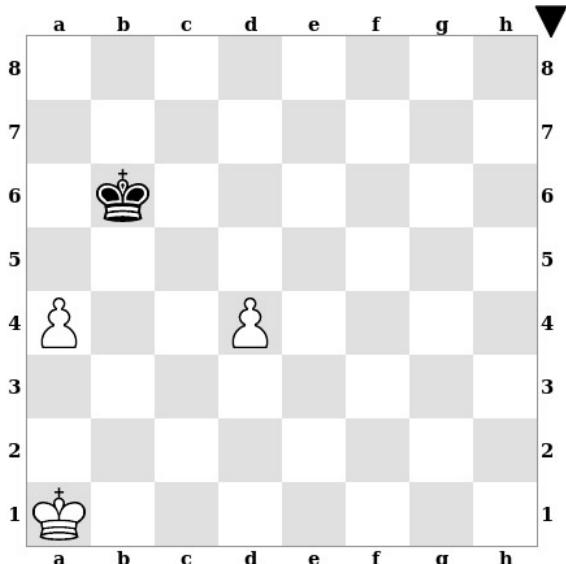
11.07 (= / =)

However, two pawns separated by one file cannot promote without their king's help, as long as the opposing king does not approach them. Here the game ends in a draw after **1.Kh3 Kb8 2.Kg2 Kb7 3.Kh3 Kb8 =**. White would only lose by playing 2.c6? Kc7 -+ or 2.a6? Ka7 -+, when Black rounds up both of White's pawns.



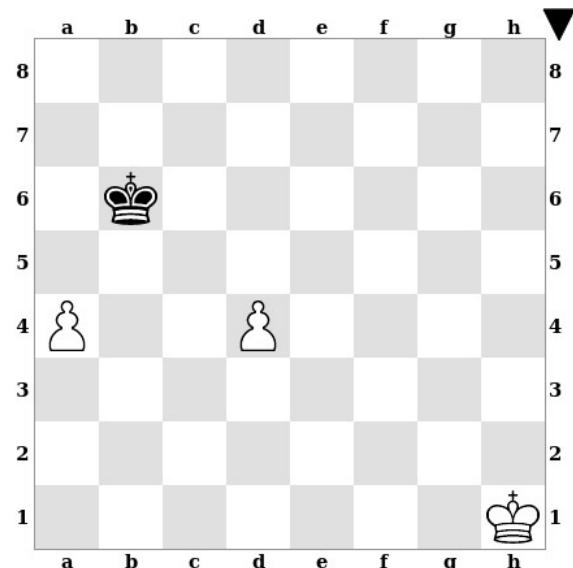
11.08 (-+ / -+)

In fact, pawns separated by one file cannot survive at all if they are forced to move. In this position, White's king can't afford to move because it would leave the square of the h3 pawn. If skipping a turn were allowed, then White could draw by skipping his turn until Black's king approached one of the pawns, at which point White would push the other one. However, since White is forced to move he is in zugzwang and quickly loses: **1.c4+ (1.Kxf4 h2 -+) 1...Kxc4 2.a4 Kb4 3.a5 Kxa5 4.Kxf4 h2 -+**.



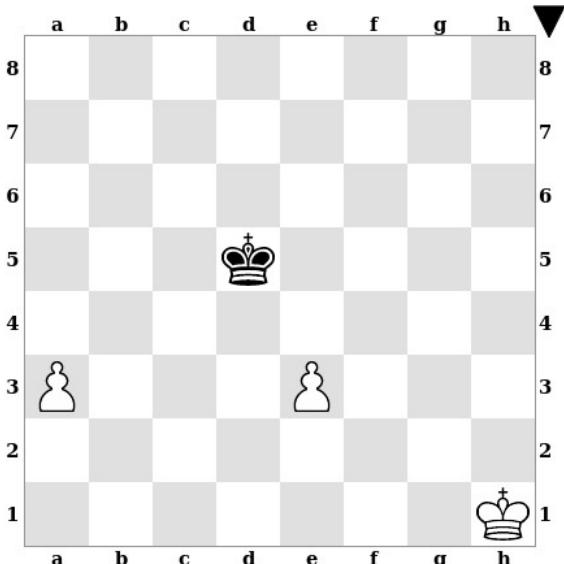
11.09 (+- / =)

Pawns separated by two files generally cannot defend themselves. Here Black draws with 1...Ka5 2.d5 (or 2.Kb2 Kxa4 3.Kc3 Kb5 4.Kd3 Kc6 5.Ke4 Kd6 =, reaching a defensive key square) 2...Kb6 3.Kb2 Kc5 4.Kc3 Kxd5 5.Kb4 Kc6 =, when Black's king gets back in time to stop White's rook pawn.



11.10 (+- / +-)

However, the stronger side can often win by sacrificing one pawn in order to get his king to the key squares of the other. This position is the same as the previous one except that White's king is in the other corner, but it turns out that this is enough to win. First, after 1...Kc6 2.Kg2 Kd5 3.a5 +- Black will have to chase down the a-pawn anyway, so he might as well target the a-pawn right from the beginning; but 1...Ka5 2.Kg2 Kxa4 3.Kf3 Kb5 4.Ke4 Kc6 5.Ke5 Kd7 6.Kd5 +-, gives White the opposition in front of his pawn.

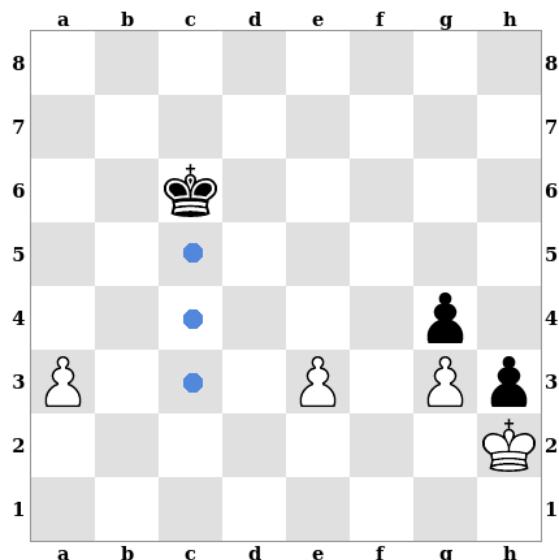


11.11 (+- / +-)

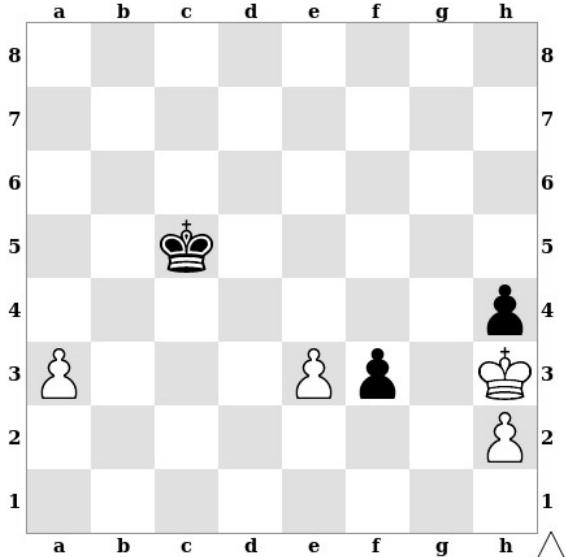
Pawns separated by three files follow the same pattern as pawns separated by one file. First, they can normally defend themselves as long as there is no zugzwang. In this position, if Black tries to attack the e-pawn then the a-pawn can advance: 1...Ke4 2.a4 Kd5 3.a5 Kc5 4.e4 +-, when the mutual square has reached the eighth rank and neither pawn can be immediately captured.

Second, such pawns can't promote on their own without the help of the king (assuming the mutual square hasn't yet reached the eighth rank). For example, this position is drawn if Black moves first; he just needs to keep his king on the three marked squares unless and until White pushes a pawn: 1...Kc5 2.Kg1 Kc4 3.Kh2 Kc5 4.Kg1 Kc4 5.Kh2 =. White would even lose if he pushed one of the pawns with Black's king on the marked squares: 2.e4? Kd4 3.a4 Kxe4 +-.

On the other hand, if White moves first he wins since the pawn's mutual square reaches the eighth rank after 1.e4 Kb5 (otherwise 2.a4 +-) 2.e5 Kc5 3.a4 +-.

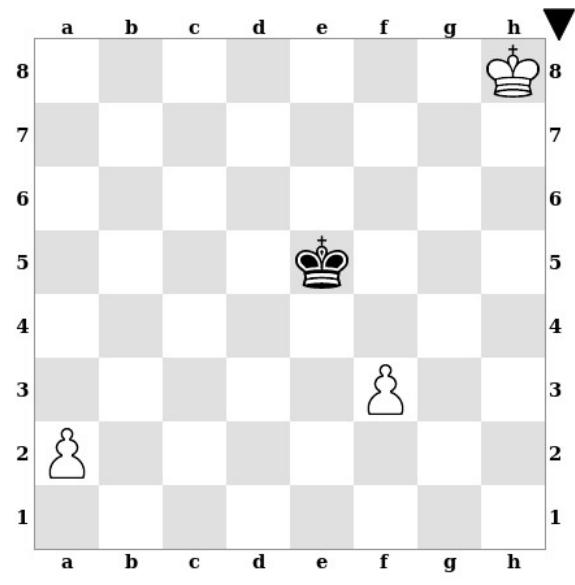


11.12 (+- / =)



11.13 (-+ / -+)

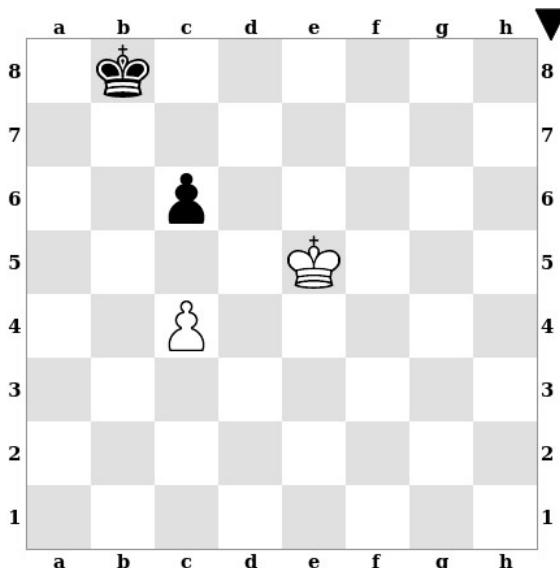
Finally, if the pawns are forced to move at the wrong time, they will both be lost. White loses here after **1.a4 Kb4 2.e4 Kxa4 3.e5 Kb5 4.e6 Kc6 -+**.



11.14 (+- / +-)

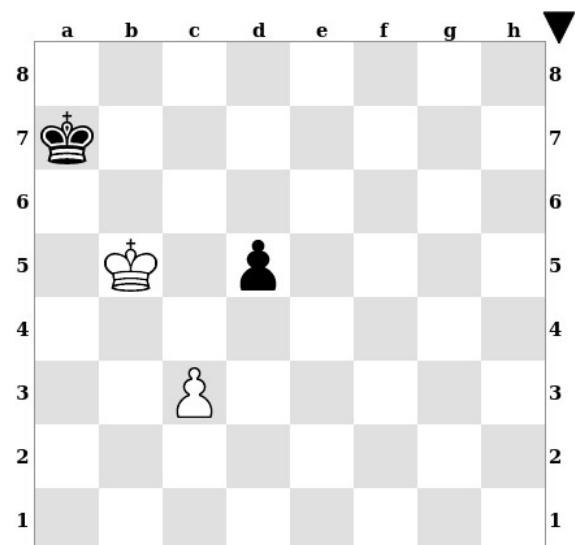
Pawns separated by four or more files almost always win. Even with Black to move here, **1...Kf4 2.a4 +-** is hopeless.

## Defensive Sacrifices



11.15 (+- / =)

In many pawn versus pawn positions, the defender can sacrifice his pawn to reach a drawn single pawn ending. In this position Black is going to lose his pawn in any case and such a sacrifice is his only chance of drawing. After 1...Kc7? 2.c5 Kd7 3.Kf6 +-, White has reached a critical square of Black's pawn. Since c6 is a key square of White's c5 pawn, this is enough to win the game. Additionally, Black's counterattack is not in time after 1...Kb7? 2.c5 Ka6 3.Ke6 Ka5 4.Kd7 Kb5 5.Kd6 +-, with mutual zugzwang. Instead, Black draws with 1...c5 2.Kd6 Kc8 3.Kxc5 Kc7 =, when he uses the opposition to stop White from reaching the key squares of his pawn.

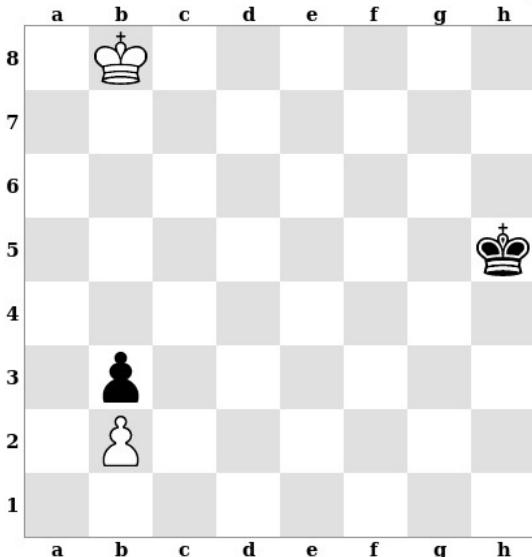


11.16 (+- / =)

In this position White will win if he is allowed to play 1.Kc5 and 2.Kxd5. Not only will he win Black's pawn in this case, but he will occupy a key square of his own pawn in the process. However, Black can draw with a pawn sacrifice, forcing both White's pawn and its key squares to come forward a rank: 1...d4 2.cxd4 Kb7 3.Kc5 Kc7 =. Note that this sacrifice would not work after 1...Kb7? 2.Kc5 d4 because White can play 3.Kxd4. He would now threaten 4.Kd5, reaching a key square, and 3...Kc6 4.Kc4 +- gives him the opposition in front of his pawn.

## King Routes

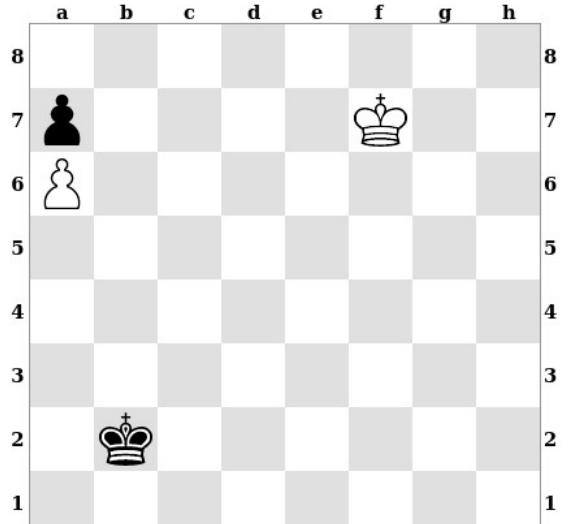
The kings often have multiple routes available to reach the same place; these routes are rarely of equal value. You might think that the straightest route will always be fastest, but this isn't true because the king's diagonal move covers more physical distance than his vertical and horizontal moves while taking the same amount of time.



11.17 (+- / =)

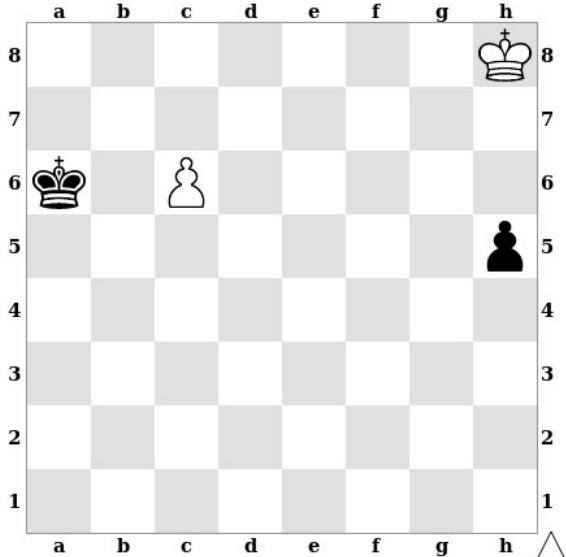
In this position, let's first consider what happens if Black tries to defend and both kings move along a straight path: 1...Kg5? 2.Kb7? Kf5 3.Kb6 Ke5? 4.Kb5? 5.Kd5 5.Kb4 Kc6 6.Kxb3 Kb5 =. This allowed Black to draw, but as the question marks indicate, both sides can improve on this. If Black begins with 1...Kg5?, White can use shoulder-charging to disrupt Black's journey: 2.Kc7 Kf5 3.Kd6. Black can't win White's pawn, as shown by 3...Ke4 4.Kc5 Kd3 5.Kb4 Kc2 6.Ka3 +-, with mutual zugzwang. Black also can't get back to defend in time: 3...Kf6 4.Kd5 Ke7 5.Kc4 Kd6 6.Kxb3 Kc5 7.Ka4 +-.

However, Black can draw with his own strange king route: 1...Kg6 2.Kc7 Kf7 3.Kd6 Ke8 4.Kc5 Kd7 5.Kb4 Kc6 6.Kxb3 Kb5 =.



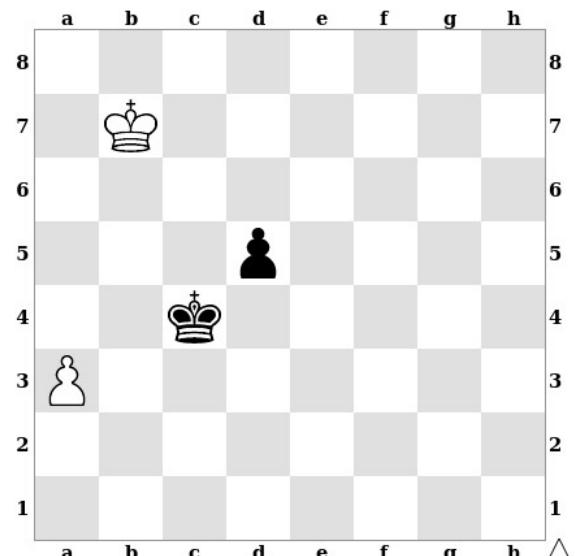
11.18 (+- / =)

In this position White is going to win Black's pawn in any case. However, the most straightforward way of doing this allows Black to draw by reaching a defensive key square of the a6-pawn just in time: 1.Ke7? Kc3 2.Kd7 Kd4 3.Kc7 Kd5 4.Kb7 Kd6 5.Kxa7 Kc7 =. Instead, White can win by taking another route and shoulder-charging: 1.Ke6 Kc3 2.Kd5 Kd3 3.Kc6 Kd4 4.Kb7 Kc5 5.Kxa7 Kc6 6.Kb8 Kb6 7.a7 +-.



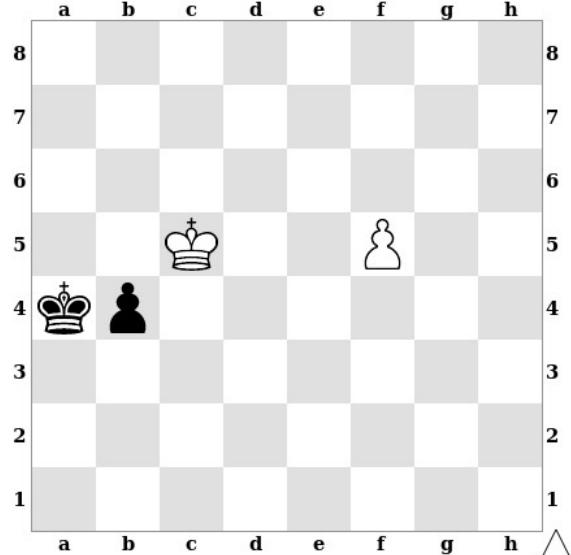
11.19 (= / -+)

The king's choice of different routes sometimes allows it to pursue multiple goals at the same time, as in this famous study by Richard Réti. It looks impossible for White to either catch Black's pawn (he is far outside its square) or to assist his own pawn in promoting. For example, White loses after 1.c7? Kb7 -+, 1.Kg8? Kb6 -+, and 1.Kh7? h4 -+. However, White can draw using a series of moves that bring his king closer to both pawns at the same time: **1.Kg7 h4 2.Kf6 Kb6 (2...h3 3.Ke7 h2 4.c7 =) 3.Ke5 h3 (3...Kxc6 4.Kf4 =) 4.Kd6 h2 5.c7 Kb7 6.Kd7 h1=Q 7.c8=Q+ =.**



11.20 (= / -+)

Similarly, it seems in this position that White's king can neither catch Black's pawn nor save his own pawn. However, after **1.a4 Kb4**, White draws with the dual purpose move **2.Kb6: 2...d4 (2...Kxa4 3.Kc5 =) 3.a5 =.**



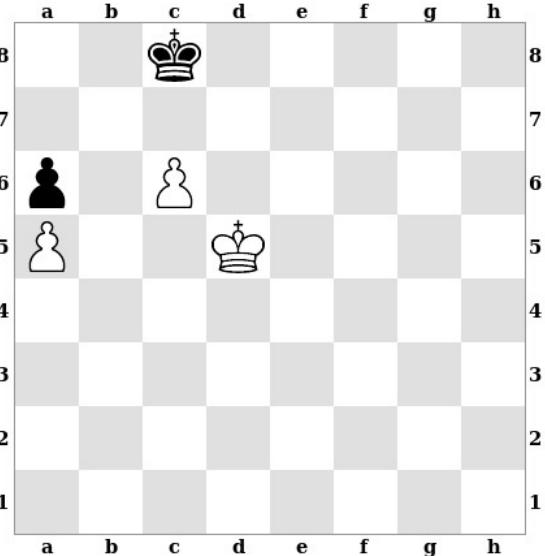
11.21 (+- / -+)

A related idea involves making king moves in order to force the opponent's king to an inconvenient square. Here White would only draw by pushing his pawn right away: 1.f6? b3 2.f7 b2 3.f8=Q b1=Q 4.Qa8+ Kb3 5.Qb7+ Kc2 =. Instead, White can win by forcing Black's king to a3, even at the cost of spending two king moves doing so: **1.Kc4 b3 2.Kc3 Ka3 3.f6 b2 4.f7 b1=Q 5.f8=Q+ Ka4 6.Qa8+ Kb5 7.Qb8+ Kc5 8.Qxb1 +-.**

## Triangulation

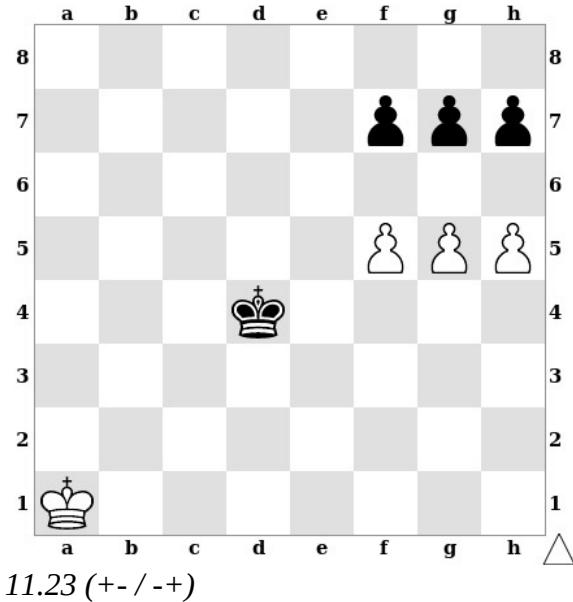
If Black moves first here, he is in zugzwang and loses quickly. After 1...Kc7 2.Kc5 Kc8 3.Kb6 +-, White will win Black's pawn. On the other hand, after **1...Kd8 2.Kd6 Kc8 3.c7 Kb7 4.Kd7 Ka7**, White only needs to avoid 5.c8=Q? =. Instead, both **5.Kc6 +-** and **5.c8=R +-** win.

Strangely, if White moves first it isn't obvious how he should win. After 1.Kd6 Kd8 2.c7+? Kc8 = White can only choose between stalemate and a drawn rook pawn ending. White also makes no progress with 1.Kc5 Kc7 2.Kd5 Kc8. To win, White must use the d4 and c4 squares to *triangulate* and recreate the starting position with Black to move: **1.Kd4 Kd8 2.Kc4 Kc8 3.Kd5 +-.** Now that Black is forced to move, he loses as already shown.



11.22 (+- / +-)

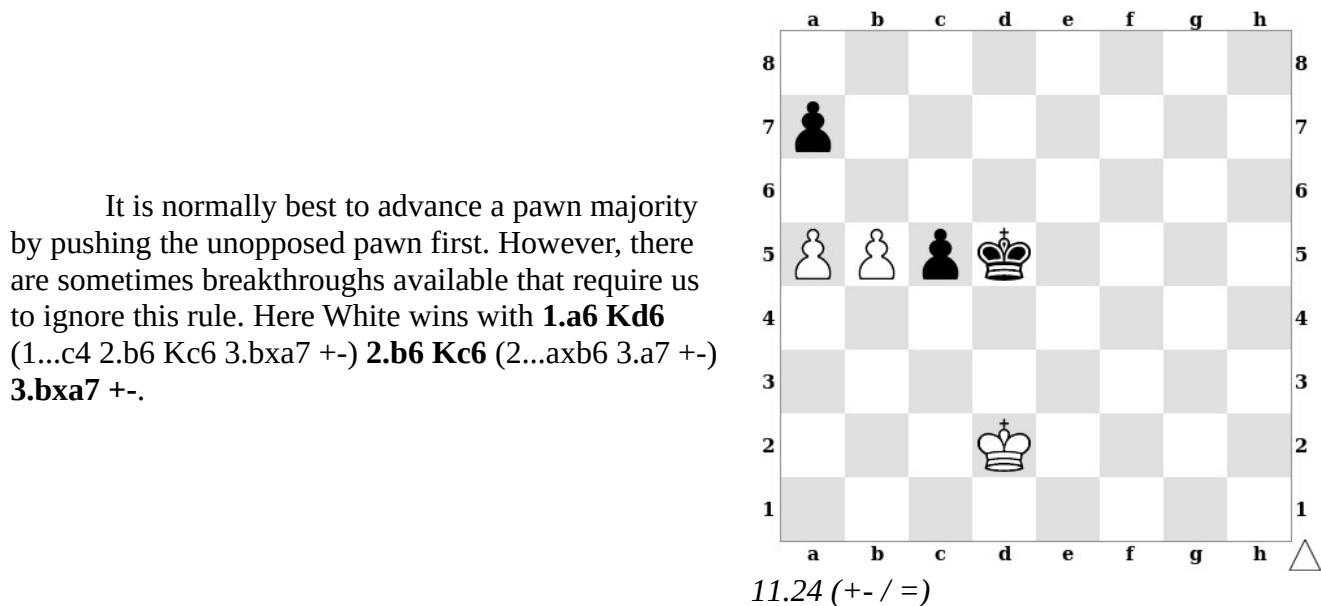
## Breakthroughs



It is sometimes possible to create a passed pawn in a surprising way with a *breakthrough*. In this position White wins with 1.g6, threatening both gxh7 and gxf7. No matter which way Black captures, White gets an unstoppable passed pawn:

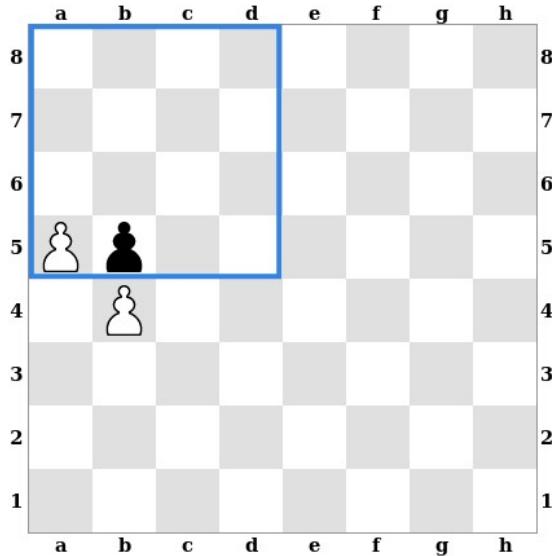
a.) 1...fxg6 2.h6 gxh6 3.f6 Ke5 4.f7 +-

b.) 1...hxg6 2.f6 gxf6 3.h6 +-



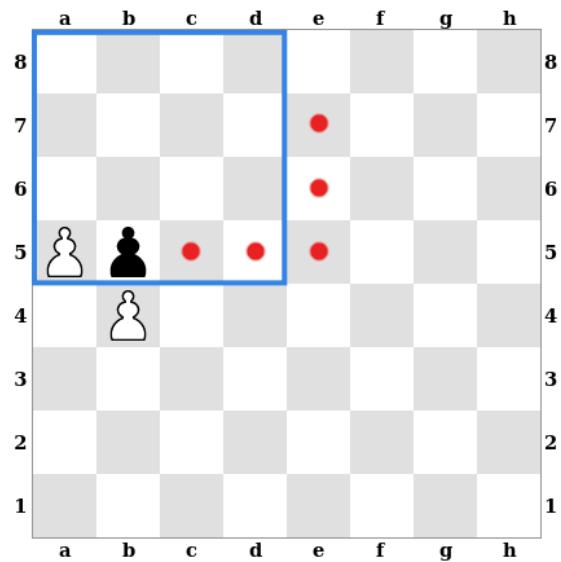
It is normally best to advance a pawn majority by pushing the unopposed pawn first. However, there are sometimes breakthroughs available that require us to ignore this rule. Here White wins with 1.a6 Kd6 (1...c4 2.b6 Kc6 3.bxa7 +-) 2.b6 Kc6 (2...axb6 3.a7 +-) 3.bxa7 +-.

## Protected Passed Pawns



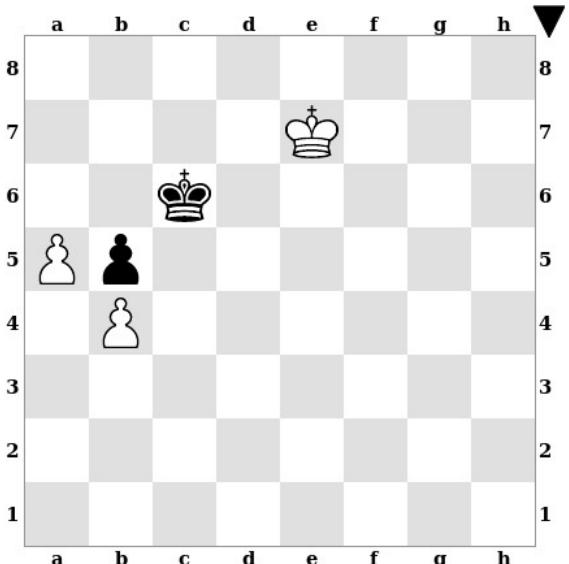
11.25

A protected passed pawn, such as White's pawn on a5 in this position, is usually a significant advantage. Such a pawn can't be captured and the defending king normally has to stay inside its square to stop it from promoting. This restricts the movement of the defending king, interfering with both its ability to counterattack and to maintain opposition.



11.26

We already know that Black's pawn has three critical squares on the fifth rank. It turns out that blocked pawns have more critical squares than this. If White's king is on e6 or e7 here, then he can force his way in to Black's territory and win Black's pawn. Without the white pawn on a5, these extra critical squares aren't useful because Black's king would almost always have a quick counterattack against White's b4 pawn. With the extra pawn on a5, however, Black's king can't counterattack and the extra critical squares are suddenly useful.



11.27 (+- / +-)

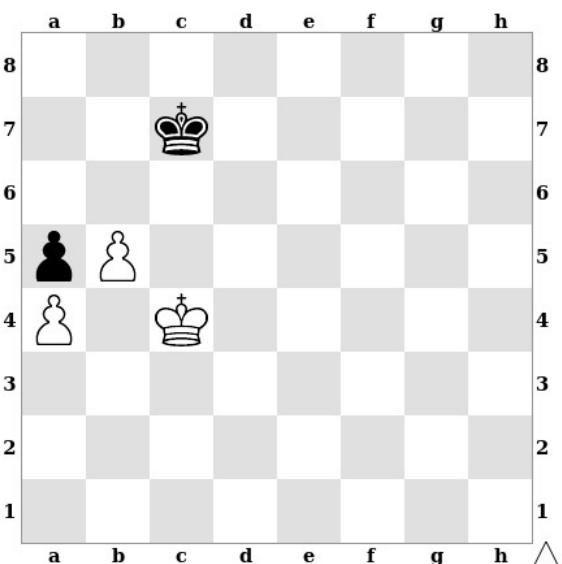
If White didn't have his a5-pawn, Black would even win with 1...Kd5 and 2...Kc4. Here, however, 1...Kd5 2.Kd7 Kc4 takes Black's king outside of the a-pawn's square and White wins with 3.a6 +-.

Black can take the opposition instead, but **1...Kc7 2.Ke6 Kc6 3.Ke5 +-** will lead to White winning Black's pawn, giving him a trivially winning position with two extra pawns.

In this position, White's protected passed pawn makes it harder for his king to get to Black's pawn. Surprisingly, taking the opposition does not help. After 1.Kc5 Kb7, White would even throw the win away with 2.b6? Ka6 =. The problem is that 3.Kc6 is stalemate and any other move loses the b-pawn, resulting in a drawn rook pawn ending.

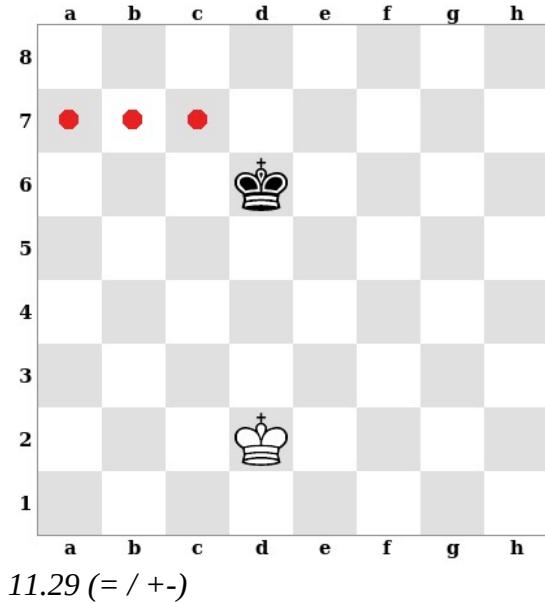
Instead, White should play **1.Kd5 Kb6 2.Kd6 Kb7 3.Kc5 Kc7 (3...Ka7 4.Kc6 Kb8 5.Kb6 +-)** **4.b6+ Kb7 5.Kb5 +-**, which leads to an easily winning position with two extra pawns.

This is yet another example of the fact that opposition does not automatically have value in any given position. Pawn endings are very concrete, and we should rely on calculation unless we know a specific rule that applies to the position.



11.28 (+- / +-)

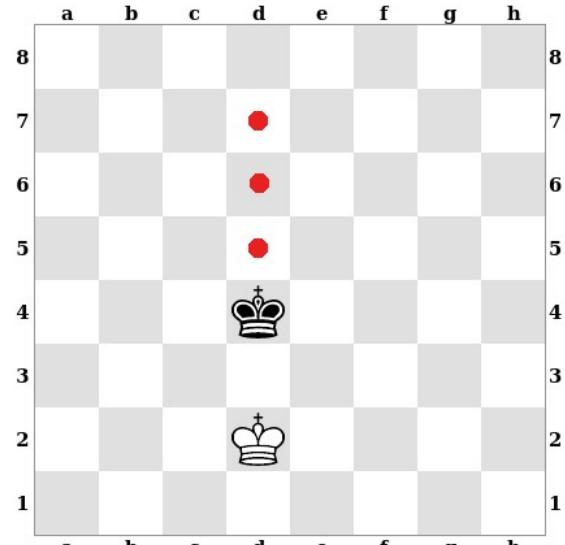
## More Opposition

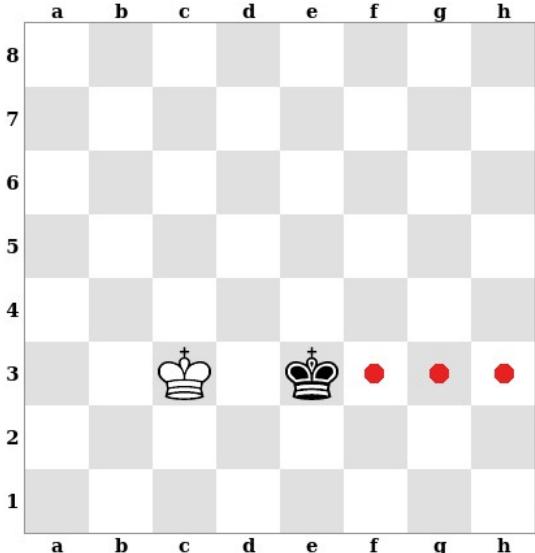


Of course, there are situations in which opposition is a valuable tool. We are going to learn some new forms of opposition, but first let's briefly review what we already know.

The kings are in opposition when they stand an odd number of squares apart on the same file or rank. Opposition tends to produce mutual zugzwang, so whoever moves second has the advantage and is said to "have the opposition." If White moves first here, then Black has the opposition and White's king can never reach the fourth rank, e.g. 1.Kc2 Kc6 2.Kb3 Kb5 =. If Black moves first, then White can use both opposition and outflanking to reach the target squares: 1...Kc6 2.Kc2 Kb6 3.Kb2 Kc6 4.Ka3 Kb7 5.Kb3 Ka7 6.Kc4 Kb6 7.Kb4 Kc6 8.Ka5 Kb7 9.Kb5 Ka7 10.Kc6 Kb8 11.Kb6 Kc8 12.Ka7 +-.

This position is similar to previous examples of opposition but with one important difference: the target squares now run parallel to the distance between the kings. If Black has the opposition here, then nothing changes; White's king can't reach the third rank: 1.Ke2 Ke4 2.Kd2 Kd4 =. More surprisingly, White can't reach the target squares even if he has the opposition. For example, after 1...Ke4 2.Kc3 Ke3 =, Black has the horizontal opposition and so White can never again reach the d-file. It doesn't change anything for White to play 2.Ke2 Kd4 3.Kf3 Kd3 =, when White can never again reach the e-file. This shows that vertical opposition isn't useful when the attacker is trying to reach squares that lie along a file.

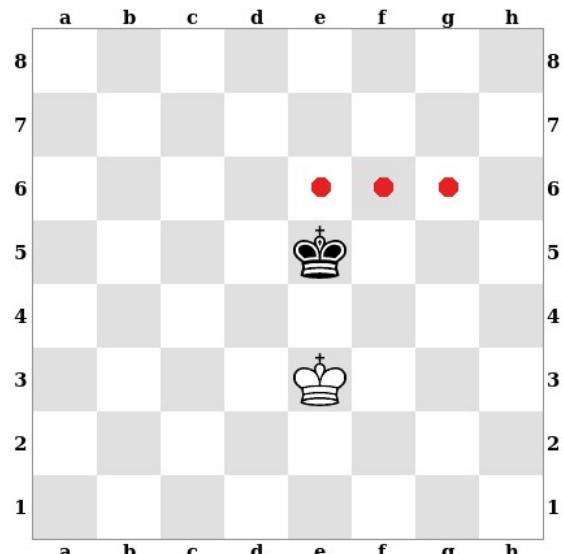




11.31 (= / =)

Similarly, the attacker can't use horizontal opposition to force his way onto target squares that lie along a rank. Black can defend the target squares here even if White has the opposition: 1...Ke4 2.Kc4 (2.Kd2 Kd4 = keeps White off of the third rank forever) 2...Ke3 3.Kd5 Kd3 =.

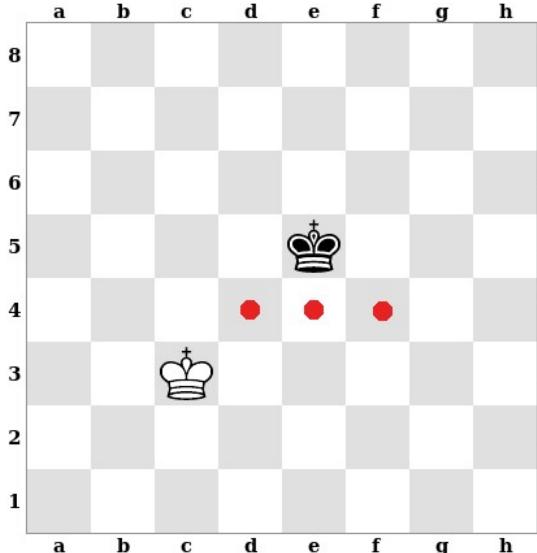
Both this example and the previous one show that when the attacker uses opposition to start outflanking, it gives the defender a chance to take the other form of opposition to the one just used. After 1...Ke4 2.Kd2, White is outflanking horizontally, and with 2...Kd4 Black takes the vertical opposition.



11.32 (= / +-)

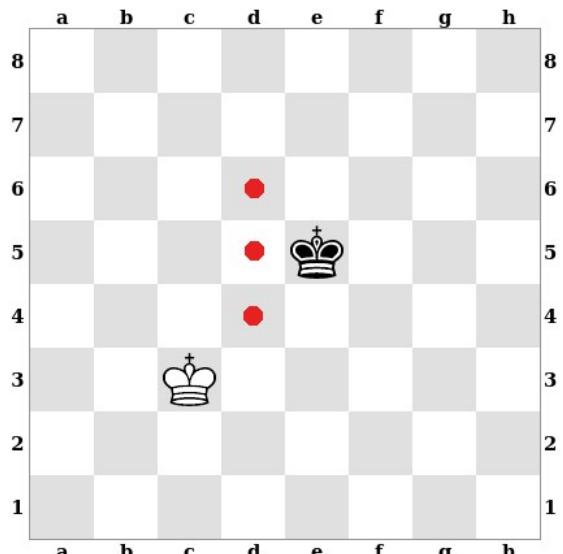
This isn't a problem for the attacker when the target squares run perpendicular to the kings' opposition, as they did in all of our earlier examples. Here, after 1...Kf5 2.Kf3 Ke5 3.Kg4, it is true that Black can take the horizontal opposition with 3...Ke4, but this does nothing to stop White from reaching g6; he can simply proceed with 4.Kg5 +-.

With the previous examples in mind, we can now look at some new forms of opposition.



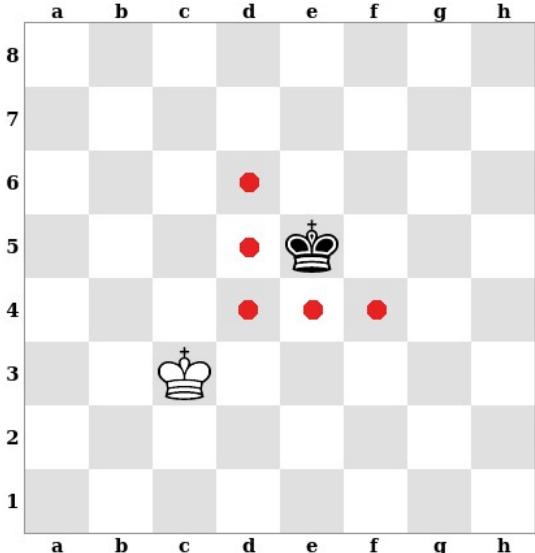
11.33 (= / =)

When the kings are separated by an odd number of squares on the same diagonal they are in *diagonal opposition*. Unfortunately, many endgame books misrepresent diagonal opposition as working exactly the same way as horizontal and vertical opposition; in fact, it isn't quite that straightforward. If Black moves first here, then White has the diagonal opposition. As a result, Black must allow White to gain either vertical or horizontal opposition. However, Black gets to choose which of the two to give White. As a previous example showed, horizontal opposition is not useful for reaching squares that lie along a rank. As a result, Black can defend the target squares here with 1...Ke4 (not 1...Kd5? 2.Kd3 +-) 2.Kc4 (2.Kc2 Ke3 3.Kc3 Ke4 =) 2...Ke5 3.Kd3 Kd5 =.



11.34 (= / =)

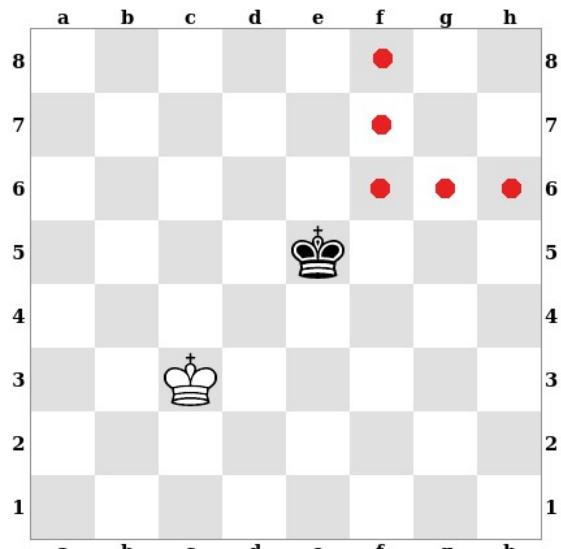
Similarly, Black can defend the target squares here even if White has the diagonal opposition: 1...Kd5 (not 1...Ke4? 2.Kc4 +-) 2.Kd3 Ke5 3.Kc4 Ke4 =.



11.35 (= / +-)

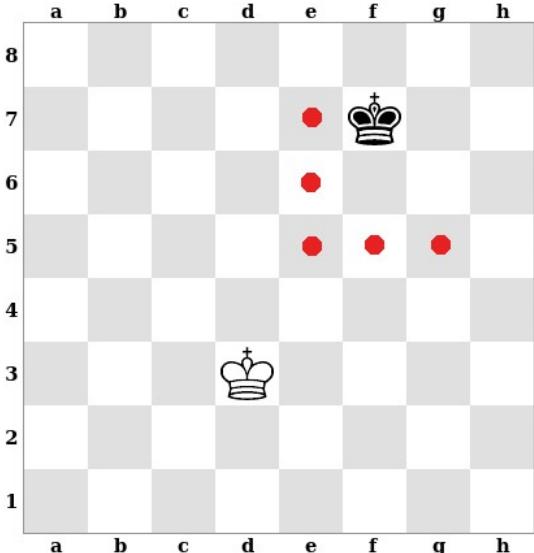
In fact, diagonal opposition is only useful for the attacker when he is trying to reach squares that extend both vertically and horizontally. This makes some intuitive sense since in this type of opposition the kings are separated by both files and ranks. If White moves first here, then Black has the diagonal opposition and can defend all of the marked squares. For example, after 1.Kc4 Ke4 = White will never reach the d-file, while after 1.Kd3 Kd5 = White will never reach the fourth rank. White also wouldn't get anywhere with 1.Kb3 Kd5 2.Kb2 Kd4 =, when Black still has diagonal opposition.

If Black moves first, then White reaches the target squares after 1...Kd5 2.Kd3 Ke5 3.Ke3 +- or 1...Ke4 2.Kc4 Ke5 3.Kc5 +-.



11.36 (= / +-)

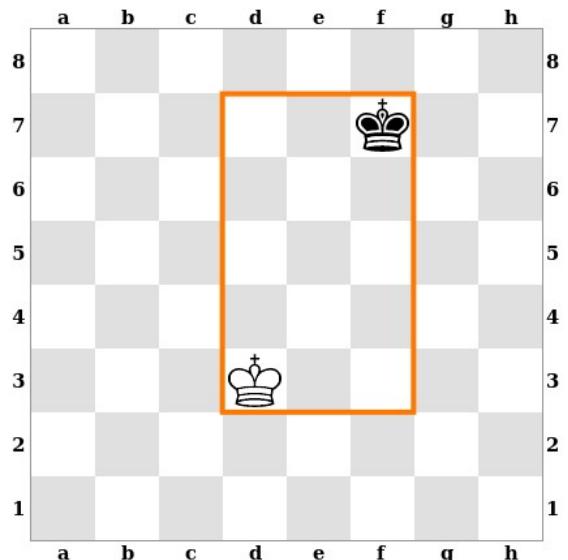
As with other kinds of opposition, the attacker can reach the target squares even if they are behind the defending king as long as he has the opposition. If Black here plays 1...Kd5 2.Kd3 +-, then White will eventually reach the target squares on the sixth rank. Similarly, 1...Ke4 2.Kc4 +- allows White's king to eventually reach the target squares on the f-file. Finally, any of 1...Ke6 2.Kc4 +-, 1...Kf5 2.Kd3 +-, and 1...Kf6 2.Kd4 +- just postpone Black's choice by giving White diagonal opposition again with the kings closer to the target squares.



11.37 (= / +-)

Finally, the kings are in *virtual opposition* whenever they are separated by both an odd number of ranks and an odd number of files. As with diagonal opposition, virtual opposition only benefits the attacker when he is pursuing target squares that extend both vertically and horizontally. In this position, the kings have one file and three ranks between them, so they are in virtual opposition. If White moves first, then Black can defend all of the target squares, e.g. 1.Ke3 Ke7 = or 1.Kd4 Kf6 =. After 1.Kd2 Kf6 = (or even 1...Kf8 =), Black defends by re-establishing virtual opposition.

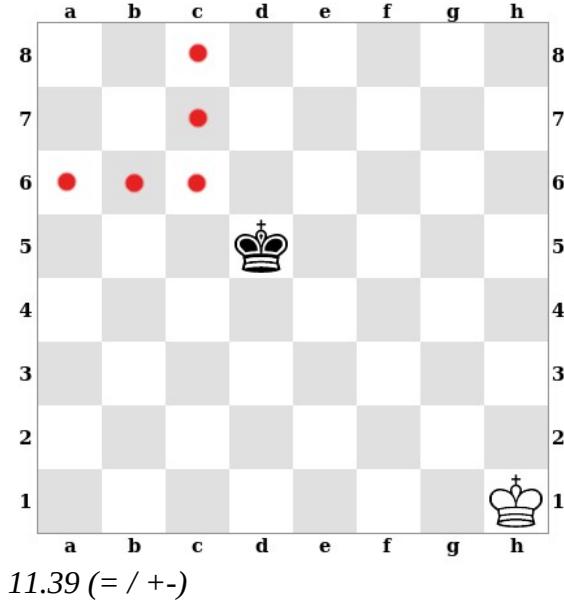
If Black moves first, then White reaches the target squares after 1...Kf6 (1...Ke7 2.Ke3 +-) 2.Kd4 +-.



11.38

Rather than counting the ranks and files between the kings, we can test for virtual opposition in a simpler way. **Rule 11.2 (Virtual Opposition Rule):** (if the kings are on different ranks and different files from one another), draw a rectangle that has the kings on two of its corners. If all four corners of the rectangle are squares of the same color, then the kings are in virtual opposition.

The four corners of the rectangle shown here are all white squares and the kings occupy two of these corners, so we know that they are in virtual opposition.



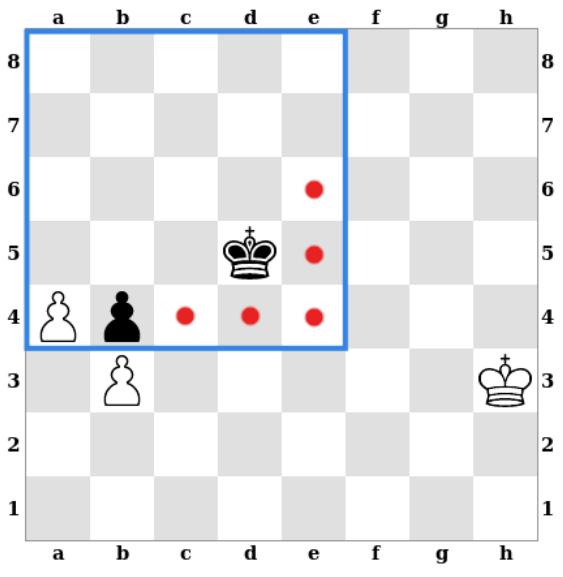
Unsurprisingly, if the attacker has virtual opposition, he can force his way onto target squares even when they are located behind the defending king. If Black moves first here, then White reaches the target squares by consistently moving toward them while re-establishing some form of opposition with each move. For example, 1...Ke5 2.Kg1 (virtual opposition) 2...Ke4 3.Kg2 (diagonal opposition) 3...Ke3 4.Kg3 (horizontal opposition) 4...Ke4 5.Kg4 Ke5 6.Kg5 Ke6 7.Kg6 Ke7 8.Kg7 Ke8 9.Kf6 (outflanking) 9...Kd7 10.Kf7 +- etc.

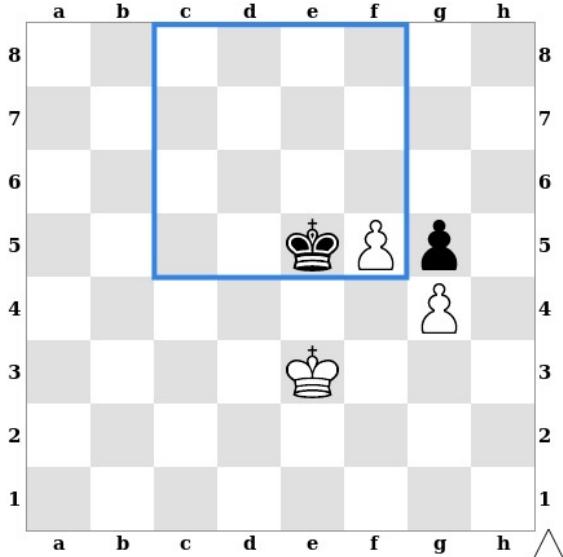
If White moves first, then Black can defend the squares by taking some form of opposition each move while moving toward White's king when possible: 1.Kh2 Kd4 (virtual opposition) 2.Kg2 Ke4 (diagonal opposition) 3.Kf2 Kf4 (vertical opposition) 4.Ke2 Ke4 =.

In this position, Black loses with a direct attack on White's b-pawn: 1...Kd4 2.Kh4 Kc3 3.a5 Kxb3 4.a6 Kc2 5.a7 b3 6.a8=Q +-, when the queen wins against the knight pawn. On the other hand, White cannot win by sacrificing his a-pawn: 1.Kg4 Ke4 2.a5 Kd5 3.Kf5 Kc5 4.Ke5 Kb5 5.Kd5 Kxa5 6.Kc5 Ka6 7.Kxb4 Kb6 =.

Since the kings are already in virtual opposition, the result depends on who moves first. If Black moves first, then he can't guard the critical squares: 1...Kd4 2.Kh4 Kd5 3.Kh5 (but not 3.Kg3? Ke5 =) 3...Kd4 (3...Kd6 4.Kg4 +-) 4.Kg6 Ke5 5.Kg5 Ke6 6.Kf4 Kd5 7.Kf5 +- etc.

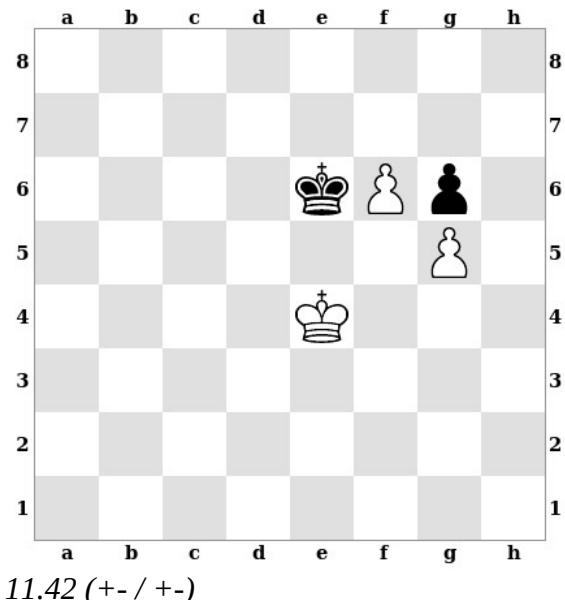
If White moves first, then Black draws by repeatedly taking the opposition while staying inside the a-pawn's square: 1.Kh4 Kd4 2.Kg3 Ke5 (not 2...Ke3? 3.a5 +-) 3.Kh3 Kd5 (virtual opposition) 4.Kg4 Ke4 5.Kg5 Ke5 =.





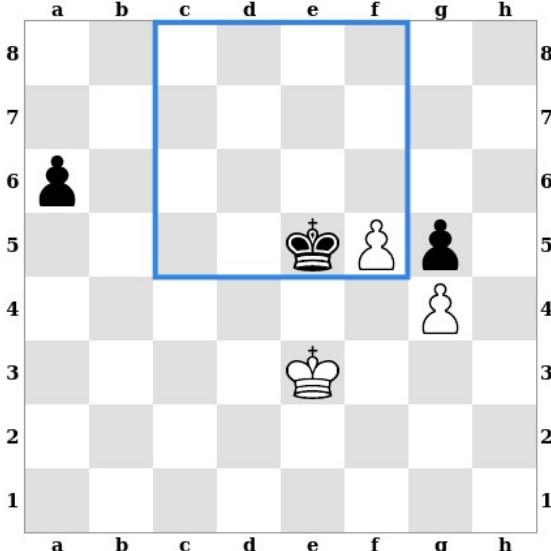
11.41 (+- / +-)

In this position, Black loses because the pawns restrict his king's movement in such a way that he can't keep the opposition. After 1.Ke2, Black would like to play the illegal move 1...Ke6, which takes distant opposition and stays in the square of the f-pawn. Since this is impossible he must give up the opposition:  
**1...Kd6 (1...Ke4 2.f6 +-)** **2.Kd2 Ke5** **3.Kc3 Kd5** **4.Kd3 Ke5** **5.Kc4 Kd6** **6.Kd4 Kd7** **7.Ke5 Ke7** **8.f6+ Kf7** **9.Kf5 +-.**



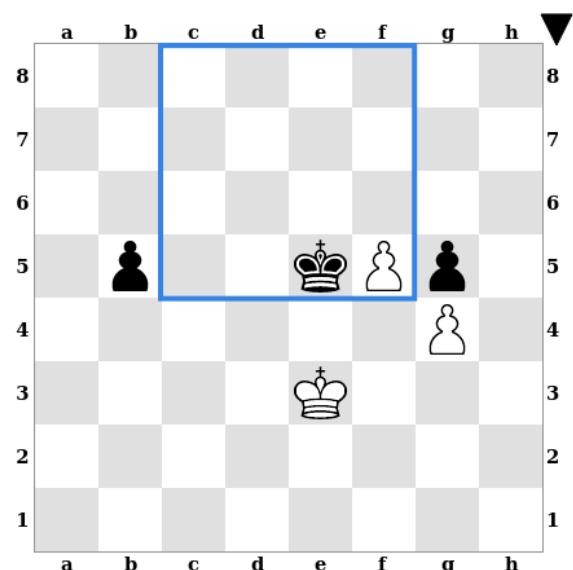
11.42 (+- / +-)

With the blocked pawn already on the fifth rank, the win becomes easier. Here White can just find the right moment to sacrifice his f-pawn in order to reach the critical squares of Black's pawn. Winning this pawn will put White's king on a key square of his own pawn, so he will have a winning position: **1.Kf4 Kd6 (1...Kf7 2.Ke5 Kf8 3.f7 Kxf7 4.Kd6 +-)** **2.f7 Ke7** **3.Ke5 Kxf7 4.Kd6 +-.** Alternatively, White could win by sacrificing his pawn right away: **1.f7 Kxf7 2.Kd5 Ke7 3.Ke5 Kf7 4.Kd6 +-.**



11.43 (+- / +-)

Here Black's king is unable to defend his a-pawn because he would need to leave the square of White's f-pawn in order to do so. After **1.Kd3 Kd5 2.Kc3 Kc5 3.Kb3 Kd5** (3...Kb5 4.f6 +-) **4.Ka4 Kc6 5.Ka5 +-.**

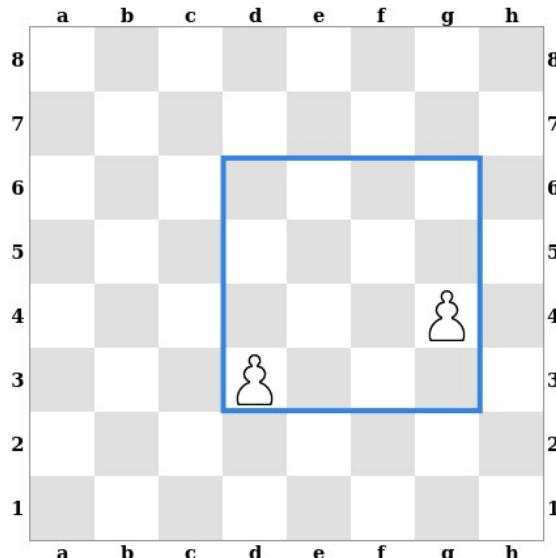


11.44 (= / =)

In this case the outside pawn is close enough that Black's king can defend it without leaving the square of White's pawn, making it possible for Black to draw: **1...Kd5 2.Kd3 Ke5** (2...Kc5? 3.Ke4 b4 [3...Kd6 is even easier for White: 4.Kd4 Kc6 5.Ke5 +-] 4.Ke5 b3 5.f6 +- wins for White because of some immediate tactics in the resulting queen endgame.) **3.Kc3 Kd5 4.Kb4 Kc6** Now Triangulation doesn't work because Black can triangulate as well: **5.Kb3 Kc5 6.Kc3 Kd5 7.Kb4 Kc6** Eventually White may sacrifice the f-pawn, but this only leads to a draw: **8.Ka5 Kc5 9.f6 Kd6 10.Kxb5 Ke6 11.Kc5 Kxf6 12.Kd5 =.**

## Conclusion

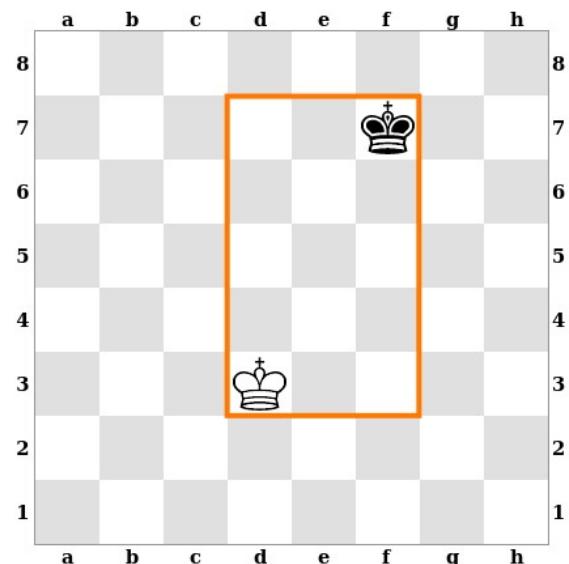
Despite this being a long chapter, there are only a couple of rules and results that you should try to remember.



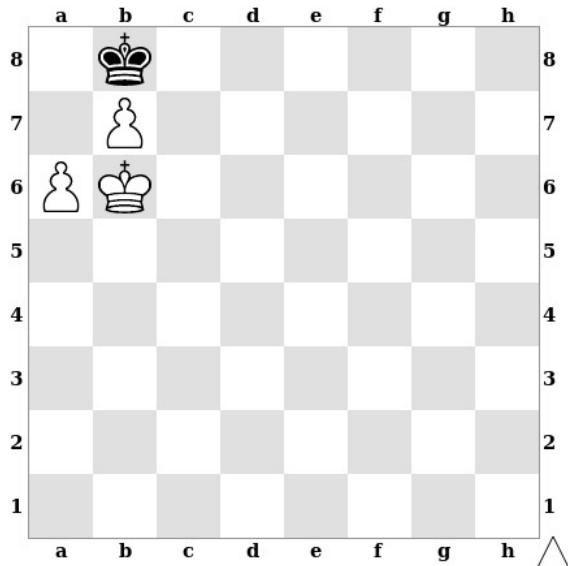
First, there is **Rule 11.1 (Rule of the Mutual Square)**: when two pawns face a lone king, begin with the less advanced pawn and draw the smallest square that includes the more advanced pawn. If this square reaches the eighth rank with the pawns to move, then the defending king cannot stop both pawns.

You should also know which pairs of pawns can defend themselves without the help of the king (ignoring cases of zugzwang). The types of pawns that can defend themselves are **connected pawns**, **pawns separated by one file**, and **pawns separated by three or more files**. **Doubled pawns** and **pawns separated by two files** cannot defend themselves.

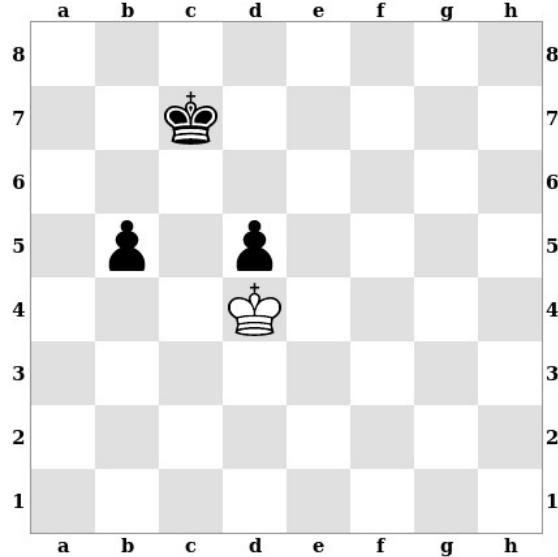
Finally, you should remember **Rule 11.2 (Virtual Opposition Rule)**: (if the kings are on different ranks and different files from one another), draw a rectangle that has the kings on two of its corners. If all four corners of the rectangle are squares of the same color, then the kings are in virtual opposition.



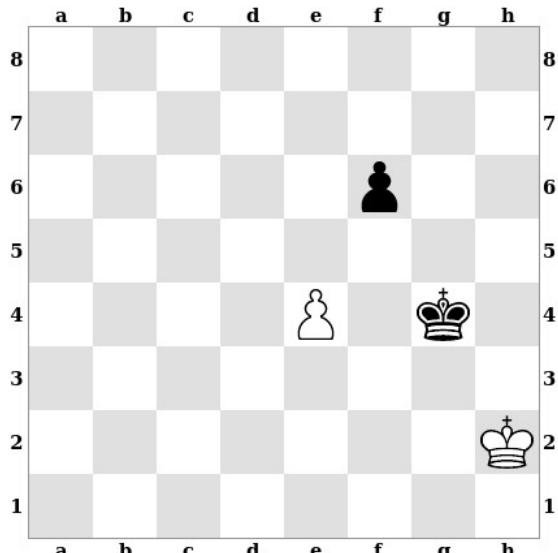
## Exercises



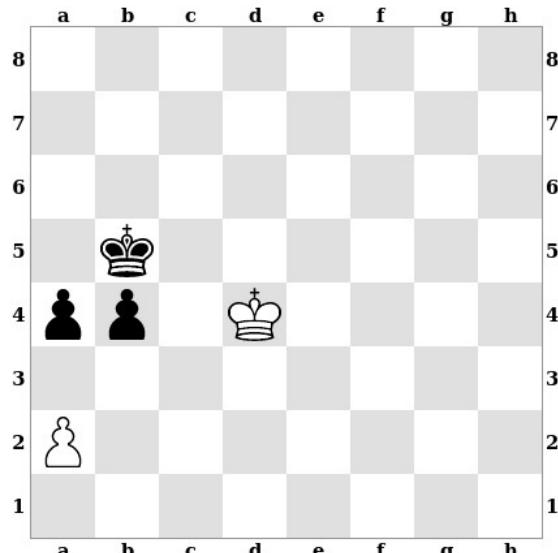
E11.01 (W)



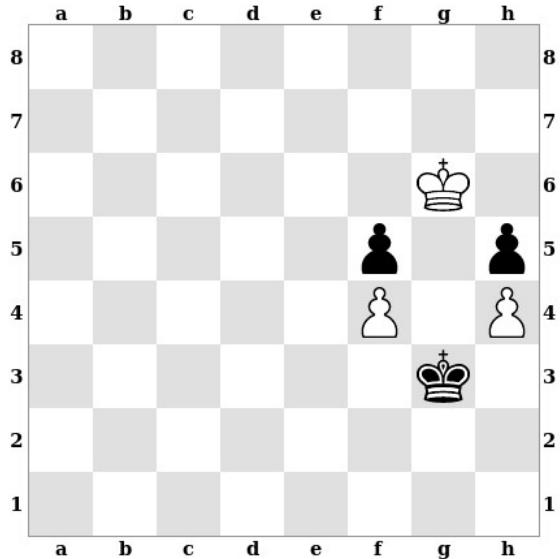
E11.02 (W/B)



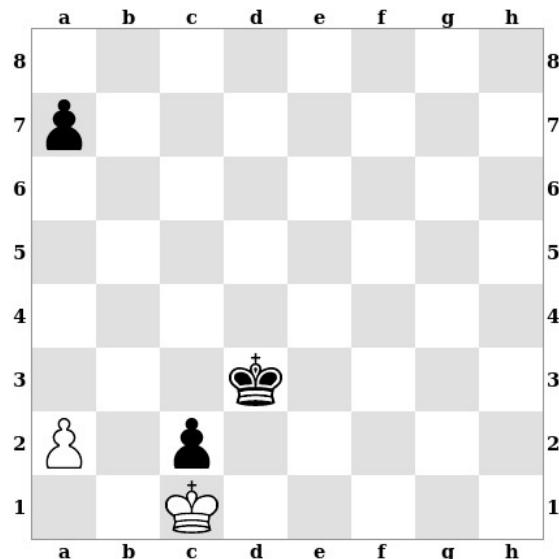
E11.03 (W/B)



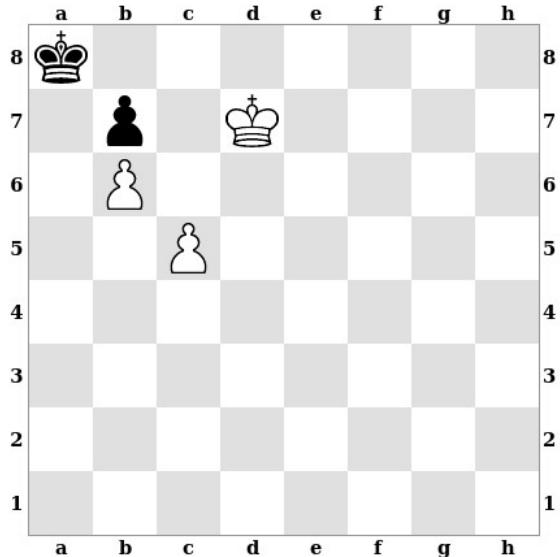
E11.04 (W/B)



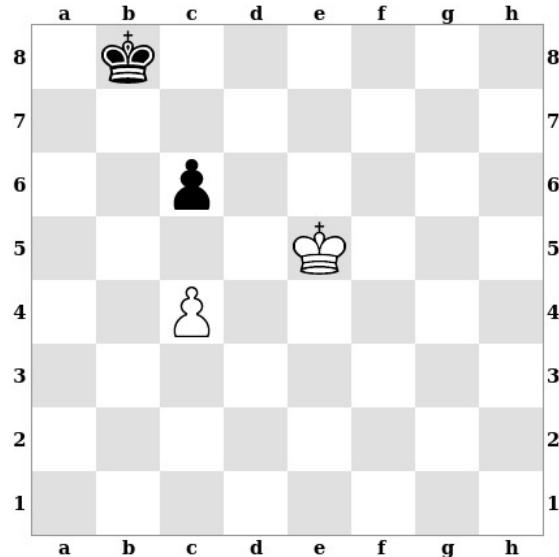
E11.05 (W/B)



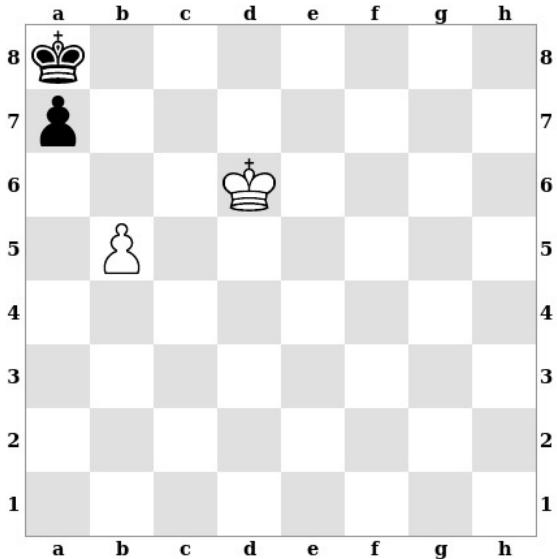
E11.06 (W/B)



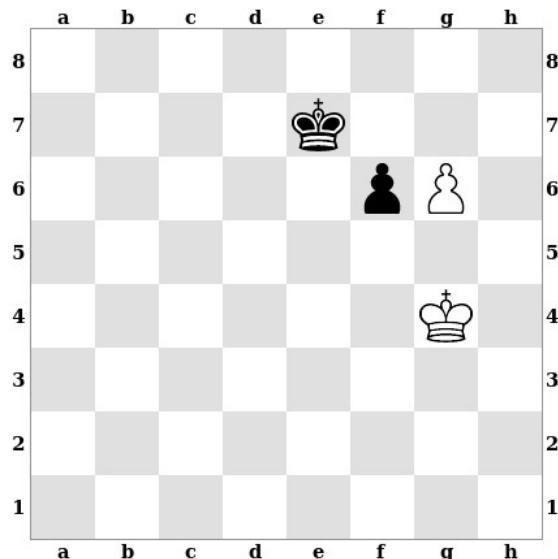
E11.07 (W/B)



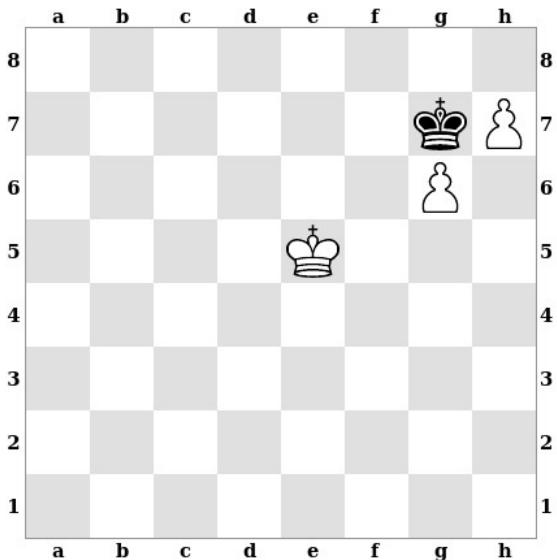
E11.08 (W/B)



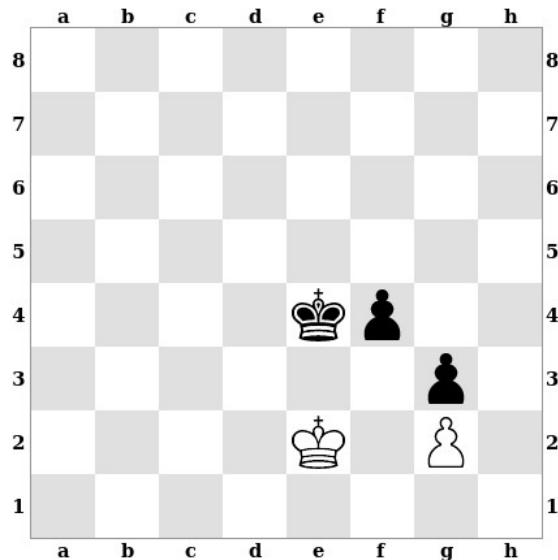
E11.09 (W/B)



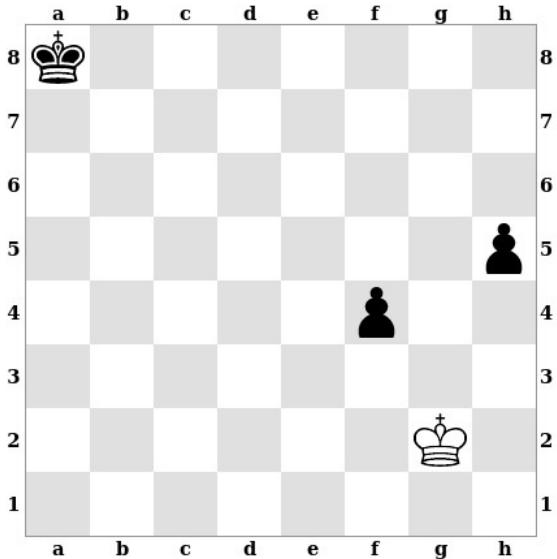
E11.10 (W/B)



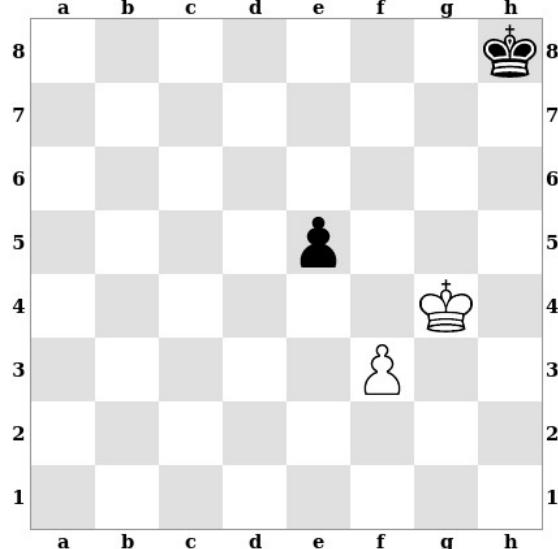
E11.11 (W/B)



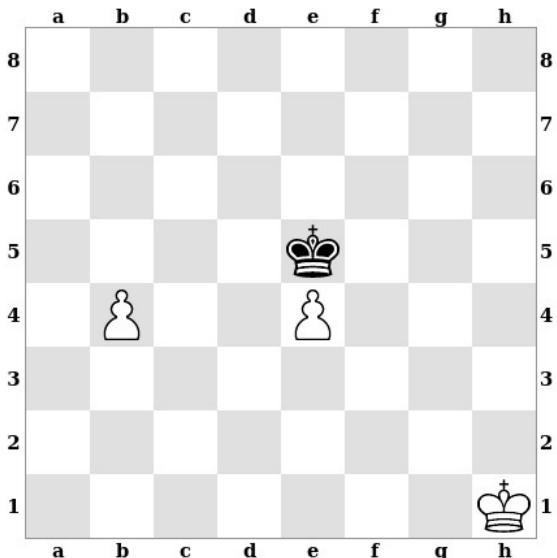
E11.12 (W/B)



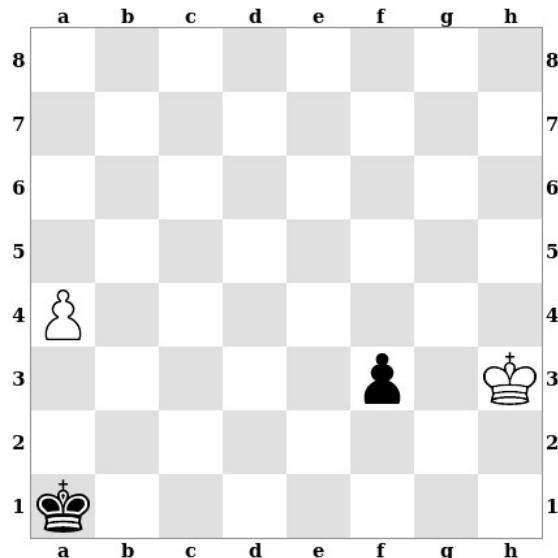
E11.13 (W/B)



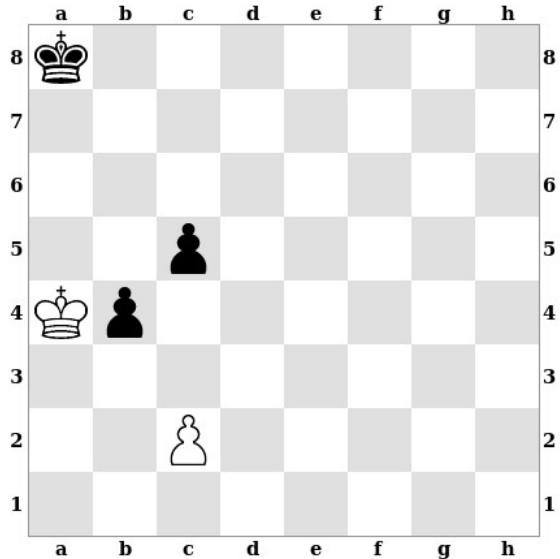
E11.14 (W/B)



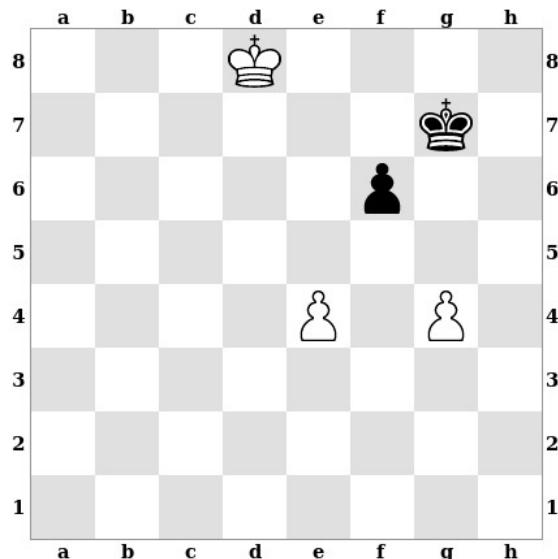
E11.15 (W/B)



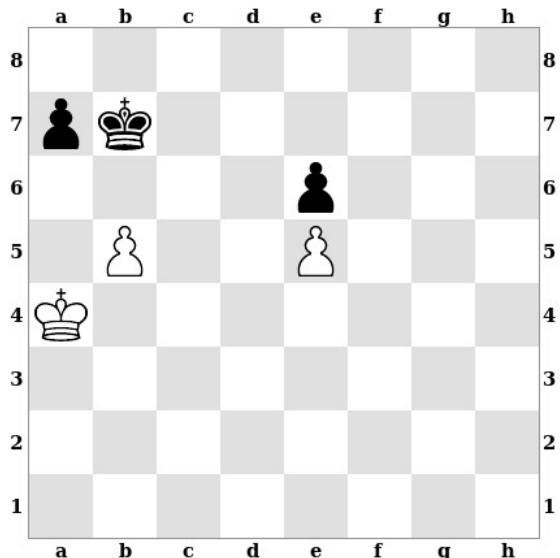
E11.16 (W/B)



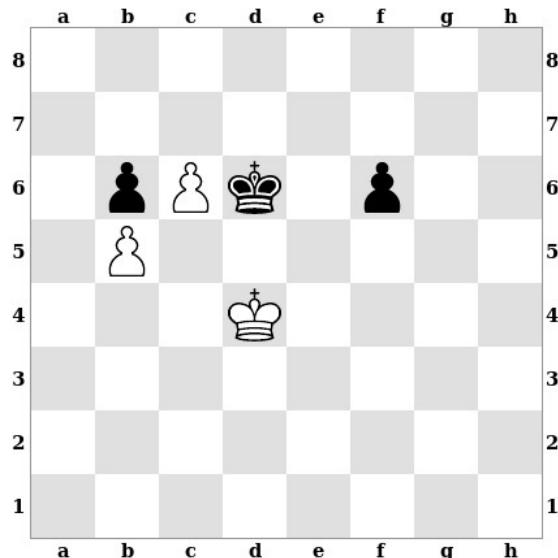
E11.17 (W/B)



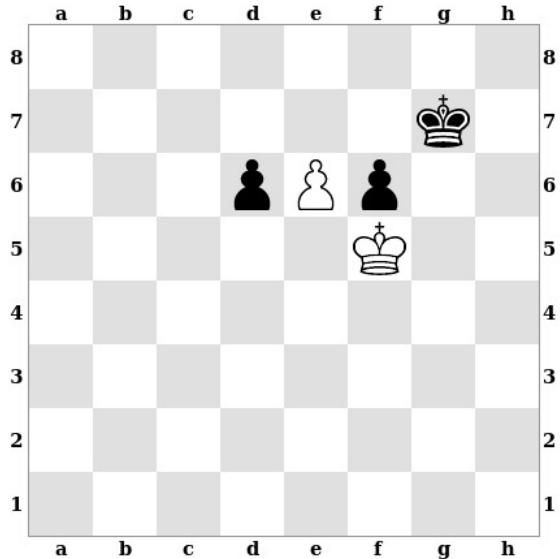
E11.18 (W/B)



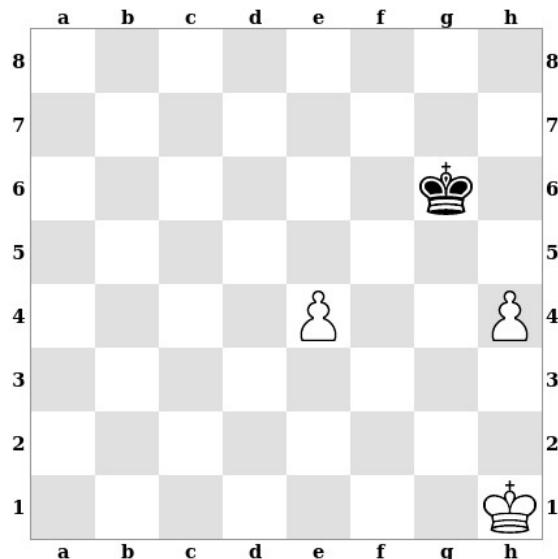
E11.19 (W/B)



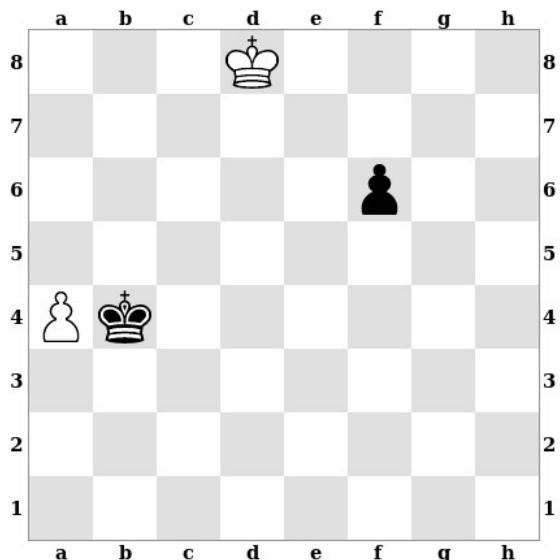
E11.20 (W/B)



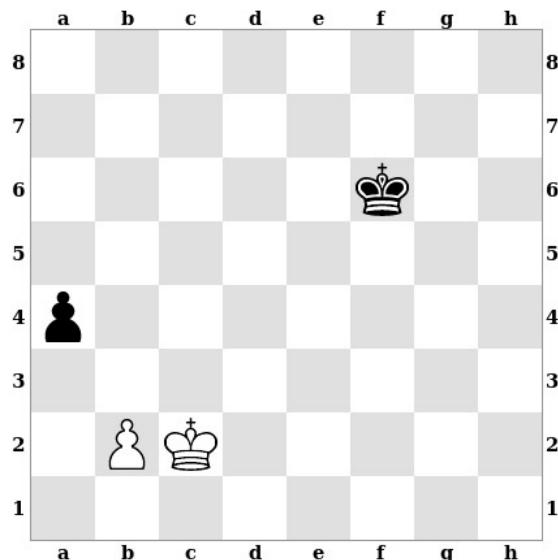
E11.21 (W/B)



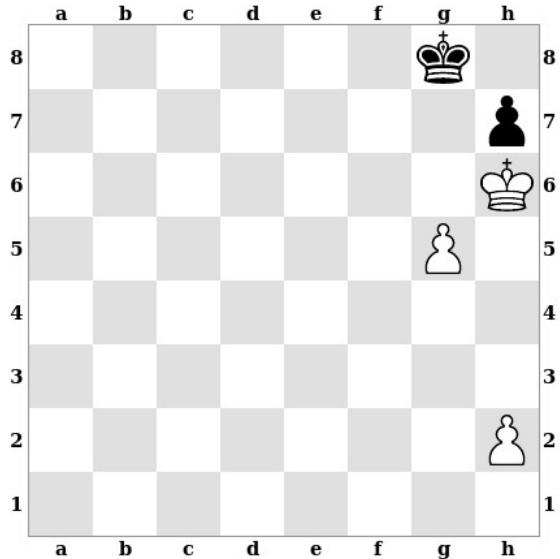
E11.22 (W/B)



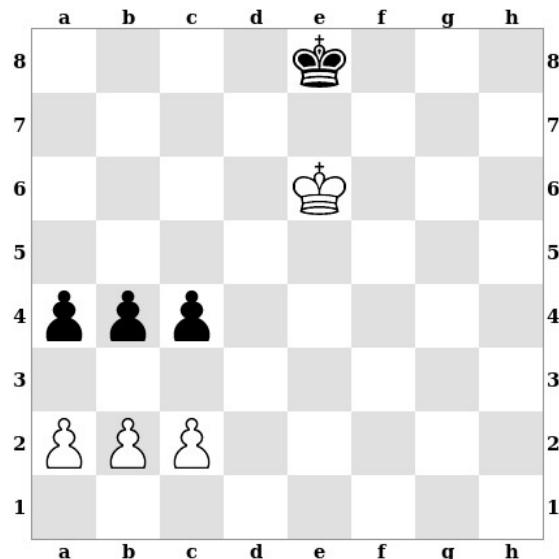
E11.23 (W/B)



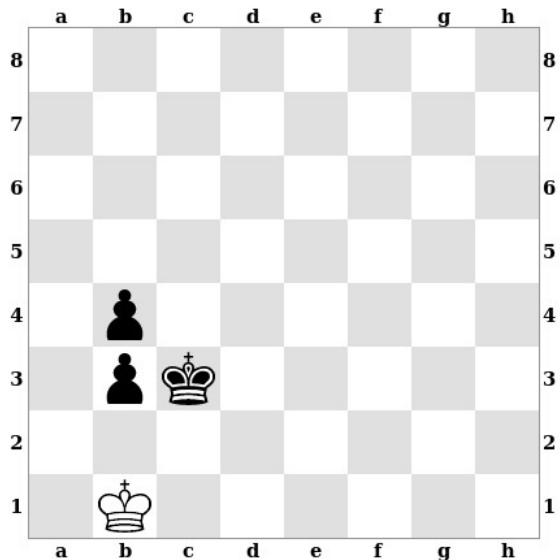
E11.24 (W/B)



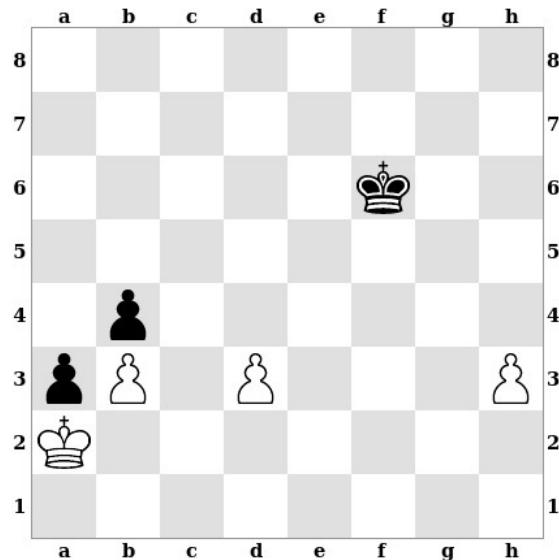
E11.25 (W/B)



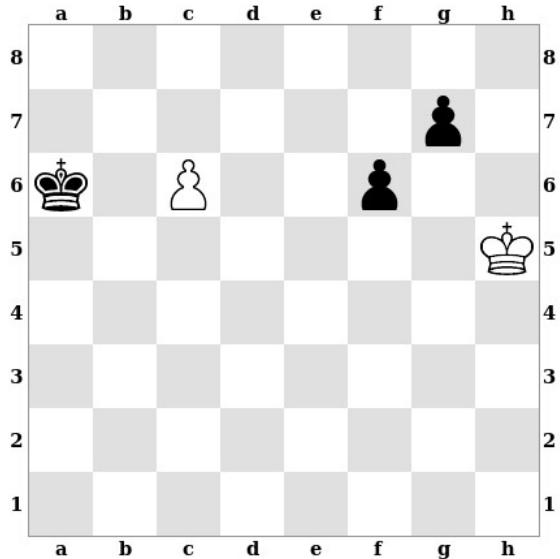
E11.26 (W/B)



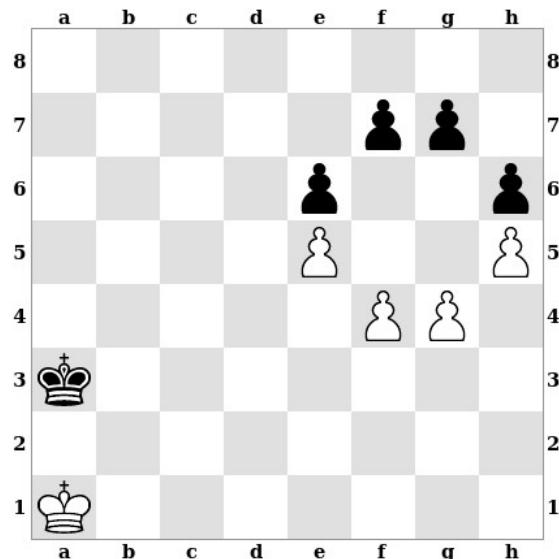
E11.27 (W/B)



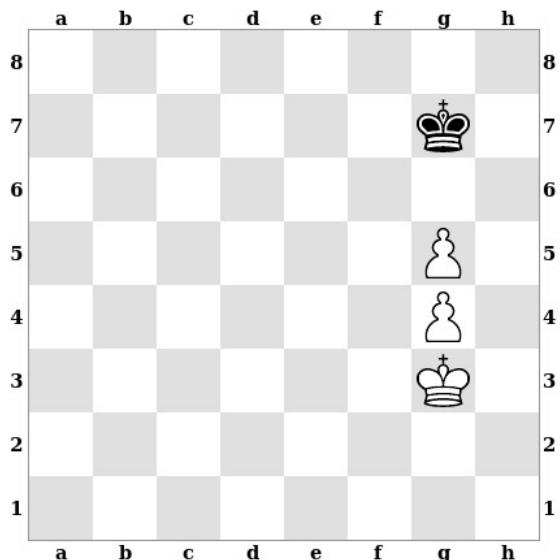
E11.28 (W/B)



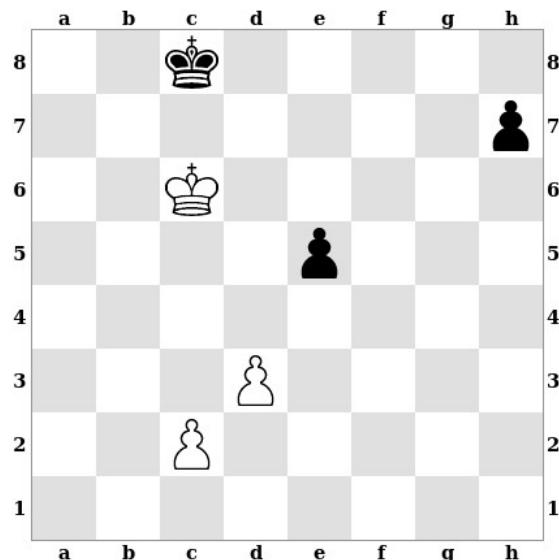
E11.29 (W/B)



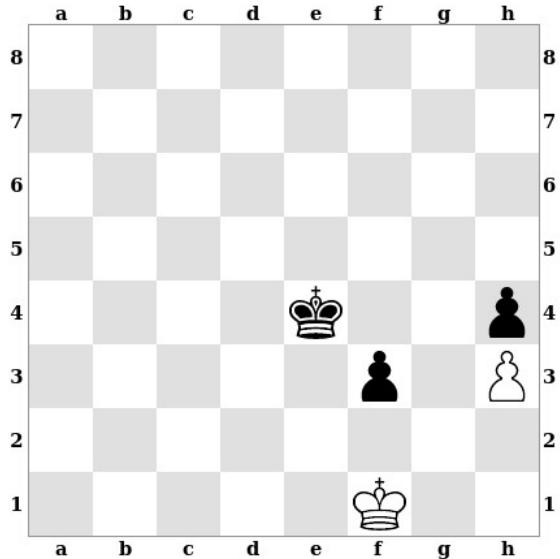
E11.30 (W/B)



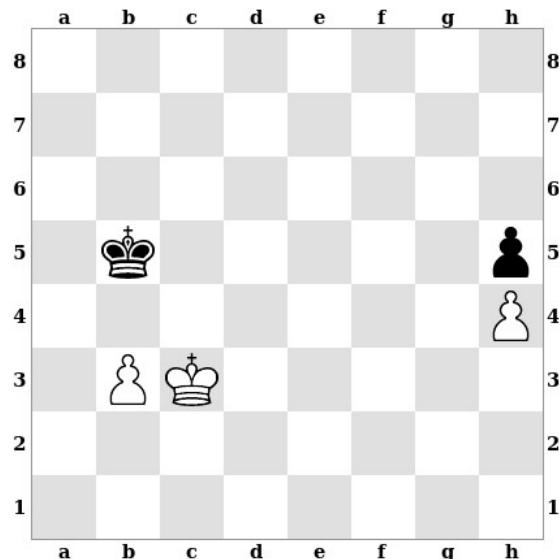
E11.31 (W/B)



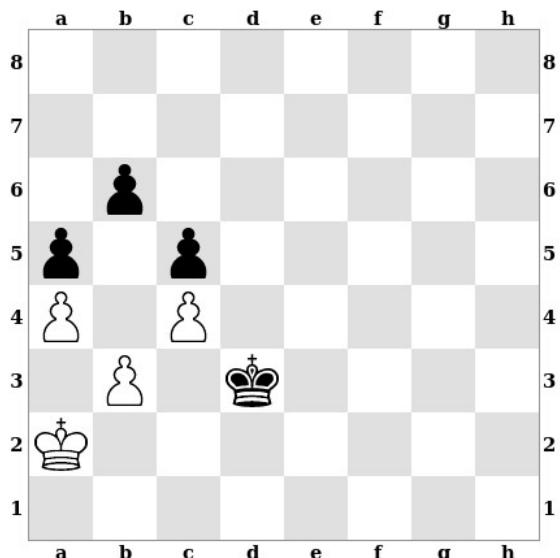
E11.32 (W/B)



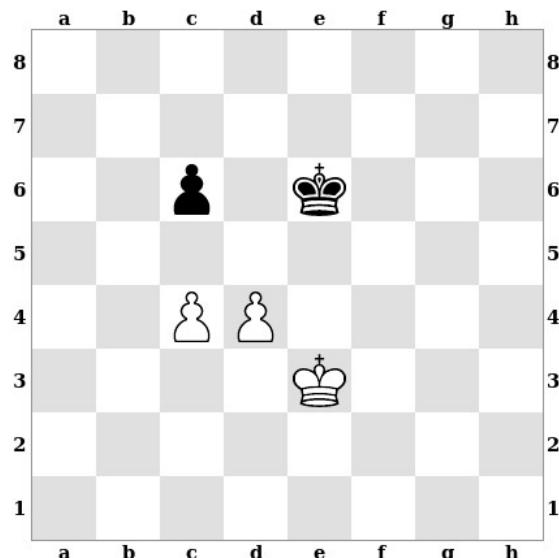
E11.33 (W/B)



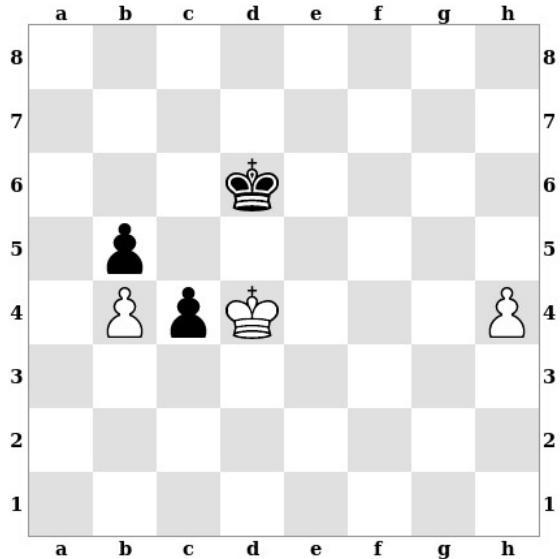
E11.34 (W/B)



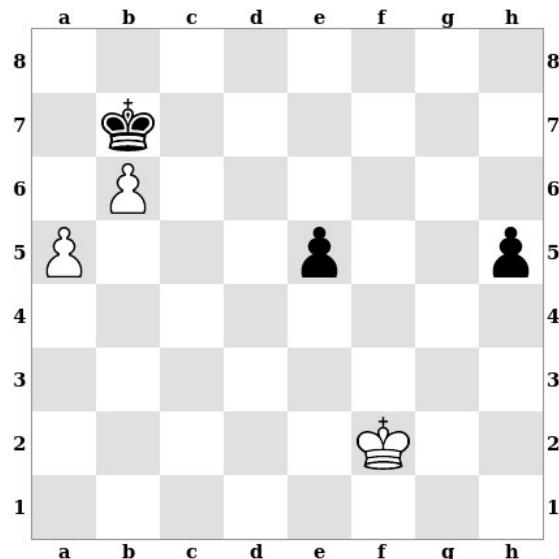
E11.35 (W/B)



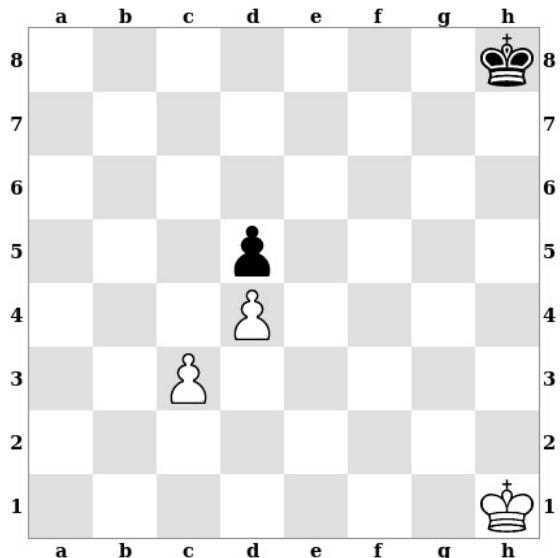
E11.36 (W/B)



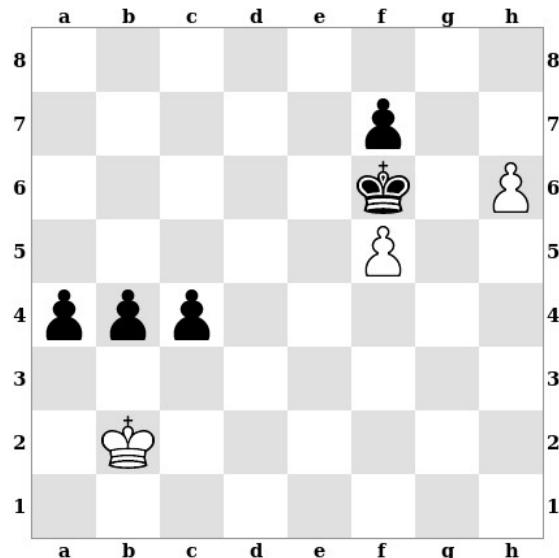
E11.37 (W/B)



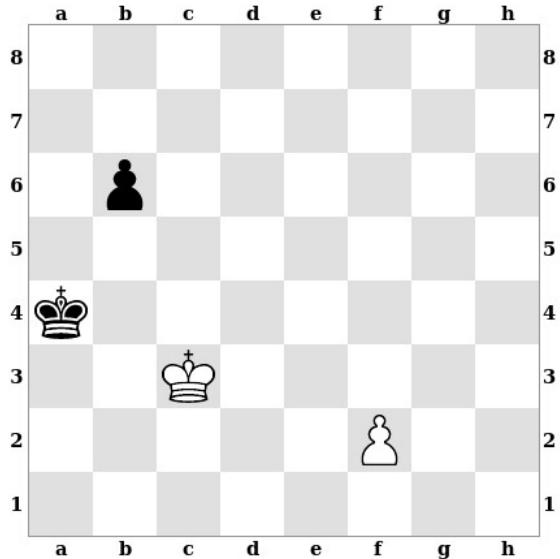
E11.38 (W/B)



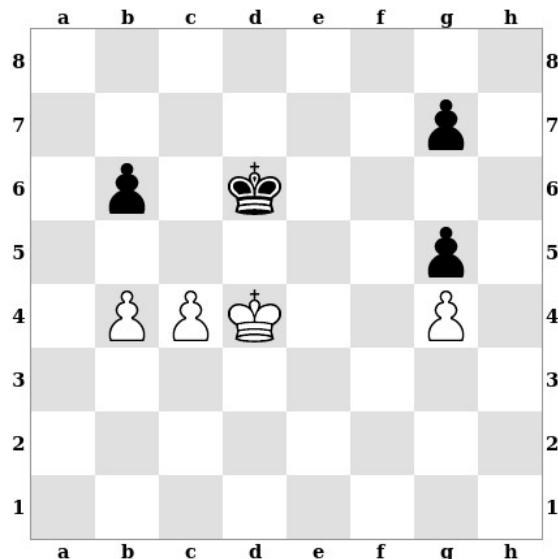
E11.39 (W/B)



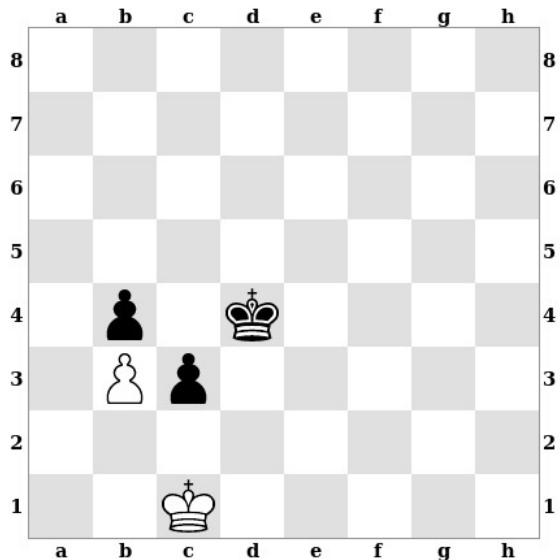
E11.40 (W/B)



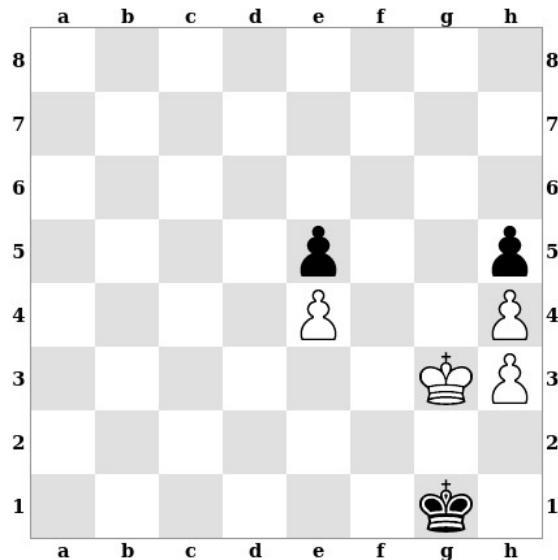
E11.41 (W/B)



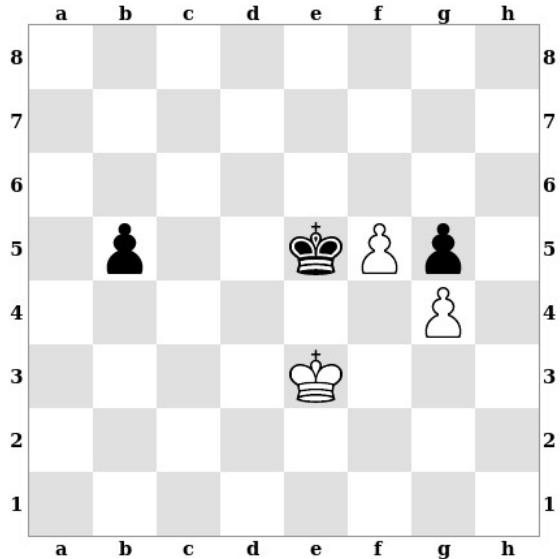
E11.42 (W/B)



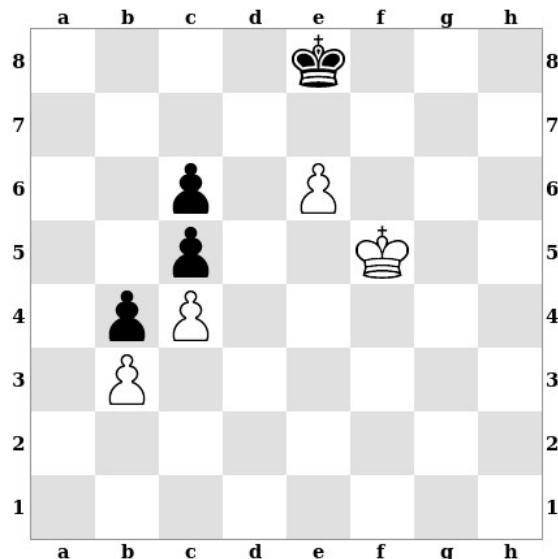
E11.43 (W/B)



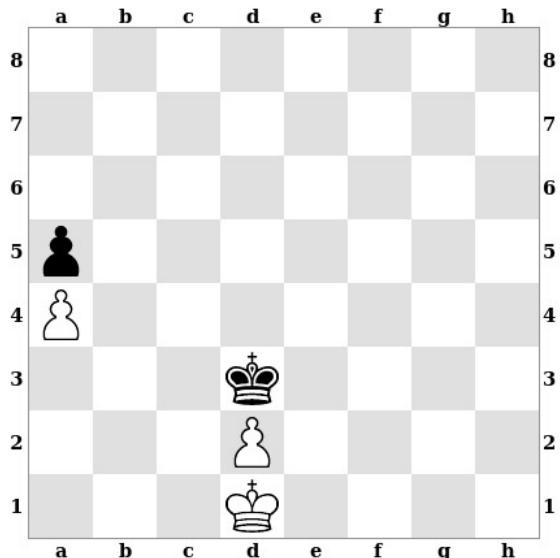
E11.44 (W/B)



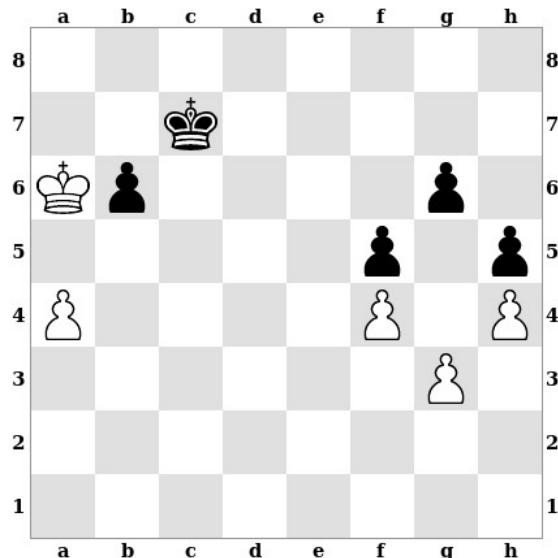
E11.45 (W/B)



E11.46 (W/B)



E11.47 (W/B)



E11.48 (W/B)

## Solutions

### E11.01

(+-) 1.a7#

### E11.02

W: (=) 1.Kc5 (1.Kxd5? Kb6 2.Kd4 Ka5 3.Kc3 Ka4 4.Kb2 Kb4 -+) 1...Kb7 (1...b4 2.Kxb4 Kd6 3.Kc3 Ke5 4.Kd3 =) 2.Kxb5 =

B: (-+) 1...Kd6 2.Kd3 Kc5 3.Kc3 d4+ 4.Kd3 Kd5 5.Kd2 Kc4 6.Kc2 d3+ 7.Kd2 Kd4 8.Kd1 Kc3 9.Kc1 d2+ 10.Kd1 b4 11.Ke2 Kc2

### E11.03

W: (=) 1.e5 (1.Kg2? Kf4 2.e5 Kxe5 3.Kf3 Kf5 -+) 1...fxe5 2.Kg2 Kf4 3.Kf2 =

B: (-+) 1...Kf4 2.Kg2 Kxe4 -+

### E11.04

W: (=) 1.Kd3 Kc5 2.Kc2 Kc4 3.Kb2 Kd3 4.Kb1 Kd2 (4...a3 5.Kc1 Kc3 6.Kb1 Kd2 7.Ka1 =) 5.Kb2 a3+ 6.Kb1 Kc3 7.Kc1 b3 8.Kb1 Kb4 9.axb3 Kxb3 10.Ka1 =

B: (-+) 1...a3 (1...b3? 2.axb3 a3 3.Kc3 Kc5 4.b4+ Kb5 5.Kb3 =) 2.Kd3 b3 (2...Kc5? 3.Kc2 Kc4 4.Kc1 Kc3 5.Kb1 =) 3.Kc3 (3.axb3 a2 -+) 3...bxa2 -+ (3...b2? 4.Kc2 =)

### E11.05

W: (+-) 1.Kg5 (1.Kxh5? Kxf4 2.Kg6 Kg4 3.h5 f4 =; 1.Kxf5? Kxh4 2.Kg6 Kg4 3.f5 h4 =) 1...Kf3 2.Kxf5 Kg3 3.Kg5 +-

B: (-+) 1...Kg4 2.Kf6 Kxf4 3.Kg6 Kg4 -+

### E11.06

(+)

W: 1.a3 (1.a4 a5 -+) 1...a6 2.a4 a5 3.Kb2 Kd2 -+

B: 1...Kc3 (1...a6? 2.a3 Kc3 3.a4 Kb4 4.Kxc2 =; 1...a5? 2.a4 =) 2.a3 (2.a4 a6 3.a5 Kb3 4.Kd2 Kb2 -+) 2...a5 3.a4 Kb3 4.Kd2 Kb2 -+

### E11.07

(+)

W: 1.c6 (1.Kc7? =) 1...bxc6 2.Kc7 c5 3.b7+ Ka7 4.b8=Q+ +-

B: 1...Kb8 2.Kd8 Ka8 3.c6 bxc6 4.Kc7 +-

### E11.08

W: (+-) 1.c5 Kc7 2.Ke6 Kc8 3.Kd6 Kb7 4.Kd7 Kb8 5.Kxc6 +-

B: (=) 1...c5 (1...Kb7? 2.c5 Ka6 3.Ke6 Ka5 4.Kd7 Kb5 5.Kd6 Ka6 6.Kxc6 +-; 1...Kc7? 2.c5 Kd7 3.Kf6 -+) 2.Kd6 Kc8 3.Kd5 Kb7 4.Kxc5 Kc7 =

### E11.09

W: (+-) 1.Kc7 (1.Kc6? a6 2.b6 Kb8 3.b7 a5 4.Kb5 Kxb7 =) 1...a6 2.b6 a5 3.b7+ Ka7 4.b8=Q+ +-

B: (=) 1...Kb7 2.Kc5 Kc7 3.Kc4 Kb6 4.Kb4 a6 =

### E11.10

W: (+-) 1.Kf5 Ke8 2.Ke6 (2.Kxf6? Kf8 =) 2...Kf8 (2...f5 3.g7 +-) 3.Kxf6 Kg8 4.g7 +-

B: (=) 1...Kf8 2.Kf4 Kg8 3.Ke4 Kf8 4.Kf4 =

### E11.11

(+)

W: 1.h8=Q+ Kxh8 2.Kf6 Kg8 3.g7 Kh7 4.Kf7 +-

B: 1...Kh8 2.Kf5 Kg7 3.h8=Q+ Kxh8 4.Kf6 +-

### E11.12

(-)

W: 1.Kf1 (1.Kd2 f3 2.Ke1 fxg2 -+) 1...Kd3 2.Ke1 Ke3 3.Kf1 Kd2 4.Kg1 Ke2 5.Kh1 f3 6.gxf3 Kf2 +-

B: 1...Kd4 2.Kf3 ( 2.Kd2 f3 -+; 2.Ke1 Ke3 3.Kf1 Kd2 -+) 2...Ke5 3.Ke2 Ke4 -+ reaching the initial position with White to move.

### E11.13

W: (=) 1.Kf3 h4 2.Kxf4 h3 3.Kg3 =

B: (-) 1...h4 2.Kh3 (2.Kg1 Kb7 3.Kg2 Kc6 -+) 2...f3 3.Kh2 Kb7 4.Kg1 h3 5.Kf2 h2 +-

### E11.14

W: (+-) 1.Kf5 e4 2.fxe4 Kg7 3.Ke6 +-

B: (=) 1...e4 (1...Kg7? 2.Kf5 e4 3.Kxe4 Kf6 4.Kf4 -+) 2.fxe4 (2.f4 Kg7 3.f5 e3 4.Kf3 Kf6 =) 2...Kg8 3.Kf4 Kf8 4.Kf5 Kf7 5.Ke5 Ke7 =

### E11.15

W: (+-) 1.b5 (1.Kg2? Kxe4 2.b5 Kd5 =) 1...Kd6 (1...Kxe4 2.b6 -+) 2.Kg2 Kc5 3.Kf3 Kxb5 4.Kf4 Kc6 5.Ke5 Kd7 6.Kf6 +-

B: (=) 1...Kxe4

### E11.16

W: (+-) 1.Kg3 Kb2 2.Kxf3 Kb3 3.a5 +-

B: (=) 1...Kb2 2.Kg3 (2.a5 Kc3 3.a6 Kd2 4.a7 f2 5.Kg2 Ke2 =) 2...Kc3 3.a5 Kd4 4.a6 Ke3 5.a7 f2 6.Kg2 Ke2 =

### E11.17

W: (+-) 1.Kb5 b3 (1...Kb7 2.Kxc5 b3 3.cxb3 -+) 2.cxb3 c4 3.Kxc4 Ka7 4.Kb5 +-

B: (=) 1...b3 (1...Kb7? 2.Kb5 b3 3.cxb3 c4 4.bxc4 -+) 2.cxb3 (2.Kxb3 Kb7 3.Kc4 Kc6 =) 2...c4 (2...Kb7? 3.Kb5 c4 4.bxc4 -+) 3.bxc4 (3.b4 Kb7 4.b5 Kb6 =) 3...Kb8 4.Kb4 Kc8 5.Kb5 Kb7 =

### E11.18

W: (+-) 1.Ke7 Kg6 2.Kf8 (2.Ke6 Kg7 3.Kf5? Kf7 4.g5 fxg5 5.Kxg5 Ke6 =) 2...Kh6 3.Kf7 Kg5 4.Kg7 Kxg4 5.Kxf6 Kf4 6.e5 +-

B: (=) 1...Kh8 2.Kd7 (2.g5 Kg7 [2...fxg5? 3.e5 g4 4.e6 g3 5.e7 g2 6.e8=Q+ +-] 3.Ke7 fxg5 4.e5 g4 5.e6 g3 6.Kd8 g2 7.e7 g1=Q 8.e8=Q =) 2...Kh7 3.Ke6 (3.g5 fxg5 [3...Kg6? 4.gxf6 Kxf6 5.Kd6 Kf7 6.e5 Ke8 7.Ke6 +-] 4.e5 g4 5.e6 g3 6.e7 g2 7.e8=Q g1=Q =) 3...Kg6 4.Ke7 Kg7 5.Ke8 Kg8 =

### E11.19

W: (+-) 1.Ka5 (1.Kb4? Kb6 2.Kc4 Kb7 3.Kc5 Kc7 4.Kb4 Kb6 5.Ka4 a6 6.bxa6 Kxa6 =) 1...Kc7 2.Ka6 Kb8 3.b6 Ka8 4.Kb5 Kb7 5.bxa7 Kxa7 6.Kc6 +-

B: (=) 1...Kb6 (1...a6? 2.Ka5 axb5 3.Kxb5 Kc7 4.Kc5 Kd7 5.Kb6 +-) 2.Kb4 Kc7 (2...Kb7? 3.Ka5 Kc7 4.Ka6 Kb8 5.b6 +-) 3.Kc5 (3.Ka5 Kb7 =) 3...Kd7 4.b6 a6 (4...a5? 5.Kb5 +-) 5.b7 (5.Kb4? Kc6 6.Ka5 Kb7 +-) 5...Kc7 6.b8=Q+ Kxb8 7.Kb6 a5 8.Kxa5 Kc7 9.Kb5 Kb7 =

### E11.20

(+-)

W: 1.Ke4 Ke6 2.Kf4 Kd6 3.Kf5 Ke7 4.c7 Kd7 5.Kxf6 Kxc7 6.Ke6 +-

B: 1...Ke6 2.Ke4 Kd6 3.Kf5 Ke7 4.c7 +-

### E11.21

W: (=) 1.e7 (1.Ke4? Kf8 2.Kd4 [2.Kd5 Ke7 +-] 2...Ke7 3.Kd5 f5 4.Kd4 Kxe6 +-) 1...Kf7 2.e8=Q+ Kxe8 3.Ke6 (3.Kxf6? Kd7 4.Kf5 Kc6 5.Ke4 Kc5 6.Kd3 Kd5 +-) 3...d5 (3...Kd8 4.Kxd6 =) 4.Kxd5 Kf7 5.Ke4 Kg6 6.Kf4 =

B: (-+) 1...d5 2.Kf4 Kf8 3.Ke3 Ke7 4.Kd4 Kxe6 +-

### E11.22

W: (+-) 1.Kg2 Kh5 (1...Kf6 2.Kf3 +-) 2.Kf3 Kxh4 3.Kf4 Kh5 4.Kf5 +-

B: (=) 1...Kh5 (1...Kf6? 2.Kg2 Ke5 3.Kf3 +-) 2.e5 (2.Kg2 Kxh4 3.Kf3 Kg5 4.Ke3 Kf6 5.Kd4 Ke6 =) 2...Kg6 3.Kg2 Kf5 4.Kf3 Kxe5 5.Kg4 Kf6 6.Kh5 Kg7 =

### E11.23

W: (=) 1.Kc7 f5 (1...Kxa4 2.Kd6 =) 2.Kb6 f4 (2...Kxa4 3.Kc5 Kb3 4.Kd4 =) 3.a5 f3 4.a6 f2 5.a7 f1=Q 6.a8=Q =

B: (-+) 1...Kxa4 2.Ke7 f5 +-

### E11.24

W: (+-) 1.Kb1 (1.Kc3? a3 2.bxa3 [2.b4 Ke6 3.Kb3 Kd6 4.Kxa3 Kc6 5.Ka4 Kb6 =] 2...Ke6 3.Kc4 Kd7 4.Kc5 Kc8 =) 1...a3 (1...Ke5 2.Ka2 Kd5 [2...a3 3.Kxa3 Kd5 4.Kb4 +-] 3.Ka3 Kc5 4.Kxa4 +-) 2.b3 (2.b4? Ke6 3.Ka2 Kd6 4.Kxa3 Kc6 5.Ka4 Kb6 =; 2.bxa3? Ke6 3.Kb2 Kd7 4.Kb3 Kc8 =) 2...Ke5 3.Ka2 Kd5 4.Kxa3 Kc5 5.Ka4 Kb6 6.Kb4 +-

B: (=) 1...a3 2.bxa3 (2.b3 Ke6 3.Kb1 Kd6 4.Ka2 Kc5 5.Kxa3 Kb5 =; 2.b4 Ke6 3.Kb3 Kd6 4.Kxa3 Kc6 5.Ka4 Kb6 =) 2...Ke6 3.Kc3 Kd7 4.Kb4 Kc8 =

### E11.25

(+-)

W: 1.h3 (1.h4? Kh8 2.h5 Kg8 3.g6 hxg6 4.hxg6 Kh8 =) 1...Kh8 2.h4 Kg8 3.h5 Kh8 4.g6 hxg6 (4...Kg8 5.g7 Kf7 6.Kxh7 +-) 5.hxg6 Kg8 6.g7 Kf7 7.Kh7 +-

B: 1...Kh8 2.h4 (2.h3? Kg8 3.h4 Kh8 4.h5 Kg8 5.g6 hxg6 6.hxg6 Kh8 =) 2...Kg8 3.h5 Kh8 4.g6 hxg6 5.hxg6 Kg8 6.g7 Kf7 7.Kh7 +-

### E11.26

W: (=) 1.b3 (1.Kd5? b3 2.cxb3 a3 3.bxa3 c3 +-) 1...cxb3 2.cxb3 (2.axb3? a3 +-) 2...axb3 (2...a3? 3.Kd6 Kd8 4.Kc5 Kc7 5.Kxb4 +-) 3.axb3 Kd8 4.Kd6 Kc8 5.Kc5 Kc7 6.Kxb4 Kb6 =

B: (-+) 1...b3 2.axb3 (2.cxb3 a3 3.bxa3 c3 +-) 2...c3 3.bxc3 a3 +-

### E11.27

W: (=) 1.Kc1 b2+ 2.Kb1 Kc4 3.Kxb2 =

B: (-+) 1...b2 2.Ka2 b1=Q+ 3.Kxb1 Kb3 4.Ka1 Kc2 5.Ka2 b3+ 6.Ka1 b2+ 7.Ka2 b1=Q+ --

### E11.28

W: (+-) 1.d4 Ke6 2.h4 +- (mutual square rule)

B: (=) 1...Kf5 (1...Ke5? 2.h4 Kf5 3.d4 +- [mutual square rule]) 2.Ka1 Kf4 3.Ka2 Kf5 (3...Ke3? 4.h4 Kf4 5.d4 +- [mutual square rule]) 4.Ka1 (4.d4? Ke4 5.h4 Kxd4 6.h5 Ke5 -+) 4...Kf4 =

### E11.29

W: (=) 1.Kg6 Kb6 2.Kxg7 f5 3.Kf6 f4 4.Ke5 f3 5.Kd6 f2 6.c7 f1=Q 7.c8=Q =

B: (-+) 1...Kb6 2.Kg6 Kxc6 -+

### E11.30

W: (+-) 1.f5 (or 1.g5 +- with the same idea) 1...Kb4 (1...exf5 2.gxf5 Kb4 3.e6 +-) 2.g5 Kc5 (2...exf5 3.g6 fxg6 [3...f6 4.e6 +-] 4.e6 +-; 2...hxg5 3.f6 +-) 3.f6 gxf6 (3...g6 4.gxh6 +-) 4.gxh6 +-

B: (-+) 1...Kb4 2.g5 Kc5 3.f5 exf5 4.g6 fxg6 5.e6 Kd6 -+

### E11.31

W: (+-) 1.Kf3 (1.Kf4? Kg6 =) 1...Kf7 (1...Kg6 2.Kf4 Kg7 3.Kf5 Kf7 4.g6+ Kg7 5.Kg5 Kg8 6.Kh6 Kh8 7.g7+ Kg8 8.g5 Kf7 9.Kh7 +-) 2.Ke4 Ke6 3.Kf4 Kf7 4.Kf5 Kg7 5.g6 Kh6 6.g7 Kxg7 7.Kg5 +-

B: (+-) 1...Kf7 2.Kf3 Ke6 3.Kf4 Kf7 4.Kf5 Kg7 5.g6 Kh6 6.g7 Kxg7 7.Kg5 +-

### E11.32

W: (+-) 1.Kd5 h5 2.Kxe5 +-

B: (-+) 1...e4 (1...h5? 2.Kd5 h4 3.Ke4 =) 2.dxe4 (2.Kd5 e3 -+) 2...h5 3.Kd5 (3.e5 h4 4.e6 Kd8 -+) 3...h4 4.Ke5 h3 -+

### E11.33

(-+)

W: 1.Ke1 (1.Kf2 Kf4 2.Kf1 Kg3 -+) 1...Ke3 2.Kf1 f2 3.Kg2 Ke2 4.Kh2 Kf3 (4...f1=Q? =) 5.Kh1 f1=Q+ -+

B: (-+) 1...Kf5 (or 1...Ke5 -+ with the same idea; Black makes no progress with 1...Kf4 2.Kf2 -+ or 1...Ke3 2.Ke1 f2? 3.Kf1 =) 2.Ke1 Ke5 3.Kf1 Ke4 -+ reaching the starting position with White to move.

### E11.34

(+-)

W: 1.Kd4 (1.b4? Kb6 2.Kc4 Kc6 3.b5+ Kb6 4.Kb4 Kb7 5.Kc5 Kc7 6.b6+ Kb7 7.Kb5 Kb8 8.Kc6 Kc8 9.Kd6 Kb7 10.Ke6 Kxb6 11.Kf6 Kc6 12.Kg6 Kd6 13.Kxh5 Ke7 14.Kg6 Kf8 =) 1...Kb4 2.Ke4

Kxb3 3.Kf5 Kc4 4.Kg5 Kd5 5.Kxh5 Ke6 6.Kg6 Ke7 7.Kg7 +-

B: 1...Kc5 2.Kd3 Kb4 3.Ke4 Kxb3 4.Kf5 +-

### E11.35

(-+)

W: 1.Kb2 Kd2 2.Kb1 Kc3 3.Ka2 Kc2 4.Ka3 Kb1 5.b4 cxb4+ 6.Kb3 Kc1 7.Ka2 Kc2 -+

B: 1...Kc2 2.Ka3 Kb1 -+

### E11.36

(+-)

W: 1.Ke4 Kf6 (1...Kd6 2.Kf5 Ke7 3.Ke5 Kd7 4.d5 cxd5 [4...c5 5.d6 Kc6 6.Ke6 Kb7 7.Kd5 +-]  
5.Kxd5 +-) 2.Kf4 (2.d5? Ke7 3.Ke5 cxd5 4.Kxd5 Kd7 =; 2.c5? Ke6 3.d5+ cxd5+ 4.Kd4 Kd7  
5.Kxd5 Kc7 =) 2...Ke6 3.Kg5 Ke7 4.Kf5 Kd6 5.Kf6 Kd7 6.Ke5 Ke7 7.c5 Kd7 8.Kf6 +-

B: 1...Ke7 2.Ke4 Ke6 3.c5 Kf6 4.d5 Ke7 5.d6+ Ke8 6.Kf5 Kd7 7.Ke5 Kd8 8.d7 Kxd7 9.Kf6 +-

### E11.37

(-+)

W: 1.Ke4 Ke6 2.Kf4 (2.Kd4 Kf5 3.Kc5 c3 -+) 2...Kf6 3.Ke4 Kg6 4.Kf4 Kh5 5.Kg3 c3 -+

B: 1...Ke6 2.Ke4 (2.Kc5 c3 -+) 2...Kf6 3.Kf4 Kg6 4.Kf3 (4.Kg4 c3 -+) 4...Kh5 5.Ke3 Kxh4 -+

### E11.38

(+-)

W: 1.Kf3 Ka6 (1...h4 2.Kg4 +-) 2.Ke4 h4 3.Kf3 Kb7 4.Kg4 e4 5.Kxh4 +-

B: 1...Ka6 (1...h4 2.Kf3 +-; 1...e4 2.Ke3 h4 3.Kxe4 +-) 2.Kf3 Kb7 (2...h4 3.Kg4 e4 4.Kxh4 +-) 3.Ke4  
h4 4.Kf3 Ka6 5.Kg4 +-

### E11.39

W: (-) 1.Kh2 Kg7 2.Kg3 Kf7 3.Kf3 Kg7 (3...Ke7 4.Kg4 Kf6 5.Kf4 Ke6 6.Kg5 +-) 4.Ke2 Kf6 5.Kd3  
Ke6 6.Kc2 Kd6 7.Kb3 Kc6 8.Ka4 Kb6 9.Kb4 Kc6 10.Ka5 +-

B: (=) 1...Kh7 2.Kg2 Kg6 3.Kf1 Kf7 4.Ke1 Ke7 5.Kd2 Kd6 6.Kc2 Kc6 7.Kb3 Kb5 8.Ka3 Ka5  
9.Kb3 Kb5 =

### E11.40

W: (-) 1.Kb1 (1.Kc2?? a3 2.Kb1 b3 3.Kc1 c3 4.Kb1 c2+ 5.Kc1 a2 6.Kb2 c1=Q+ 7.Kxc1 a1=Q+ -+)  
1...a3 (1...b3 2.Kb2 +-) 2.Ka2 c3 3.Kb3 a2 4.Kxa2 c2 5.Kb2 b3 6.Kc1 Kxf5 7.h7 +-

B: (-) 1...b3 2.Kb1 (2.Kc3 a3 -+) 2...a3 3.Kc1 c3 4.Kb1 c2+ 5.Kc1 a2 6.Kb2 c1=Q+ 7.Kxc1  
a1=Q+ -+

### E11.41

W: (+) 1.Kd4 (1.f4? Kb5 2.Kd4 Kc6 3.Ke5 Kd7 4.Kf6 b5 5.f5 b4 6.Kg7 b3 7.f6 b2 8.f7 b1=Q  
9.f8=Q =) 1...Kb5 (1...b5 2.f4 b4 3.f5 b3 4.Kc3 Ka3 5.f6 b2 6.f7 b1=Q 7.f8=Q+ Ka4 8.Qa8+  
Kb5 9.Qb8+ +-) 2.Kd5 Ka6 (2...Ka4 3.f4 b5 4.f5 b4 5.Kc4 b3 6.Kc3 Ka3 7.f6 b2 8.f7 b1=Q  
9.f8=Q+ +-) 3.f4 (3.Kc6? b5 4.Kc5 Ka5 5.f4 b4 6.Kc4 b3 7.Kxb3 Kb5 =) 3...Kb7 4.f5 Kc8  
5.Ke6 Kd8 6.Kf7 b5 7.Kg7 b4 8.f6 b3 9.f7 b2 10.f8=Q+ +-

B: (=) 1...Kb5 2.Kd4 Kc6 3.Ke5 Kd7 4.Kf6 b5 5.Ke5 (5.f4? b4 -+) 5...Ke7 =

### E11.42

(+)

W: 1.c5+ bxc5+ 2.bxc5+ Kc6 (2...Ke6 3.Ke4 g6 4.Kd4 Ke7 5.Ke5 Kf7 6.Kd6 Ke8 7.Ke6 +-) 3.Kc4 (3.Ke5? Kxc5 4.Kf5 Kd4 5.Kxg5 Ke4 6.Kg6 Kf4 7.g5 Kg4 =) 3...g6 4.Kd4 Kd7 5.Ke5 Ke7 6.c6 Ke8 7.Kf6 +-

B: 1...Ke6 2.c5 bxc5+ 3.bxc5 Kd7 4.Kd5 Kc7 5.Ke6 Kc6 6.Kf7 Kxc5 7.Kxg7 Kd5 8.Kf6 +-

### E11.43

(-)

W: 1.Kc2 Ke3 2.Kc1 c2 3.Kxc2 Ke2 4.Kc1 Kd3 5.Kb2 Kd2 6.Kb1 Kc3 7.Ka2 Kc2 8.Ka1 Kxb3 9.Kb1 Ka3 10.Ka1 b3 11.Kb1 b2 12.Kc2 Ka2 +-

B: 1...c2 2.Kxc2 Ke3 -+

### E11.44

(+)

W: 1.Kf3 Kh2 2.Ke3 Kxh3 3.Kd3 Kxh4 4.Kc4 Kg4 5.Kd5 Kf4 +-

B: 1...Kh1 2.Kf3 Kh2 +-

### E11.45

(=)

W: 1.Kd3 Kd5 2.Kc3 Kc5 3.Kb3 Kd5 4.Kb4 Kc6 5.Kb3 Kc5 6.Kc3 Kd5 7.Kb4 Kc6 8.Ka5 Kc5 9.f6 (9.Ka6? b4 10.f6 Kd6 +-) 9...Kd6 10.Kxb5 Ke6 11.Kc5 Kxf6 12.Kd5 Kf7 13.Ke5 Kg8 14.Kf5 Kf7 15.Kxg5 Kg7 =

B: 1...Kd5 (1...b4? 2.Kd3 Kd5 3.Kc2 Kd6 4.Kb3 Kc5 5.Ka4 Kc4 6.f6 b3 7.Ka3 Kc3 8.f7 b2 9.f8=Q b1=Q 10.Qc5+ Kd2 11.Qxg5+ +-) 2.Kd3 Ke5 (2...Kc5? 3.Ke4 b4 [3...Kd6 4.Kd4 Kc6 5.Ke5 b4 6.Kd4 Kb5 7.f6 Ka4 8.f7 b3 9.f8=Q b2 10.Qb8 +-) 4.Ke5 b3 5.f6 b2 6.f7 b1=Q 7.f8=Q+ Kc4 8.Qc8+ Kb3 9.Qb7+ Kc2 10.Qxb1+ Kxb1 11.Kf5 +-) 3.Kc3 Kd5 4.Kb4 Kc6 =

### E11.46

(+)

W: 1.Ke4 (White makes no progress with 1.Ke5 Ke7 2.Kf5 Ke8 + or 1.Kf6 Kf8 2.e7+? Ke8 3.Kf5 Kxe7 4.Ke5 Kd7 5.Kf6 Kd6 6.Kf5 Kd7 =) 1...Kd8 2.Kf4 Ke8 3.Kf5 Ke7 4.Ke5 Ke8 5.Kd6 Kd8 6.Kxc6 Ke7 7.Kxc5 Kxe6 8.Kxb4 +-

B: 1...Ke7 2.Ke5 Ke8 3.Kd6 +-

### E11.47

(+)

W: 1.Ke1 Kc4 (1...Kd4 2.Ke2 Ke4 3.d3+ Kd4 4.Kd2 Kd5 5.Kc3 Kc5 6.d4+ Kd5 7.Kd3 Kd6 8.Kc4 Kc6 9.d5+ Kd6 [9...Kb6 10.Kd4 Kc7 11.Kc5 Kd7 12.Kb5 Kd6 13.Kxa5 Kxd5 14.Kb6 +] 10.Kd4 Kd7 11.Kc5 Kc7 12.Kb5 Kd6 13.Kxa5 Kxd5 14.Kb6 +-) 2.Ke2 Kb4 3.Kd3 Kxa4 4.Kc4 Ka3 5.d4 Kb2 6.d5 a4 7.d6 a3 8.d7 a2 9.d8=Q a1=Q 10.Qd2+ Kb1 (10...Ka3 11.Qb4+ Ka2 12.Qb3# ) 11.Kb3 +-

B: 1...Kc4 (1...Kd4 2.Ke2 +-) 2.Ke2 +-

**E11.48**

**W:** (+-) 1.Ka7 Kc6 2.Kb8 Kd5 (2...b5 3.a5 b4 4.a6 b3 5.a7 b2 6.a8=Q+ +-) 3.Kb7 Kc5 (3...Ke4 4.Kxb6 Kf3 5.a5 Kxg3 6.a6 Kxf4 7.a7 Kg4 8.a8=Q Kxh4 9.Qf3 +-) 4.Kc7 Kb4 5.Kxb6 Kxa4 6.Kc5 Kb3 7.Kd4 Kc2 8.Ke5 Kd3 9.Kf6 Ke3 10.Kxg6 +-

**B:** (-+) 1...Kc6 2.Ka7 (2.a5 b5 3.Ka7 b4 4.a6 b3 5.Kb8 b2 6.a7 b1=Q+ 7.Kc8 Qb7+ -+) 2...b5 3.axb5+ Kxb5 4.Kb7 Kc5 5.Kc7 Kd5 6.Kd7 Ke4 7.Ke6 Kf3 8.Kf6 Kxg3 9.Kxg6 Kg4 +-