The Exponential Distribution and the Central Limit Theorem

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August 14, 2016

Project Instructions

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda.

- Set lambda = 0.2 for all of the simulations.
- You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials. You should:

- Show the sample mean and compare it to the theoretical mean of the distribution.
- Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
- Show that the distribution is approximately normal.

Overview

Here we want to run a simulation consisting in generating 40 variables from an exponential function with given parameters and repeating this for 1000 times. Then we compute the mean for each simulation(1000 means in total)

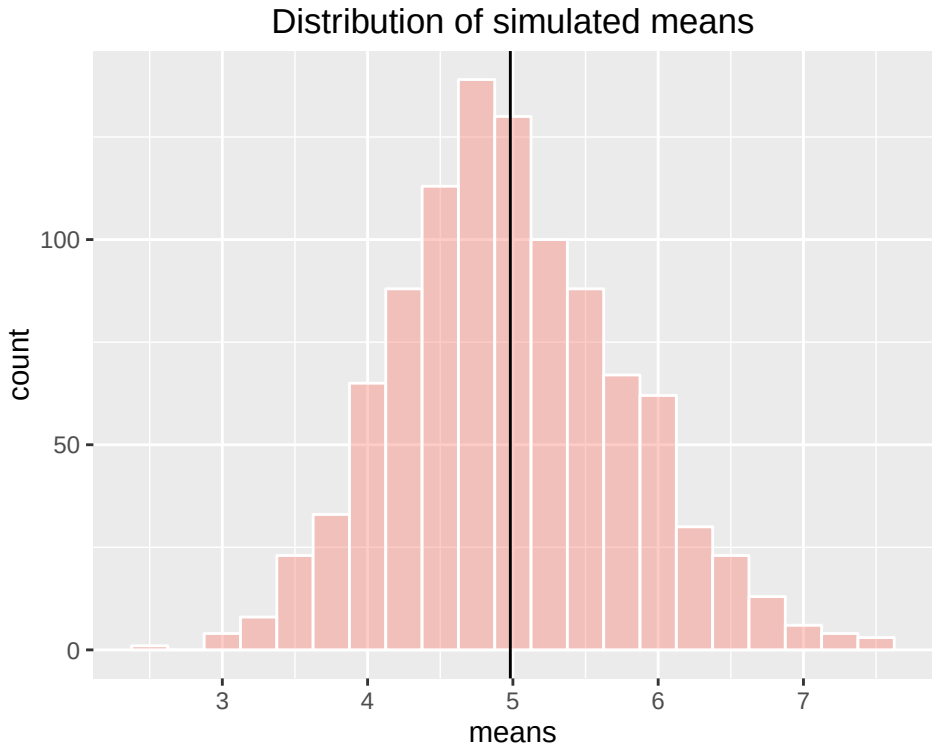
We run the simulation and store the results in a matrix object, *sim.distrib*

We compute the means and store the results in a dataframe which is what the dplyr and ggplot2 packages take as input and it's also the typical datastructure in R. So we create a dataframe, *sim_mns*.

Sample mean vs theoretical mean

Here we want to compare the theoretical mean for an exponetial distribution, given by mu = 1/lambda = 5, to the mean of our simulated distribution.

```
## simul.mean
##   4.982365
```



From the plot we can see that the distribution of the means is centered around the mean of our simulated distribution, that is **4.982365** (the black vertical line) which is very close to the theoretical mean 1/lambda = **5**

Sample Variance versus Theoretical Variance

```
## [1] 0.01570633
```

```
## [1] 0.015625
```

As we can see they're very close, 0.01570633 and 0.015625, respectively.

Normality of the Distribution

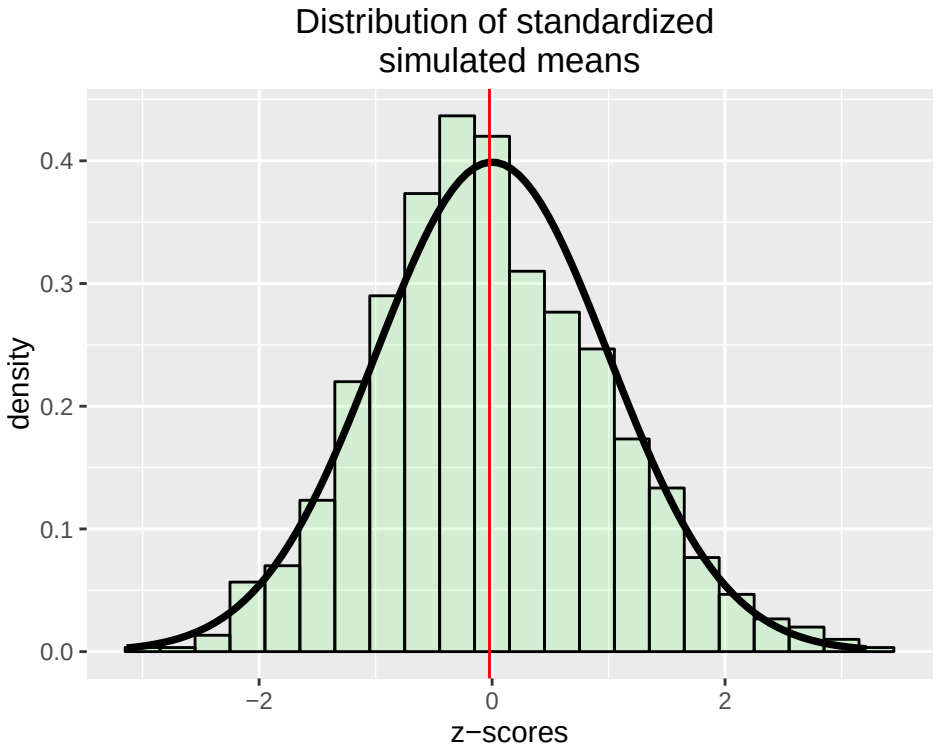
From the Central limit theorem we know that the distribution of averages of normalized variables becomes that of a standard normal distribution as the sample size increases.

Here we want to normalize our sample means. To do this we need to transform each mean in our simulated dataset according to the following formula:

z-score = (xbar - 1/lambda) / (1/lambda / sqrt(n))

Based on the CLT the result should be a normal distribution centered at(more or less) zero. To see if this is the case we create a plot comparing the density of our transformed sample means distribution with the density of the standard normal distribution.

```
##      z_mean
## -0.02230626
```



From the plot we see that the normalized distribution of sample means is approximately the same as the standard normal distribution as we can see comparing it to the density function, the black bell-shaped curve. Also, the mean is -0.02230626, very close to zero, which is the mean of the standardized normal distribution. This is consistent with what is stated in the Central Limit Theorem.

APPENDIX

```
#Load packages
library(dplyr, warn.conflicts = F)
library(ggplot2)

#Exponential function parameters
lambda <- 0.2
n <- 40
num.of.sim <- 1000

#set the seed
set.seed(119983)
```

Simulation + Plot 1

```
#Create a 1000x40 matrix containing the results of the simulation
sim.distrib <- matrix(data=rexp(n * num.of.sim, lambda), nrow=num.of.sim)
```

```r
#compute the mean for each of the 1000 simulations(rows)
sim_mns <- data.frame(means=apply(sim.distrib, 1, mean))

#Convert dataframe to tbl_df object for more convenient printing
sim_mns <- tbl_df(sim_mns)

#compute the mean of the simulated means
(mean_sim <- sim_mns %>% summarize(simulated.mean = mean(means)) %>% unlist())

#Plot sample means distribution with the calculated...bla bla...
sim_mns  %>%
  ggplot(aes(x = means) ) + geom_histogram(alpha=0.4, binwidth= .25,
                                           fill = "salmon", col = "white") +
  geom_vline(xintercept = mean_sim, color="black", size = 0.5) +
  ggtitle("Simulated distribution of means")
```

Sample Variance versus Theoretical Variance

```r
#Compute the variance of the sample means
sd.samp <- sim_mns  %>% select(means)  %>% unlist() %>%  sd()
(var.samp <- sd.samp ^ 2 / 40)

#Theoretical variance of the exponential distribution
(((1/lambda))/(40))^2
```

Plot 2

```r
#Compute mean of our normalized means
(z_mean <- sim_mns %>%  mutate(z_score = (means - 1/lambda) /
                                 (1/lambda / sqrt(n)))  %>% select(z_score) %>%
   summarise(z_mean = mean(z_score)) %>% unlist())

#create Z scores from the sample means and plot the distribution
sim_mns %>%  mutate(z_score = (means - 1/lambda) / (1/lambda / sqrt(n))) %>%
  ggplot(aes(x = z_score)) +
   geom_histogram(alpha=0.1,  binwidth = 0.3, fill="green", color="black",
                  aes(y = ..density..)) +
   stat_function(fun = dnorm, size = 1.3) +
   geom_vline(xintercept = z_mean, color="red", size = 0.5) +
   ggtitle("Distribution of standardized \nsimulated means") +
   xlab("z-scores")
```