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INCORPORATING REFERENCE PRICE EFFECTS INTO A THEORY OF CONSUMER CHOICE

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Although there has been a good deal of research on incorporating the effects of reference price formation into empirical models of consumer buying behavior, little formal theoretical work had been undertaken to date. This paper incorporates reference price effects into the traditional economic theory of consumer choice, and examines the effects of reference price formation on the results of the traditional theory, its marketing implications, and the implications for empirical models which examine the effects of reference price formation on actual consumer behavior. Several implications of the theoretical model are empirically tested using weekly retail egg sales data from Southern California. This analysis indicates that reference price formation does have significant effects on consumer behavior. Furthermore, these effects are asymmetric with consumers two and a half times more responsive to egg price increases that are in excess of the reference price than they are to comparable egg price decreases.

(Buyer Behavior; Choice Models; Pricing)

1. Introduction

Recently, there has been a good deal of work on incorporating different psychological theories of price perception in empirical models of consumer purchase behavior (Rinne 1981; Winer 1986; Kalyanaram and Little 1988; Raman and Bass 1988; Lattin and Bucklin 1989; and Kalwani, Yim, Rinne, and Sugita 1990). Chief among these theories is that consumers use past price levels and other context variables for a product in order to form an internal "reference price" that is used to judge the current actual product price level.¹ Either explicitly or implicitly, these empirical models assume that deviations between the actual price and the reference price of a product convey utility and thus influence consumer purchasing behavior.

Although there has been a good deal of empirical work, little theoretical work has been done on the effects of reference price formation on consumer behavior. This paper incorporates reference price formation into the traditional economic theory of consumer choice. The primary purpose of this undertaking is threefold: first, to examine the implications of reference price formation on the results of traditional economic theory; second to gain insights into the marketing implications of reference price induced behavior

¹ The notion that consumers form reference prices in order to judge actual product price levels is an application of the psychological theory of adaptation-level formation (Helson 1964). The first studies to apply adaptation-level theory to product price levels were conducted by Emory (1970) and Della Bitta and Monroe (1974). These and other early reference price studies are discussed in Rao's (1984) review of pricing research in marketing.

on product pricing; and third, to provide guidance in formulating empirical models which incorporate the effects of reference prices. This paper also provides a contribution to the marketing literature by suggesting ways in which the findings and concepts of behavioral science research can be used to enrich economic-theoretic models of consumer choice, thereby making these models a more effective tool for developing marketing theory.

Like the previous empirical studies, the theoretical model developed in the next section is based on the assumption that consumers compare the actual price with their reference price for a product. This comparison leads consumers to perceive a gain if the actual price is less than the reference price or a loss if the actual price is in excess of the reference price. The coding of a possible purchase as involving gains or losses (which in turn causes the act of purchasing the good to convey utility in the case of gains or disutility in the case of losses) is an example of a framing effect (Kahneman and Tversky 1979, Tversky and Kahneman 1981, and first suggested in a pricing context by Thaler 1985). Moreover, research on the utility effects of gains and losses (see Galanter and Pliner 1974, Tversky 1977, and Fishburn and Kochenberger 1979) indicates that for a gain and loss of the same size, losses loom larger than gains, a phenomenon known as loss aversion. Consequently, consumers may react very differently to a given product price when it results in a gain compared to when it results in a loss.

In the model, it is assumed that consumers are fully informed about the quality of products under consideration, and do not judge quality by price. However, the literature on the effects of judging quality by price (for example, Kalman 1968 and Rao and Gautschi 1982) indicates that if consumers do engage in this practice, the effects of reference price formation will be mitigated. The reason for this is that a high product price for a good will probably convey both losses (which will reduce purchases) and the perception of high quality (which may increase purchases). Conversely, a low price is likely to convey both gains (that should increase purchases) and the perception of low quality (which may decrease purchases). When both the effects of reference price formation and judging quality by price are operative, which effect dominates becomes an empirical question, and is likely to vary across product classes.

The level of a gain (loss) associated with a product can be viewed as an attribute of the product. However, in contrast to the attributes of standard characteristics models (Lancaster 1971 and Ratchford 1975), gains and losses are not fixed, instead, they are contingent on current actual and reference price levels. It is this feature of the model which drives many of the theoretical results and separates it from previous work.

In the next section, a theoretical model in which the consumer's utility function is defined to take account of both consumption levels and perceived gains and losses is presented. The theoretical model differs from previous work in this area which specified utility (value) functions that included either consumption levels, or gains and losses, but not both. The section also discusses how reference-price-induced framing effects alter the findings of the traditional economic theory of consumer choice, provides some insight on the marketing implications of reference price formation for pricing policy, and extends the general theoretical model to an examination of brand choice behavior.

In §3 the relationship between the findings of the theoretical analysis and recent empirical studies of reference price effects is explored. §4 presents an application of the theoretical model to product demand analysis, using retail egg demand in Southern California as a specific example. The final section contains a discussion of possible areas of future research.

2. Reference Price Effects and the Economic Theory of the Consumer

Three assumptions are made in the theoretical analysis. First, in each period a consumer is assumed to maximize utility subject to a predetermined level of total expenditure for

the period. This assumption is formally known as temporal separability, and implies that a consumer's actions in one time period do not directly influence behavior in other periods. This assumption is probably a reasonable abstraction for analyzing consumer behavior for frequently purchased goods, while weakening this assumption would greatly complicate the analysis without providing much additional insight. However, this assumption is probably overly restrictive for modeling consumer purchases of durable goods.

The second assumption is that consumers are well informed about the quality of the products they consider, and do not judge a product's quality level by its price. Therefore, price only has an effect on behavior through a consumer's budget constraint and by conveying utility through possible losses or gains. Again, this is probably a reasonable assumption for frequently purchased products that are familiar to most consumers, but may not be reasonable for infrequently purchased durable goods such as electronic equipment or automobiles.

Finally, a given reference price is assumed to be an adaptation level for the product's price that is based primarily on past price levels for the product.² Reference prices are formed before the consumer chooses what to purchase, and are viewed as being exogenous at the time of choice. Furthermore, they do not influence the consumer's budget constraint, rather they influence behavior by directly entering the utility function.

The Consumer's Utility Function

The difference between the actual price of a good and the reference price constitutes a marginal (or per unit) gain or loss experienced by the consumer in the purchase of the good. Specifically, a marginal loss for good i can be described as

$$l_i = I_i(P_i - RP_i), \quad (1a)$$

and a marginal gain as

$$g_i = (1 - I_i)(RP_i - P_i). \quad (1b)$$

where RP_i is the reference price for good i , P_i is the actual price, and

$$I_i = \begin{cases} 1, & P_i > RP_i, \\ 0, & \text{otherwise,} \end{cases}$$

indicates if the consumer perceives the price of the good as being in the domain of losses or not. The marginal gain and loss are defined so that the consumer never experiences both a loss and a gain for the same good at a given point in time.

It is possible that the consumer evaluates and experiences the utility effects of gains and losses on a per unit basis. As a result, the consumer may monotonically rescale marginal gains and losses from their original linear scale to a nonlinear one. The rescaled values of marginal gains and losses can be viewed as "effective" per unit gains and losses, and can be represented as

$$E_i(l_i, g_i) = \begin{cases} E_{li}(l_i), & P_i > RP_i, \\ 0, & P_i = RP_i, \\ E_{gi}(g_i), & P_i < RP_i, \end{cases} \quad (2)$$

where it is assumed that $E_{li}(l_i) > 0$, $E_{gi}(g_i) > 0$, $E_{li}(\cdot)$ approaches zero as l_i approaches zero, and $E_{gi}(\cdot)$ approaches zero as g_i approaches zero.

² Other factors such as suggested retail prices are also likely to play a role in reference price formation.

The effective total loss or gain the consumer experiences with respect to good i depends on the amount of the good that is purchased, given the effective per unit loss or gain. Thus, the total effective loss or gain for good i is

$$L_i = E_{li}x_i \quad (3a)$$

$$G_i = E_{gi}x_i, \quad (3b)$$

where L_i is the total loss, G_i is the total gain, and x_i is the amount of good i purchased. Consequently, by choosing the level of consumption for good i , the consumer simultaneously determines both the level of utility associated with actual consumption, and the level of utility (disutility) associated with purchasing the good.

For this analysis, it is assumed that in each period a consumer attempts to maximize the utility function

$$\max_x U(x, L, G) \quad (4)$$

subject to the budget constraint

$$\sum_{i=1}^n P_i x_i = M, \quad (5)$$

where x is an n -vector of consumption levels, L is an n -vector of perceived losses with individual elements given by (3a), G is an n -vector of perceived gains with individual elements given by (3b), and M is the consumer's predetermined expenditure level for the period.³ It is assumed that the derivatives of the utility function are $U_x > 0$; $U_{xx} < 0$; $U_G > 0$; $U_{GG} < 0$; $U_L < 0$; and $U_{LL} > 0$. These assumptions (which are in line with the findings of Galanter and Pliner 1974, Tversky 1977, and Fishburn and Kochenberger 1979) result in a utility function that is concave for consumption and gains, and convex for losses.

The first-order conditions for the interior maximum of (4) subject to (5) are

$$\frac{\partial U}{\partial x_i} + E_{li} \frac{\partial U}{\partial L_i} + E_{gi} \frac{\partial U}{\partial G_i} - \lambda P_i = 0, \quad (6a)$$

$$M - \sum_{i=1}^n P_i x_i = 0. \quad (6b)$$

Two important features of the model are evident in (6a). First, the definition of gains and losses implies that the marginal utility for a good (i.e., the first derivative of the utility function with respect to the quantity of that good) depends on both the consumption level for the good and the level of either the gain or loss associated with the purchase of the good. The second feature is that if either all prices equate to their respective reference prices or if

$$\frac{\partial U}{\partial L_i} = \frac{\partial U}{\partial G_i} = 0$$

for all goods (i.e., if gains and losses have no effect on utility), then the model reduces to the traditional economic model of consumer behavior.

³ In the model, utility from consumption is defined on the products themselves, rather than the consumption characteristics (i.e., characteristics excluding gains and losses) of the products, but this turns out to be an insignificant point. Instead, the utility function could be written as $V(z, L, G)$ where z is a vector of consumption characteristics derived from purchased products, and given by the vector valued production function $z = f(x)$. However, in this case the utility function can be rewritten as $V(f(x), L, G) = U(x, L, G)$, assuming that $f(\cdot)$ is fixed.

Assuming that an interior solution exists, the implicit function theorem indicates that the Marshallian demand functions which constitute the solution to (6) are of the form

$$x_i = x_i(P, I(P - RP), (1 - I)(RP - P), M). \quad (7)$$

Consequently, specified Marshallian demand functions should include not only actual prices and income (as suggested by the traditional analysis), but also marginal gain and loss terms.⁴ The implications of equation (7) for previous empirical work which incorporated the effects of reference price formation on buying behavior will be considered in the next section.

An Indifference Curve Analysis

One important property of the utility function defined in (3), and the assumptions made on the derivatives of the utility function, is that the marginal rate of transformation (the ratio of marginal utilities) between two goods at a given point depends on the level of perceived gains and losses for each good. More formally, the slope of an indifference curve through a single point (x_i^*, x_j^*) in good i and j space depends on whether or not $P_i < RP_i$, $P_i = RP_i$, or $P_i > RP_i$. In particular,

$$\frac{\left[\frac{\partial U}{\partial x_i} + E_{li} \frac{\partial U}{\partial L_i} \right]_{x_i=x_i^*}}{v_j|_{x_j=x_j^*}} < \frac{\left[\frac{\partial U}{\partial x_i} \right]_{x_i=x_i^*}}{v_j|_{x_j=x_j^*}} < \frac{\left[\frac{\partial U}{\partial x_i} + E_{gi} \frac{\partial U}{\partial G_i} \right]_{x_i=x_i^*}}{v_j|_{x_j=x_j^*}}, \quad \text{where} \quad (8)$$

$$v_j = \frac{\partial U}{\partial x_j} + E_{lj} \frac{\partial U}{\partial L_j} + E_{gj} \frac{\partial U}{\partial G_j}.$$

The above inequality indicates that an indifference curve through a point in two-good space is steeper (assuming that good i is on the vertical axis) when good i is in the domain of losses, than when $P_i = RP_i$, and the indifference curve through the point is less steep when good i is in the domain of gains. Consequently, the indifference surface is not fixed in space as in the traditional economic theory of the consumer, but is instead contingent upon the relationship between the actual prices of products and their reference prices.

Below the relationship between perceived gains and losses and the indifference surface is used to examine gain and loss effects on consumer behavior using an indifference curve analysis. This analysis provides some further insights on the effects of perceived gains and losses on traditional economic theory as well as their marketing implications.

Figure 1 provides a simple illustration of the inequality contained in equation (8). In the figure the indifference curve marked G is the indifference curve which indicates that $P_i < RP_i$, U is the indifference curve which indicates that $P_i = RP_i$, and L is the indifference curve which indicates that $P_i > RP_i$. The locations of G and L are not unique, but depend on the size of any marginal gains and losses. The straight diagonal line in the figure is the consumer's budget constraint. The indifference curves are drawn so that they are oriented around the point in the space (x_i^*, x_j^*) that corresponds to the utility maximizing consumption levels when $P_i = RP_i$. The point x_i^g, x_j^g corresponds to the utility maximizing bundle when $P_i < RP_i$, which occurs at the point where the unpictured (for the sake of visual clarity) indifference curve G' is tangent to the budget constraint. The point (x_i^l, x_j^l) corresponds to the utility maximizing consumption levels when $P_i > RP_i$, and occurs where the unpictured indifference curve L' is tangent to the budget constraint. One important thing to notice is that even though the prices are the same in the three instances, $x_i^g > x_i^* > x_i^l$ and $x_j^g < x_j^* < x_j^l$. Consequently, consumption of good i depends on the relationship between its actual price and its reference price, with

⁴ Later in this section the theoretical analysis is extended to the case of discrete choice where there is a noninterior solution to the utility maximization problem.

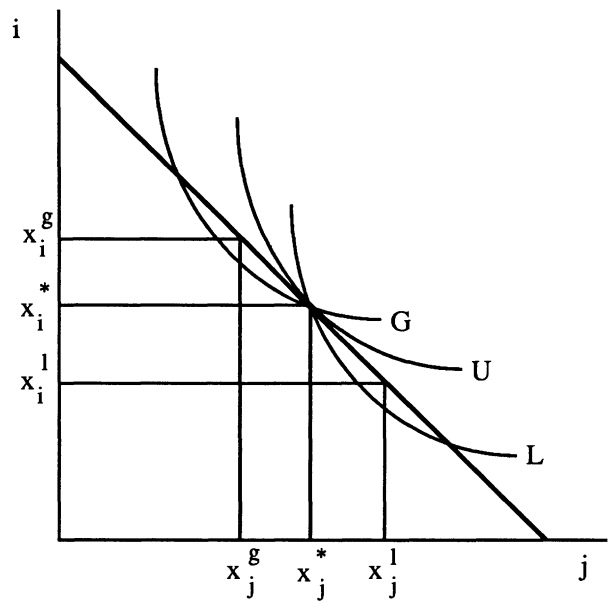


FIGURE 1. The Marginal Rate of Substitution Between Goods i and j .

consumption of the good lower when its actual price exceeds its reference price (the case of perceived losses) than when its reference price exceeds its actual price (the case of perceived gains). Since reference prices are partially based on past prices, the figure points out that it is not only the current level of prices that determines consumer purchase levels (as is the case in traditional economic theory), but also the path of past prices. Thus, even static models of the consumer depend on previous events.

Figure 2 examines the effect of an increase in the price of good i from P_i^0 to P_i^1 when the initial price equals the reference price. The effect of a price change can be decomposed into both the traditional income (the movement from point A to point B in the figure)

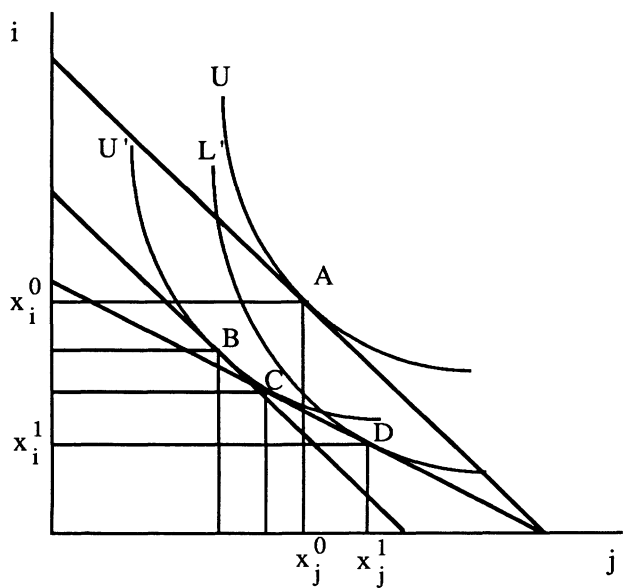


FIGURE 2. The Effect of Increasing the Price of Good i .

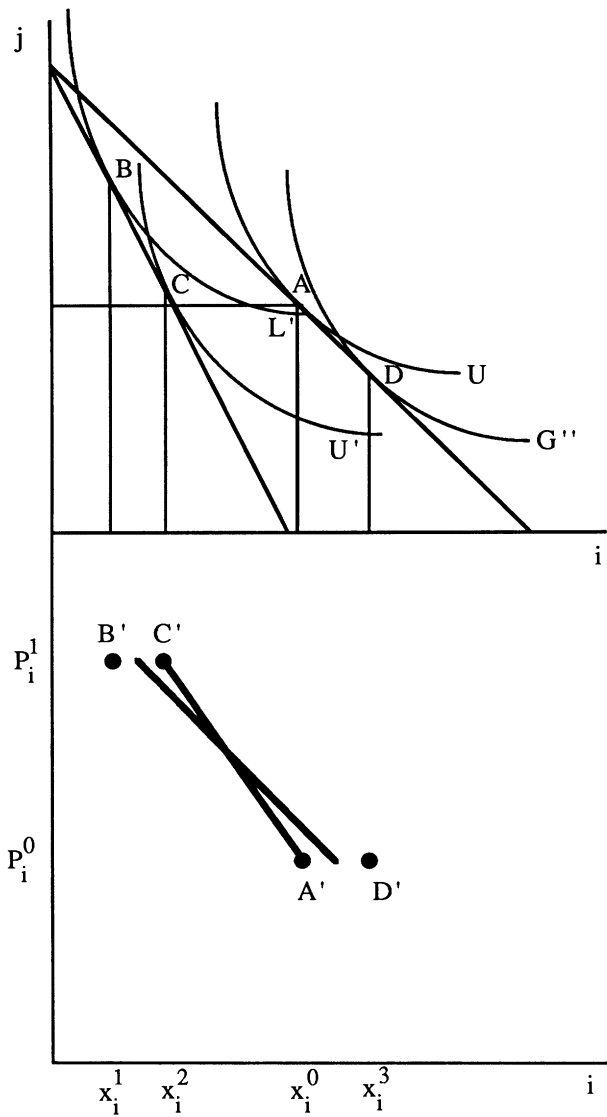


FIGURE 3. An Increase Followed by a Decrease in the Price of Good i .

and substitution (the movement from point B to point C) effects, and a gain/loss effect (the movement from point C to point D). This decomposition of the effect of a price change will be formalized in Proposition 1. Two features of the figure should be noted. First, the loss effect enhances the reduction in the consumption of good i beyond the substitution and income effects. Second, when only the income and substitution effects are considered, goods i and j are gross complements (consumption of both decreases as P_i increases), but with the loss effect included, they are gross substitutes.

Figure 3 considers the demand for good i by an individual consumer when there is an increase in the price of good i followed by a decrease in the price back to its original level. The upper portion of the figure is the consumer's indifference map and the lower is her demand schedule (designated by solid dots). The figure is based on the assumptions that initially the price of good i equals the reference price and that the reference price adjusts very quickly, so that in the period after the price change the actual price equals the reference price (i.e., perceived gains and losses occur only in the period of a price

change). Neither assumption is crucial; the basic result would still hold if there were any adjustment of the reference price in the period after the price change. The movement from point A to point C in the upper portion of the figure (with corresponding points A' and C' in the lower portion) is called the “true” price effect (which is due to changes in the consumer’s budget constraint) and gives the effect the price change would have if perceived losses and gains had no effect on utility. The primary thing to notice in Figure 3 is that an estimated demand function (denoted by the less steep of the two diagonal lines in the lower part of the figure) that does not account for gain and loss effects will be misspecified. The estimated own-price parameter is a combination of both the “true” price effect (the second and steeper slanted line in the lower diagram) and gain/loss effects. Furthermore, the extent of this misspecification increases as the gain/loss effects increase.

Probably the most important implication of reference price induced behavior from a pricing policy perspective is illustrated in Figure 4 (which is the same as the bottom portion of Figure 3 with different markings). In the figure, the line $C'D'$ is the demand curve for good i when the reference price for the product is at the level P_i^0 , and the line $A'B'$ is the demand curve when the reference price is at the level P_i^1 . Since $P_i^0 > P_i^1$, the figure illustrates the point that increases in the reference price for a product causes its demand curve to shift outwards. This finding provides a theoretical basis for Thaler’s (1985) conclusion that marketers benefit from techniques, such as high suggested retail prices, that raise the reference price level for a product. Moreover, it is consistent with speculation in the general merchandise retailing industry (*Washington Post*, December 3, 1989) that the recent increase in the use of promotional pricing by some retailers (used to maintain cash-flow in order to service debt) has eroded consumer demand by lowering the prices they anticipate paying for products (i.e., their reference prices).

A Generalized Slutsky Equation

The analysis of Figure 2 indicated that a price change for good i can be decomposed into substitution, income, and gain/loss effects. Below, this notion is formalized in Proposition 1.

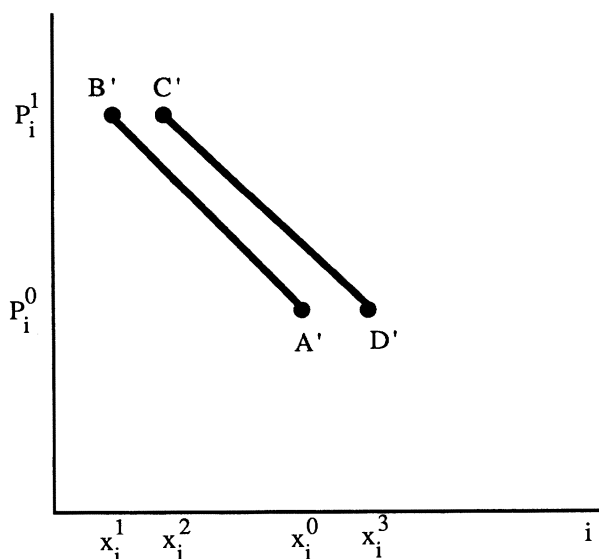


FIGURE 4. Demand Schedules for Good i at Reference Prices p_i^0 and P_i^1 .

PROPOSITION 1 (A Generalized Slutsky Equation). *The effect of an own-price change can be decomposed into a substitution, income, and gain/loss effect of the form*

$$\frac{\partial}{\partial P_i}(x_i) = \frac{\partial}{\partial P_i}(h_i) - x_i \frac{\partial x_i}{\partial M} + \left[(1 - I_i) \frac{\partial M}{\partial g_i} - I_i \frac{\partial M}{\partial l_i} \right] \frac{\partial x_i}{\partial M}.$$

In the proposition,

$$\begin{aligned} \frac{\partial}{\partial P_i}(x_i) &= \frac{\partial x_i}{\partial P_i} + I_i \frac{\partial x_i}{\partial l_i} - (1 - I_i) \frac{\partial x_i}{\partial g_i} \quad \text{and} \\ \frac{\partial}{\partial P_i}(h_i) &= \frac{\partial h_i}{\partial P_i} + I_i \frac{\partial h_i}{\partial l_i} - (1 - I_i) \frac{\partial h_i}{\partial g_i} \end{aligned}$$

are respectively the partial derivatives of the Marshallian and Hicksian demand functions for good i with respect to its own price. The proof of this and the remaining two propositions are contained in the appendix.

One important thing to note is that the first term of the right-hand side of the generalized Slutsky equation is not the same as its counterpart in the traditional Slutsky equation. Instead, it is a combination of the traditional substitution effect and gain/loss effects. The second term is a pure income effect and the third is a gain/loss effect.

Kinked Demand Curves

One implication of loss aversion is that demand curves at the individual level will be kinked at the point where the actual price is equal to the reference price. Specifically, price response will be more elastic for price increases than for price decreases. A kink in the demand function exists if the derivative of the function does not exist at the point where the actual price equals the reference price. Proposition 2 formalizes the idea that reference price effects may cause demand functions to be kinked.

PROPOSITION 2. *The own price derivative of the demand function of good i exists at the point $P_i = RP_i$ if and only if*

$$\sum_{j=1}^n D_{ji} w_{ji} |_{P_i > RP_i} = \sum_{j=1}^n D_{ji} w_{ji} |_{P_i < RP_i^*}$$

In the proposition, D_{ji} is the j, i th cofactor of the bordered Slutsky substitution matrix,

$$\begin{aligned} w_{ji} |_{P_i > RP_i} &= \frac{\partial E_{li}}{\partial l_i} \left(\frac{\partial^2 U}{\partial x_j \partial L_i} + E_{lj} \frac{\partial^2 U}{\partial L_j \partial L_i} + E_{gj} \frac{\partial^2 U}{\partial G_j \partial L_i} \right), \quad \text{and} \\ w_{ji} |_{P_i < RP_i} &= - \frac{\partial E_{gi}}{\partial g_i} \left(\frac{\partial^2 U}{\partial x_j \partial G_i} + E_{lj} \frac{\partial^2 U}{\partial L_j \partial G_i} + E_{gj} \frac{\partial^2 U}{\partial G_j \partial G_i} \right). \end{aligned}$$

Although the condition in Proposition 2 may hold for some products in other random circumstances, the most likely situation that will cause the condition to hold is when

$$w_{ji} |_{P_i > RP_i} = w_{ji} |_{P_i < RP_i} \quad (9)$$

for all i and j . Equation (9) essentially states that the derivative of the demand function will exist at the point $P_i = RP_i$ if the gain effect equals the negative of the loss effect. However, loss aversion is likely to cause the left-hand side of (9) to exceed the right-hand side, resulting in a demand curve for an individual consumer that is more elastic for price increases compared to decreases. This finding has strong managerial implications. In particular, it suggests that optimal price adjustments, brought on by such things as rising production costs, will differ when a firm raises the price of its products compared to when it lowers its price.

Reference Price Effects and Money Illusion

One important result of the traditional economic theory of the consumer is that Marshallian demand functions are homogeneous of degree zero in prices and income. This result is also known as the “no money illusion” axiom of consumer theory because it implies that a proportional change in all prices and income will have no effect on the quantities of goods a consumer demands. Proposition 3 shows that this standard result may not hold if reference price effects influence consumer behavior.

PROPOSITION 3. *The Marshallian demand functions based on the utility function (4) are not in general homogeneous of degree zero in prices and income. The demand function for good i will be homogeneous of degree zero if and only if*

$$\sum_{j,h=1}^n x_j P_j \frac{D_{hi}}{D} w_{hj} = 0.$$

In the proposition, D is the determinant and D_{hi} is h , i th cofactor of the bordered Slutsky substitution matrix, and

$$w_{hj} = I_j \frac{\partial E_{lj}}{\partial l_j} \left[\frac{\partial^2 U}{\partial x_h \partial L_j} + E_{lh} \frac{\partial^2 U}{\partial L_h \partial L_j} + E_{gh} \frac{\partial^2 U}{\partial G_h \partial L_j} \right] \\ - (1 - I_j) \frac{\partial E_{gj}}{\partial g_j} \left[\frac{\partial^2 U}{\partial x_h \partial G_j} + E_{lh} \frac{\partial^2 U}{\partial L_h \partial G_j} + E_{gh} \frac{\partial^2 U}{\partial G_h \partial G_j} \right].$$

There are two important implications which follow from Proposition 3. The first is that testing to see if homogeneity holds for a system of demand equations does not necessarily constitute a test of the utility maximization hypothesis. Proposition 3 indicates that the demand functions of a utility-maximizing consumer who receives utility and disutility from reference price induced perceived gains and losses may not be homogeneous. Consequently, if reference price effects do occur as hypothesized in the theoretical model, they provide a partial explanation as to why most studies (for a review see Chapter 3 of Deaton and Muellbauer 1980) that have tested for the existence of homogeneity have rejected the condition. The second implication of Proposition 3 is for empirical demand analysis. Specifically, the common practice of deflating prices by a price index, which is justified using homogeneity, is likely to be invalid in the presence of reference price effects. A related issue is that the increasingly common practice of imposing the homogeneity condition on demand systems, such as the Rotterdam system (Clements and Selvanathan 1988), probably does not increase the efficiency of the resulting estimates (which is the reason why it is done). Instead, its most likely effect is to increase the level of misspecification bias.

An Extension to Brand Choice Behavior

In this subsection the theoretical framework is extended to examine the behavior of a consumer that chooses a single brand from a specific product category. This extension is consistent with McFadden's (1974) random utility model (a model extensively used in the marketing literature), and allows for econometric examination of brand choice behavior. One assumption typically used in traditional random utility models is that the marginal utility from consuming a given brand is constant. As a result, the marginal rate of substitution between two brands is also constant, and indifference curves between the two brands are straight parallel lines. However, equation (8) indicates that gains and losses with respect to a reference price for a brand alters (through changes in the marginal utility for that brand) the marginal rate of substitution between the brand and other brands. Consequently, the model developed here allows the marginal utility from a brand to vary due to reference price effects.

Following Allenby and Rossi (1990), it is assumed that brands within a product category form a group that is weakly separable from the consumption of other products. Implicitly, this assumes that consumers use a two-stage budgeting process in which they first determine how much of their income to spend in total on the product category of interest, as well as on all other products outside of the category. Given this predetermined level of expenditure (which is designated as m) for the product category, a consumer then chooses which brand of the product and how much of it to purchase in order to maximize (subject to m) a subutility function defined on the brands in the category.⁵

As is the case in many discrete choice models, it is assumed that the subutility function for brands in the category takes the linear form

$$u(x) = \sum_{i=1}^b \phi_i x_i, \quad (10)$$

where x is a b -vector of the quantity demanded for each of the b brands in the category, x_i is the quantity of brand i demanded, and ϕ_i is the marginal utility from purchasing brand i . The ϕ_i can be viewed as a utility-based weighted average of each brand's underlying attributes (Allenby and Rossi 1990 and Hanemann 1984). However, as was pointed out in the introduction, any gains or losses associated with purchasing a particular brand can be viewed as an attribute of the brand that depends on the relationship between the actual and reference price for the brand at the time of purchase. Thus, the marginal utility for brand i is a function of any marginal gains or losses associated with purchasing the brand of the form $\phi_i(l_i, g_i)$, where $\partial\phi_i/\partial l_i < 0$ and $\partial\phi_i/\partial g_i > 0$, and l_i and g_i are defined in (1).⁶ Consistent with the theoretical analysis just presented, $\phi_i(l_i, g_i)$ can be written as

$$\phi_i(l_i, g_i) = \alpha_i + \beta_i E_{li}(l_i) + \gamma_i E_{gi}(g_i), \quad (11)$$

where α_i is a utility-based weighted average of brand i 's consumption attributes, $\beta_i E_{li}(l_i)$ captures the utility effects of any losses associated with the purchase of brand i , and $\gamma_i E_{gi}(g_i)$ are the utility effects of any gains.

Based on this discussion, the consumer is assumed to solve the choice problem

$$\max_{x_i} \sum_{i=1}^b \phi_i(l_i, g_i) x_i, \quad (12)$$

subject to

$$\sum_{i=1}^b P_i x_i = m. \quad (13)$$

The solution to (12) subject to (13) consists of selecting the brand, j , which maximizes the ratio $\phi_j(l_j, g_j)/P_j$. This, except on a set of measure zero, results in the selection of a single brand. Since the level of l_j or g_j determines the value of this ratio, reference price formation influences which brand the consumer chooses.

In order to make this choice model empirically operational, the random utility model of McFadden (1974) can be employed. Under this model, it is assumed that the utility function is not fully observable to the researcher. Thus, although it is assumed that consumers solve deterministic maximization problems, a researcher only observes a distribution of utility levels over consumers. A convenient way to model the stochastic nature of utility is to multiply the marginal utility that a consumer receives for each

⁵ In this presentation it is assumed that only a single brand in the category is purchased. This assumption is consistent with both empirical observation and underlies most of the existing brand choice literature.

⁶ If the utility function is concave in gains and convex in losses, then $\partial^2\phi_i/\partial l_i^2 > 0$ and $\partial^2\phi_i/\partial g_i^2 < 0$.

brand by a random error component. This allows the utility function for an individual consumer to be written as

$$u(x) = \sum_{i=1}^b \phi_i(l_i, g_i) \nu_i x_i. \quad (14)$$

where ν_i is the random component of the marginal utility from brand i and $\phi_i(l_i, g_i)$ is the deterministic component.

A consumer will choose brand j if

$$\frac{\phi_j(l_j, g_j) \nu_j}{P_j} \geq \frac{\phi_i(l_i, g_i) \nu_i}{P_i}, \quad (15)$$

for all i . Taking logarithms and rearranging allows (15) to be written as

$$(\log \phi_j(l_j, g_j) - \log P_j) + \nu'_j \geq (\log \phi_i(l_i, g_i) - \log P_i) + \nu'_i, \quad (16)$$

where ν'_i is the logged value of ν_i . Choosing a distribution for the ν'_i allows the model to be empirically estimated. In particular, if the ν'_i are assumed to follow a Weibull distribution, then the well-known multinomial logit estimation procedure can be used (McFadden 1974). Selection of a functional form for $\phi_i(l_i, g_i)$ completes the specification. One (but not the only) specification that approximates the findings of the literature on the utility effects of gains and losses (Galanter and Pliner 1974, Tversky 1977, and Fishburn and Kochenberger 1979) is

$$\phi_i(l_i, g_i) = \alpha_i + \beta_{1i} l_i^{\beta_{2i}} + \gamma_{1i} g_i^{\gamma_{2i}}, \quad (17)$$

where β_{1i} is negative, γ_{1i} is positive, and β_{2i} and γ_{2i} take on values between zero and one.⁷

Summary

The theoretical analysis just presented provides insight into five effects of reference price formation on consumer behavior. First, it indicates that empirical demand functions and brand choice models need to include not only prices, but also separate perceived marginal gain and loss terms. Second, the effect of a price change on the consumption of a good (or probability of choice) can be decomposed into gain and loss effects. Third, demand curves may be kinked at the point where the price and reference price for a good equate. Furthermore, the asymmetry between gains and losses makes the demand for a product more elastic for own-price increases than for own-price decreases. In the case of the discrete choice model, gain/loss asymmetry implies that the own-price marginal probability of selecting a brand is greater in absolute value (i.e., more negative) for own-price increases than for own-price decreases. These results suggest a need for firms to differentiate up-side versus down-side pricing policy. Fourth, the demand curve for a given product shifts outward (the probability of purchase increases in a discrete choice model) as the reference price for the good increases. Conversely, the demand for a product will be eroded (i.e., the demand curve will shift inward) as the reference price for that product decreases. Finally, the theoretical analysis indicates that if reference price effects are operative, demand functions may not be homogeneous of degree zero in prices and income, calling into question the use of price deflators and the imposition of homogeneity restrictions in empirical demand analysis.

⁷ As β_{2i} and γ_{2i} approach one, utility is linear in gains and losses. Conversely, as these two parameters approach zero, gains and losses have a discrete effect on brand choice.

3. Empirical Studies of Reference Price Formation

Recent empirical studies on the effects of reference price formation on buying behavior have used one of two different approaches. The first is the use of household-level logit brand choice models for brands within a product category, and has been used by Rinne (1981), Winer (1986), Kalyanaram and Little (1988), Lattin and Bucklin (1989), and Kalwani, Yim, Rinne, and Sugita (1990). These models assume that the utility received by a consumer from purchasing one brand is partially due to the reference price for that product as well as other factors.⁸ The second approach has been to examine the effect of reference price formation on the market share of a brand within a product category, and has been used by Rinne (1981) and Raman and Bass (1988). The estimated models use traditional market share formulations with variables that incorporate reference price concepts appended to the estimated equations or as a variable that determines which of two different regimes a given observation belongs to in a switching regression framework.

All but one of these studies is strongly supportive of the hypothesis that reference prices influence consumer purchasing behavior. Analyses of the potential asymmetry between gains and losses have been conducted by Rinne, Kalyanaram and Little, Raman and Bass, and Kalwani et al. In both the studies of Rinne and Kalyanaram and Little, it appears that consumers do react more strongly to perceived losses than to perceived gains (as suggested by loss aversion), but the asymmetry does not tend to be statistically significant. However, Raman and Bass and Kalwani et al. find that the asymmetry is both in the right direction (i.e., stronger for perceived losses) and is significant.

The one study that is apparently not supportive of reference price effects is that of Lattin and Bucklin (1989). The contradictory findings are particularly surprising given the generally confirmatory results of Kalwani et al. Kalyanaram and Little, and Winer since all four studies use data for caffeinated ground coffee.⁹ However, it can be argued that Lattin and Bucklin's results are actually consistent with the notion that reference price effects are nonlinear, being strongly concave in gains and convex in losses.¹⁰

One possible reason why past studies have had difficulty in statistically detecting asymmetric reference price effects is the choice of variables used to capture those effects. The theoretical analysis provides insight into which variables should be included in empirical analyses of reference price effects. In particular, estimated models should include actual product price levels and marginal gain and loss terms. The only existing empirical study that includes this exact set of variables is that of Kalwani et al., and they do in fact find that reference price effects are asymmetric and consistent with the theoretical analysis.

4. An Application to Retail Egg Demand in Southern California

In this section, several implications of the theoretical model presented in §2 are empirically tested. The analysis uses data on weekly per capita retail demand for eggs in Southern California for the period July 11, 1981 to July 9, 1983. A complete description and a listing of the data is contained in Putler (1988). The modeling approach used is the estimation of a per capita quantity demand function, which is frequently used in applied demand work in economics (for example, Houthakker and Taylor 1970). This approach differs from the brand choice and market share methodologies used in previous reference price studies, which were considered inappropriate for a homogeneous product such as eggs where there is little brand competition.

⁸ None of the empirical specifications used in these past brand choice studies conforms with the theoretical model of brand choice behavior presented in the last section.

⁹ Lattin and Bucklin and Kalyanaram and Little both use the IRI academic data set for the category.

¹⁰ Which is consistent with the work of Galanter and Pliner (1974), Tversky (1977), and Fishburn and Kochenberger (1979) on the utility of gains and losses, and implies that the values of β_{2i} and γ_{2i} in equation (17) are near zero.

Aggregate data are being used to test a model of individual behavior in this instance and, thus, present a possible problem for the analysis. However, despite this problem, there are several characteristics of the data which minimize the problem, and make it reasonable for a test of reference price effects.¹¹ First, the weekly observation period for these data provides a better opportunity for analyzing the effects of price changes compared to the monthly, quarterly, or yearly data typically used for this type of analysis in economics. Data of longer observation intervals are likely to suffer from temporal aggregation problems which mask the nature of short-run price changes, and include within interval reference price adjustments. Second, the average retail price of a dozen large eggs is very volatile, experiencing an average week-to-week price change of over 4.5 percent, with weekly changes as large as nearly 32 percent. Finally, Gabor and Granger (1961) found that 93.4 percent of consumers could recall a price last paid for eggs. Uhl and Brown (1971) found that consumers identified 75.5 percent of all egg price changes, fourth highest among the twelve products studied, even though egg prices were among the most volatile. The combination of high levels of price volatility and price awareness makes eggs an ideal choice for a test of the effects of reference prices.

Reconciling the Data with the Theoretical Model

Two issues need to be considered before the theoretical model developed in §2 can be applied to the Southern California egg sales data. The first, which has already been mentioned, is aggregation. In this instance, there is a need to aggregate over different product reference prices, all of which are assumed to follow some distribution across consumers. In particular, it is assumed that a consumer's reference price for product i is given by

$$RP_i^h = \overline{RP}_i + \epsilon_i, \quad (18)$$

where RP_i is the population mean of the reference price distribution, and ϵ_i is a normally distributed random variable with mean zero and variance σ^2 . These assumptions result in a distribution of reference prices over all consumers that is normally distributed with mean RP_i and variance σ^2 .

The approach taken in this analysis is exact linear aggregation, and was chosen in order to allow for asymmetric gain and loss effects.¹² A demand specification that is consistent with both exact linear aggregation and the representative consumer hypothesis can be based on individual utility functions of the form

$$\prod_{i=1}^n [x_i^h - \alpha_i^h - \alpha_{Li}I_i(P_i - RP_i^h) - \alpha_{Gi}(1 - I_i)(RP_i^h - P_i)]^{\beta_i}, \quad (19)$$

which is a modified version of the well-known Klein-Rubin utility function.¹³ A per capita aggregate demand function derived from this specification of individual preferences can be written as

$$\bar{x}_i = \bar{\delta}_i + \delta_{Li}\bar{L}_i + \delta_{Gi}\bar{G}_i + \beta_i \frac{\bar{M}}{P_i} + \sum_{\substack{j=1 \\ j \neq i}}^n \frac{P_j}{P_i} (\bar{\kappa}_j + \kappa_{Lj}\bar{L}_j + \kappa_{Gj}\bar{G}_j), \quad (20)$$

where $\delta_i = (1 - \beta_i)\alpha_i$; $\delta_{Li} = (1 - \beta_i)\alpha_{Li}$; $\delta_{Gi} = (1 - \beta_i)\alpha_{Gi}$; $\kappa_j = \beta_i\alpha_j$; $\kappa_{Lj} = -\beta_i\alpha_{Lj}$; $\kappa_{Gj} = -\beta_i\alpha_{Gj}$; \bar{L}_i is the average level of losses experienced by consumers for good i , and is given by

¹¹ In the next subsection the problem of aggregation is considered.

¹² Other aggregation approaches do not lend themselves to asymmetric reference price effects. Chapter 6 of Deaton and Muellbauer (1980) contains a discussion of the issues surrounding aggregation.

¹³ Under the representative consumer hypothesis, an aggregate demand function is specified that is consistent with the behavior of an idealized "representative" consumer that has a specific representation of preferences.

$$\bar{L}_i = (P_i - \bar{RP}_i) - \int_{-\infty}^{P_i - \bar{RP}_i} \epsilon_i \phi(\epsilon_i) d\epsilon_i;$$

and G_i is the average level of gains experienced by consumers for the same good, and is given by

$$\bar{G}_i = (\bar{RP}_i - P_i) + \int_{P_i - \bar{RP}_i}^{\infty} \epsilon_i \phi(\epsilon_i) d\epsilon_i.$$

The integral terms in these expressions are first moments on either side of a truncated normal distribution, which can be expressed as functions of normal cumulative and probability density function values (Maddala 1983).

Although the per capita demand function contained in (20) is consistent with the representative consumer hypothesis, the function itself places a large number of restrictions on consumer preferences and is inflexible (Deaton and Muellbauer 1980).¹⁴ This inflexibility leads to the possibility that reference price effects could be confounded with misspecification of the price response parameters. In an attempt to determine if this is a serious problem, a second demand specification is also applied to the data which is based on a flexible functional form that is consistent with exact linear aggregation, but is not consistent with the representative consumer hypothesis. This equation consists of a transcendental logarithmic (Christensen, Jorgenson, and Lau 1975), or translog, price response function with appended intercept and log income terms. The specification of this per capita aggregate demand function is completed by including linear average gain and loss terms, and can be written as

$$\bar{x}_i = \psi + \sum_{j=1}^n \rho_j \log P_j + \frac{1}{2} \sum_{j,k=1}^n \mu_{jk} \log P_j \log P_k + \sum_{j=1}^n \nu_j \bar{L}_j + \sum_{j=1}^n \omega_j \bar{G}_j + \eta \log \bar{M}. \quad (21)$$

The second issue that needs to be addressed before applying the theoretical framework to the egg sales data is making a determination of which product price variables should be included in the estimated demand function. In the theoretical model a consumer's demand for a good depends on an unspecified number of product prices. The theoretical construct of weakly separable preferences (Deaton and Muellbauer 1980) indicates that only prices for goods within a separable group, along with total group expenditures, influence the demand for a product in the group. Unfortunately, theory does not provide a means of determining which goods fall into a particular group. However, previous time series studies that examine egg demand in the United States, using both demand system (Huang and Haidacher 1983 and Huang 1985) and single equation (Chavas and Johnson 1980 and Henson 1976) approaches, consistently indicate that the prices of meat, poultry, and cereal products significantly influence egg demand. Cross-section studies (Darrah and Hanna 1968 and Henson 1976) are consistent with the time-series findings, indicating that most consumers view eggs as a breakfast food and as an inexpensive source of protein. Based on these findings, the statistical analysis assumes that eggs, beef, pork, chicken, and cereal products form a weakly separable group. In particular, price and marginal gain and loss variables for seven-bone roast (a comparatively inexpensive beef roast), bacon, whole fryers, and breakfast cereal (in the form of a price index) were included along with the egg price variables. The prices of other types of meat (e.g., hamburger, ham, etc.) were not included since the price of one product in a meat category (e.g., seven-bone roast in the beef category) was found to have a high level of correlation with other products in that category (e.g., regular hamburger).

¹⁴ More flexible representations of individual preferences are not consistent with exact linear aggregation.

Modeling the Reference Price Variables

Adaptation level theory (Helson 1964) indicates that consumers use past product price levels and other factors such as suggested retail prices in forming reference prices. Most previous studies (Della Bitta and Monroe 1974, Rinne 1981, Winer 1986, Kalyanaram and Little 1988, and Raman and Bass 1988) have modeled reference prices using only past price levels.¹⁵ In this paper, the mean of the reference price distribution for a good (\overline{RP}_i in equation (18)) is assumed to come from an extrapolative expectations model of the form

$$\overline{RP}_{it} = \sum_{j=t-1}^{t-5} \frac{\gamma^{(t-j)}}{\Gamma} P_{ij}, \quad (22)$$

where γ is an estimated parameter subject to the constraint that $0 < \gamma \leq 1$, t is a time subscript, and

$$\Gamma = \sum_{j=t-1}^{t-5} \gamma^{(t-j)}.$$

This reference price formation model is a distributed lag model of the previous five weekly prices.¹⁶ The weights geometrically decay, with the rate of the decay determined by the value of γ , and are normalized to sum to one. In order to make model estimation manageable, it is assumed that the value of γ in (22) and the variance of ϵ_i in (18) are the same for all products.

Other Variables

In addition to the price and reference price variables, several other variables, most of which account for seasonal differences in egg demand patterns, are included in the analysis. Egg sales are typically higher in the first week of each month (Siebert, Antle, and Faber 1985). To account for this, a variable which indicates whether or not a given week is the first week of the month is included. Monthly dummy variables (using December as a benchmark) are also included to account for any other seasonal effects. Egg sales are abnormally high (over 150 percent above their average level) in the week before and the week of Easter and then sharply decline (falling to only 75 percent of their average level) in the week following Easter. Given the atypical behavior associated with Easter, Easter weeks and the week on either side of an Easter week have been excluded from the analysis (a total of six weeks).

A 25-industry average weekly earnings figure is used as a proxy for group expenditures (M) since these figures are unavailable for the Southern California area. Implicitly, this assumes two things. First, that earnings are an acceptable proxy for total expenditure, which seems reasonable. Second, that preferences are homothetic with respect to the separable group, which is probably overly restrictive.

Model Estimation

The estimated demand equations are obtained by appending the seasonality terms to both the Klein-Rubin (20) and the translog (21) specifications. The final form of the Klein-Rubin demand equation is

¹⁵ Kalwani et al. (1990) include other context variables in their formulation of reference prices.

¹⁶ Altering the number of weeks that enter into the distributed lag model has little effect on the estimation results.

$$\bar{x}_e = \bar{\delta}_e + \delta_{Le}\bar{L}_e + \delta_{Ge}\bar{G}_e + \beta_e \frac{\bar{M}}{P_e} + \sum_{\substack{j \in br, \\ ba, wf, c}} \frac{P_j}{P_e} (\bar{\kappa}_j + \kappa_{Lj}\bar{L}_j + \kappa_{Gj}\bar{G}_j) + \theta FIRST + \sum_{i=1}^{11} \nu_i MO_i, \quad (23)$$

while the final form of the translog demand equation is

$$\bar{x}_e = \psi + \sum_{\substack{j \in e, br, \\ ba, wf, c}} \rho_j \log P_j + \frac{1}{2} \sum_{\substack{j, k \in e, br, \\ ba, wf, c}} \mu_{jk} \log P_j \log P_k + \sum_{\substack{j \in e, br, \\ ba, wf, c}} \nu_j \bar{L}_j + \sum_{\substack{j \in e, br, \\ ba, wf, c}} \omega_j \bar{G}_j + \eta \log \bar{M} + \xi FIRST + \sum_{i=1}^{11} \zeta_i MO_i. \quad (24)$$

In the two equations, the subscripts e , br , ba , wf , and c are respectively for eggs, seven-bone roast, bacon, whole fryers, and breakfast cereal; *FIRST* is a dummy variable which indicates whether or not it is the first week of the month; and MO_i ($i = 1, 11$) are monthly dummy variables.

Given the national nature of wholesale egg price determination, it seems unlikely that the estimated equations suffer from simultaneous equation bias, eliminating the need for a simultaneous equation estimator.¹⁷ However, a Hausman specification test (Hausman 1978) was used to test the validity of the above assumption, and indicated that the assumption was statistically valid for both demand specifications.

To conserve space, the individual parameter estimates for the two specifications are not presented. However, statistical tests on the effects of reference price formation for both specifications are reported in the next subsection.

Results

Two questions on the effect of reference price formation on purchasing behavior are considered. First, does reference price formation have any effect on consumer purchases of eggs? Second, if reference price formation does influence behavior, is consumer response to perceived gains and losses symmetric or asymmetric? The questions are addressed by testing two different hypotheses using both specifications of the estimated egg demand function.

The first question is addressed by testing the hypothesis that all marginal gain and loss terms are equal to zero (a total of ten restrictions). A likelihood ratio test is used to test this hypothesis. The test statistic for the Klein-Rubin demand function is 22.26 ($p < 0.05$), while the value of this test statistic is 32.71 for the translog specification ($p < 0.01$). Consequently, the null hypothesis that reference price formation has no effect on retail egg demand can be rejected for both specifications, providing strong support for the notion that reference price formation influences consumer demand for eggs.

The results of testing for the asymmetry of gain and loss effects are mixed. A strong definition of asymmetry can be examined by testing the hypothesis that the loss terms for each product can be set equal to the negative of the gain terms, a five-restriction test. When this test of asymmetry is applied to the translog demand specification the calculated test statistic is 17.59 ($p < 0.01$), but the same restrictions cannot be rejected for the Klein-Rubin specification (chi-squared = 4.93, $p > 0.10$). This suggests that the data do not support this strong definition of asymmetry for the Klein-Rubin specification. However, the marginal loss variable for eggs is significant at the five percent level, while the

¹⁷ The structure of the egg market is described in Putler (1988).

marginal gain variable has the correct sign, but is not statistically significant for the Klein-Rubin specification, indicating the existence of own-price asymmetry.

These contradictory findings indicate that statistical inferences about the asymmetry of reference price effects depend on the functional form used in specifying the demand equation. It appears that the use of a restrictive demand specification may result in a misspecification bias that masks the asymmetric nature of reference price effects. The use of a more flexible functional form mitigates this bias, allowing the asymmetry of reference price effects to be statistically detected.

In order to determine the magnitude of the influence of reference price effects on retail egg sales, the own-price elasticities (taken at the sample mean for all variables, with reference prices equal to actual prices for all products except eggs) for both egg price increases (losses) and decreases (gains) were calculated. The calculated own-price elasticity for a price increase is -0.78 , and the estimated elasticity for a price decrease is -0.33 . The ratio of the price increase elasticity to the price decrease elasticity is a measure of the relative dominance of losses to gains. In this case, the ratio of the two elasticities is 2.4 to 1. The ratio indicates that consumer response is nearly two and a half times greater for egg price increases than for egg price decreases.

Overall, the empirical analysis is consistent with the theory that consumers experience and act on perceived gains and losses caused by comparing an internal reference price with the actual price of a product. The analysis of whether or not consumers react differently to perceived gains compared to perceived losses is mixed, but is generally supportive of asymmetric reference price effects. Furthermore, the estimated models suggest that the response in egg sales to a given loss caused by an egg price increase is nearly two and a half times stronger than the response in egg sales to an egg price decrease that results in a comparable gain. This finding is consistent with the notion that consumers exhibit loss aversion when confronted with perceived losses and gains.

5. Directions for Future Research

This study along with the others that have been done on this topic have just begun to uncover the important issues related to the effects of reference price formation on consumer purchasing behavior.

The theoretical model presented in this paper is based on the assumption that deviations between the actual price and the reference price convey utility in and of themselves and that loss aversion causes the observed asymmetric behavior. The question arises as to whether or not this is the only mechanism that can or does cause the observed behavior. In particular, could the observed behavior be due to a set of shopping heuristics used by consumers in an attempt to lower information processing and gathering efforts? Such a heuristic might lead consumers to use reference prices in creating a "shopping list" which they are more likely to deviate from if the actual price of a product on the list exceeds its reference price. The important issues here are: What are the differences between an "information processing" versus a "utility conveying" theory of reference price effects; and which theory is better able to predict actual consumer behavior.

A second important area of concern is in the area of the reference price formation mechanism. To date, there has not been a truly systematic analysis of how reference prices are formed. Presumably, such a study would need to be based on experimental investigations. The models of reference price formation that have been used in recent empirical studies using market data are largely based on the economic expectation formation literature, and not the adaptation-level formation literature of psychophysics from which the concept originally arose. A question which should be addressed is which literature provides better insights into actual reference price formation.

A third area, which has received little attention either empirically or theoretically, is the impact of reference price effects on retailers' and manufacturers' optimal pricing

policies. This line of research will be of particular value for industries in which input prices are volatile and have a large effect on the cost structures of firms. Addressing these issues will require the use of dynamic models that also incorporate the expectation formation processes of the firms themselves.¹⁸

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¹⁸ This paper was received May 12, 1989, and has been with the author 14 months for 3 revisions. Processed by Brian Ratchford, Area Editor.

Appendix: Proofs of the Propositions

Shephard's Lemma

Frequently, proofs of propositions in consumer economics can be greatly simplified by enlisting the powerful tools of duality. In particular, the derivation of the generalized Slutsky equation can be greatly simplified by taking advantage of a modified version of Shephard's Lemma. Therefore, this modified version of Shephard's Lemma, along with a brief description of duality, will be presented before moving on to the proof of Proposition 1.

Analogous to traditional economic theory, the consumption levels for each good which maximizes the consumer's utility (given actual prices, reference prices, and expenditure levels) correspond to the quantities which minimize the expenditure necessary to allow the consumer to reach that utility level. Formally, the consumer's expenditure minimization problem is

$$\min_x \sum_{i=1}^n P_i x_i \quad (\text{A1})$$

subject to

$$U(x, L, G) = U^*. \quad (\text{A2})$$

Hicksian demand functions of the form

$$x_i = h_i(P, I(P - RP), (1 - I)(RP - P), U^*) \quad (\text{A3})$$

constitute the solutions to the expenditure minimization problem. Substituting the Hicksian demand functions into the objective function yields the expenditure function

$$M = M(P, I(P - RP), (1 - I)(RP - P), U^*). \quad (\text{A4})$$

One important property of the expenditure function contained in (A4), known as Shephard's Lemma, is that the Hicksian demand functions can be recovered directly from this function, as is shown in Proposition A1.

PROPOSITION A1 (Shephard's Lemma). *The Hicksian demand function for good i is given by*

$$\frac{\partial M}{\partial P_i} = h_i(P, I(P - RP), (1 - I)(RP - P), U^*) = x_i.$$

PROOF. Define the function $w(P)$ as

$$w(P) = \sum_{i=1}^n P_i x_i^* - M(P, I(P - RP)^*, (1 - I)(RP - P)^*, U^*), \quad (\text{A5})$$

where x_i^* is the cost minimizing consumption level of good i when prices, marginal losses, marginal gains, and utility are respectively at the levels P^* , $I(P - RP)^*$, $(1 - I)(RP - P)^*$, and U^* . The function $w(P)$ is always nonnegative, and attains its minimum (at zero) when $P = P^*$. Consequently,

$$\frac{\partial w(P)}{\partial P_i} = x_i^* - \frac{\partial M}{\partial P_i}(P^*, I(P - RP)^*, (1 - I)(RP - P)^*, U^*) = 0, \quad (\text{A6})$$

which implies that

$$x_i^* = \frac{\partial M}{\partial P_i} (P^*, I(P - RP)^*, (1 - I)(RP - P)^*, U^*). \quad (A7)$$

Since P^* , $I(P - RP)^*$, $(1 - I)(RP - P)^*$, and U^* are arbitrary, the proposition follows. Q.E.D.

PROOF OF PROPOSITION 1. At the optimum, given P^* , $I(P - RP)^*$, and $(1 - I)(RP - P)^*$,

$$x_i(P, I(P - RP), (1 - I)(RP - P), M(P, I(P - RP), (1 - I)(RP - P), U^*))$$

$$\equiv h_i(P, I(P - RP), (1 - I)(RP - P), U^*). \quad (A8)$$

The total differential of this identity with respect to P_i is

$$\frac{\partial}{\partial P_i} (x_i) + \left[\frac{\partial M}{\partial P_i} + I_i \frac{\partial M}{\partial I_i} - (1 - I_i) \frac{\partial M}{\partial g_i} \right] \frac{\partial x_i}{\partial M} = \frac{\partial}{\partial P_i} (h_i). \quad (A9)$$

Using Shephard's Lemma and rearranging allows (A9) to be written as

$$\frac{\partial}{\partial P_i} (x_i) = \frac{\partial}{\partial P_i} (h_i) - x_i \frac{\partial x_i}{\partial M} + \left[(1 - I_i) \frac{\partial M}{\partial g_i} - I_i \frac{\partial M}{\partial I_i} \right] \frac{\partial x_i}{\partial M}. \quad \text{Q.E.D.} \quad (A10)$$

PROOF OF PROPOSITION 2. The derivative for good i exists at the point $RP_i = P_i$ if and only if

$$\lim_{P_i - RP_i \rightarrow 0^+} \frac{\partial x_i}{\partial P_i} = \lim_{P_i - RP_i \rightarrow 0^-} \frac{\partial x_i}{\partial P_i}. \quad (A11)$$

Through the use of comparative static analysis, it can be shown that (see Putler 1988 and Kalman 1968)

$$\frac{\partial x_i}{\partial P_i} = D^{-1} \left(\lambda D_{ii} - \sum_{j=1}^n D_{ji} w_{ji} x_i + x_i D_{n+1i} \right), \quad (A12)$$

where D is the determinant of the bordered Slutsky substitution matrix

$$\begin{bmatrix} v_{11} & \cdots & v_{1n} & -P_1 \\ \vdots & & \vdots & \vdots \\ v_{n1} & \cdots & v_{nn} & -P_n \\ -P_1 & \cdots & -P_n & 0 \end{bmatrix},$$

D_{ij} is the cofactor of the ij th element of D , and

$$v_{ij} = \frac{\partial^2 U}{\partial x_i \partial x_j} + E_{li} \frac{\partial^2 U}{\partial L_i \partial x_j} + E_{gi} \frac{\partial^2 U}{\partial G_i \partial x_j} + E_{lj} \left[\frac{\partial^2 U}{\partial x_i \partial L_j} + E_{li} \frac{\partial^2 U}{\partial L_i \partial L_j} + E_{gi} \frac{\partial^2 U}{\partial G_i \partial L_j} \right]$$

$$+ E_{gj} \left[\frac{\partial^2 U}{\partial x_i \partial G_j} + E_{li} \frac{\partial^2 U}{\partial L_i \partial G_j} + E_{gi} \frac{\partial^2 U}{\partial G_i \partial G_j} \right].$$

Substituting (A12) into (A11) indicates that the limits on both the left- and right-hand sides of (A11) depend on the limits of D , D_{ij} , and w_{ji} . The limits of D and D_{ij} , in turn, depend on the limits of v_{ij} , which are

$$\lim_{P_i - RP_i \rightarrow 0^+} v_{ij} = \lim_{P_i - RP_i \rightarrow 0^+} \frac{\partial^2 U}{\partial x_i \partial x_j} + E_{li} \frac{\partial^2 U}{\partial L_i \partial x_j} + E_{lj} \left(\frac{\partial^2 U}{\partial x_i \partial L_j} + E_{li} \frac{\partial^2 U}{\partial L_i \partial L_j} \right) + E_{gj} \left(\frac{\partial^2 U}{\partial x_i \partial G_j} + E_{li} \frac{\partial^2 U}{\partial L_i \partial G_j} \right)$$

$$= \frac{\partial^2 U}{\partial x_i \partial x_j} + E_{lj} \frac{\partial^2 U}{\partial x_i \partial L_j} + E_{gj} \frac{\partial^2 U}{\partial x_i \partial G_j}, \quad \text{and} \quad (A13)$$

$$\lim_{P_i - RP_i \rightarrow 0^-} v_{ij} = \lim_{P_i - RP_i \rightarrow 0^-} \frac{\partial^2 U}{\partial x_i \partial x_j} + E_{gi} \frac{\partial^2 U}{\partial G_i \partial x_j} + E_{lj} \left(\frac{\partial^2 U}{\partial x_i \partial L_j} + E_{gi} \frac{\partial^2 U}{\partial G_i \partial L_j} \right) + E_{gj} \left(\frac{\partial^2 U}{\partial x_i \partial G_j} + E_{gi} \frac{\partial^2 U}{\partial G_i \partial G_j} \right)$$

$$= \frac{\partial^2 U}{\partial x_i \partial x_j} + E_{lj} \frac{\partial^2 U}{\partial x_i \partial L_j} + E_{gj} \frac{\partial^2 U}{\partial x_i \partial G_j}. \quad (A14)$$

Therefore,

$$\lim_{P_i - RP_i \rightarrow 0^+} D = \lim_{P_i - RP_i \rightarrow 0^-} D \quad \text{and} \quad (\text{A15})$$

$$\lim_{P_i - RP_i \rightarrow 0^+} D_{ij} = \lim_{P_i - RP_i \rightarrow 0^-} D_{ij}. \quad (\text{A16})$$

The limits of w_{ji} are

$$\lim_{P_i - RP_i \rightarrow 0^+} w_{ji} = \frac{\partial E_{ii}}{\partial L_i} \left(\frac{\partial^2 U}{\partial x_j \partial L_i} + E_{ij} \frac{\partial^2 U}{\partial L_j \partial L_i} + E_{gj} \frac{\partial^2 U}{\partial G_j \partial L_i} \right) = w_{ji} |_{P_i > RP_i} \quad \text{and} \quad (\text{A17})$$

$$\lim_{P_i - RP_i \rightarrow 0^-} w_{ji} = -\frac{\partial E_{gi}}{\partial G_i} \left(\frac{\partial^2 U}{\partial x_j \partial G_i} + E_{lj} \frac{\partial^2 U}{\partial L_j \partial G_i} + E_{gj} \frac{\partial^2 U}{\partial G_j \partial G_i} \right) = w_{ji} |_{P_i < RP_i}. \quad (\text{A18})$$

Consequently,

$$\lim_{P_i - RP_i \rightarrow 0^+} \frac{\partial x_i}{\partial P_i} = D^{-1} \left(\lambda D_{ii} - \sum_{j=1}^n D_{ji} x_i w_{ji} |_{P_i > RP_i} + x_i D_{n+1i} \right) \quad \text{and} \quad (\text{A19})$$

$$\lim_{P_i - RP_i \rightarrow 0^-} \frac{\partial x_i}{\partial P_i} = D^{-1} \left(\lambda D_{ii} - \sum_{j=1}^n D_{ji} x_i w_{ji} |_{P_i < RP_i} + x_i D_{n+1i} \right). \quad (\text{A20})$$

Thus, for (A11) to hold, it requires that

$$\sum_{j=1}^n D_{ji} w_{ji} |_{P_i > RP_i} = \sum_{j=1}^n D_{ji} w_{ji} |_{P_i < RP_i}. \quad \text{Q.E.D.} \quad (\text{A21})$$

PROOF OF PROPOSITION 3. By Euler's Theorem, a Marshallian demand function is homogeneous of degree zero if and only if

$$\sum_{j=1}^n \frac{\partial x_i}{\partial P_j} P_j + \frac{\partial x_i}{\partial M} M = 0. \quad (\text{A22})$$

Using (A12) and the result that (see Putler 1988 and Kalman 1968)

$$\frac{\partial x_i}{\partial M} = -\frac{D_{n+1i}}{D} \quad (\text{A23})$$

allows the condition in (A22) to be written as

$$\sum_{j=1}^n P_j \left(\frac{\lambda D_{ji}}{D} - \sum_{h=1}^n \frac{D_{hi}}{D} w_{hj} x_j + x_j \frac{D_{n+1i}}{D} \right) - \frac{D_{n+1i}}{D} M = 0. \quad (\text{A24})$$

Equation (A24) can be rearranged into the form

$$\frac{\lambda}{D} \sum_{j=1}^n D_{ji} P_j - \sum_{j,h=1}^n x_j P_j \frac{D_{hi}}{D} w_{hj} + \frac{D_{n+1i}}{D} \sum_{j=1}^n x_j P_j - \frac{D_{n+1i}}{D} M = 0. \quad (\text{A25})$$

This implies that

$$\frac{\lambda}{D} \sum_{j=1}^n D_{ji} P_j - \sum_{j,h=1}^n x_j P_j \frac{D_{hi}}{D} w_{hj} = 0 \quad (\text{A26})$$

must hold. The first term is an expansion of D by alien cofactors, and, consequently, vanishes (Kalman 1968). Thus, the condition becomes

$$\sum_{j,h=1}^n x_j P_j \frac{D_{hi}}{D} w_{hj} = 0. \quad \text{Q.E.D.} \quad (\text{A27})$$

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