



# Applying complementary energy methods to estimate ground reaction forces of an exoskeleton

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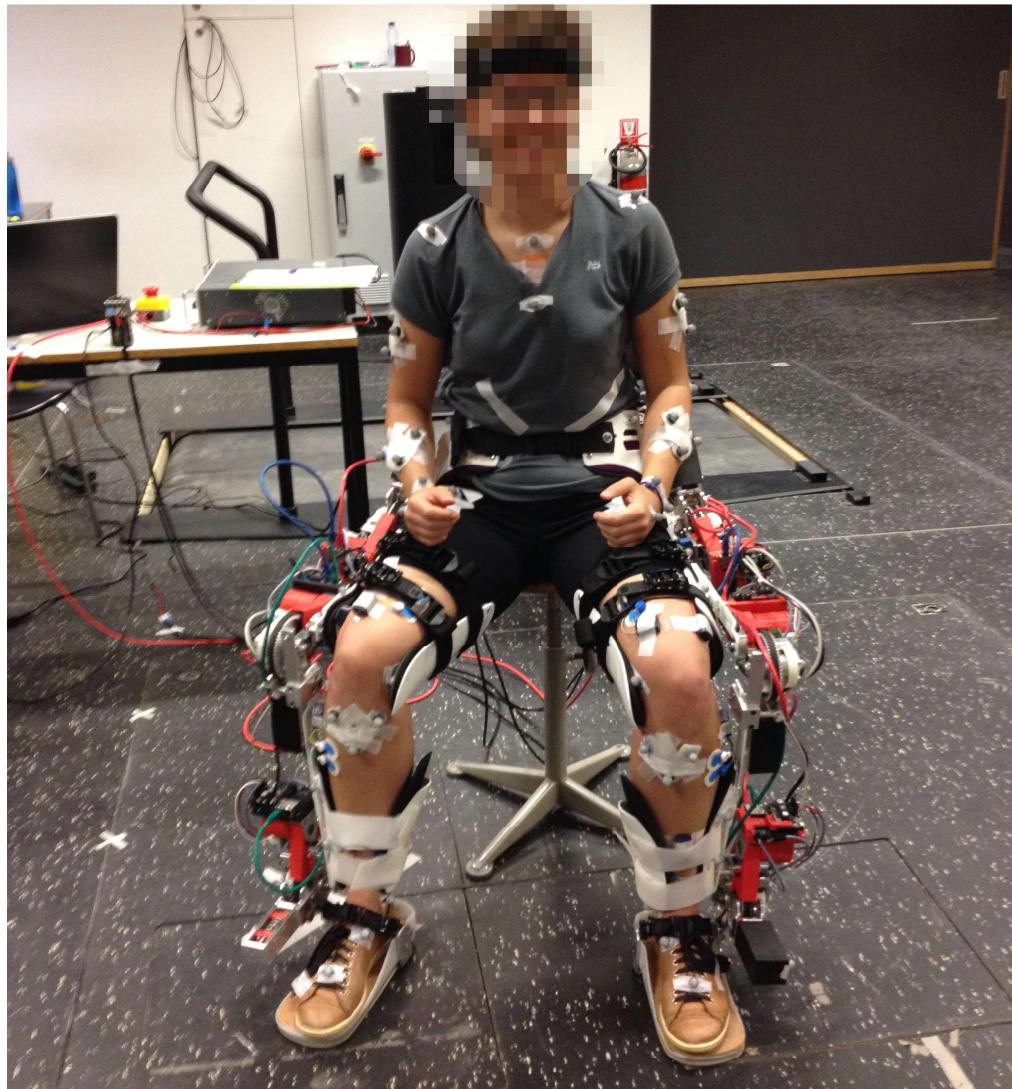
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05/02/2018

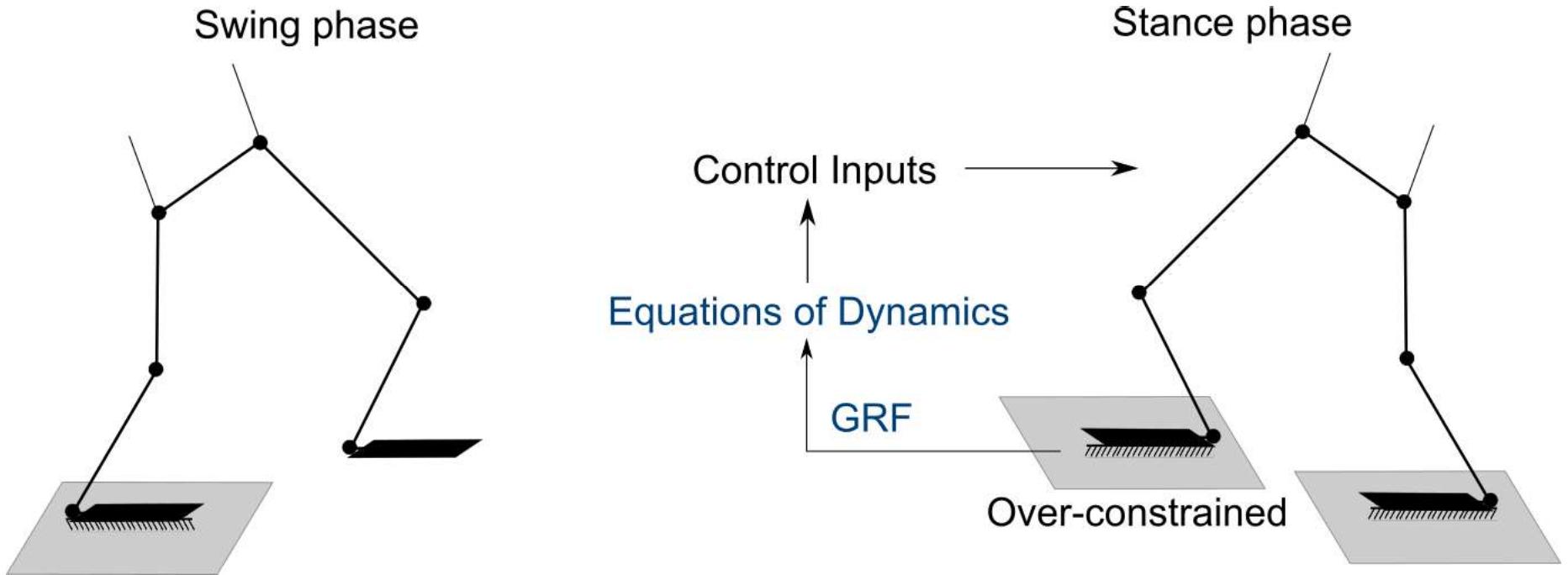


# Introduction

Lower limb exoskeleton ⇒ mobility assistance/ rehabilitation



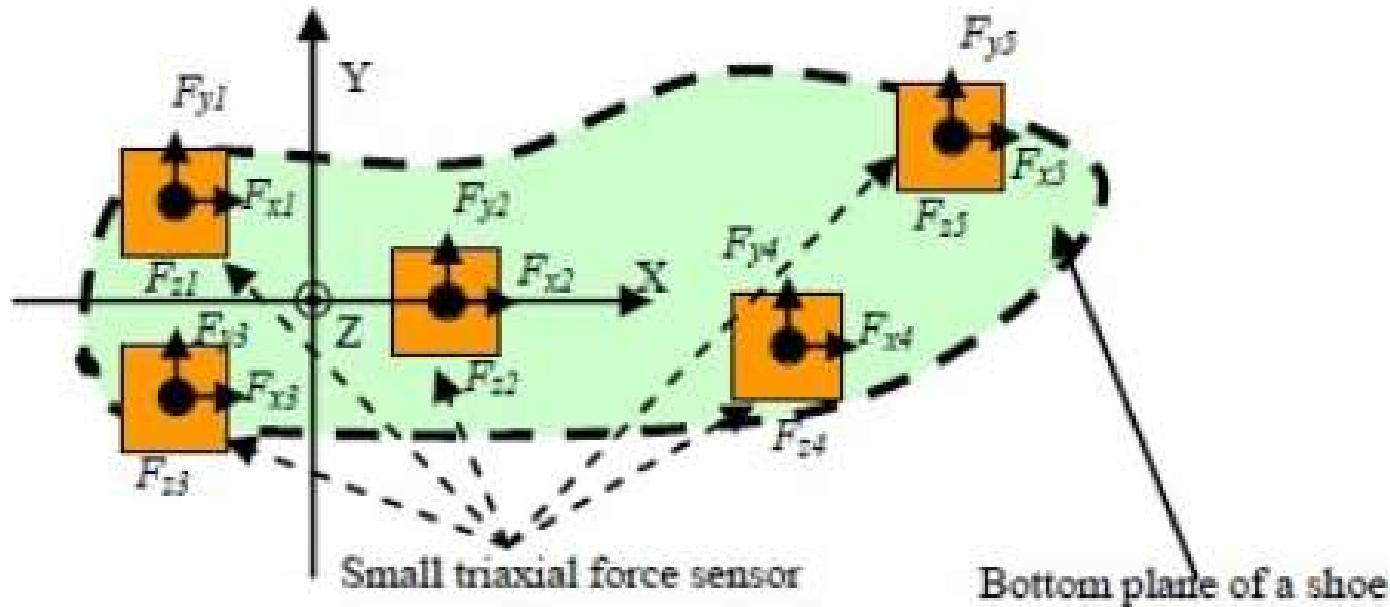
# Introduction



- Method proposed to obtain physically inspired solution
- Applied on stand-alone exoskeleton  $\Rightarrow$  similar to humanoids and other legged robots

# Introduction

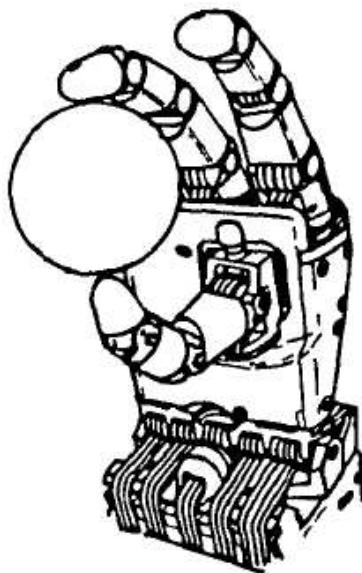
## Research Background



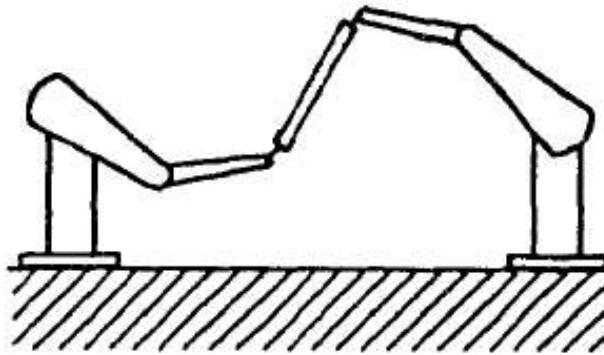
Measuring the GRFs using force cells [Tao et al., 2012]

# Introduction

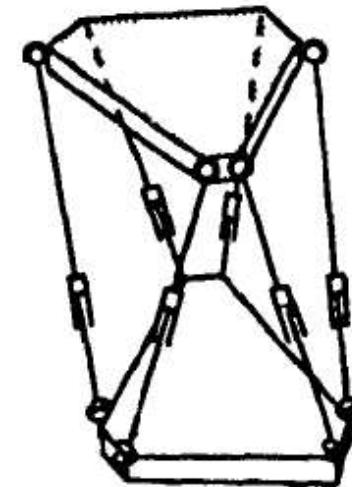
## Research Background



Grasping



Cooperative Robots



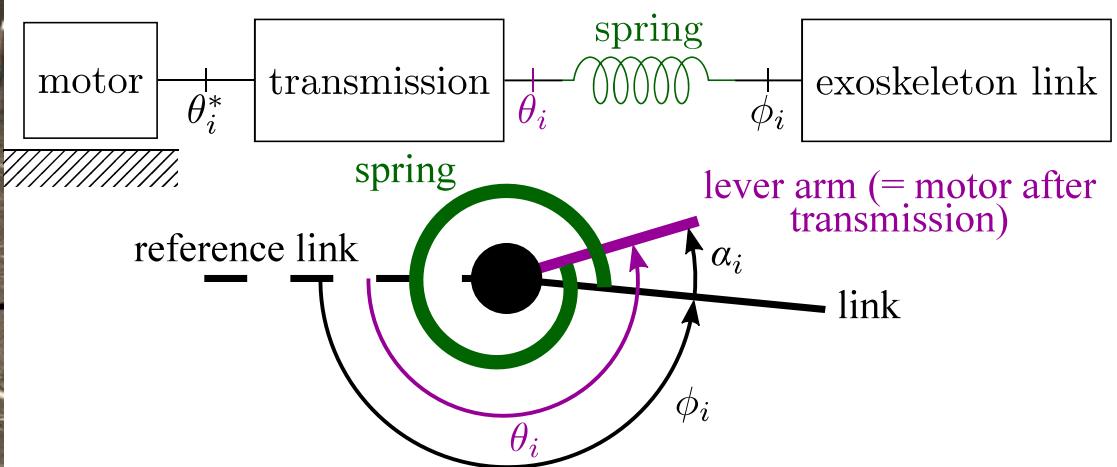
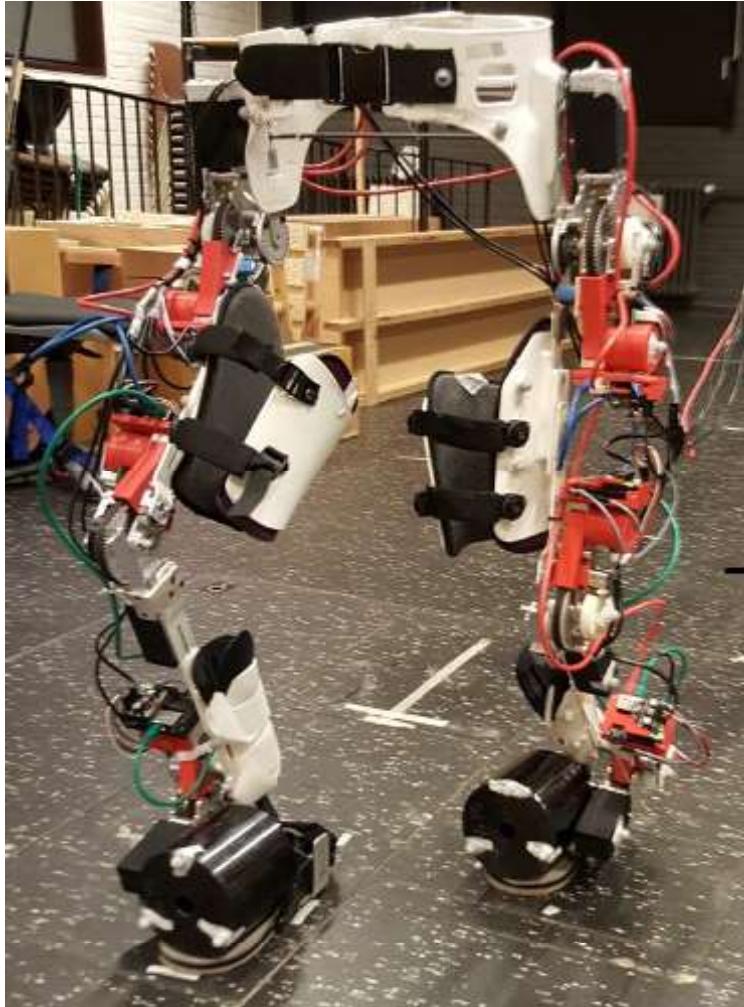
Parallel  
Manipulator

[Nahon, 1993]

Estimating the GRFs using the optimization techniques

# Exoskeleton

## Design

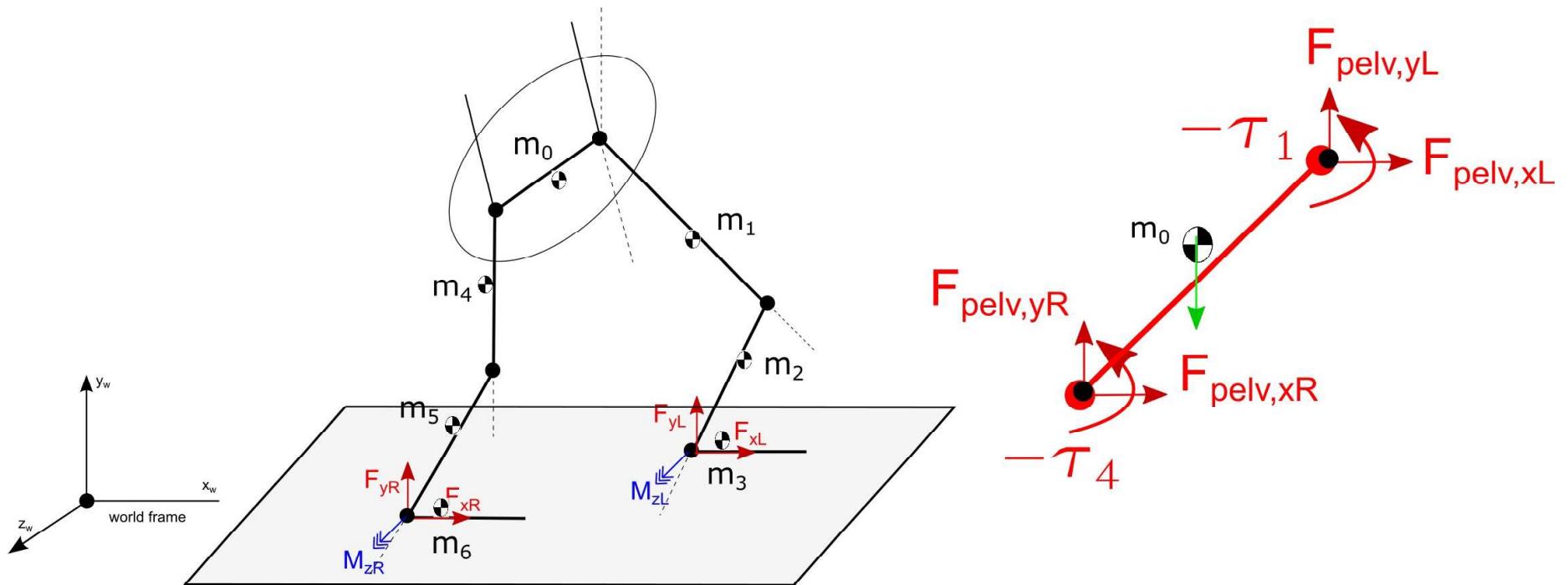


Serial Elastic Actuator

MIRAD Exoskeleton

# Exoskeleton

## Stiffness Model

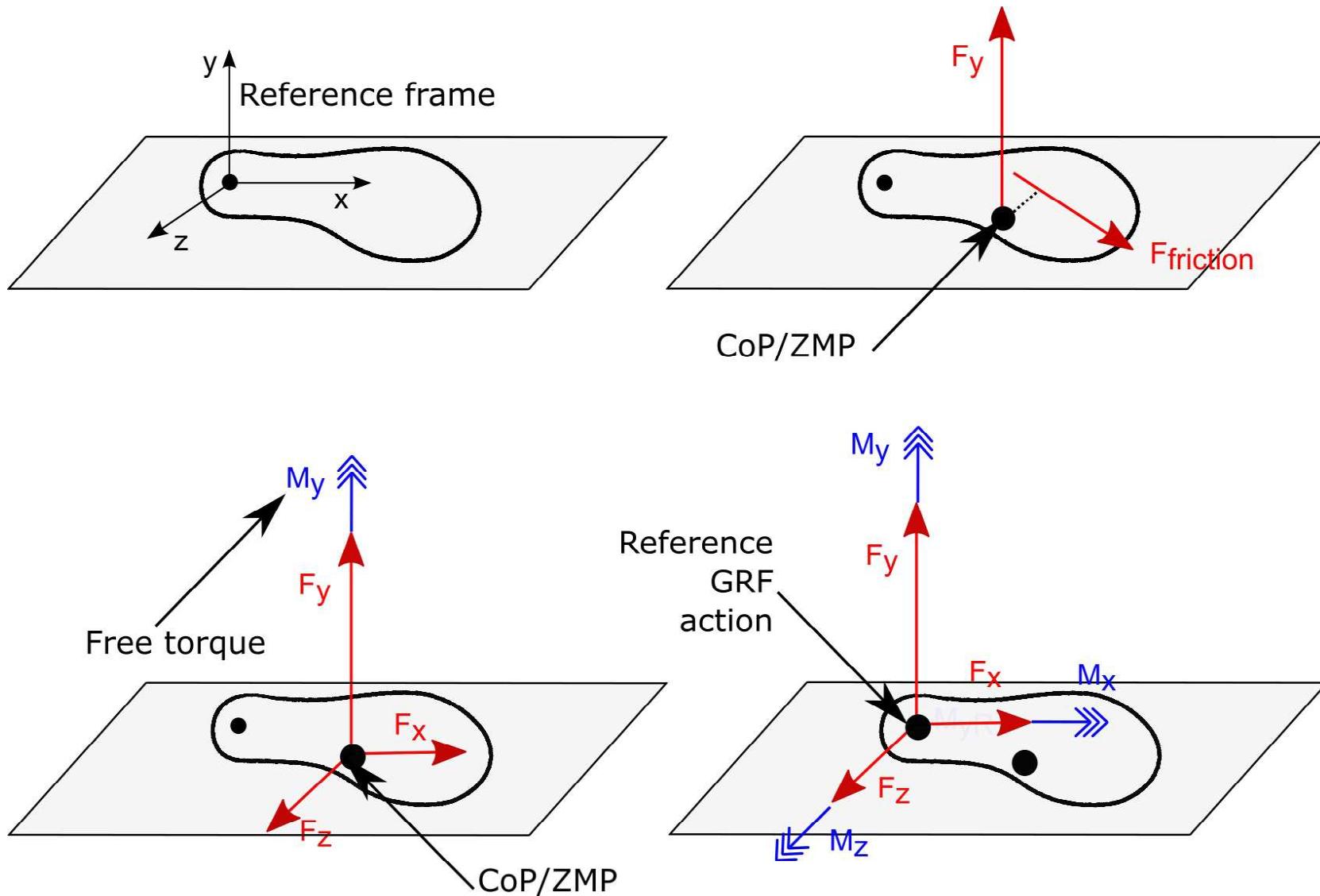


Compliances in the system

- Joint actuators
- Pelvis joining the two legs
- Links are many times stiffer (neglected)

# Exoskeleton

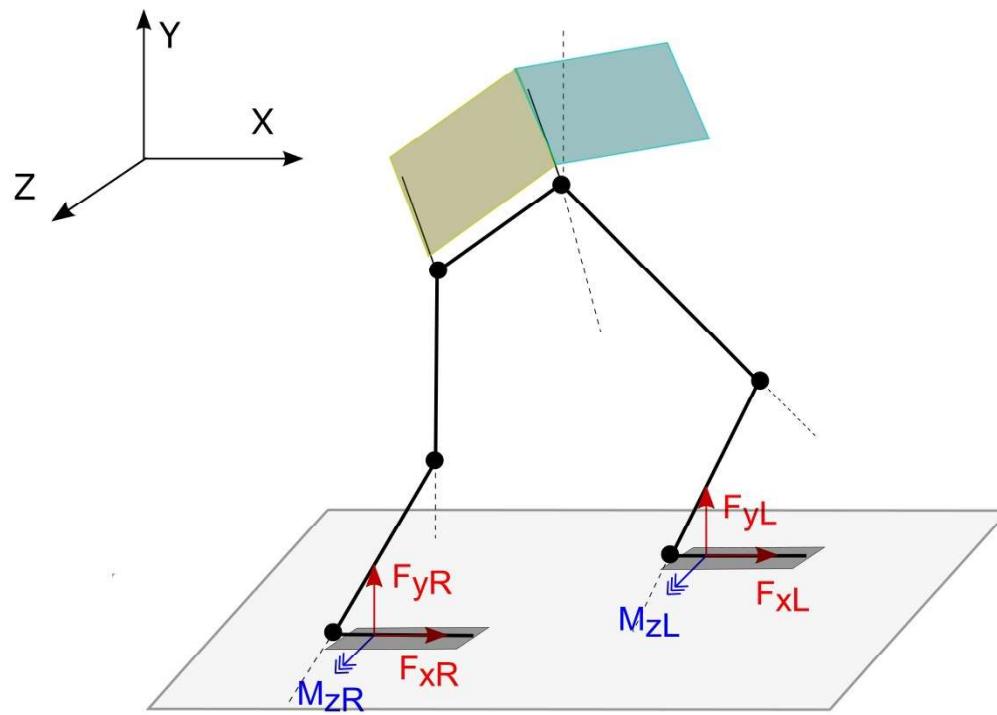
GRF model at a foot



# Exoskeleton

## Dynamic Model

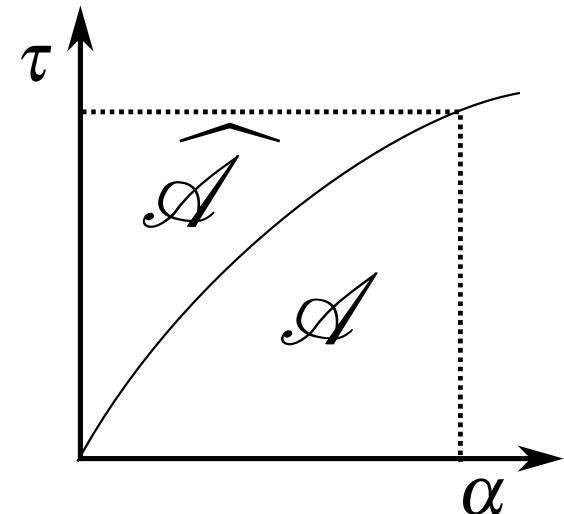
$$\boldsymbol{M}(\boldsymbol{q}) \ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{s}^T \boldsymbol{\tau}_{spr} + \boldsymbol{J}_c(\boldsymbol{q})^T \boldsymbol{w}_c$$



# Solution Strategy

## Complementary Energy Methods

- Equations of compatibility can be derived from the generalized force approaches which work on a scalar quantity called complementary energy



Torque versus deflection curve

- Corollary of the principle of minimum complementary energy: Castigliano's theorem, Crotti-Engesser theorem

$$\begin{aligned}\hat{\mathcal{A}} &= \sum_{i=1}^n \int_0^{\tau_i} \alpha_i(\tau_i) d\tau_i \\ \frac{\partial \hat{\mathcal{A}}}{\partial w_j} &= \sum_{i=1}^n \alpha_i(\tau_i) \frac{\partial \tau_i}{\partial w_j} = 0\end{aligned}$$

# Solution Strategy

Applied to the exoskeleton

- Equilibrium Equations

$$\boldsymbol{M}_{flb} \ddot{\boldsymbol{q}} + \boldsymbol{C}_{flb} \dot{\boldsymbol{q}} + \boldsymbol{g}_{flb} = \boldsymbol{J}_{Lf, flb}^T \boldsymbol{w_L} + \boldsymbol{J}_{Rf, flb}^T \boldsymbol{w_R}$$

$$\boldsymbol{M}_L \ddot{\boldsymbol{q}} + \boldsymbol{C}_L \dot{\boldsymbol{q}} + \boldsymbol{g}_L = \boldsymbol{\tau}_{spr, L} + \boldsymbol{J}_{Lf, L}^T \boldsymbol{w_L}$$

$$\boldsymbol{M}_R \ddot{\boldsymbol{q}} + \boldsymbol{C}_R \dot{\boldsymbol{q}} + \boldsymbol{g}_R = \boldsymbol{\tau}_{spr, R} + \boldsymbol{J}_{Rf, R}^T \boldsymbol{w_R}$$

- Compatibility Equations

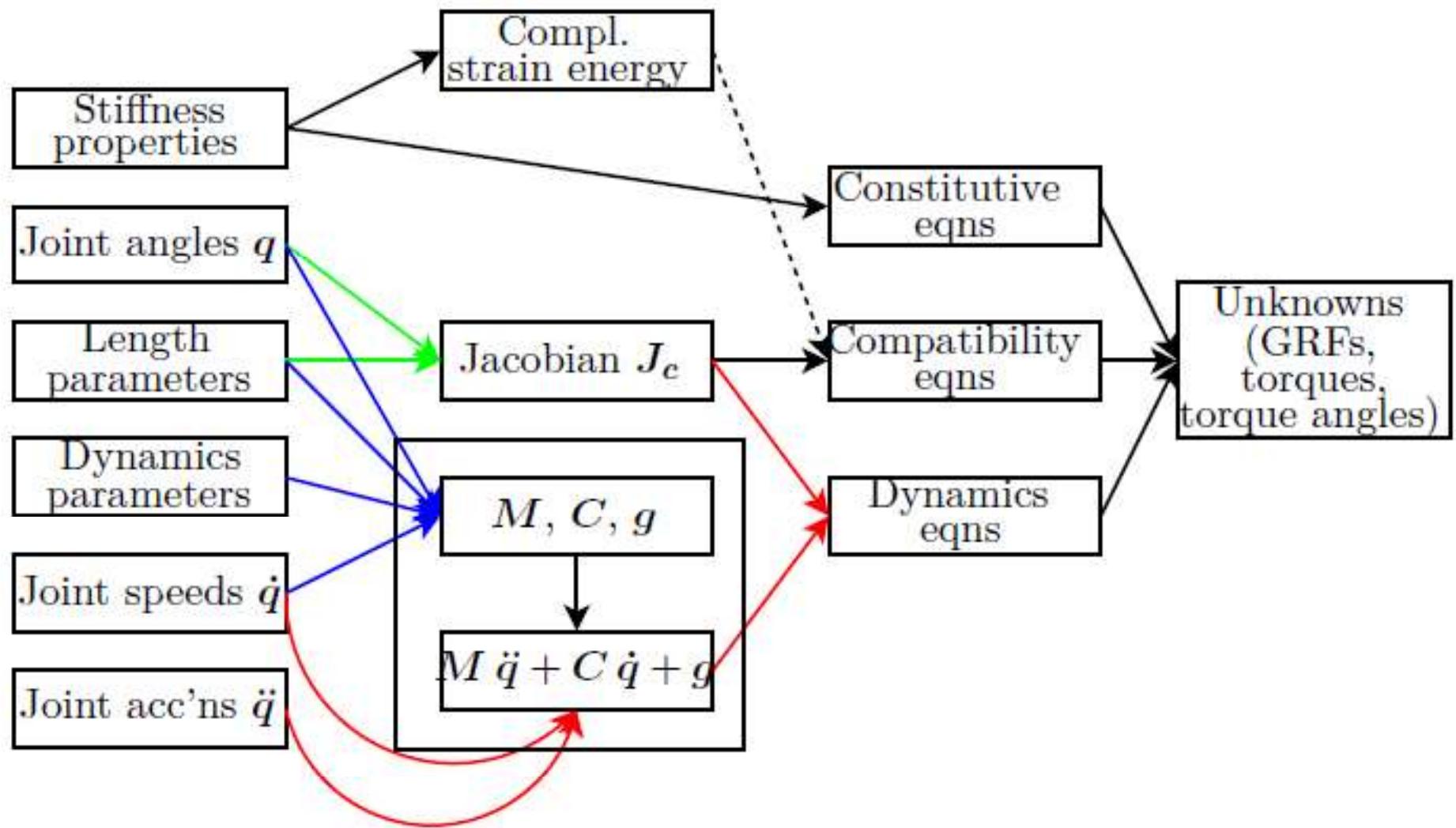
$$((-\boldsymbol{J}_{Lf, L}^T)(\boldsymbol{J}_{Lf, flb}^T)^{-1}(-\boldsymbol{J}_{Rf, flb}^T))^T \boldsymbol{\alpha_L} + (-\boldsymbol{J}_{Rf, R}^T)^T \boldsymbol{\alpha_R} + [..] = 0_{3 \times 1}$$

- Constitutive Laws

$$\boldsymbol{\tau_i} = k_{0,i} + k_{1,i} \boldsymbol{\alpha_i} + k_{2,i} \boldsymbol{\alpha_i}^2 + k_{3,i} \boldsymbol{\alpha_i}^3, \text{ for } i = 1:6$$

# Solution Strategy

## Estimation Algorithm



# Solution Strategy

## Optimization Formulation

$$\begin{aligned} \min_{\mathbf{w}_L, \mathbf{w}_R, \boldsymbol{\tau}_{spr,L}, \boldsymbol{\tau}_{spr,R} \boldsymbol{\alpha}} & \sum_{i=1}^6 \left( \int_0^{\tau_i} \alpha_i d\tau_i \right) \\ & + \frac{1}{2K_{pelv,x}} (2F_{x,R} - \overline{\tau_{dyn,flb,x}})^2 + \frac{1}{2K_{pelv,x}} (2F_{y,R} - \overline{\tau_{dyn,flb,y}})^2 \end{aligned}$$

subject to

$$\mathbf{M}_{flb} \ddot{\mathbf{q}} + \mathbf{C}_{flb} \dot{\mathbf{q}} + \mathbf{g}_{flb} = \mathbf{J}_{Lf,flb}^T \mathbf{w}_L + \mathbf{J}_{Rf,flb}^T \mathbf{w}_R$$

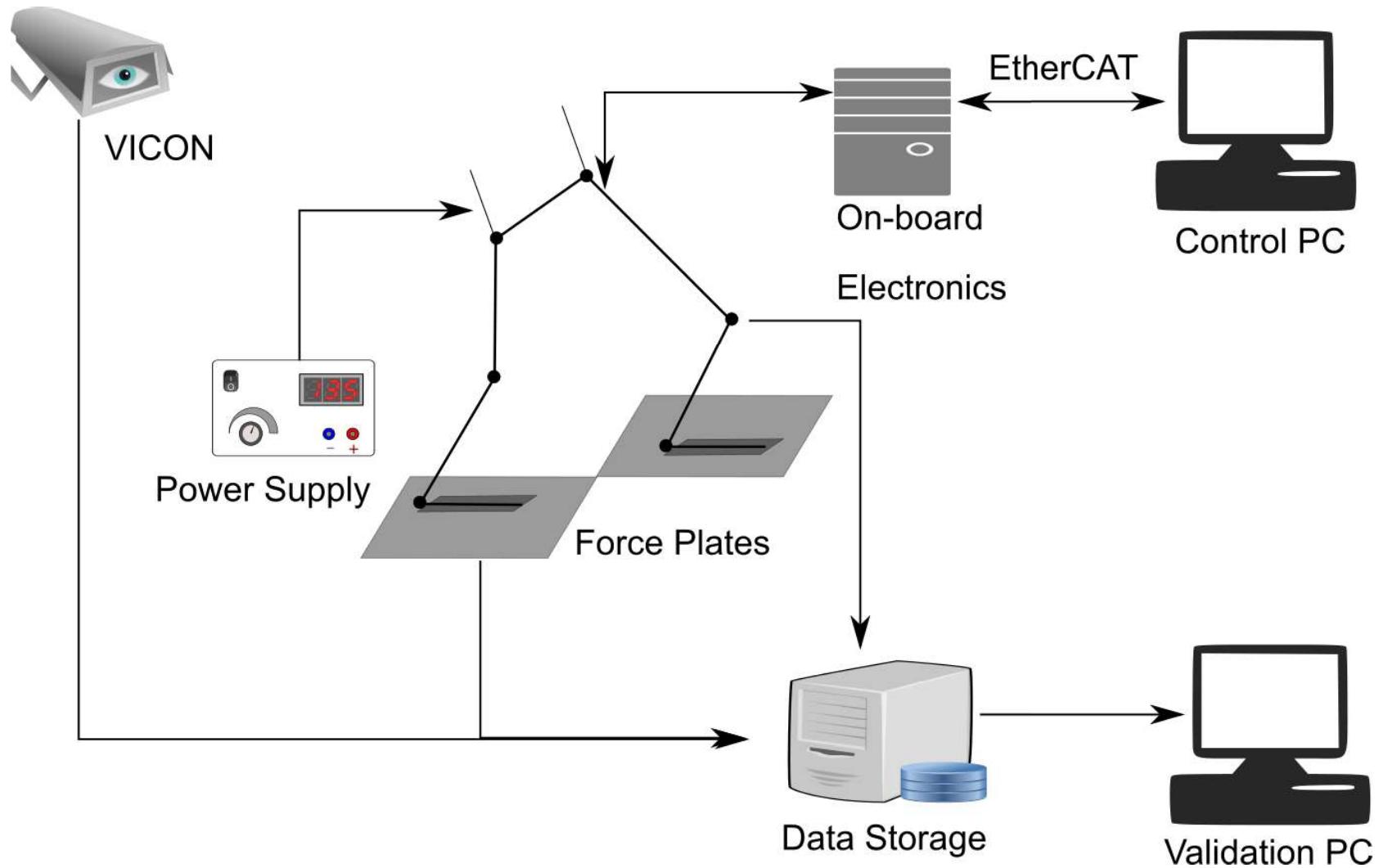
$$\mathbf{M}_L \ddot{\mathbf{q}} + \mathbf{C}_L \dot{\mathbf{q}} + \mathbf{g}_L = \boldsymbol{\tau}_{spr,L} + \mathbf{J}_{Lf,L}^T \mathbf{w}_L$$

$$\mathbf{M}_R \ddot{\mathbf{q}} + \mathbf{C}_R \dot{\mathbf{q}} + \mathbf{g}_R = \boldsymbol{\tau}_{spr,R} + \mathbf{J}_{Rf,R}^T \mathbf{w}_R$$

$$\boldsymbol{\tau}_i = k_{0,i} + k_{1,i} \cdot \boldsymbol{\alpha}_i + k_{2,i} \cdot \boldsymbol{\alpha}_i^2 + k_{3,i} \cdot \boldsymbol{\alpha}_i^3 \text{ for } i = 1:6 .$$

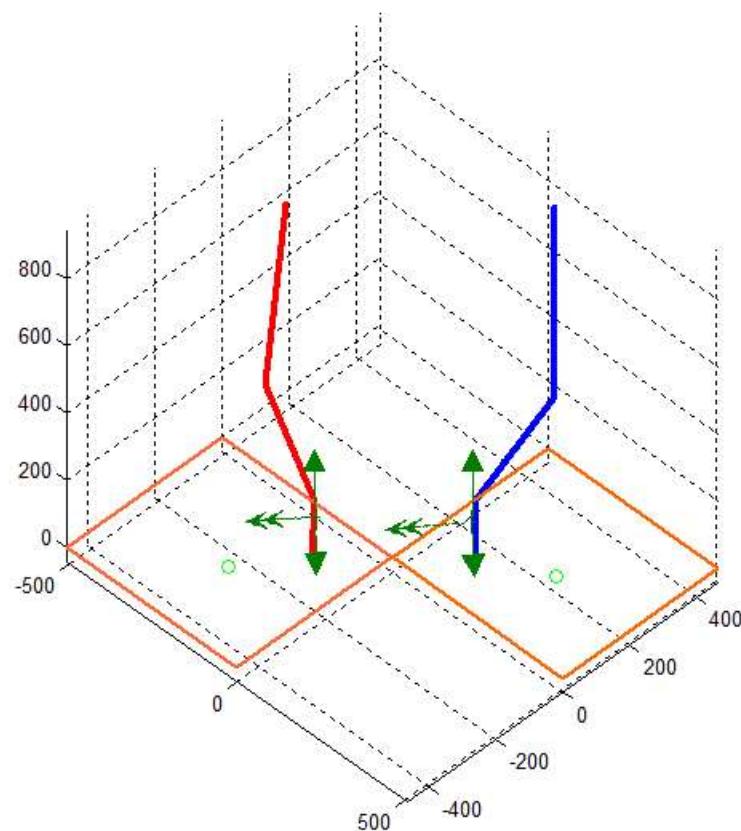
# Validation

## Set-up

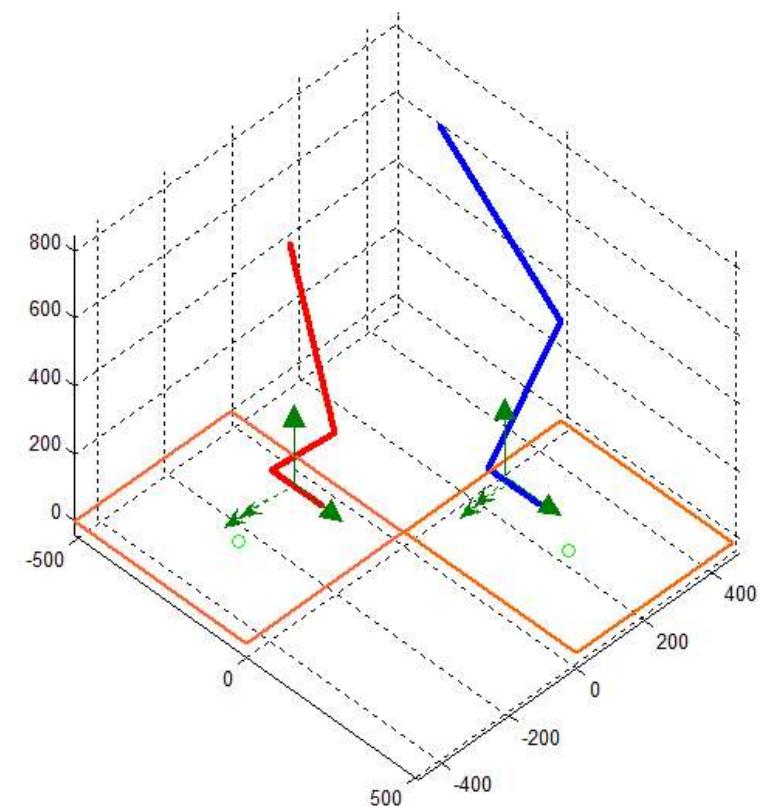


# Validation

## Procedure



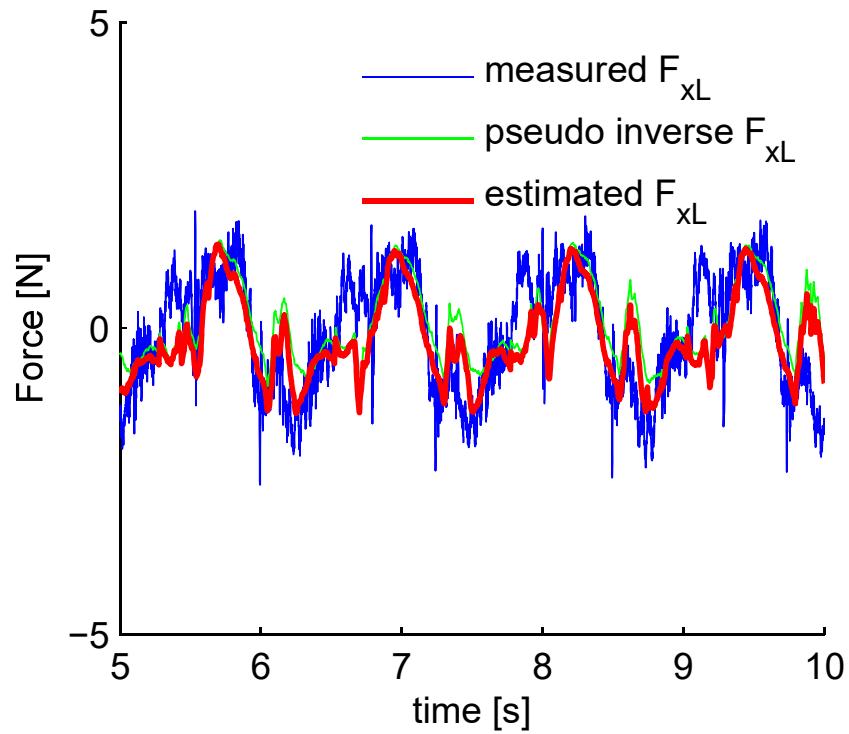
Configuration 1



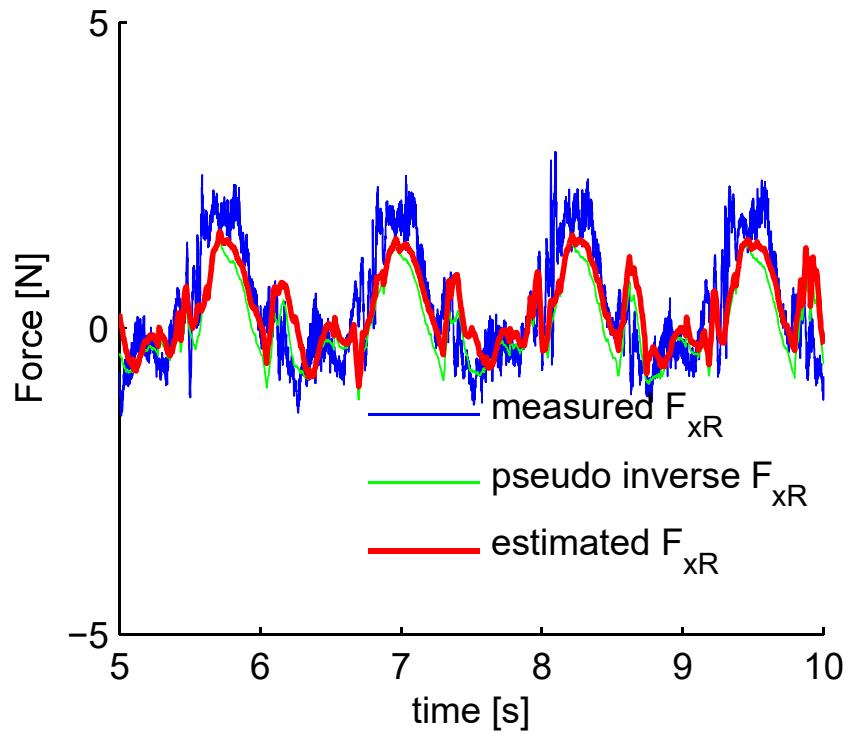
Configuration 2

# Results: Configuration 1

GRF components  $F_{xL}$  and  $F_{xR}$



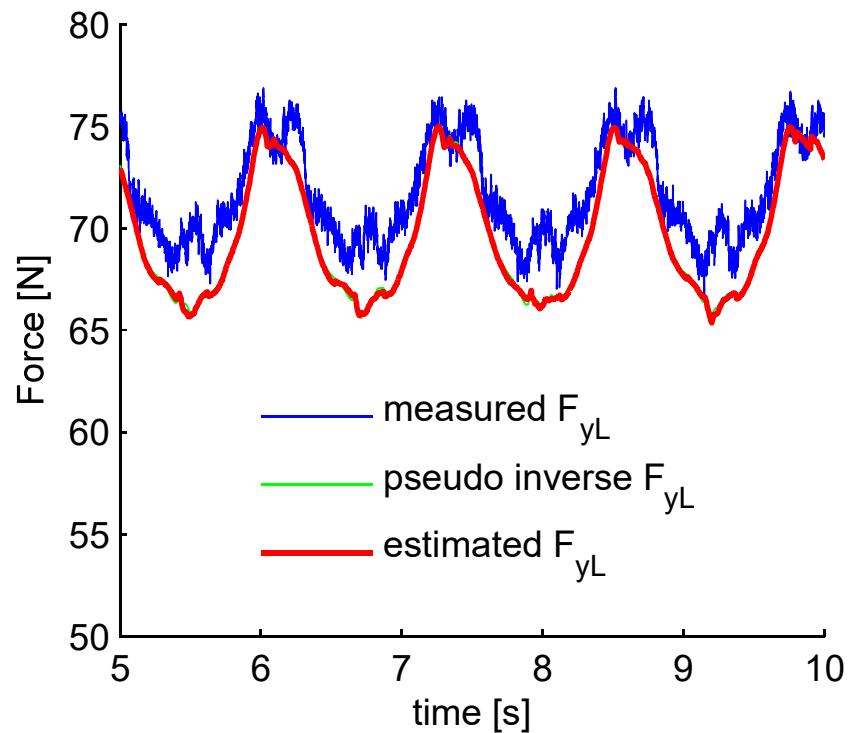
Horizontal force at left foot  $F_{xL}$



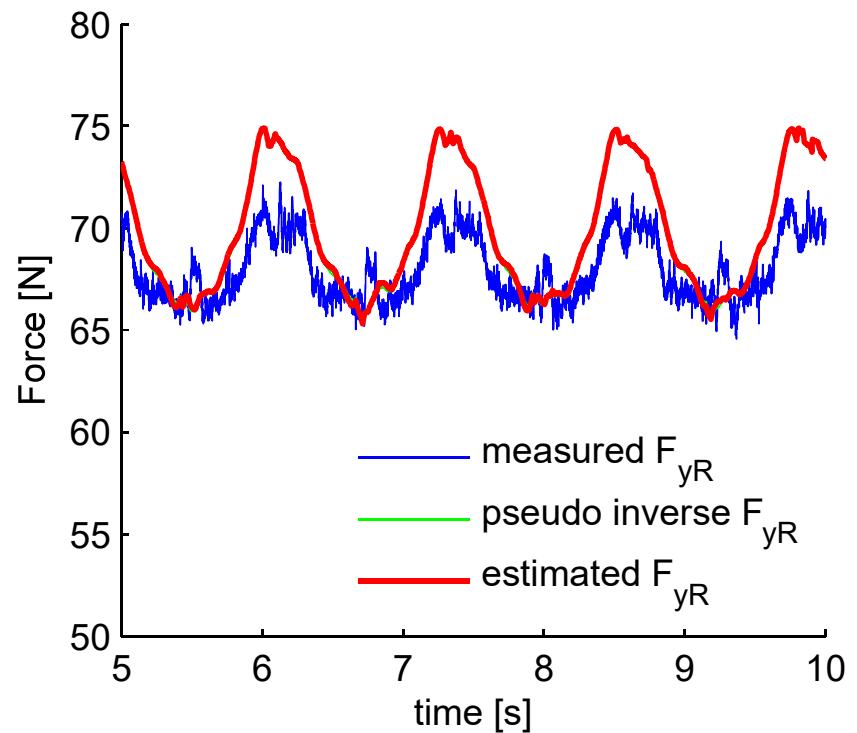
Horizontal force at right foot  $F_{xR}$

# Results: Configuration 1

GRF components  $F_{yL}$  and  $F_{yR}$



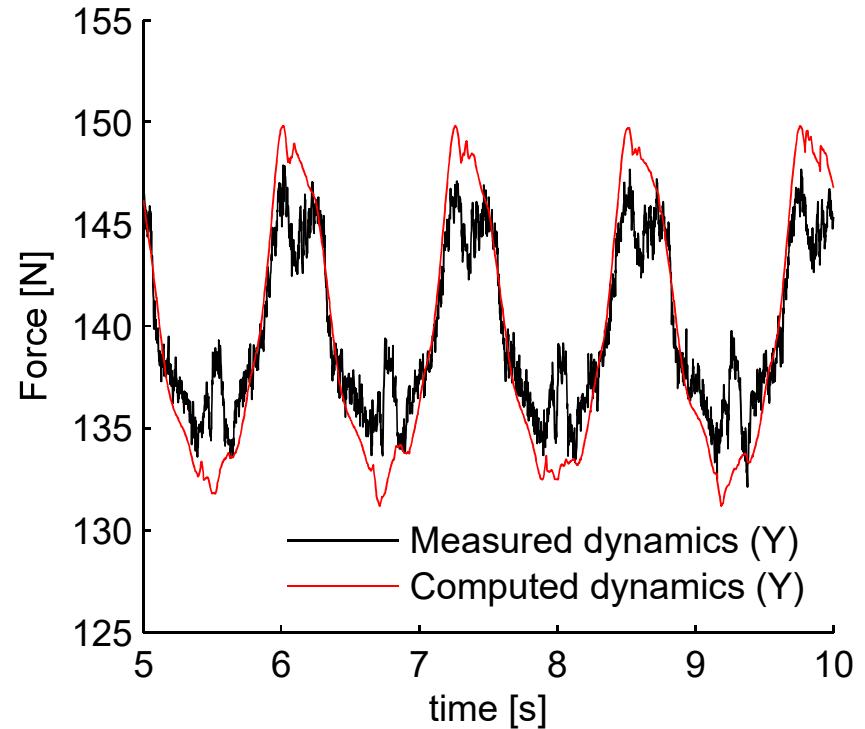
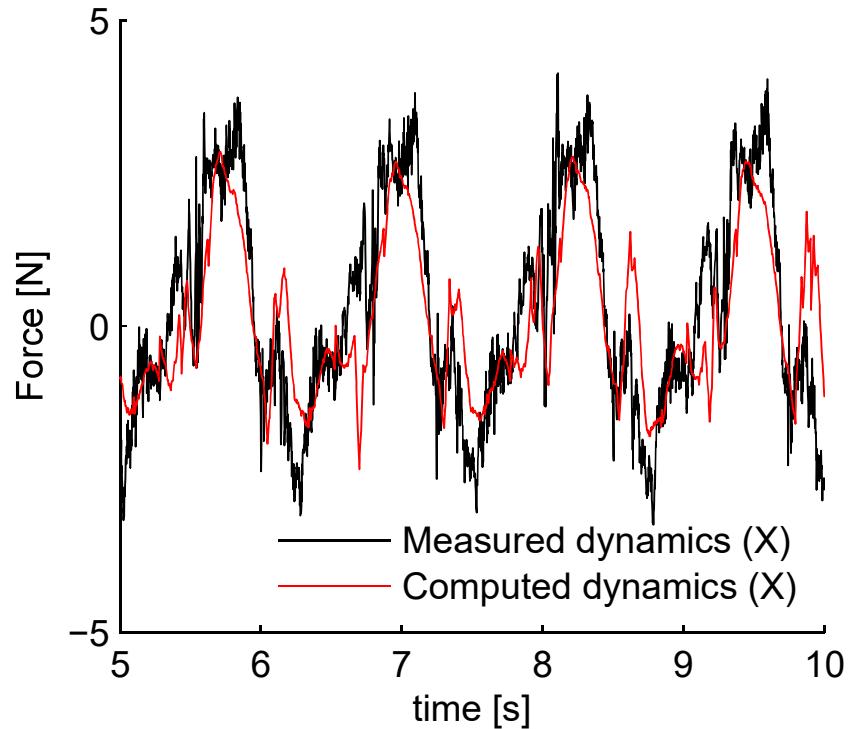
Vertical force at the left foot  $F_{yL}$



Vertical force at right foot  $F_{yR}$

# Results: Configuration 1

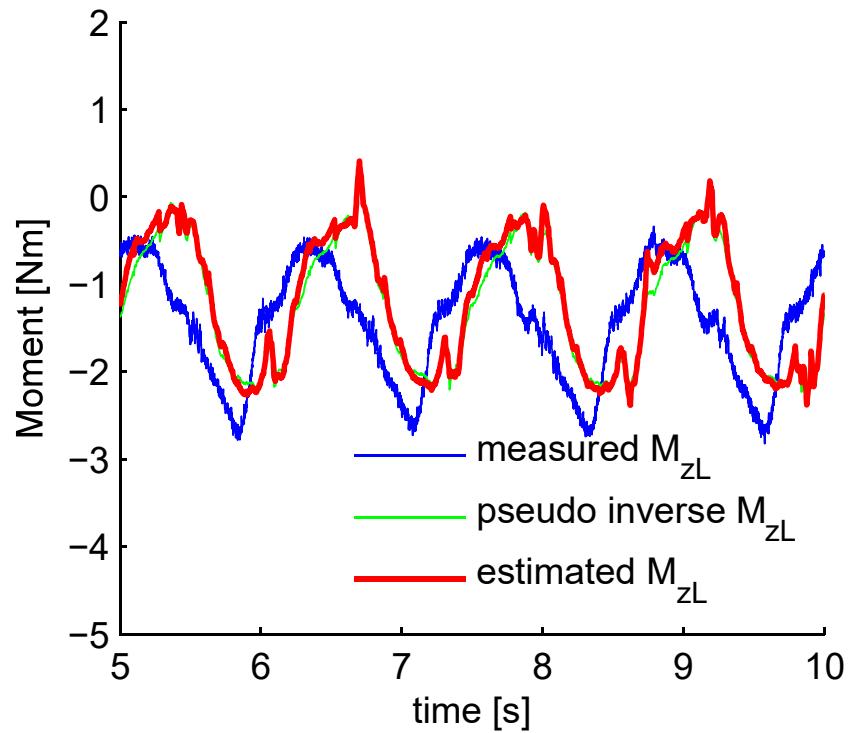
Analysis of errors in  $F_{yL}$  and  $F_{yR}$



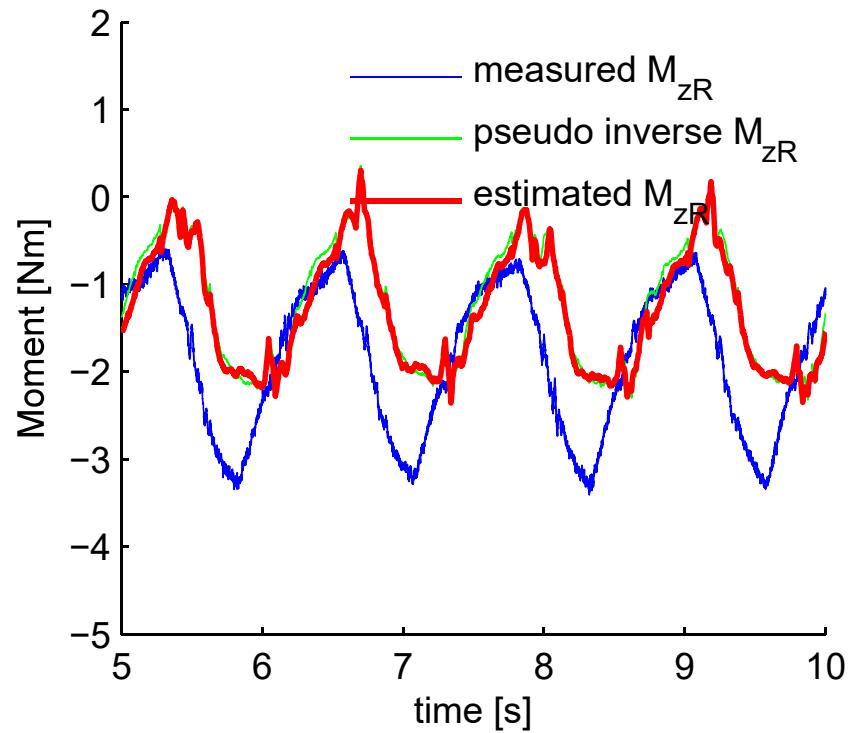
- $\boldsymbol{M}_{flb} \ddot{\boldsymbol{q}} + \boldsymbol{C}_{flb} \dot{\boldsymbol{q}} + \boldsymbol{g}_{flb} = \boldsymbol{\tau}_{dyn, flb} = \boldsymbol{J}_{Lf, flb}^T \boldsymbol{w_L} + \boldsymbol{J}_{Rf, flb}^T \boldsymbol{w_R}$

# Results: Configuration 1

GRF components  $M_{zL}$  and  $M_{zR}$



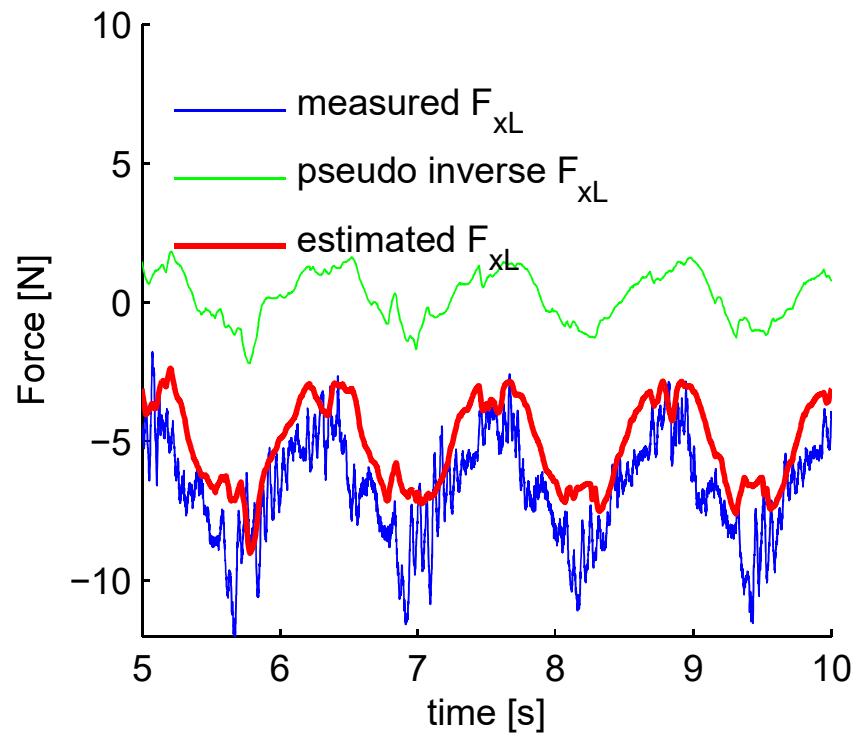
Moment about Z-axis at left foot  $M_{zL}$



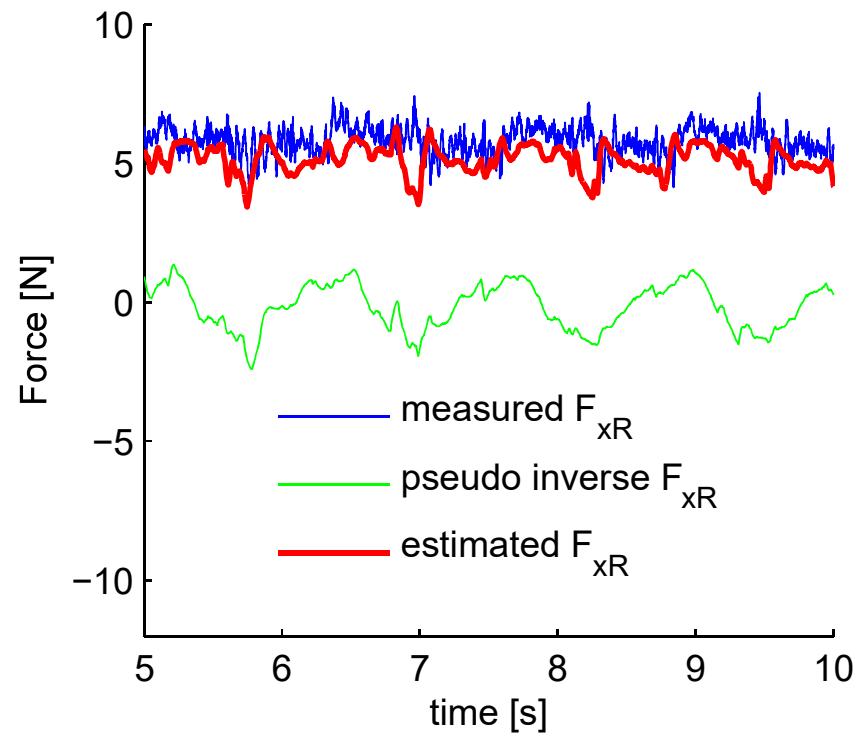
Moment about Z-axis at right foot  $M_{zR}$

# Results: Configuration 2

GRF components  $F_{xL}$  and  $F_{xR}$



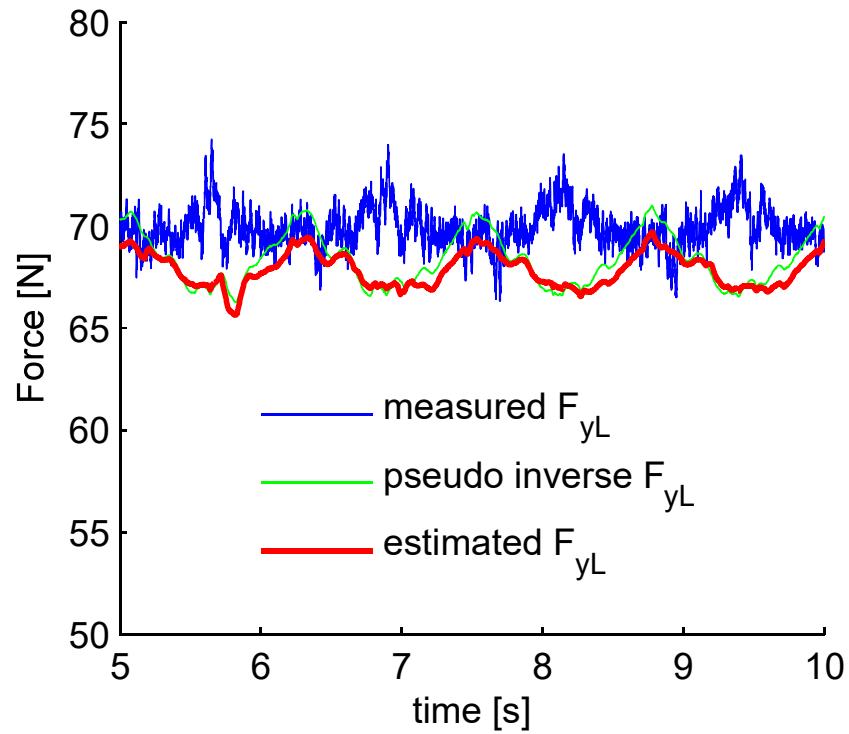
Horizontal force at left foot  $F_{xL}$



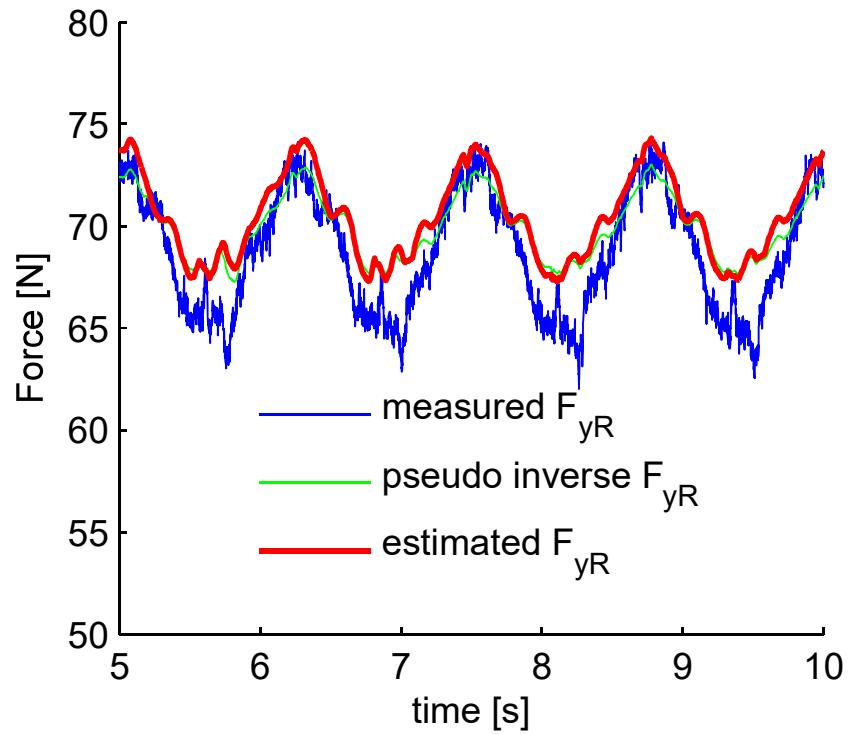
Horizontal force at right foot  $F_{xR}$

# Results: Configuration 2

GRF components  $F_{yL}$  and  $F_{yR}$



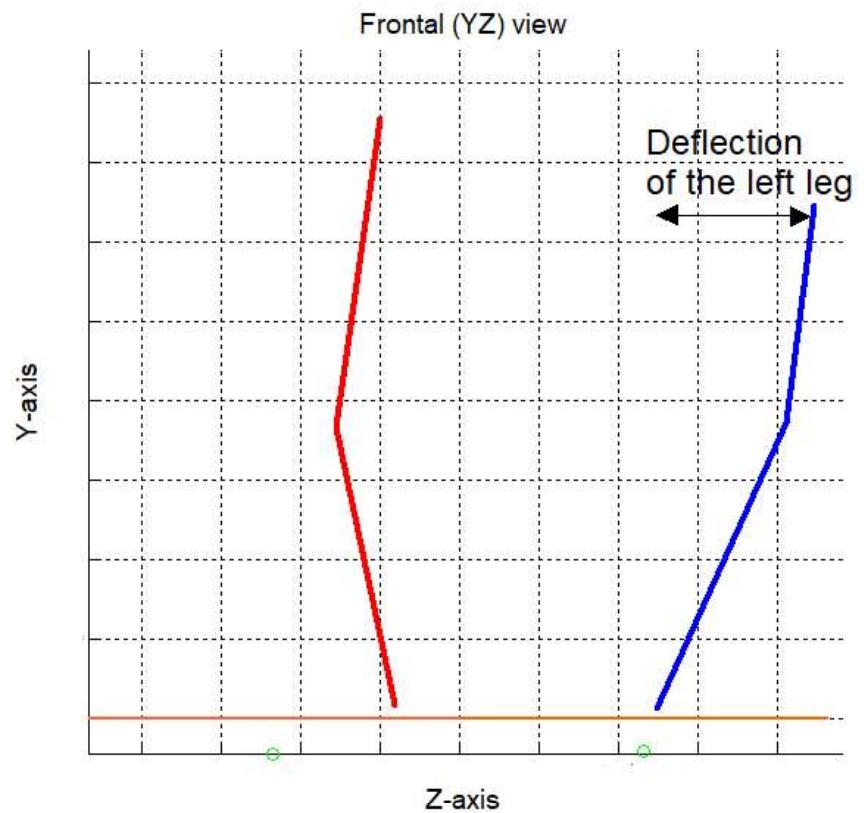
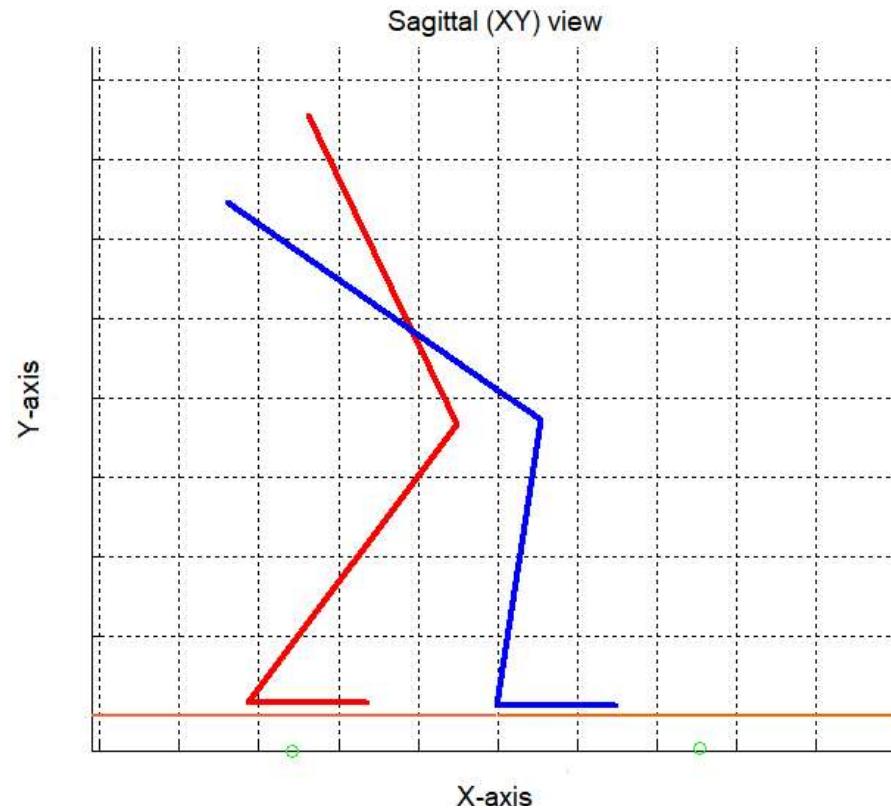
Vertical force at the left foot  $F_{yL}$



Vertical force at right foot  $F_{yR}$

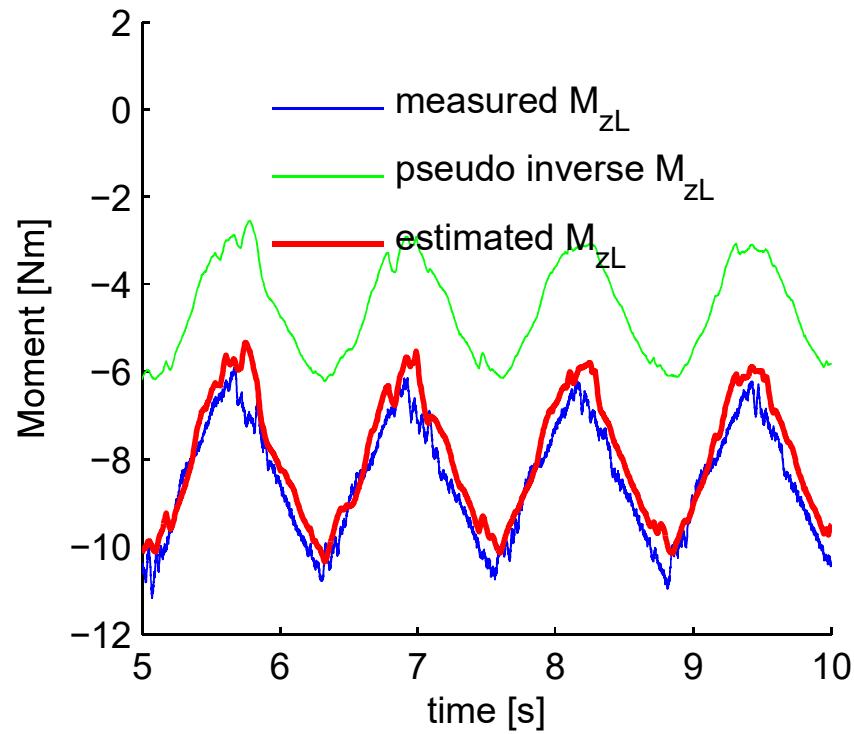
# Results: Configuration 2

Analysis of errors in  $F_{yL}$  and  $F_{yR}$

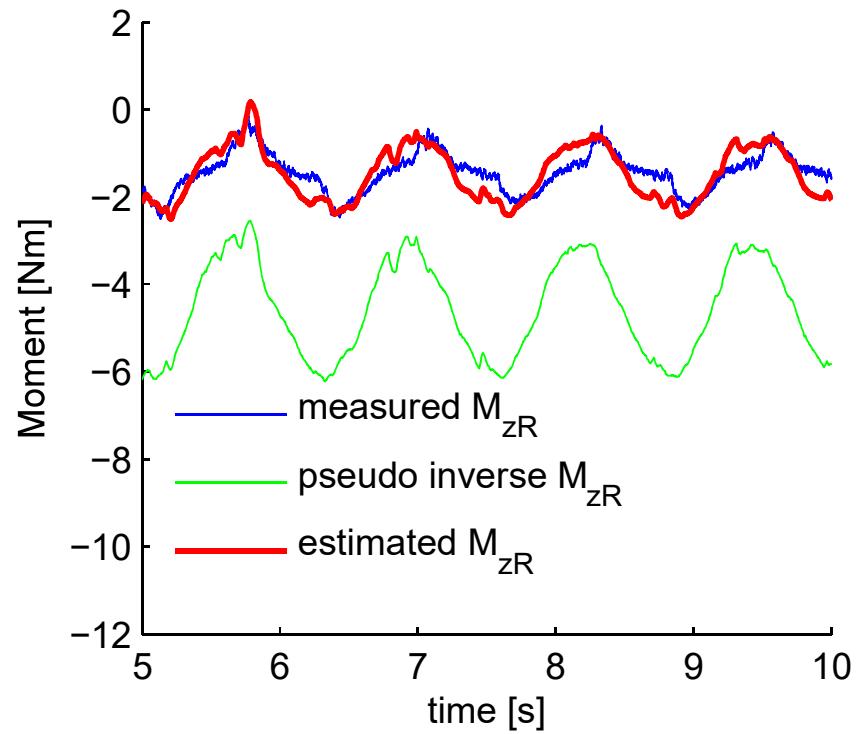


# Results: Configuration 2

GRF components  $M_{zL}$  and  $M_{zR}$



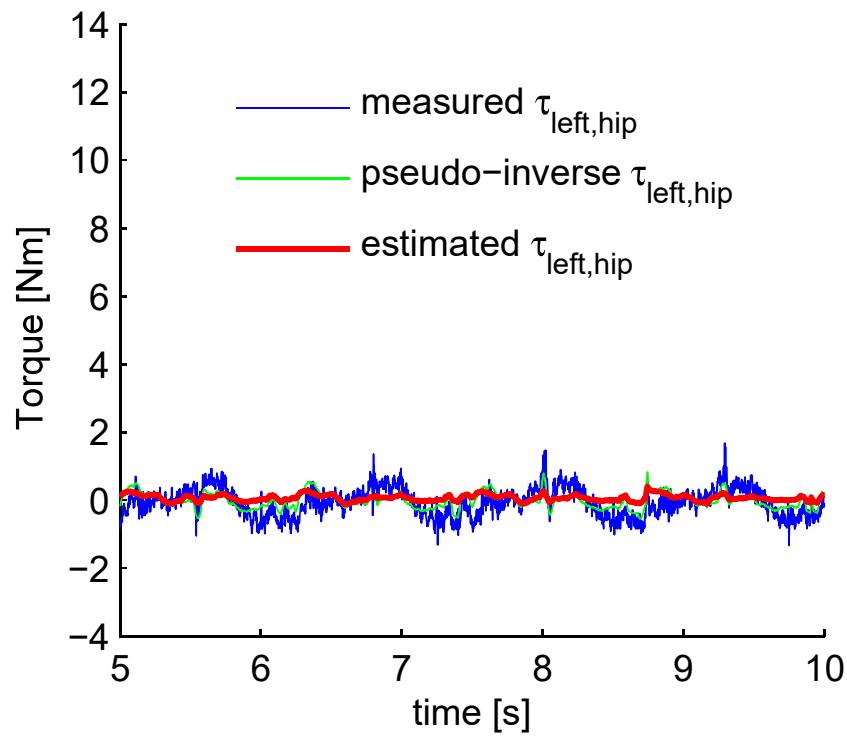
Moment about Z-axis at left foot  $M_{zL}$



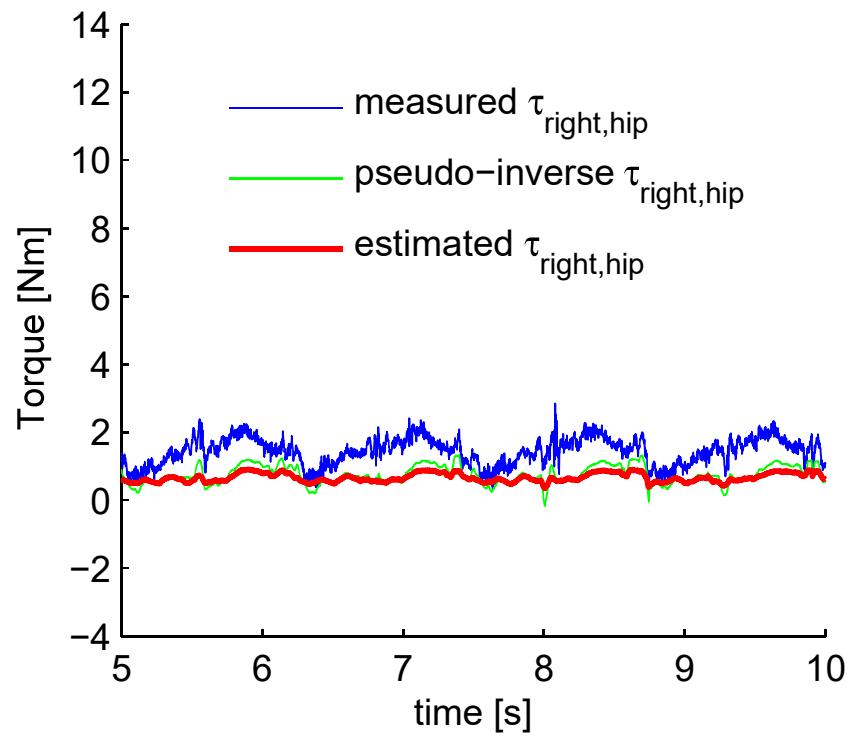
Moment about Z-axis at right foot  $M_{zR}$

# Results: Configuration 1

## Torques at the hip



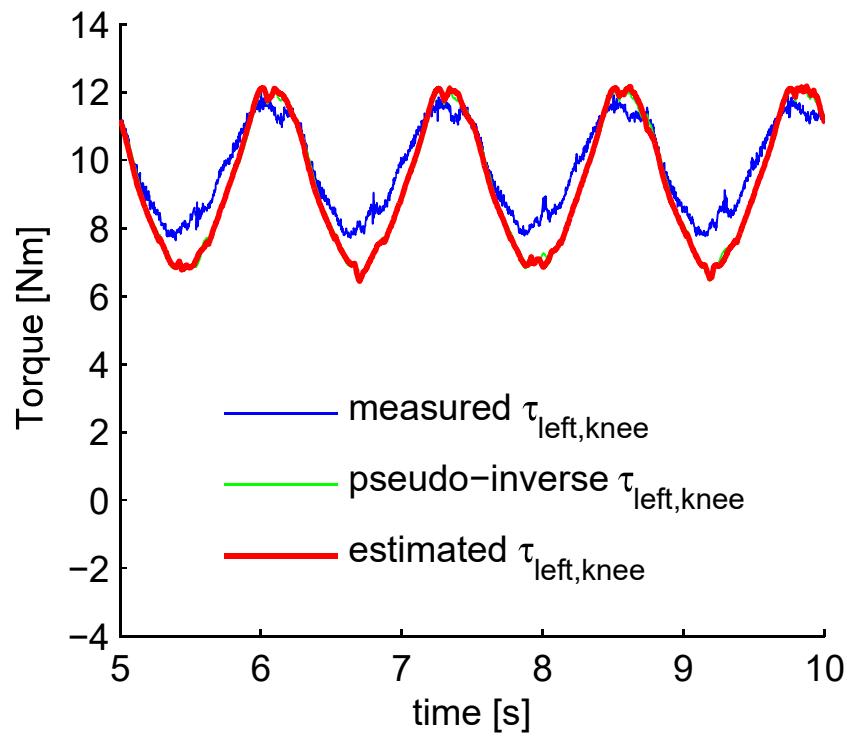
Joint torque at left hip  $\tau_{L,hip}$



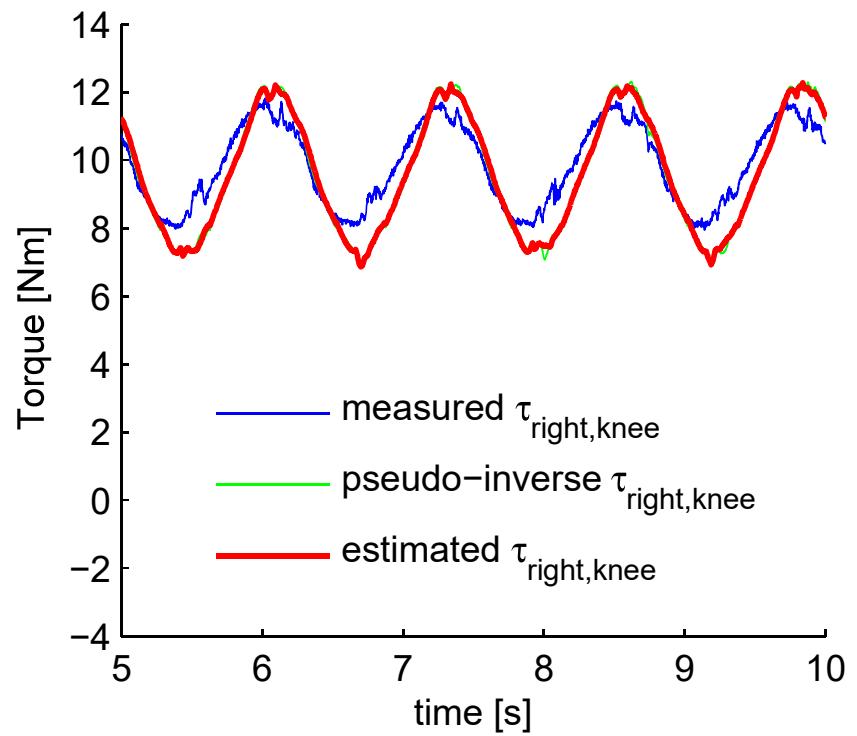
Joint torque at right hip  $\tau_{R,hip}$

# Results: Configuration 1

## Torques at the knee



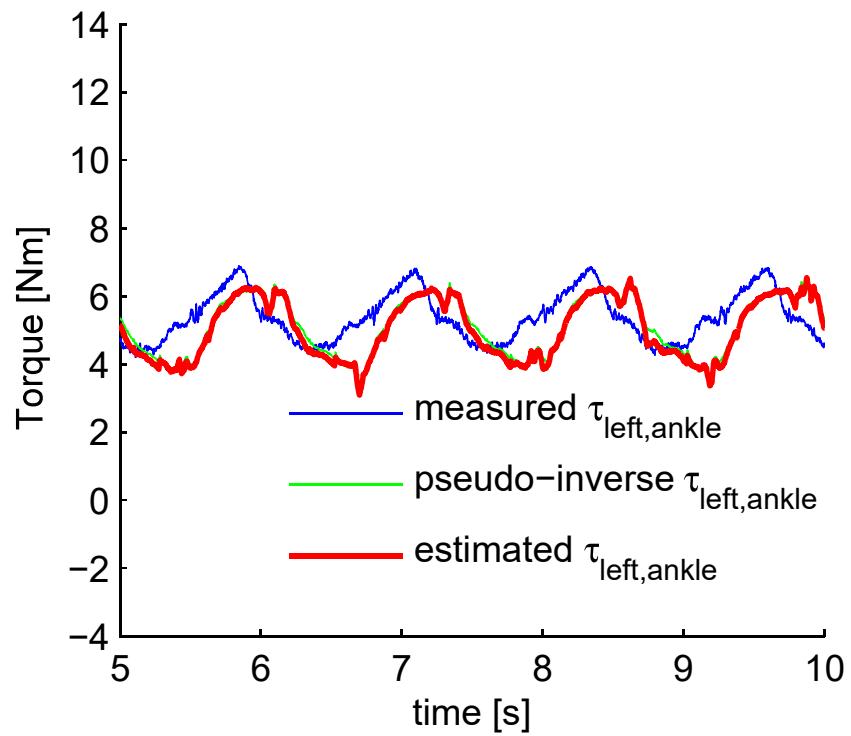
Joint torque at left knee  $\tau_{L,\text{knee}}$



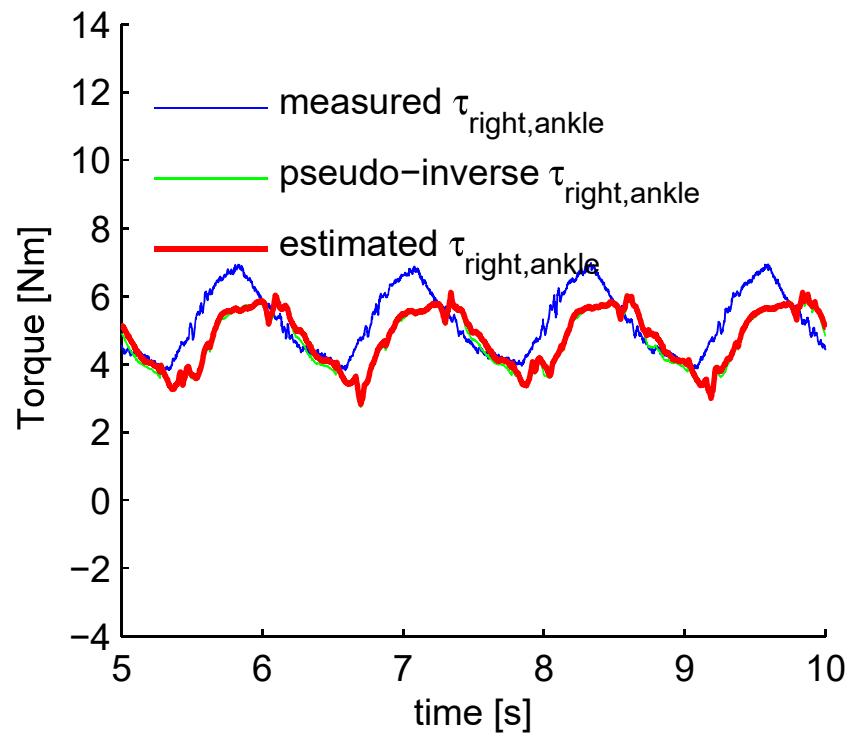
Joint torque at right knee  $\tau_{R,\text{knee}}$

# Results: Configuration 1

## Torques at the ankle



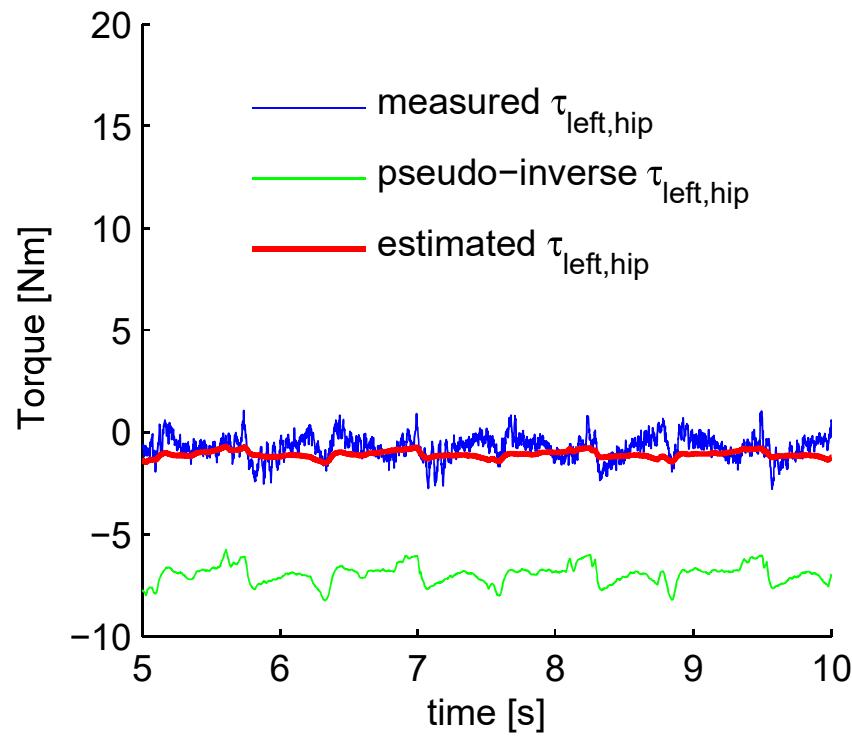
Joint torque at left ankle  $\tau_{L,\text{ankle}}$



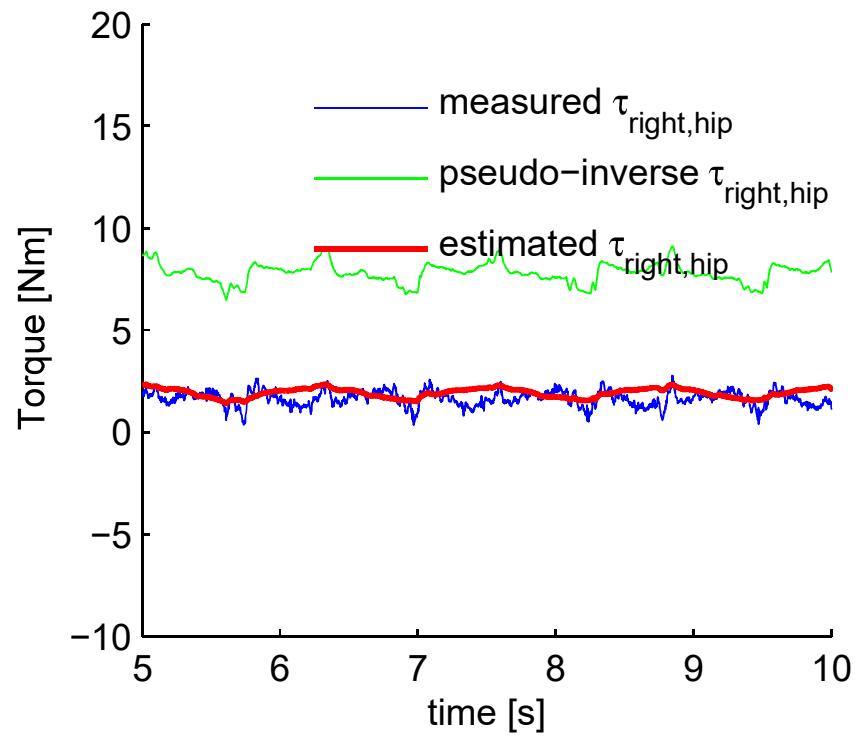
Joint torque at right ankle  $\tau_{R,\text{ankle}}$

# Results: Configuration 2

## Torques at the hip



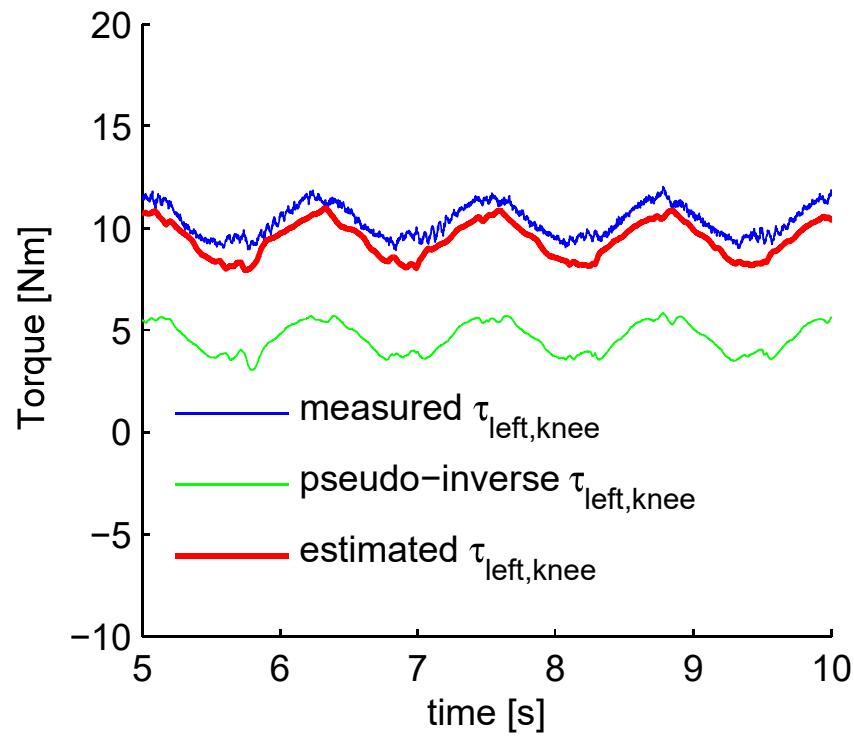
Joint torque at left hip  $\tau_{L,\text{hip}}$



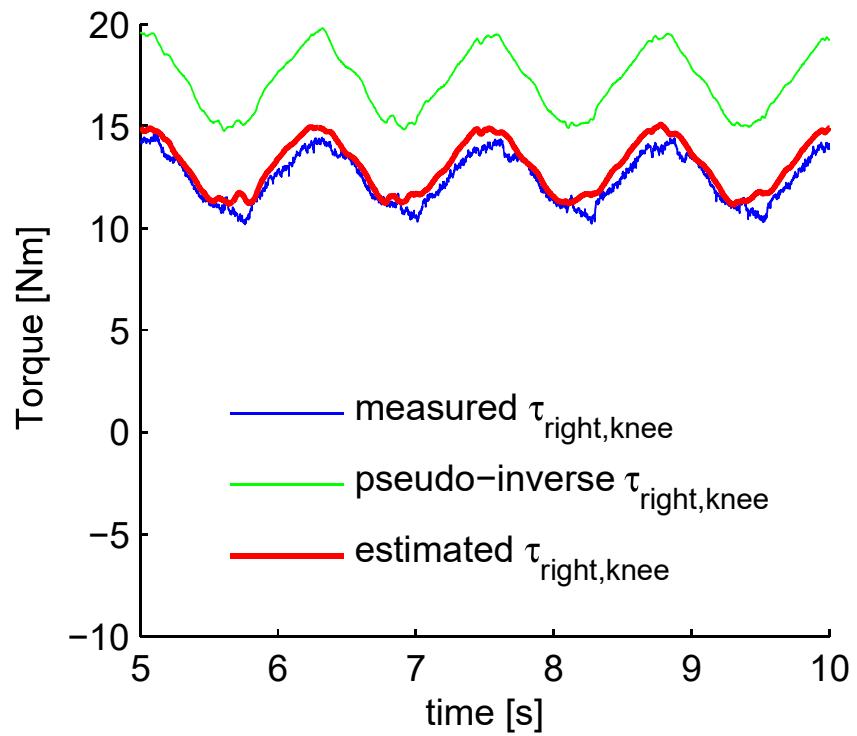
Joint torque at right hip  $\tau_{R,\text{hip}}$

# Results: Configuration 2

## Torques at the knee



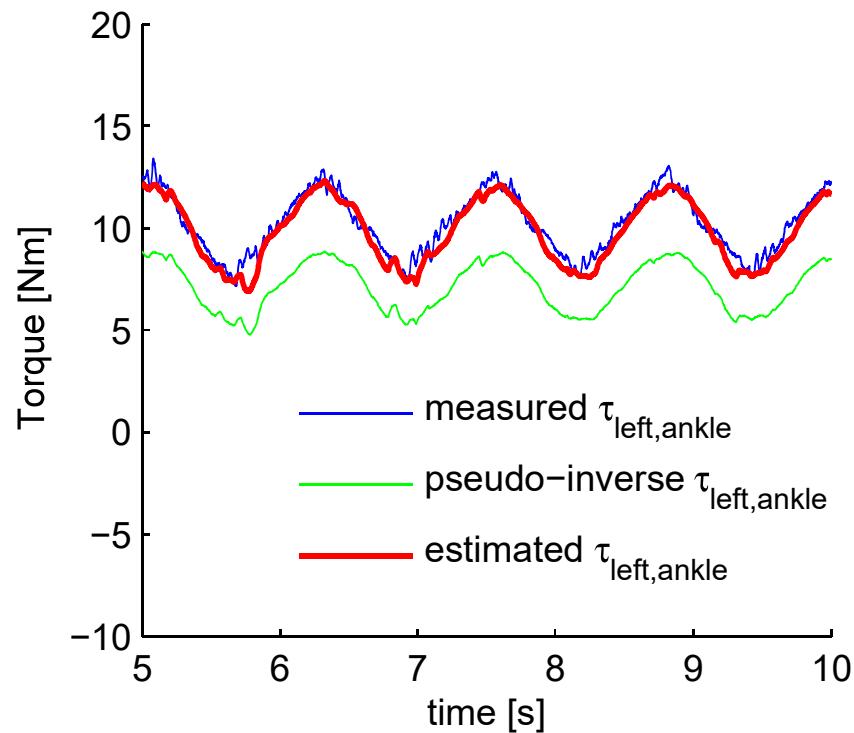
Joint torque at left knee  $\tau_{L,\text{knee}}$



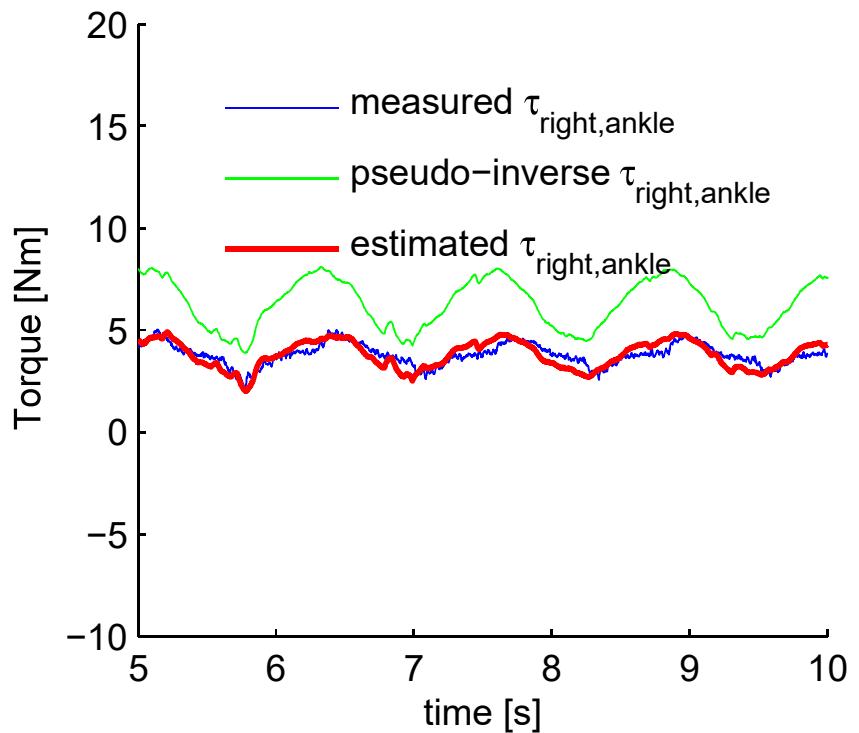
Joint torque at right knee  $\tau_{R,\text{knee}}$

# Results: Configuration 2

## Torques at the ankle



Joint torque at left ankle  $\tau_{L,\text{ankle}}$



Joint torque at right ankle  $\tau_{R,\text{ankle}}$

# Conclusion

- Under-determinacy can be resolved in a physically consistent way by taking the stiffness model into account
- The method is quick to execute in a real-time controller
- The GRF estimation is sensitive to modelling errors
- However, the effect of modelling errors are shown to be small
- The method is shown to work under the dynamic conditions
- Energy stored in the system is minimized  $\Rightarrow$  safe for human interaction

Thank you!