1062CH05

ARITHMETIC PROGRESSIONS

0.1 Introduction

You must have observed that in nature, many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple and on a pine cone etc.

We now look for some patterns which occur in our day-to-day life. Some such examples are :

(i) Reena applied for a job and got selected. She has been offered a job with a starting monthly salary of 8000, with an annual increment of 500 in her salary. Her salary (in) for the 1st, 2nd, 3rd, . . . years will be, respectively

8000, 8500, 9000, ...

(ii) The lengths of the rungs of a ladder decrease uniformly by 2 cm from bottom to top (see Fig. 5.1). The bottom rung is 45 cm in length. The lengths (in cm) of the 1st, 2nd, 3rd, . . ., 8th rung from the bottom to the top are, respectively

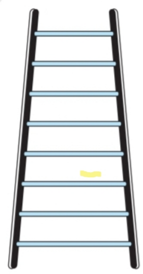

Fig. 5.1

45, 43, 41, 39, 37, 35, 33, 31

(iii) In a savings scheme, the amount becomes $\frac{5}{4}$ times of itself after every 3 years. The maturity amount (in) of an investment of 8000 after 3, 6, 9 and 12 years will be, respectively :

10000, 12500, 15625, 19531.25

(i) The number of unit squares in squares with side 1, 2, 3, . . . units (see Fig. 5.2) are, respectively

$$1^2, 2^2, 3^2, \ldots .$$

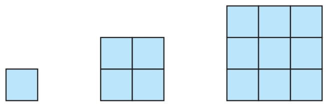

Fig. 5.2

(ii) Shakila puts ‘ 100 into her daughter’s money box when she was one year old and increased the amount by ‘ 50 every year. The amounts of money (in ‘) in the box on the 1st, 2nd, 3rd, 4th, . . . birthday were

$$100, 150, 200, 250, \ldots, \text{respectively.}$$

(iii) A pair of rabbits are too young to produce in their first month. In the second, and every subsequent month, they produce a new pair. Each new pair of rabbits produce a new pair in their second month and in every subsequent month (see Fig. 5.3). Assuming no rabbit dies, the number of pairs of rabbits at the start of the 1st, 2nd, 3rd, . . . , 6th month, respectively are:

$$1, 1, 2, 3, 5, 8$$

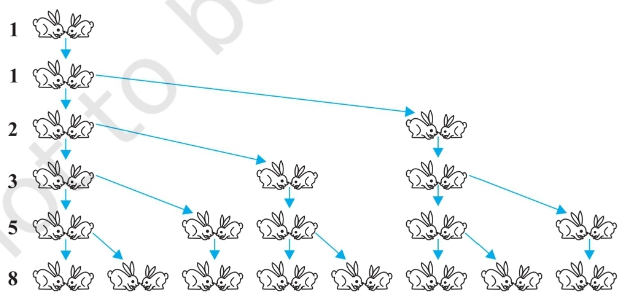

Fig. 5.3

In the examples above, we observe some patterns. In some, we find that the succeeding terms are obtained by adding a fixed number, in other by multiplying with a fixed number, in another we find that they are squares of consecutive numbers, and so on.

In this chapter, we shall discuss one of these patterns in which succeeding terms are obtained by adding a fixed number to the preceding terms. We shall also see how to find their nth terms and the sum of n consecutive terms, and use this knowledge in solving some daily life problems.

0.2 5.2 Arithmetic Progressions

Consider the following lists of numbers:

(i) 1, 2, 3, 4, ...

(ii) 100, 70, 40, 10, ...

(iii) $-3, -2, -1, 0, \ldots$

(iv) 3, 3, 3, 3, ...

(v) $-1.0, -1.5, -2.0, -2.5, \ldots$

Each of the numbers in the list is called a term.

Given a term, can you write the next term in each of the lists above? If so, how will you write it? Perhaps by following a pattern or rule. Let us observe and write the rule.

In (i), each term is 1 more than the term preceding it.

(i) The number of unit squares in squares with side 1, 2, 3, ... units (see Fig. 5.2) are, respectively

$$1^2, 2^2, 3^2, \ldots .$$

(ii) Shakila puts 100 into her daughter's money box when she was one year old and increased the amount by ‘ 50 every year. The amounts of money (in ‘) in the box on the 1st, 2nd, 3rd, 4th, ... birthday were

100, 150, 200, 250, ..., respectively.

(iii) A pair of rabbits are too young to produce in their first month. In the second, and every subsequent month, they produce a new pair. Each new pair of rabbits produce a new pair in their second month and in every subsequent month (see Fig. 5.3). Assuming no rabbit dies, the number of pairs of rabbits at the start of the 1st, 2nd, 3rd, ..., 6th month, respectively are:

Let us denote the first term of an AP by a_1, second term by a_2, . . . , nth term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, \ldots, a_n$.

So, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$.

Some more examples of AP are:

(a) The heights (in cm) of some students of a school standing in a queue in the morning assembly are 147, 148, 149, . . . , 157.

(b) The minimum temperatures (in degree celsius) recorded for a week in the month of January in a city, arranged in ascending order are −3.1, −3.0, −2.9, −2.8, −2.7, −2.6, −2.5

(c) The balance money (in rupees) after paying 5% of the total loan of 1000 every month is 950, 900, 850, 800, . . . , 50.

(d) The cash prizes (in rupees) given by a school to the toppers of Classes I to XII are, respectively, 200, 250, 300, 350, . . . , 750.

(e) The total savings (in rupees) after every month for 10 months when 50 are saved each month are 50, 100, 150, 200, 250, 300, 350, 400, 450, 500.

It is left as an exercise for you to explain why each of the lists above is an AP.

You can see that $a, a + d, a + 2d, a + 3d, \ldots$ represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an AP.

Note that in examples (a) to (e) above, there are only a finite number of terms. Such an AP is called a finite AP. Also note that each of these Arithmetic Progressions (APs) has a last term. The APs in examples (i) to (v) in this section, are not finite APs and so they are called infinite Arithmetic Progressions. Such APs do not have a last term.

Now, to know about an AP, what is the minimum information that you need? Is it enough to know the first term? Or, is it enough to know only the common difference? You will find that you will need to know both – the first term a and the common difference d.

For instance if the first term a is 6 and the common difference d is 3, then the AP is $6, 9, 12, 15, \ldots$ and if a is 6 and d is -3, then the AP is $6, 3, 0, -3, \ldots$

Similarly, when

$a = -7, d = -2$, the AP is $-7, -9, -11, -13, \ldots$

$a = 1.0, d = 0.1$, the AP is $1.0, 1.1, 1.2, 1.3, \ldots$

$a = 0, d = 1\frac{1}{2}$, the AP is $0, 1\frac{1}{2}, 3, 4\frac{1}{2}, 6, \ldots$

$a = 2, d = 0$, the AP is $2, 2, 2, 2, \ldots$

So, if you know what a and d are, you can list the AP. What about the other way round? That is, if you are given a list of numbers can you say that it is an AP and then find a and d? Since a is the first term, it can easily be written. We know that in an AP, every succeeding term is obtained by adding d to the preceding term. So, d found by subtracting any term from its succeeding term, i.e., the term which immediately follows it should be same for an AP.

For example, for the list of numbers: $6, 9, 12, 15, \ldots,$

We have $a_2 - a_1 = 9 - 6 = 3,$

$a_3 - a_2 = 12 - 9 = 3,$

$a_4 - a_3 = 15 - 12 = 3$

Here the difference of any two consecutive terms in each case is 3. So, the given list is an AP whose first term a is 6 and common difference d is 3.

For the list of numbers: $6, 3, 0, -3, \ldots,$

$a_2 - a_1 = 3 - 6 = -3$

$a_3 - a_2 = 0 - 3 = -3$

$a_4 - a_3 = -3 - 0 = -3$

Similarly this is also an AP whose first term is 6 and the common difference is -3.

In general, for an AP $a_1, a_2, \ldots, a_n$, we have $d = a_{k+1} - a_k$ where a_{k+1} and a_k are the $(k+1)$th and the kth terms respectively.

To obtain d in a given AP, we need not find all of $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$. It is enough to find only one of them.

Consider the list of numbers $1, 1, 2, 3, 5, \ldots$. By looking at it, you can tell that the difference between any two consecutive terms is not the same. So, this is not an AP.