

## Modular exponentiation with repeated squaring

Mod-Exp ( $a, b, n$ )

- 1  $c = 0$
- 2  $d = 1$
- 3 let  $\langle b_k, b_{k-1}, \dots, b_0 \rangle$  be the binary representation of  $b$
- 4 for  $i = k$  down to 0
- 5      $c = 2c$
- 6      $d = (d \cdot d) \bmod n$
- 7     if  $b_i = 1$
- 8          $c = c + 1$
- 9      $d = (d \cdot a) \bmod n$
- 10 return  $d$ .

If the inputs  $a, b$ , and  $n$  are  $\beta$ -bit numbers, then the total number of arithmetic operations required is  $O(\beta)$  and the total number of bit operations required is  $O(\beta^3)$ . Example:  $2^{(0)_2} \equiv 1 \pmod{10}$ ,  $2^{(1)_2} \equiv 2 \pmod{10}$ ,  $2^{(10)_2} \equiv 2^2 \equiv 4 \pmod{10}$ ,  $2^{(100)_2} \equiv 4^2 \equiv 6 \pmod{10}$ ,  $2^{(101)_2} \equiv 2 \cdot 6 \equiv 2 \pmod{10}$ ,  $2^{(1010)_2} \equiv 2^2 \equiv 4 \pmod{10}$ .