Control Theory 1

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Contents

Chapter 1

Introduction

What´s about Control Theory (CT)?
CT is:

- a basically engineering disciplin
- the science of influencing dynamic processes
- provides methodes for building controllers in the area of
  - process automation
  - process control engineering
  - general: for all technically influenced processes

The technology of CT consists of:

- Elektronic
- Hydraulic, Pneumatic

CT demands i.g. a mathematical-physical model of the process. A typically single control loop consists of:

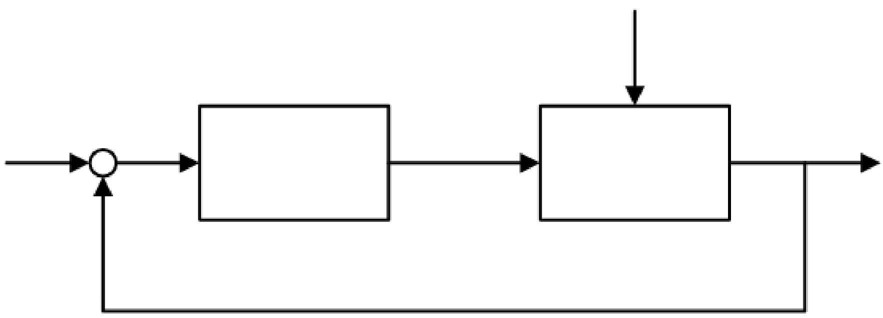


Figure 1.1: Control loop with (single) feedback path

There is no control without a feedback - only "steering", i.e. open loop control. Feedback control includes a ”return-line” . . . with danger of instability.

The job of control engineer:

1. the control loop must be stable. Because there is a feedback path in a control loop, the feedback system can make higher and higher oscillation, i.e. can be instable
2. the (steady state) error should go to zero, i.e. the actual value converges to the desired value, e.g.: voltage-control (also pressure-, temperatur-control)
3. the settling of the control loop should be "short"
4. the overshoot of the actual value should be "little"
5. the controller has to be realized and constructed

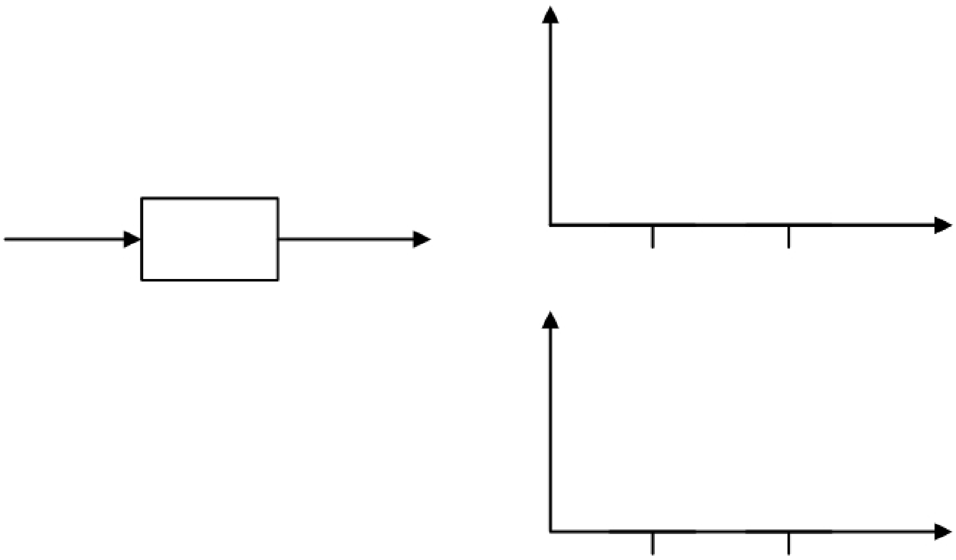


Figure 1.2: System step response

Example:

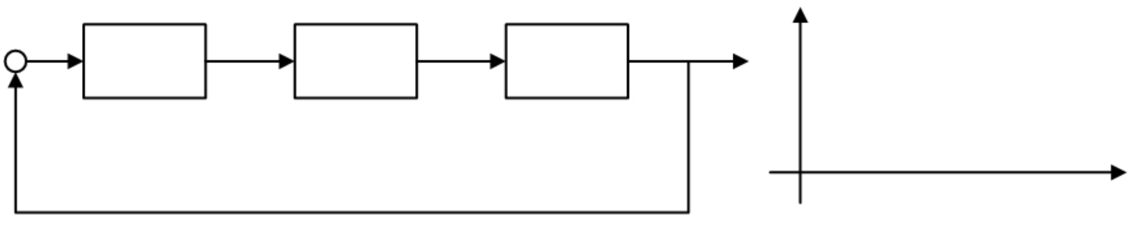


Figure 1.3: Loudspeaker-feedback

For establishing the stability of a (closed) control loop, we analyse the characteristic of the (open) control loop (cutted feeback) and forecast - by using certain stability tests (e.g.: RL, Nyquist) - to the stability of the closed control loop.
Question: Why don´t we establish the stability of a control loop in the closed form?

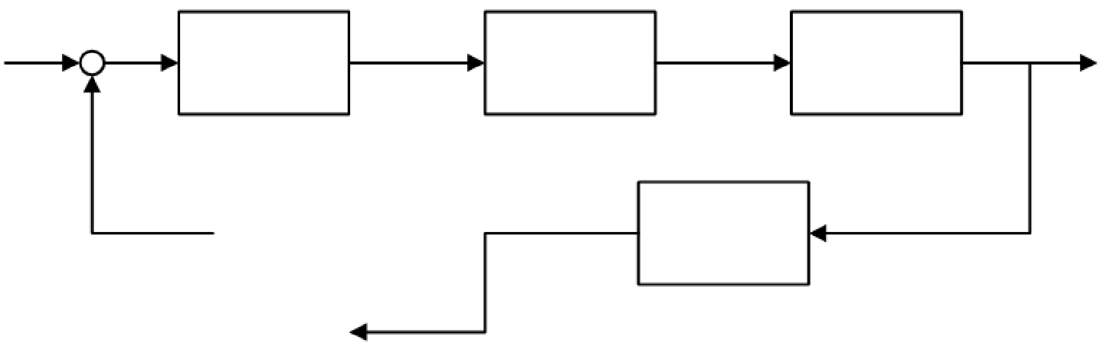


Figure 1.4: open control loop

Process *with* or *without* self-regulation:

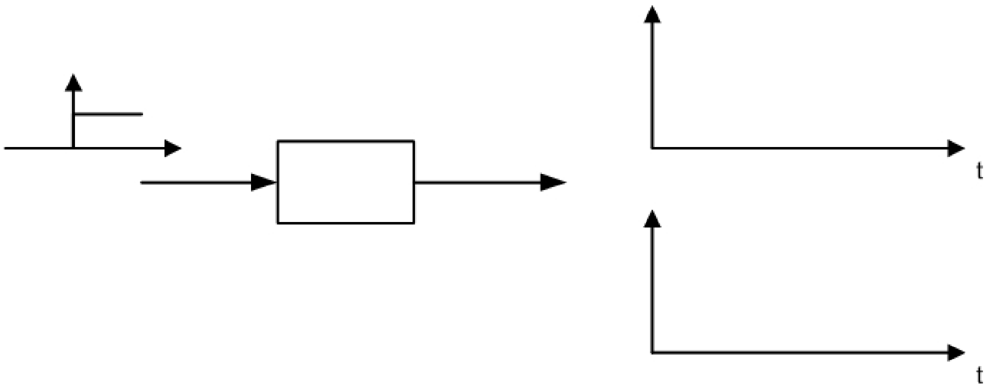


Figure 1.5: Different self-regulation

Exercise:

Dead-time T_t = 0,5 sek; w = 2

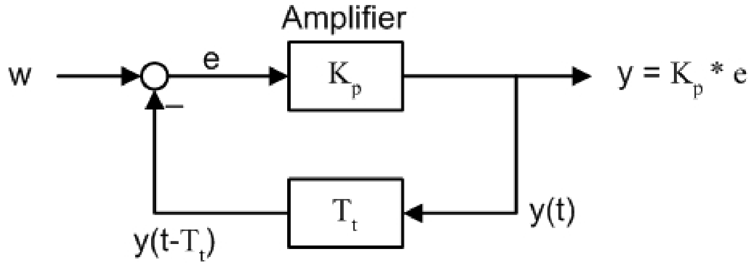


Figure 1.6: Control loop with dead-time system

Analyse the feedback system with K_p given:
$K_p = 0,95; 1; 1,05$

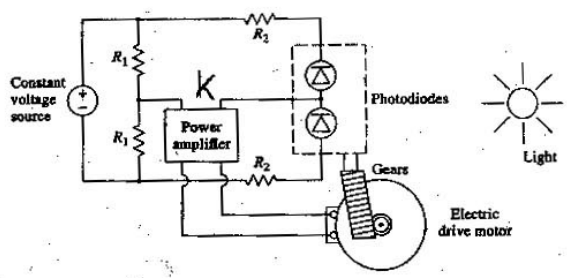
Exercise: Modelbuilding = Making abstraction

Modelbuilding

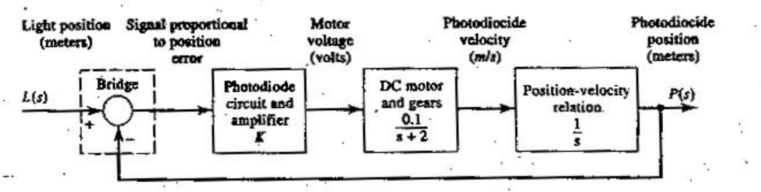
This example system is designed to follow, in one dimension, a moving light source. As pictured in Figure 4.26(a), when equal light intensities are detected by the two photodiodes, the electrical bridge is balanced, and zero voltage is applied to the drive motor. When one photodiode receives more light than the other, the bridge is unbalanced, and a nonzero voltage is amplified and applied to the drive motor, which then moves the photodiodes toward the equal-light-intensity position. Similar systems are used for precision machine tool alignment, where the light is reflected from a calibrated scale or transmitted through a tiny hole in the tool or the work. Variations of this system are used to track the sun or another star in navigation systems, to follow aircraft in collision avoidance systems, and to track the recording path on optical videodisks.

For small signals, a block diagram of the system is shown in Figure 4.26(b). The system transfer function is, in terms of the gain constant K,

$$T(s) = \frac{\dfrac{0.1K}{s(s+2)}}{1 + \dfrac{0.1K}{s(s+2)}} = \frac{0.1K}{s^2 + 2s + 0.1K}$$



(a)



(b)

Figure 4.26 Light-source tracking system. (a) Physical arrangement. (b) Block diagram model.

which is stable for all $K > 0$. A relative stability of two units for a system means that the natural component of system response decays with time as $\exp(-2t)$, that is, with a $\frac{1}{2}$-sec time constant. This degree of stability cannot be achieved with the system. This is evident from the root locus plot for Figure 4.27(a), where it is seen that the system's relative stability (the distance from the imaginary axis to the nearest pole) is always equal to one unit. System response to a unit step change in light position, for various representative values of K, are shown in Figure 4.27(b). For each, the settling time is relatively long, in consequence of the small degree of relative stability.

Adding position and velocity feedback.

The performance of this system can be improved substantially by the addition of velocity feedback as well as the position feedback. A tachometer coupled to the drive motor shaft will produce a voltage nearly proportional to the motor speed, which in turn is proportional to the photodiode velocity. Adding a fraction of this voltage to the bridge voltage (which is amplified to drive the motor) results in the

Figure 1.7: Sun tracking system

Chapter 2

Transfer function

A feedback system consists of dynamically systems (in opposite of: statically, e.g.: systems with nonlinear functions). Dynamical systems are mathematically described by differential-equations. This is because we *can´t* calculate feedback systems in the time domain.

Example:

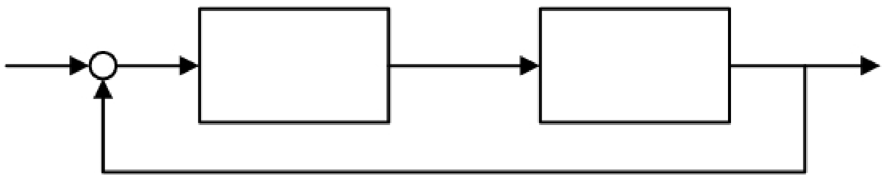


Figure 2.1: Unpossible analyse in the time domain

with $u = 1/5 \int e \Rightarrow \dot{y} - 4y = 1/5 \int e$
and $e = w - y$
there is $\dot{y} - 4y = 1/5 \int (w - y)dt$

Problem: Calculate y(t), if w(t) - desired value - the input signal is given.

$\Rightarrow$ y(t)=.....?

In general *it´s not* possible to solve the problem analytically. However we get help by a mathematical trick: the Laplace-Transformation. By using this we get rid of the time domain.

2.1 Laplace-Transformation, Transferfunction

When we do the $\mathscr{L}$-Transformation for a differential-equation we obtain simple algebraical equations. The $\mathscr{L}$-Transformation is defined as:

$$\mathscr{L}\{y(t)\} = \int\limits_0^\infty y(t)e^{-pt}dt \tag{2.1}$$

$$Shortform : \mathscr{L}\{y(t)\} = Y(p) \tag{2.2}$$

$$With\ p = \sigma + jw$$

Now we use the Laplace-Transformation for a differentiated function $\dot{y}(t)$

$$\mathscr{L}\{\dot{y}(t)\} = \int\limits_0^\infty \dot{y}(t)e^{-pt}\ dt$$

using partial integration $\int uv' = uv - \int u'v\ dt$ we obtain

$$\begin{aligned}
\Rightarrow \mathscr{L}\{\dot{y}(t)\} &= y(t)e^{-pt}\Big|_0^\infty - \int\limits_0^\infty -p\ e^{-pt}y(t)\ dt \\
\Rightarrow \mathscr{L}\{\dot{y}(t)\} &= 0 - y(t=0)\ +\ p\underbrace{\int\limits_0^\infty y(t)e^{-pt}\ dt}_{\mathscr{L}\{y(t)\}=Y(p)} \\
\Rightarrow \mathscr{L}\{\dot{y}(t)\} &= p\ \underbrace{Y(p)}_{Laplacefunction}\ -\ \underbrace{y(t=0)}_{initial\ value\ von\ y(t)}
\end{aligned}$$

As you can see, there is no differentiated function. If the initial value is set to zero (this doesn´t change the dynamical character of a system, of course) => $y(t = 0) = 0$ we obtain

$$\mathscr{L}\{\dot{y}(t)\} = p\ Y(p)$$

When we´d like to transform the second derivation, i.e. $\mathscr{L}\{\ddot{y}(t)\}$ we obtain (using partial integration again)

$$\mathscr{L}\{\ddot{y}(t)\} = p^2\ Y(p) - p\ y(t=0) - \dot{y}(t=0)$$

When initial values are set to zero, we obtain

$$\mathscr{L}\{\ddot{y}(t)\} = p^2\ Y(p)$$

When we repeat this procedure for the n-th derivation we see an easy relation between the time-domain and the (algebraical) Laplace-domain

$$\mathscr{L}\{y^n(t)\} = p^n \; Y(p) \tag{2.3}$$

REMARK: The Laplace-Transformation changes differential equations into algebraical equations.

Example:

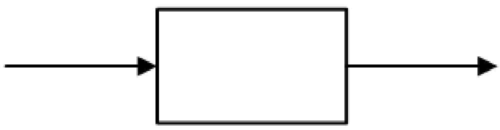


Figure 2.2: Transferfunction

$$\begin{aligned} H(s) &= 5\dddot{\vartheta} - 0,8\ddot{\vartheta} + 3,2\vartheta = u + 7\dot{u} \\ temperatur &= \vartheta \\ voltage &= u \end{aligned}$$

Application of the Laplace-Transformation:

$$\begin{aligned} &\Rightarrow \;\; \mathscr{L}\{5\dddot{\vartheta} - 0,8\ddot{\vartheta} + 3,2\vartheta\} = \mathscr{L}\{u + 7\dot{u}\} \\ &\Rightarrow \;\; 5p^3\vartheta(p) - 0,8p^2\vartheta(p) + 3,2\vartheta(p) = u(p) + 7p\;u(p) \\ &\Rightarrow \;\; \vartheta(p)(5p^3 - 0,8p^2 + 3,2) = u(p)\;(1 + 7p) \\ &\Rightarrow \;\; \vartheta(p) \;=\; \underbrace{\frac{1 + 7p}{5p^3 - 0,8p^2 + 3,2}}_{Transferfunction\;\,F(p)} u(p) \\ &\Rightarrow \;\; F(p) = \frac{\vartheta(p)}{u(p)} \end{aligned} \tag{2.4}$$

When u(t) is e.g. a step-function (like a switching on of power) as shown in the figure, you *needn´t* put the time-function as U(p), but the transformed function, of course.

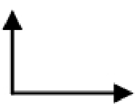


Figure 2.3: Step function

The step function $u(t) = u_0\;\sigma(t)$ is related to the time domain.
Hence:

$$\begin{aligned} \mathscr{L}\{u(t)\} &= \int_0^\infty u(t)e^{-pt}\;dt \\ u(t = 0 \to \infty) = \;\; constant &= U_0 \end{aligned}$$

$$\Rightarrow \int_0^\infty U_0 e^{-pt}\, dt = -U_0 \frac{1}{p}\, e^{-pt}\Big|_0^\infty = -\frac{U_0}{p}(0-1) = \frac{U_0}{p}$$

$$\Rightarrow \mathscr{L}\{u(t) = U_0\ \sigma(t)\} = \frac{1}{p}\ U_0 \tag{2.5}$$

$$Sigmafunction\ \sigma(t) \quad \circ\!\!-\!\!\bullet \quad \frac{1}{s} \tag{2.6}$$

$$Ramp\ t \quad \circ\!\!-\!\!\bullet \quad \frac{1}{s^2} \tag{2.7}$$

$$Delta - Impuls\ \delta(t) \quad \circ\!\!-\!\!\bullet \quad 1 \tag{2.8}$$

$$Timedelay\ \delta(t-t_0) \quad \circ\!\!-\!\!\bullet \quad 1\ e^{-pt_0} \tag{2.9}$$

Korrespondenztabelle

Nr.	$f(s)$	$f(t)$ (für $t < 0$ ist $f(t) = 0$)
1	1	$\delta(t) = \begin{cases} \infty & \text{für } t = 0 \\ 0 & \text{für } t \neq 0 \end{cases}$
2	$\frac{1}{s}$	1
3	$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$
4	$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
5	$\frac{1}{s(s+\alpha)}$	$\frac{1}{a}\left(1-e^{-\alpha t}\right)$
6	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$
7	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$
8	$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{e^{-\beta t}-e^{-\alpha t}}{\alpha-\beta}$
9	$\frac{1}{(s+\alpha)^n}$ für $n > 0$	$\frac{t^{n-1}}{(n-1)!} \cdot e^{-\alpha t}$
10	$\frac{1}{s(s+\alpha)^n}$	$\frac{1}{a^n}\left[1-\left(\sum_{v=0}^{n-1}\frac{(\alpha t)^v}{v!}\right)\cdot e^{-\alpha t}\right]$
11	$\frac{1}{s^2+s\cdot 2\alpha+\beta^2}$	$\frac{1}{2w}\cdot\left(e^{s_1 t}-e^{s_2 t}\right)$ $D=\frac{\alpha}{\beta}>1$ $\frac{1}{\omega}\cdot e^{-\alpha t}\cdot\sin\omega t$ $(D<1)$
12	$\frac{s}{s^2+s\cdot 2\alpha+\beta^2}$	$\frac{1}{2w}\cdot\left(s_1 e^{s_1 t}-s_2 e^{s_2 t}\right)$ $D=\frac{\alpha}{\beta}>1$ $e^{-\alpha t}\cdot\left(\cos\omega t-\frac{\alpha}{\omega}\cdot\sin\omega t\right)$ $(D<1)$
13	$\frac{1}{s(s^2+s\,2\alpha+\beta^2)}$	$\frac{1}{\beta^2}\cdot\left(1+\frac{s_2}{2w}\cdot e^{s_1 t}-\frac{s_1}{2w}\cdot e^{s_2 t}\right)$ $D=\frac{\alpha}{\beta}>1$ $\frac{1}{\beta^2}\cdot\left[1-(\cos\omega t+\frac{\alpha}{\omega}\cdot\sin\omega t)\cdot e^{-\alpha t}\right]$ $(D<1)$

In den Beziehungen 10, 11 und 12 ist: $w=\sqrt{\alpha^2-\beta^2}$; $\omega=\sqrt{\beta^2-\alpha^2}$; $s_{1,2}=-\alpha\pm w=-\alpha\pm j\omega$

2.2 Signal flow graphs (Block diagram algebra) Final value theorem, Example: Aircraft roll control

Linearized control loops are *not* analyzed and designed in the time domain, but in the Laplace domain.

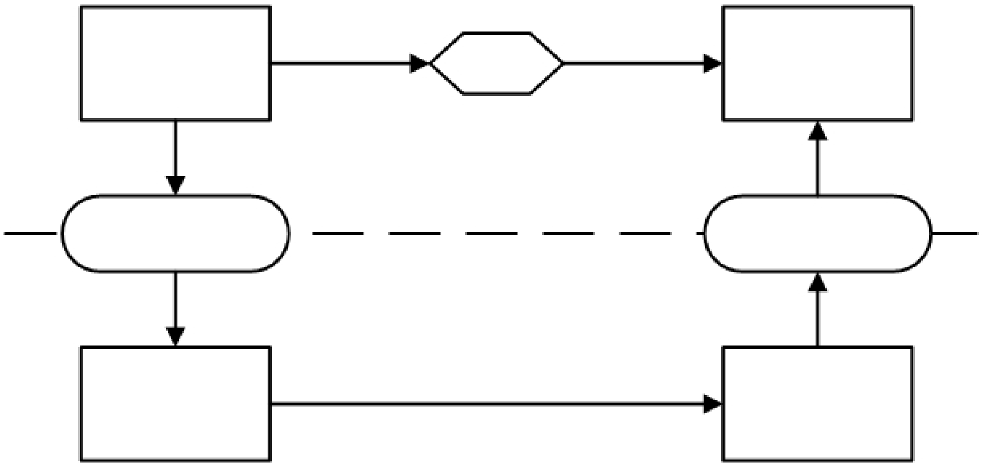


Figure 2.4: Time domain and Laplace domain

The following block diagrams are very frequently:

(a) In series (in cascade):

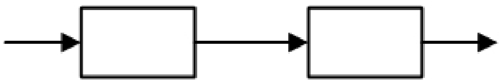


Figure 2.5: In cascade

Question: What´s the relation between $x_2(p)$ and $x_1(p)$?
We see:

$$\begin{aligned} x_2 &= F_2\ x_h \\ x_h &= F_1\ x_1 \\ \Rightarrow x_2 &= F_2\ F_1\ x_1 \end{aligned}$$

REMARK: Transfer functions must be multiplied, if systems are connected in series (cascade).

(b) In parallel (in tandem):

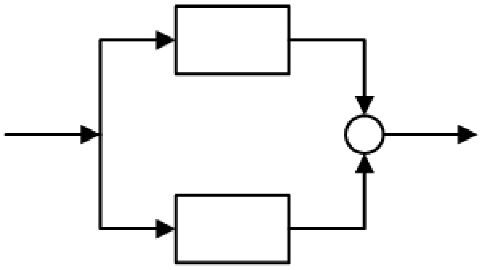


Figure 2.6: In tandem

$$
\begin{aligned}
x_2 &= F_1\, x_1 + F_2\, x_1 \\
\Rightarrow x_2 &= x_1\, (F_1 + F_2)
\end{aligned}
$$

REMARK: Transfer functions must be added, if systems are connected in parallel (tandem).

(c) Feedback configuration:

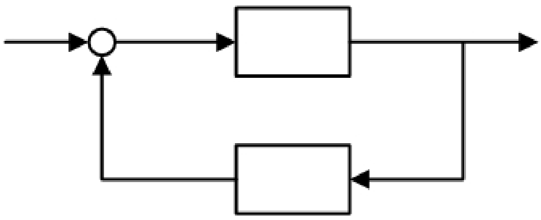
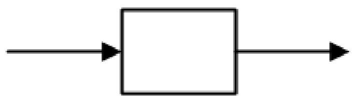


Figure 2.7: Feedback configuration

$$
\begin{aligned}
e &= x_1 - F_2\, x_2 \\
x_2 &= F_1\, e \\
x_2 &= F_1\, (x_1 - F_2\, x_2) \\
x_2 &= F_1\, x_1 - F_1\, F_2\, x_2 \\
x_2(1 + F_1\, F_2) &= F_1\, x_1 \\
\Rightarrow x_2 &= \frac{F_1}{1 + F_1 F_2}\, x_1
\end{aligned}
$$



F_1 is called forward transmittance, F_2 feedback transmittance in this arrangement. A negative (positive) sign on the feedback summation results in a positive (minus) algebraic sign in the denominator.

Final value theorems

If you aren´t interested in the transient response y(t) for all time t = 0.....∞, but only in the final value you can calculate this by using the final value theorem of Laplace.

We receive this theorem by:

$$\mathscr{L}\{\dot{y}(t)\} = \int_0^\infty \dot{y}(t)e^{-pt}\,dt = p\,Y(p) - y(t=0)$$

Let p converge to zero

$$\begin{aligned}
&\Rightarrow \quad \int_0^\infty \dot{y}(t)dt = \lim_{p\to 0}\; p\,Y(p) - y(t=0)\\
&with \quad y(t)\Big|_0^\infty = y(t\to\infty) - y(0)\\
&\Rightarrow \quad y(t\to\infty) - y(0) = \lim_{p\to 0}\; p\,Y(p) - y(0)\\
&\Rightarrow \quad y(t\to\infty) = \lim_{p\to 0}\; p\,Y(p)
\end{aligned}$$

Final value theorem:

$$y(t\to\infty) = \lim_{p\to 0}\; p\,Y(p) \tag{2.10}$$

Example:

$$\frac{\varphi_i}{\varphi_s} = \frac{7,2}{21,6 + 7,2p + 9,8p^2 + p^3} \qquad aircraftrollcontrolwithk = 0,5$$

Calculate the final value when φ_s is a unit step input.

Exercise:

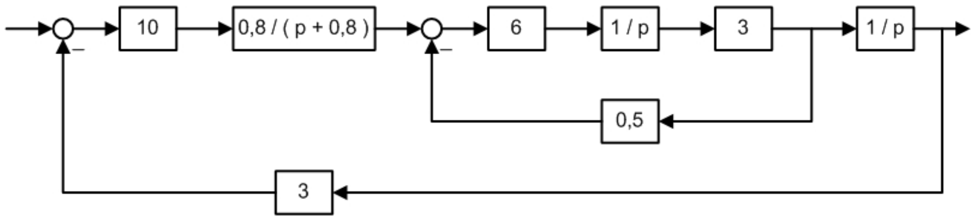


Figure 2.8: Aircraft roll control

Calculate the transfer function of $\frac{\varphi_i}{\varphi_s}$

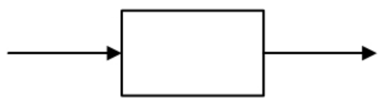
2.3 Relation: Laplace- and time domain ("Eigen" or natural response, forced response)

Laplace functions are introduced to make mathematical solutions easier. They are the "key" to enter a "room" in reality, but they aren't the room itself. So each Laplace function corresponds to a time function. Only an insider knows the meaning of Laplace-Functions in real world i.e. in the time domain.

So we have to learn directly the meaning of Laplace functions without translating them into the time domain.

First we analyze system responses without input signals, because there are non zero initial conditions, and second with an unit step as input.

(a) Natural response (Eigen-response):



If w=0 and the mass-spring system isn´t let in rest (e.g. you lift the mass by your hand or pull it down) the system response shows the natural response (swinging up and down, undamped). Each system has it´s own eigen-response or natural response or resonance frequency. The natural response is obtained mathematically by setting the input signal to zero and solving the given differential equation for non zero initial conditions.

Exercise 1:

$$\frac{x_a}{x_e} = \frac{1}{p}$$

Exercise 2:

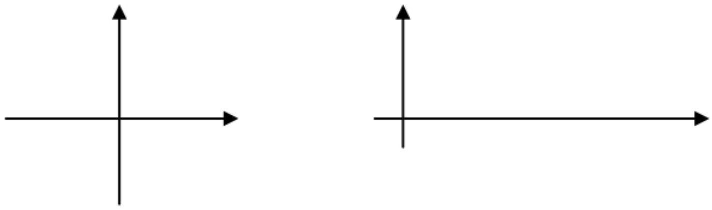
$$\frac{x_a}{x_e} = \frac{k}{1 + p\ T}$$

Exercise 3:

$$\frac{x_a}{x_e} = \frac{1}{p^2 + w^2}$$

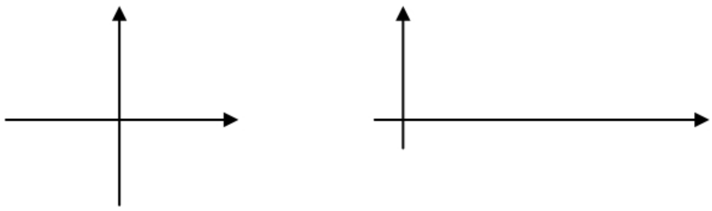
Exercise 1 shows:

$$F(p) = \frac{1}{p}$$



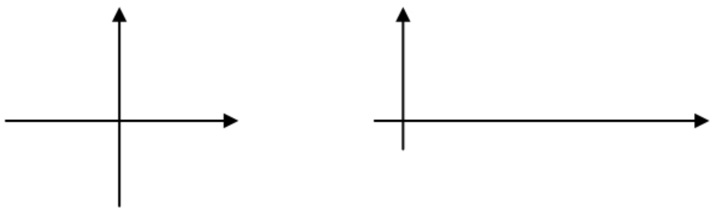
Exercise 2 shows:

$$F(p) = \frac{k}{1 + p\ T}$$



Exercise 3 shows:

$$F(p) = \frac{1}{p^2 + w^2}$$



So we see:

When systems have real poles on the left hand side (in the p-plane) they will show monoton decreasing natural responses. When there are real poles on the right hand side it will respond with monoton increasing natural responses.

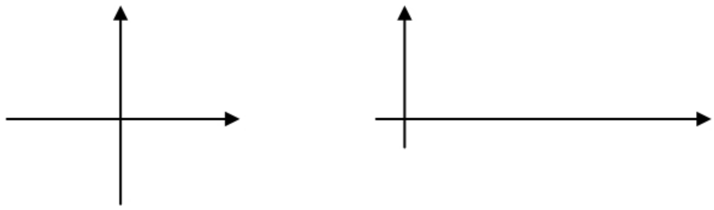


Figure 2.9: Real poles

When systems have complex poles on the left hand side (in the p-plane) they will show oscillatory decreasing natural responses. When there are complex poles on the right hand side it will respond with oscillatory increasing natural responses.

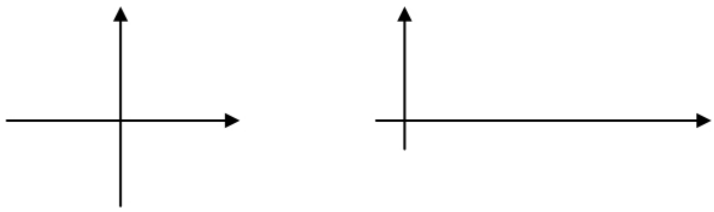


Figure 2.10: Complex poles

And: the greater the distance between the poles and the imaginary axes the quicker the system response.

6.7 *Eigenschaften wichtiger Übertragungsglieder im Frequenzbereich* 205

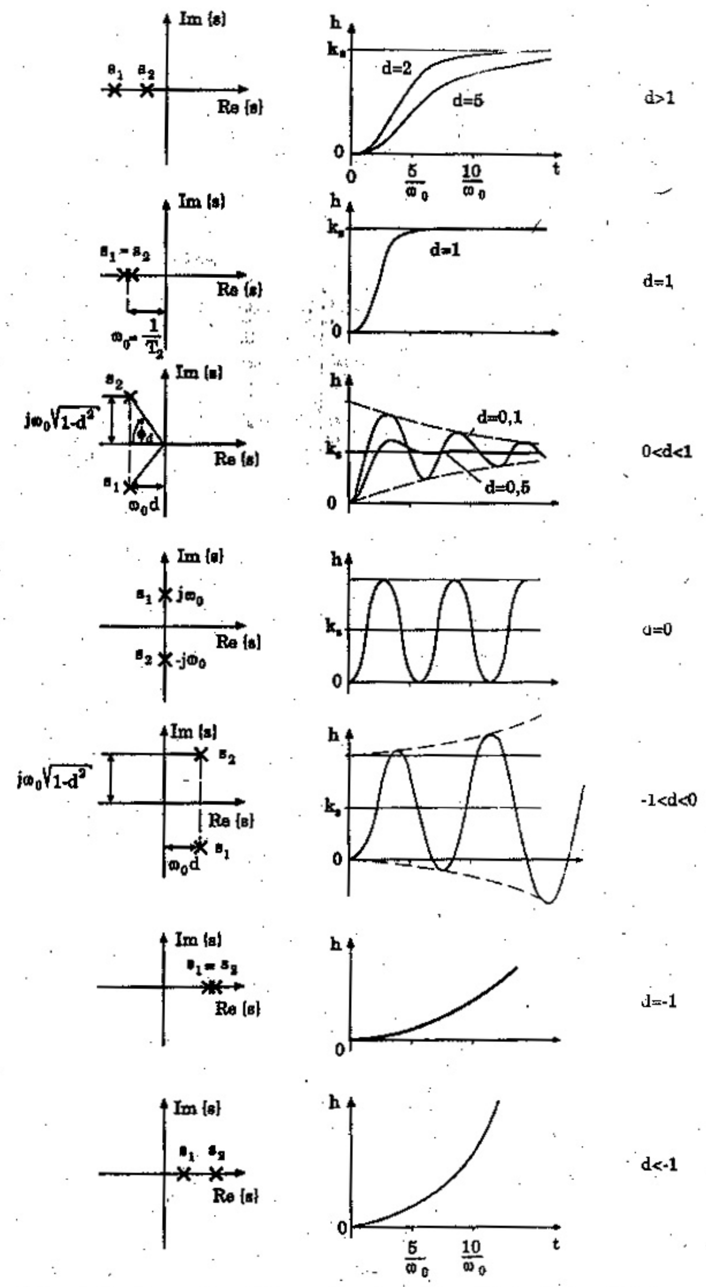


Abb. 6.33: Lage der Pole und prinzipieller Verlauf der Übergangsfunktion eines PT_2–Gliedes

Figure 2.11: Relation: Poles - Natural responses

(b) Forced response:

Let´s consider now to systems with input signals, e.g. an unit step impulse. They show us forced responses, because their natural behavior can´t open.

Exercise 1:

$$\frac{x_a}{x_e} = \frac{1}{p}$$

Exercise 2:

$$\frac{x_a}{x_e} = \frac{1}{1+p}$$

Exercise 3:

$$\frac{x_a}{x_e} = \frac{1}{1+T_1p+T_2p^2}$$

2.4 Systemtypes, Step response, Impulse function

Step response = reaction of a system with unit step signal as input.

Impulse function = reaction of a system with an δ-impulse as input.

The type letters P, I, D are understood to design a system type in an additiv way not as factors or multiplieres.

Delaying systems are designed by the letters $(T_1, T_2, ...)$ in a factorial way.

Exercise:

Systemtypes:

(a) Draw the block diagramm of a PIT_1 system.

(b) Specify the system types of the following transfer functions.

$$F_1 = \frac{2}{1+4\ p}\ \frac{1}{p}$$

$$F_2 = \frac{3+2\ p}{p}$$

$$F_3 = (5+2\ p)\ \frac{1}{3+5\ p}$$

Tabelle der wichtigsten Regelkreisglieder

	Regel-kreis-glied	Differential-gleichung	Frequenz-gang
1	P	$x_a = K_P \cdot x_e$	$F = \frac{x_a}{x_e} = K_P$
2	P_{T1}	$T_1 \cdot \frac{dx_a}{dt} + x_a = K_P \cdot x_e$	$\frac{K_P}{1+T_1 \cdot p}$
3	P_{T_2}	$T_2^2 \cdot \frac{d^2x_a}{dt^2} + T_1 \frac{dx_a}{dt} + x_a = K_P x_e$	$\frac{K_P}{1+T_1 p + T_2^2 p^2}$
4	I	$x_a = K_I \cdot \int x_e \cdot dt$	$\frac{K_I}{p}$
5	I_{T_1}	$T \cdot \frac{dx_a}{dt} + x_a = K_I \cdot \int x_e \cdot dt$	$\frac{K_I}{p(1+T \cdot p)}$
6	D	$x_a = K_D \cdot \frac{dx_e}{dt}$	$K_D \cdot p$
7	D_{T_1}	$T \cdot \frac{dx_a}{dt} + x_a = K_D \cdot \frac{dx_e}{dt}$	$\frac{K_D \cdot p}{1+T \cdot p}$

Sprung-antwort

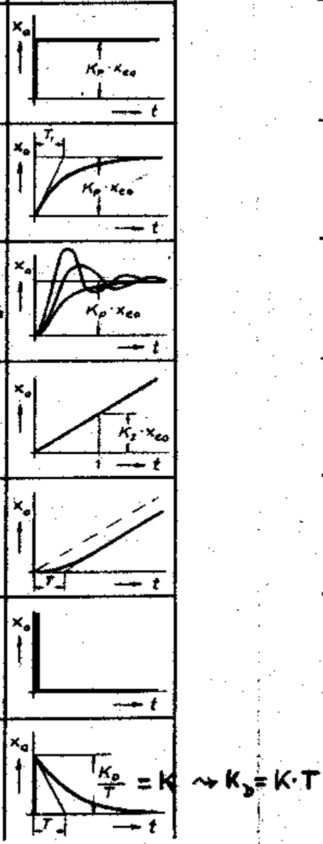
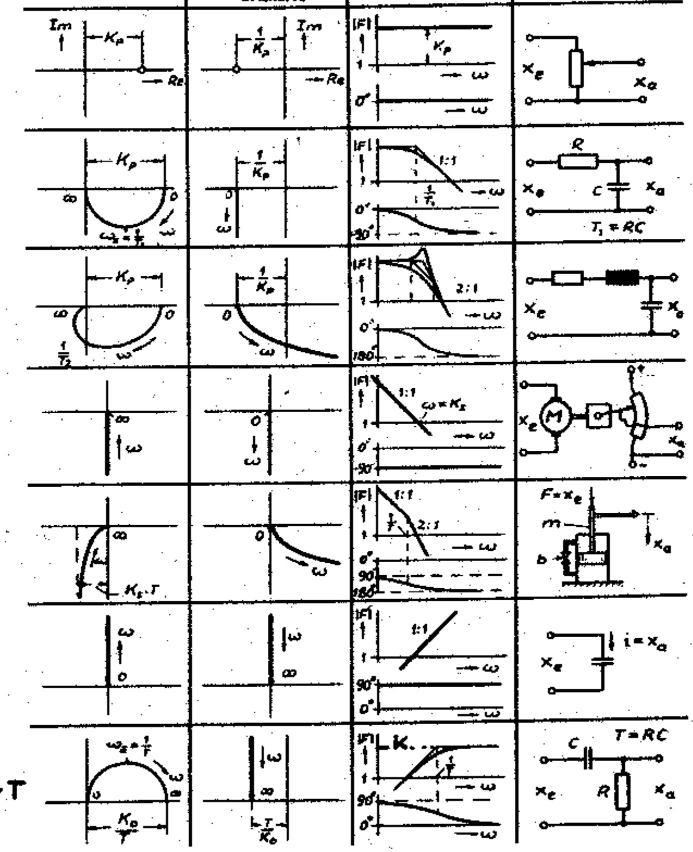


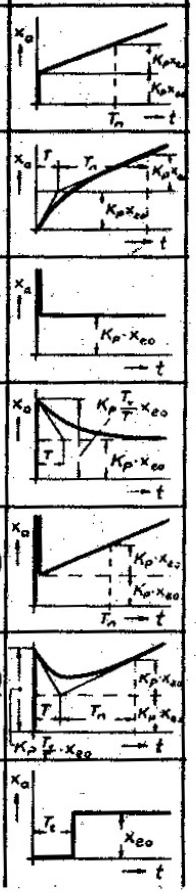
Tabelle der wichtigsten Regelkreisglieder

Ortskurve | Negativ-inverse Ortskurve | Bode-diagramm | Techn. Beispiele

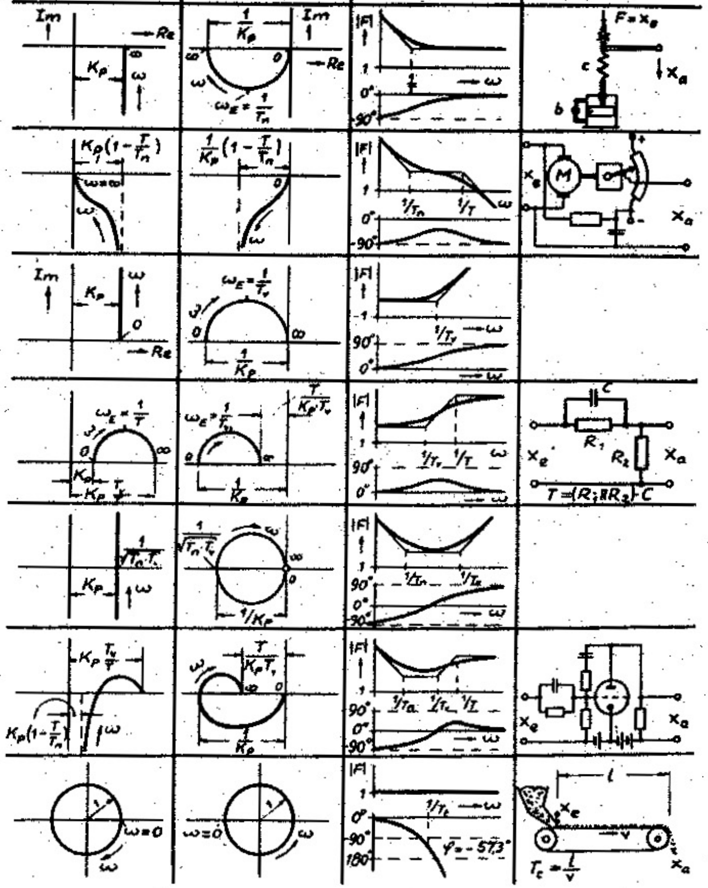


	Regelkreisglied	Differentialgleichung	Frequenzgang
8	PI	$x_a = K_P \cdot x_e + K_I \cdot \int x_e \cdot dt$ $x_a = K_P \left[x_e + \frac{1}{T_n} \int x_e dt \right]$	$K_P(1 + \frac{1}{T_n \cdot p})$
9	PI_{T_1}	$T \cdot \frac{dx_a}{dt} + x_a = K_P x_e + K_I \int x_e \cdot dt$ $T \cdot \frac{dx_a}{dt} + x_a = K_P \left[x_e + \frac{1}{T_n} \int x_e dt \right]$	$\frac{K_P(1 + \frac{1}{T_n \cdot p})}{1 + T \cdot p}$
10	PD	$x_a = K_P \cdot x_e + K_D \cdot \frac{dx_e}{dt}$ $x_a = K_P \left[x_e + T_v \cdot \frac{dx_e}{dt} \right]$	$K_P(1 + T_v \cdot p)$
11	PD_{T_1}	$T \cdot \frac{dx_a}{dt} + x_a = K_P \cdot x_e + K_D \cdot \frac{dx_e}{dt}$ $T \cdot \frac{dx_a}{dt} + x_a = K_P \left[x_e + T_v \cdot \frac{dx_e}{dt} \right]$	$\frac{K_P(1 + T_v \cdot p)}{1 + T \cdot p}$
12	PID	$x_a = K_P \cdot x_e + K_I \int x_e dt + K_D \frac{dx_e}{dt}$ $x_a = K_P \left[x_e + \frac{1}{T_n} \int x_e dt + T_v \frac{dx_e}{dt} \right]$	$K_P(1 + \frac{1}{T_n p} + T_v p)$
13	PID_{T_1}	$T \cdot \frac{dx_a}{dt} + x_a$ $= K_P \left[x_e + \frac{1}{T_n} \int x_e dt + T_v \frac{dx_e}{dt} \right]$	$\frac{K_P(1 + \frac{1}{T_n p} + T_v p)}{1 + T \cdot p}$
14	T_t	$x_a(t) = x_e(t - T_t)$	e^{-pT_t}

Sprungantwort



Ortskurve | Negativ-inverse Ortskurve | Bodediagramm | Techn. Beispiel



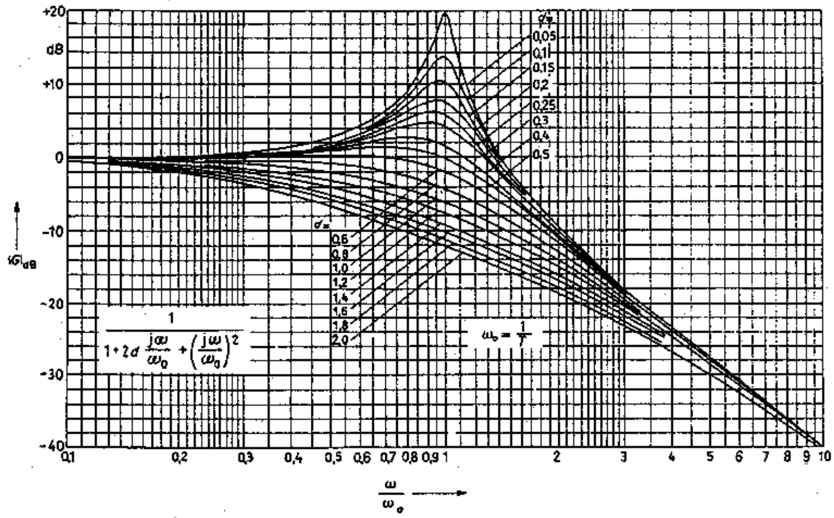
Das Verzögerungssystem 2. Ordnung

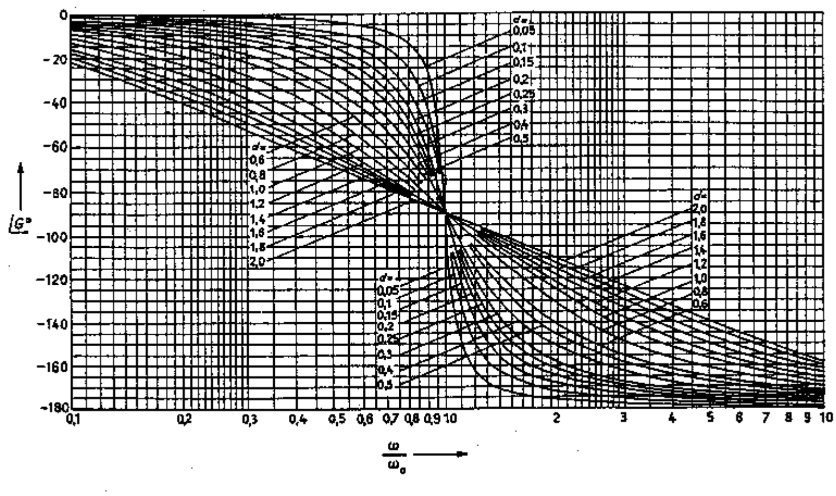
$$G(j\omega) = \frac{K}{1 + 2dj\dfrac{\omega}{\omega_0} + \left(j\dfrac{\omega}{\omega_0}\right)^2},$$

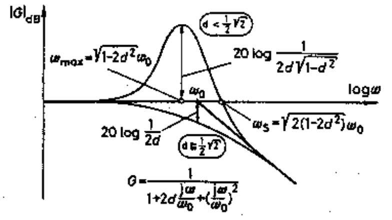
$$G(j\omega) = \frac{K}{1 + 2dTj\omega + T^2(j\omega)^2}$$

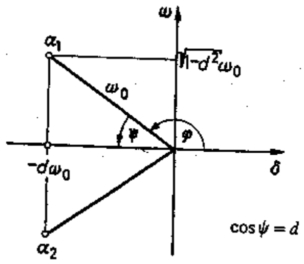
Laplace-Übertragungsfunktion

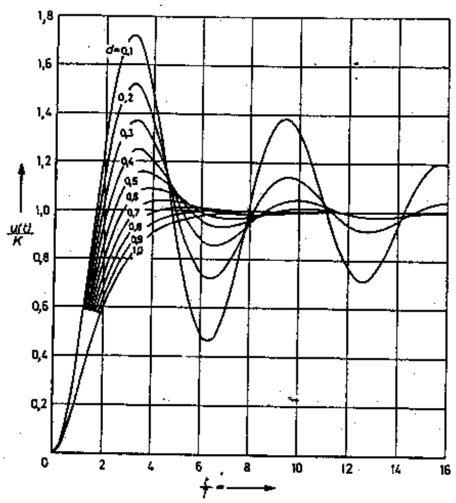
$$G(p) = \frac{K}{1 + 2dTp + T^2p^2}$$

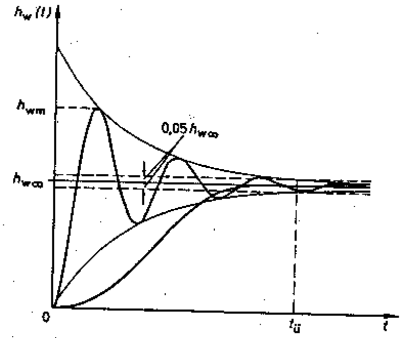








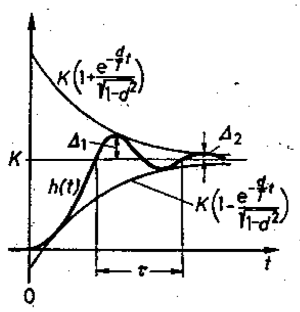




$$t_ü = \frac{1}{d\omega_0}\left(3 + \frac{1}{2}d^2\right)$$

genügende Näherung

$$t_ü \approx \frac{3}{d\omega_0} = 3\,\frac{T}{d}.$$



$$\vartheta = \ln \frac{\Delta_1}{\Delta_2} = 2{,}303 \log_{10} \frac{\Delta_1}{\Delta_2}.$$

Dann ist

$$d = \frac{\vartheta}{\sqrt{4\pi^2 + \vartheta^2}}.$$

2.5 Stability of control loops

Stability of systems will be analyzed without input signals, only with initial displacement or value. So, a system is called ”stable”, if its natural response decreases, i.e. if its poles lies in the left hand side in the s-plan.

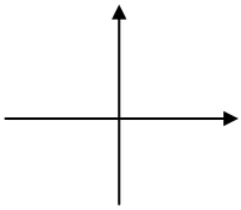


Figure 2.12: Stability

Exercise:

An unstable system (inverted pendelum) has to be stabilized by control.

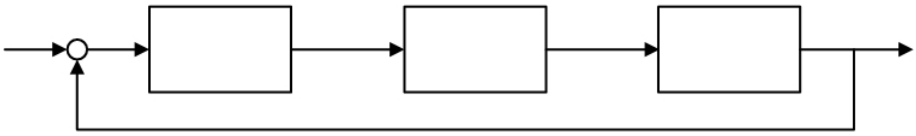


Figure 2.13: Control loop of an inverted pendelum

Exercise:

Stability:

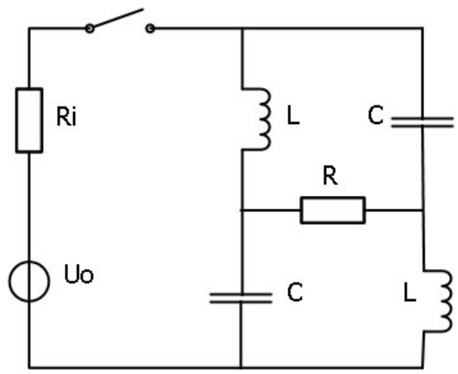
Are the following systems stable, i.e. which is the corresponding condition for T?

$$F_1 = \frac{5 + 0,3p}{2 + 5p + 0,7p^2} \; \frac{1}{p}$$

$$F_2 = \frac{3p}{2 + T\ p} \qquad T = ?\ (for\ a\ stabilized\ system)$$

Exercise:

Calculate the transfer function $\frac{U_d}{U_o} = ?$



National Semiconductor

November 2000

LM35
Precision Centigrade Temperature Sensors

General Description

The LM35 series are precision integrated-circuit temperature sensors, whose output voltage is linearly proportional to the Celsius (Centigrade) temperature. The LM35 thus has an advantage over linear temperature sensors calibrated in ˚ Kelvin, as the user is not required to subtract a large constant voltage from its output to obtain convenient Centigrade scaling. The LM35 does not require any external calibration or trimming to provide typical accuracies of ±¼˚C at room temperature and ±¾˚C over a full −55 to +150˚C temperature range. Low cost is assured by trimming and calibration at the wafer level. The LM35's low output impedance, linear output, and precise inherent calibration make interfacing to readout or control circuitry especially easy. It can be used with single power supplies, or with plus and minus supplies. As it draws only 60 µA from its supply, it has very low self-heating, less than 0.1˚C in still air. The LM35 is rated to operate over a −55˚ to +150˚C temperature range, while the LM35C is rated for a −40˚ to +110˚C range (−10˚ with improved accuracy). The LM35 series is available packaged in hermetic TO-46 transistor packages, while the LM35C, LM35CA, and LM35D are also available in the plastic TO-92 transistor package. The LM35D is also available in an 8-lead surface mount small outline package and a plastic TO-220 package.

Features

- Calibrated directly in ˚ Celsius (Centigrade)
- Linear + 10.0 mV/˚C scale factor
- 0.5˚C accuracy guaranteeable (at +25˚C)
- Rated for full −55˚ to +150˚C range
- Suitable for remote applications
- Low cost due to wafer-level trimming
- Operates from 4 to 30 volts
- Less than 60 µA current drain
- Low self-heating, 0.08˚C in still air
- Nonlinearity only ±¼˚C typical
- Low impedance output, 0.1 Ω for 1 mA load

Typical Applications

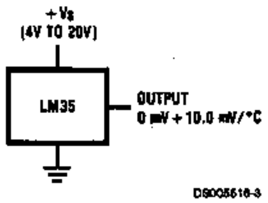
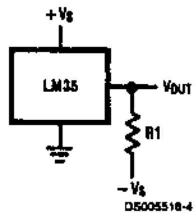


FIGURE 1. Basic Centigrade Temperature Sensor (+2˚C to +150˚C)

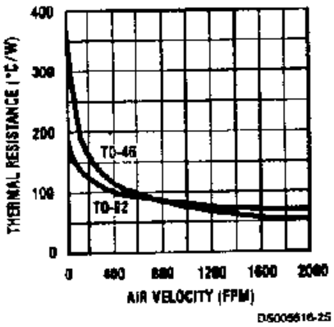


Choose $R_1 = -V_S/50\ \mu A$
V_{OUT} = +1,500 mV at +150˚C
= +250 mV at +25˚C
= −550 mV at −55˚C

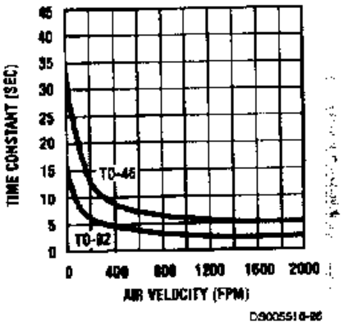
FIGURE 2. Full-Range Centigrade Temperature Sensor

Typical Performance Characteristics

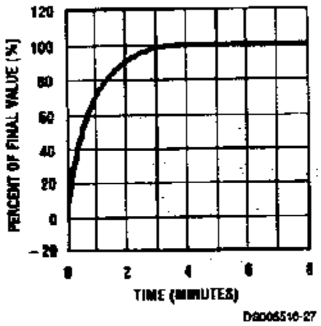
Thermal Resistance Junction to Air



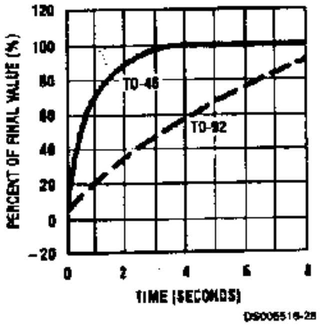
Thermal Time Constant



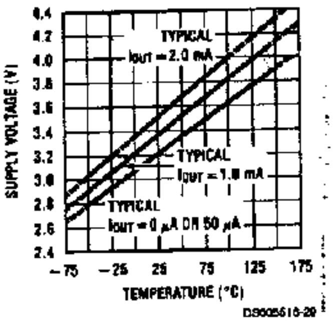
Thermal Response in Still Air



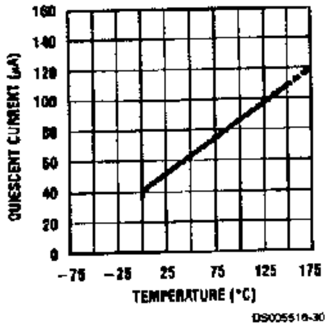
Thermal Response in Stirred Oil Bath



Minimum Supply Voltage vs. Temperature

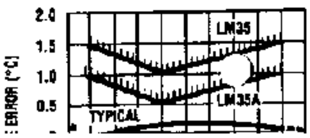


Quiescent Current vs. Temperature (In Circuit of *Figure 1.*)

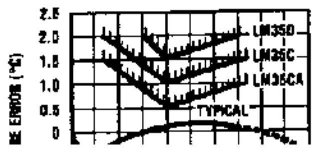


Quiescent Current vs. Temperature (In Circuit of *Figure 2.*)

Accuracy vs. Temperature (Guaranteed)



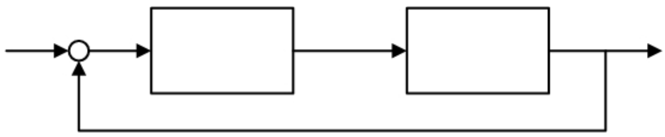
Accuracy vs. Temperature (Guaranteed)



Chapter 3

Design of controllers - a theoretical approach

3.1 Direct design of controller - Compensation controller



$$\frac{y}{w} = F_w = \frac{F_R\ F_S}{1 + F_R\ F_S\ 1}$$

If F_w is given and F_S is known, we get the controller as:

$$\begin{aligned} F_w(1 + F_R\ F_S) &= F_R\ F_S \\ F_w + F_w\ F_R\ F_S &= F_R\ F_S \\ F_w &= F_R(F_S - F_w\ F_S) \end{aligned}$$

$$F_R = \frac{F_w}{F_S(1 - F_w)} = \frac{1}{F_S}\ \frac{F_w}{1 - F_w} \tag{3.1}$$

This relation may calculate F_R, but we get F_R which is mostly not realizable. Reason: the power of the nominator will be greater than the power of the denominator.

Example 1:

$$F_S = \frac{2}{1 + 0,4p} \qquad F_w = K$$

Example 2:

$$F_S = \frac{2p}{p^2+4} \qquad F_w = \frac{1}{1+a_1p+a_2p^2} \quad (PT_2 - System)$$

In general it´s *not* possible to get a practical (realizable) controller for each desired behavior of the control loop F_w.

That´s the reason because very intelligent engineers have developed certain procedures to design controllers by using the open loop control system instead of using F_w, the closed loop control system. The idea is to design controllers in some steps - in an iterative way - until the behavior of the (closed) control system is well satisfied.

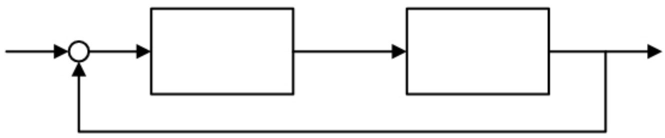
Wellknown procedures to design controllers are:

- RL-Method (root-locus), if F_S is mathematically known)
- Nyquist-criterium (chap. 6), if F_S is messured)

3.2 Root-Locus (RL, Evans 1942)

3.2.1 Motivation

Given is the following control loop:



The transfer function of the open loop control system should be:

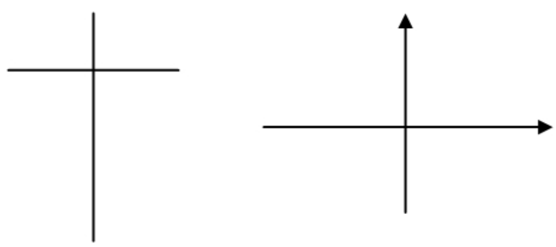
$$F_o = K_R \, \frac{K_1}{p + K_2}$$

So, we get the transfer function of the closed loop system as:

$$F_w = \frac{F_{forward}}{1 + F_{forward} \, F_{backward}} = \frac{F_{forward}}{1 + F_o} \tag{3.2}$$

We know that the eigen-motions of the closed loop system are defined by the eigen values, i.e. by the pole placement of $\Rightarrow 1 + F_o = 0$

$$\begin{aligned} 1 + K_R \, \frac{K_1}{p + K_2} &= 0 \\ p + K_2 + K_R \, K_1 &= 0 \\ p &= -K_R \, K_1 - K_2 \end{aligned}$$

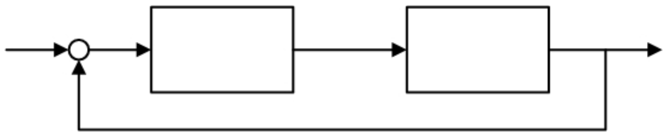


The greater K_R, the more left the pole will be placed. This curve (here: a line) is called RL, because it´s the connection of all eigen values or poles when K_R increases.

Hint: You see, that the RL starts in the point $p = -K_2$ when $K_R = 0$. This fact dues to the pole $p = -K_2$ of the open loop F_o. It´s no accident, it´s the beginning of some interested RL rules.

Hence we suppose the RL could start in the poles of F_o. We will see!

Example:



3.2.2 Sketching of the RL

All sketching rules are based on the solution of the characteristic equation of the RL: $1 + F_o = 0$. The first two rules are easy to remember, the others can only noticed by formulars.

Rule 1: The RL begins at the poles of the open control system and ends at the zeros of F_o. If there are more poles than zeros (as usual in practice) the RL branches ends at infinity.

Rule 2: Where does the RL begin and end? Answer: The RL always lies in a section of the real axis to the left of an odd number of poles and zeros.

Rule 3: The asymptotes of RL are centered at a point on the real axis given by:

$$\delta_w = \frac{\sum poles \ - \sum zeros}{n - m} \tag{3.3}$$

n=number of poles of F_o
m=number of zeros of F_o
δ_w= asymptote centroid

Rule 4: The angles of the asymptotes with respect to the real axis is:

$$\varphi_k = \frac{2k-1}{n-m}\, 180^o \qquad k = 0,\ 1,\ ...,\ n-m-1 \tag{3.4}$$

φ_k=angle of asymptote

Rule 5: The breakaway point on the real axis (if any) is given by the equation:

$$\sum \frac{1}{p_A - p_\mu} = \sum \frac{1}{p_A - p_\nu} \tag{3.5}$$

p_A=breakaway point
p_μ=poles of F_o
p_ν=zeros of F_o

Example:

$$F_o = \frac{K_I}{p} \, \frac{2,5}{1 + 0,5p}$$

3.2.3 Example: Aircraft roll control

Sketch the RL of the control system given by:

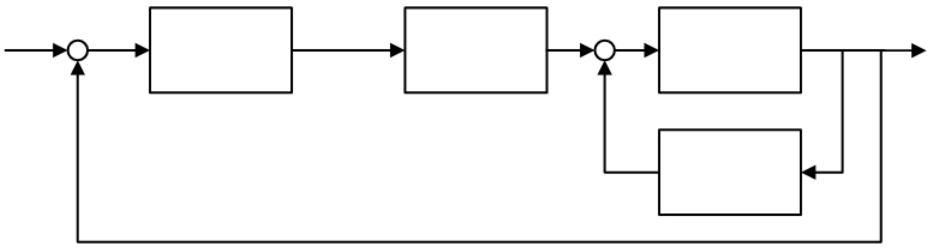


Figure 3.1: Aircraft roll control

$$F_o = K\ \frac{0,8}{s+0,8}\ \frac{\frac{18}{s^2}}{1+\frac{18}{s^2\ 0,5}}$$

Exercise:

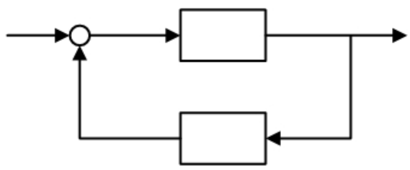
Use a PD-Controller to stabilise the roll control system.

$$F_R = K(1 + T_v\ p)$$

3.2.4 Parameter-RL

The sketching rules given above are the graphical representation of the solution of $1 + F_o = 0$ respectively $1 + K\frac{Z_o}{N_o} = 0$. An equation with this structure needn´t come necessarily from control theorie but also from others disciplines like mathematics, chemie or something else. Always if you want to solve an equation in parameter varies (here: K) the rules given above are valid
So, instead of K there can be another parameter, e.g. a time constant T of the process. In this case the RL gives us an valuable insight into the behavior of the control system when T varies. We have only to formulate $1+K\frac{Z_o}{N_o} = 0$ into the appropriate structure.

Example:



3.2.5 Résumé: RL

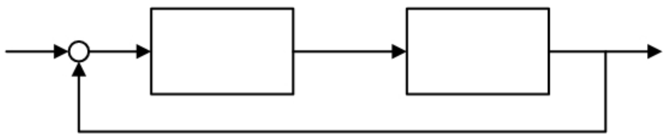
- The RL procedure is important to analyze the dynamical behavior (stability) of control systems.
- The RL procedure allows an easy and rapid prototyping of different controllers. It´s a powerful, not to say the most powerful, tool in designing controllers.
- The RL procedure is only applicable when the model of the process is given. It belongs to the class of theoretical tools to design controllers.
- Disadvantage:

  When there is a dead time in the open loop, it´s difficult to use the RL procedure.

  Explanation:

  Dead time $\rightarrow e^{-pT_t}$
  So, zeros and poles of F_o are not calculatable because of the factor e^{-pT_t} in the open loop transfer funcion.

  Example:



$$\begin{aligned} z.B. \quad F_{Str.}(p) &= \frac{5}{p}\, e^{-pT_t} \qquad (IT_t - Typ) \\ F_o &= F_R\, F_{Str.} \end{aligned}$$

When T_t is relatively low, you can use a tricky representation of:

$$e^{-pT_t} = \frac{e^{-p\frac{T_t}{2}}}{e^{+p\frac{T_t}{2}}} \approx \frac{1 - p\frac{T_t}{2}}{1 + p\frac{T_t}{2}} \tag{3.6}$$

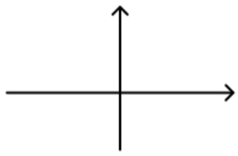
$$e^x = \frac{1}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \tag{3.7}$$

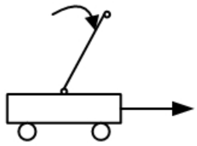
yield to
$\Rightarrow$ zero: $p_1 = +\frac{2}{T_t}$
$\Rightarrow$ pole: $p_2 = -\frac{2}{T_t}$

Exercise:

Use the RL procedure to design a controller which stabilize the unstable system.

$$\frac{\varphi}{\dot{x}} = \frac{r\ M\ p}{\Theta\ p^2 - m\ g\ r}$$





3.3 Stability criterion of Hurwitz

The stability criterion of Hurwitz allows to locate the poles of the closed feedback system, *without* solution of the characteristic equation $1 + F_o = 0$ (which is i.g. not analytically resolvable).

The Hurwitz criterion is based onto $1 + F_o$ resp. $N(p)$ the denominator of the closed loop system

$$a_n\ p^n + a_{n-1}\ p^{n-1} + \ \dots\ + a_2\ p^2 + a_1\ p + a_o = 0$$

Hurwitz has established: All poles, i.e. all solutions of the equation above, lie in the left half p-plane, if it´s valid:

(a) necessary criterion:
all coefficients $a_o,\ \dots, a_n$ exist and are greater than zero.

(b) sufficient criterion:
all so called north-east-sup-determinants must be greater then zero.

Hurwitz north-east-sup-determinants: (n=the order of N(p))

$$H = \begin{vmatrix} a_{n-1} & a_{n-3} & \cdots \\ a_n & a_{n-2} & \cdots \\ 0 & a_{n-1} & a_{n-3} \\ 0 & a_n & a_{n-2} \end{vmatrix}$$

Example:

$$\underbrace{4}_{a_3} p^3 + \underbrace{2}_{a_2} p^2 + \underbrace{8}_{a_1} p + \underbrace{15}_{a_o} = 0$$

Exercise:

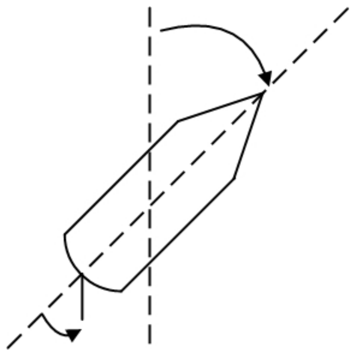
Ship-heading:

Use the Hurwitz criterion to determine the relation between K_R and K_I to stabilize the feedback heading system.

$$\frac{\varphi}{\delta} = \frac{1}{8p(1+30p)}$$

The used controller may be from type PI:

$$\frac{y}{e} = K_R + \frac{K_I}{p}$$



Chapter 4

Empirical design of controller

For a special class of dynamical systems (delayed behaviour) there are some rules of thumb to design controller.

- **Rules of Ziegler-Nichols**
- **Rules of Chien-Hrones-Reswick**
- and others ... **Comparison of some empirical rules**

8.2.3.1 Empirische Einstellregeln nach Ziegler und Nichols

Viele industrielle Prozesse weisen Übergangsfunktionen mit rein aperiodischem Verhal
gemäß Bild 8.2.7 auf, d.h. ihr Verhalten kann durch PT_n-Glieder sehr gut beschrie
werden. Häufig können diese Prozesse durch das vereinfachte mathematische Modell

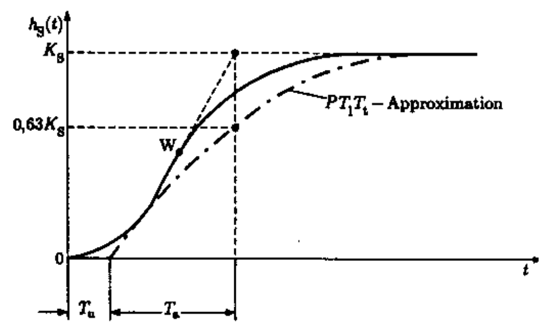
$$G_S(s) = \frac{K_S}{1+Ts}e^{-T_t s}, \qquad (8.2$$

das ein Verzögerungsglied 1. Ordnung und ein Totzeitglied enthält, hinreichend gut
proximiert werden. Bild 8.2.7 zeigt die Approximation eines PT_n-Gliedes durch ein
artiges PT_1T_t-Glied.

Dabei wird durch die Konstruktion der Wendetangente die Übergangsfunktion $h_S(t)$
folgenden drei Größen charakterisiert: K_S (Übertragungsbeiwert oder Verstärkungsfak
der Regelstrecke), T_a (Anstiegszeit) und T_u (Verzugszeit). Für eine grobe Approximat
nach Gl. (8.2.32) wird dann meist $T_t = T_u$ und $T = T_a$ gesetzt.

Für Regelstrecken der hier beschriebenen Art wurden zahlreiche Einstellregeln für St
dardregler in der Literatur [Opp72] angegeben, die teils empirisch, teils durch Simulat
an entsprechenden Modellen gefunden wurden. Die wohl am weitesten verbreiteten em
rischen Einstellregeln sind die von *Ziegler* und *Nichols* [ZN42]. Diese Einstellregeln w
den anhand ausgedehnter Untersuchungen von Reglereinstellungen empirisch abgeleit
wobei die Übergangsfunktion des geschlossenen Regelkreises je Schwingungsperiode ei

wurf im Zeitbereich



ld 8.2.7. Beschreibung der Übergangsfunktion $h_S(t)$ eines Prozesses durch die drei Größen
(Übertragungsbeiwert oder Verstärkungsfaktor der Regelstrecke), T_a (Anstiegszeit) und T_u
rzugszeit)

Tabelle 8.2.8 Reglereinstellwerte nach Ziegler und Nichols

	Reglertypen	Reglereinstellwerte K_R	T_I	T_D
Methode I	P	$0{,}5\ K_{R\,krit}$	-	-
	PI	$0{,}45K_{R\,krit}$	$0{,}85T_{krit}$	-
	PID	$0{,}6\ K_{R\,krit}$	$0{,}5\ T_{krit}$	$0{,}12T_{krit}$
Methode II	P	$\frac{1}{K_S}\frac{T_a}{T_u}$	-	-
	PI	$\frac{0{,}9}{K_S}\frac{T_a}{T_u}$	$3{,}33\ T_u$	-
	PID	$\frac{1{,}2}{K_S}\frac{T_a}{T_u}$	$2T_u$	$0{,}5\ T_u$

plitudenabnahme von ca. 25 % aufwies. Bei der Anwendung der Einstellregeln nach
gler und Nichols kann zwischen folgenden zwei Fassungen gewählt werden:

) *Methode des Stabilitätsrandes (I):* Hierbei geht man in folgenden Schritten vor:

1. Der jeweils im Regelkreis vorhandene Standardregler wird zunächst als reiner P-Regler geschaltet.
2. Die Verstärkung K_R dieses P-Reglers wird solange vergrößert, bis der geschlossene Regelkreis Dauerschwingungen ausführt. Der dabei eingestellte K_R-Wert wird als kritische Reglerverstärkung $K_{R\,krit}$ bezeichnet.
3. Die Periodendauer T_{krit} (kritische Periodendauer) der Dauerschwingung wird gemessen.
4. Man bestimmt nun anhand von $K_{R\,krit}$ und T_{krit} mit Hilfe der in Tabelle 8.2.8 angegebenen Formeln die Reglereinstellwerte K_R, T_I und T_D.

b) *Methode der Übergangsfunktion (II):* Häufig wird es allerdings bei einer industriellen Anlage nicht möglich sein, den Regelkreis zur Ermittlung von $K_{R\,krit}$ und T_{krit} im grenzstabilen Fall zu betreiben. Im allgemeinen bereitet jedoch die Messung der Übergangsfunktion $h_S(t)$ der Regelstrecke keine großen Schwierigkeiten. Daher scheint in vielen Fällen die zweite Form der Ziegler-Nichols Einstellregeln, die direkt von der Steigung der Wendetangente K_S/T_a und der Verzugszeit T_u der Übergangsfunktion ausgeht, als zweckmäßiger. Dabei ist zu beachten, dass die Messung der Übergangsfunktion $h_S(t)$ nur bis zum Wendepunkt W erforderlich ist, da die Steigung der Wendetangente bereits das Verhältnis K_S/T_a beschreibt. Anhand der Messwerte T_u und K_S/T_a sowie mit Hilfe der in Tabelle 8.2.8 angegebenen Formeln lassen sich dann die Reglereinstellwerte einfach berechnen.

T_u: equivalent dead-time T_a: build-up time

Chien-Hrones-Reswick Compensation

Many engineers believe that the method developed in 1952 by Chien, Hrones, and Reswick (CHR) provides a better way to select a compensator for process-control applications. The method operates upon the shape of the step response of the open-loop plant. Since the plant is often type 1, the steady state value of the output of the open-loop plant driven by a unit step input is given by the DC gain $G_p(0)$. That result follows from the $\underbrace{\text{final-value theorem}}_{= \text{EWS}}$. The procedure can be described in two steps.

Form - shape

1. As in Figure 3.21, a line is drawn through the linear portion of the open-loop plant unit step response just after $t = 0$ resulting in values for T_g and T_u.

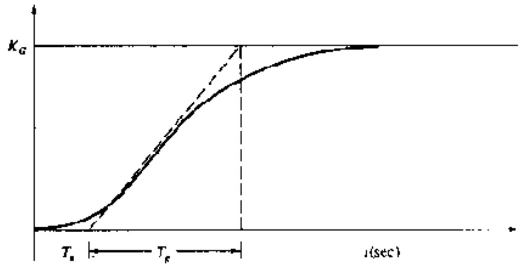


Figure 3.21 Unit step response of an open-loop plant.

2. The values for the compensator chosen depend on the ratio R,

$$R = T_g/T_u$$

as shown in Table 3.6(a).

There are two types of CHR compensators, one that should provide overdamped closed-loop behavior and one that should provide 20% overshoot. These goals are not necessarily met exactly, but the response is usually close to the desired values. In Table 3.6(b), $K_g = G_p(0)$ is the DC gain of the open-loop plant.

***Table 3.6** Chien-Hrones-Reswick Compensator*

(a) Values for $R = T_g/T_u$

Compensator	R
P	$R > 10$
PI	$7.5 < R < 10$
PID	$3 < R < 7.5$
Higher order	$R < 3$

b. CHR Compensation

Compensator	Overdamped	20% Overshoot
P	$K_p = 0.3R/K_g$	$K_p = 0.7R/K_g$
PI	$K_p = 0.35R/K_g$ $T_i = 1.2T_g$	$K_p = 0.6R/K_g$ $T_i = T_g$
PID	$K_p = 0.6R/K_g$ $T_i = T_g$ $T_d = 0.5T_u$	$K_p = 0.95R/K_g$ $T_i = 1.35T_g$ $T_d = 0.47T_u$

As an example of this method, consider the process control plant of Equation (3.17), which was considered previously. The first step in this process is to find the unit step response for the open-loop plant. A final time of 1.6 seconds is used. Figure 3.22 shows this time response. The line drawn through the linear portion of Figure 3.22 yields the following values:

$$T_u = 0.20 \text{ sec}$$

$$T_g = 1.304 - 0.2 = 1.104 \text{ sec}$$

$$R = 1.104/0.2 = 5.52$$

Using Table 3.6(a), a PID compensator is selected, and from Table 3.6(b), the values for K_p, T_i, and T_d are found for either the overdamped or 20% overshoot case. Note that K_g is the DC gain of the open-loop plant, which for Equation (3.17) is unity. These values are easily calculated for the PID compensator.

EWS

PID Compensator	
Overdamped	$K_p = 3.31$ $T_i = 1.104$ $T_d = 0.10$
20% Overshoot	$K_p = 5.24$ $T_i = 1.49$ $T_d = 0.094$

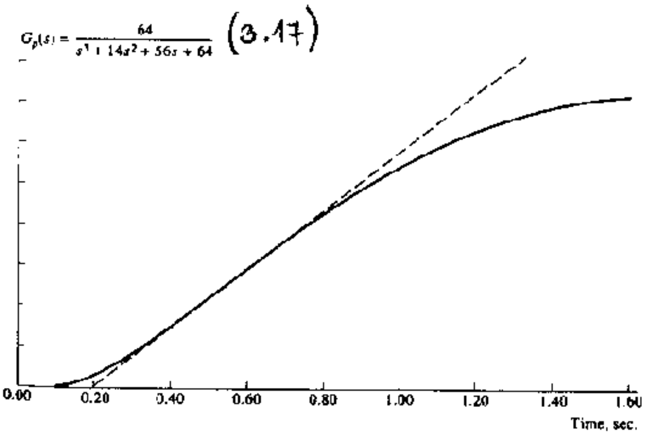


Figure 3.22 Open-loop plant response.

T_u: equivalent dead-time T_g: build-up time

4. Simulationsergebnisse und Anwendungsbeispiel

Die Qualität der Regelung, die mit der T_Σ-Regel erzielt werden kann, wird an 4 Beispielen gezeigt.
⟨1⟩ Eine P-T_1-ähnliche Strecke:

$$F_1(p) = \frac{1}{(1+20p)(1+8p)(1+2p)}$$

⟨2⟩ Eine Strecke höherer Ordnung:

$$F_2(p) = \frac{1}{(1+6p)^5}$$

⟨3⟩ Eine Strecke mit deutlich ausgeprägter Totzeit:

$$F_3(p) = \frac{1}{(1+5p)^3}\, e^{-15p}$$

⟨4⟩ Eine Strecke mit Nullstelle, deren Sprungantwort aber auch s-förmig ist:

$$F_4(p) = \frac{1+40p}{(1+58p)(1+6p)^2}$$

Alle Strecken haben die gleiche Summenzeitkonstante $T_\Sigma = 30$.

Als Vergleich wird angegeben, welches Regelergebnis man mit den Einstellregeln [1 bis 3] erhält.

Für die *Ziegler-Nichols*-Regeln wurde in der Simulation der Schwingversuch mit dem P-Regler durchgeführt und so $K_{R,\mathrm{krit}}$ und T_{krit} bestimmt.

Für die *Chien-Hrones-Reswick*-Regeln wurden aus den Sprungantworten die Wendetangenten und daraus T_u und T_t bestimmt. Es wurden die Regeln für gutes Führungsverhalten und aperiodisches Einschwingen benutzt.

Für die Regel von *Latzel* wurden die notwendigen Werte aus der Sprungantwort abgelesen. Um den Vergleich mit der neuen Regel, die als Ziel 5% Überschwingen hat, zu ermöglichen, wurde aus den in [3] angegebenen Werten für 10% und 20% Überschwingen linear auf 5% extrapoliert.

Die Summenzeitkonstanten wurden nicht als bekannt angenommen, sondern sie wurden mit der Methode von *Strejc* aus $t_{20\%}$ und $t_{80\%}$ ermittelt. ($t_{20\%}$ und $t_{80\%}$ sind die Zeitwerte, wenn die Sprungantwort 20% bzw. 80% ihres Endwertes erreicht hat). Für die 4 Strecken ⟨1 bis 4⟩ ergaben sich: $T_{\Sigma 1} = 30{,}4$; $T_{\Sigma 2} = 30{,}85$; $T_{\Sigma 3} = 30{,}4$; $T_{\Sigma 4} = 25{,}21$. Bei den ersten drei Strecken erkennt man eine sehr gute Übereinstimmung mit dem tatsächlichen Wert. Bei der Strecke ⟨4⟩ ist das so bestimmte T_Σ zu klein. Ursache ist: Erst nach $t_{80\%}$ ist die langsame Eigenbewegung in der Sprungantwort sichtbar. Das zu kleine T_Σ begünstigt allerdings das Regelverhalten, da der Regler diese langsame Eigenbewegung auch nur gering zu berücksichtigen braucht.

Bild 5 zeigt die Sprungantwort des Regelkreises für Strecke ⟨1⟩ mit PI-Regler: Zunächst fällt die schwach gedämpfte Schwingung bei *Ziegler-Nichols* auf. Da die Strecke ⟨1⟩ erst bei großem K_R des P-Reglers instabil wird, wird auch bei dem PI-Regler eine große Verstärkung eingestellt. Der Regler von *Chien, Hrones, Reswick* ist zwar schnell, aber er zeigt ein deutliches Unterschwingen. Der Regler von *Latzel* zeigt das beste Verhalten. Fast gleichwertig ist die schnelle Variante der T_Σ-Regel. Man sieht, daß bei dieser P-T_1-ähnlichen Strecke die schnelle Variante einen deutlichen Gewinn gegenüber der nor-

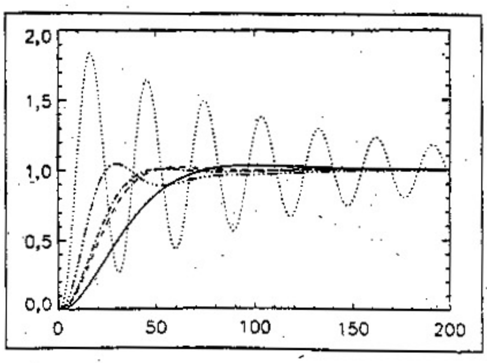


Bild 5: Strecke ⟨1⟩ (P-T_1-ähnlich) mit PI-Regler.
Bedeutung der Linien (gilt auch für Bilder 6 bis 9):
——— T_Σ-Regel, normal,
– – – – T_Σ-Regel, schnell,
· · · · · · *Ziegler, Nichols*,
– · · · – · *Chien, Hrones, Reswick*,
– · – · – · *Latzel*.

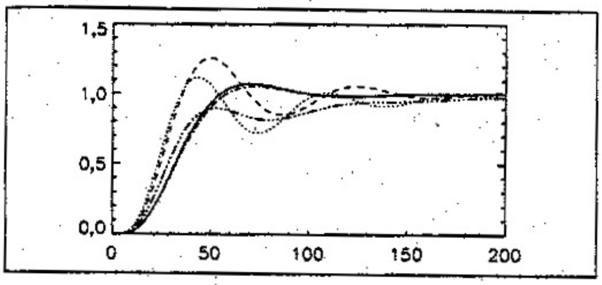


Bild 6: Strecke ⟨2⟩ (höhere Ordnung) mit PI-Regler.

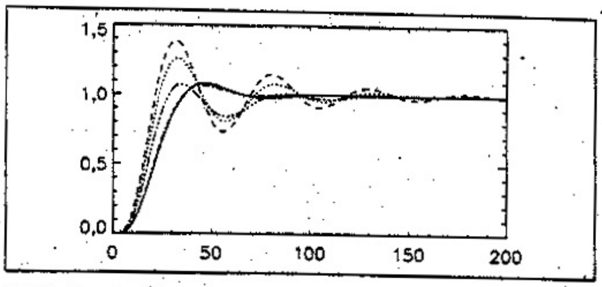


Bild 7: Strecke ⟨2⟩ (höhere Ordnung) mit PID-Regler.

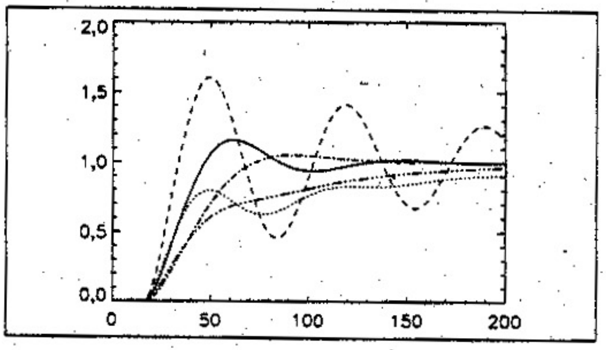


Bild 8: Strecke ⟨3⟩ (ausgeprägte Totzeit) mit PI-Regler.

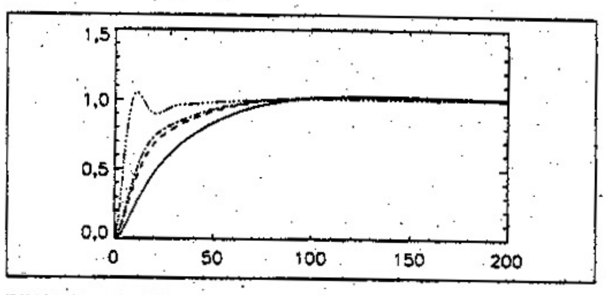


Bild 9: Strecke ⟨4⟩ (mit Nullstelle) mit PI-Regler.

malen, vorsichtigen Einstellung bringt.

Bild 6 zeigt die Strecke ⟨2⟩ mit PI-Regler. Wie man sieht, muß hier die normale Variante der T_Σ-Regel gewählt werden. Der mit der schnellen Regel eingestellte Regler greift zu heftig ein, was zu einer Schwingung führt. Auch der *Ziegler-Nichols*-Regler schwingt. Der *Chien-Hrones-Reswick*-Regler kriecht langsam von unten in den Sollwert. Er zeigt auch hier ein Unterschwingen. Die Kurve des *Latzel*-Reglers fällt fast mit der des normalen T_Σ-Reglers zusammen.

Bild 7 zeigt den PID-Regler an der Strecke ⟨2⟩. Das Einschwingverhalten ist allgemein schneller als bei dem PI-Regler, aber prinzipiell gelten hier auch die Aussagen von Bild 6.

Bild 8: Strecke mit dominierender Totzeit und PI-Regler. Das Verhältnis T_1/T_t ist ein Maß für die Regelbarkeit einer Strecke. Ab $T_1/T_t \le 3$ spricht man von schwer regelbar, wobei diese Zahl gilt, wenn T_1 und T_t mit der Wendetangentenmethode bestimmt werden. Für Strecke ⟨3⟩ gilt $T_1/T_t = 0{,}97$. Diese schwere Regelbarkeit ist im Bild 8 zu sehen. Lediglich der Regler nach *Latzel* erfüllt die Anforderung von 5% Überschwingen. Bei der normalen T_Σ-Einstellung muß man die Verstärkung etwas zurücknehmen. Im Sinne der Anforderung an die neue Einstellregel, nämlich in die Nähe guter Werte zu führen und ein Feineinstellen dann beim Betrieb der Regelung durchzuführen, kann hier die T_Σ-Regel aber doch als brauchbar bezeichnet werden.

Bild 9 zeigt die Strecke mit Nullstelle mit PI-Regler. Diese Strecke wurde zum Test der Regel von *Latzel* ausgewählt. Denn dort werden die Regelstrecken durch P-T_n-Glieder mit n gleichen Zeitkonstanten approximiert. Außerdem gibt *Latzel* die Reglerparameter nur für Strecken mit $t_{10\%}/t_{90\%} \ge 0{,}137$ an, was der Ordnung $n = 2$ entspricht. ($t_{10\%}$ und $t_{90\%}$ sind die Zeitwerte, wenn die Sprungantwort 10% bzw. 90% ihres Endwertes erreicht hat.) Hier gilt aber: $t_{10\%}/t_{90\%} = 0{,}051$. Als Regler wurde nun derjenige für $t_{10\%}/t_{90\%} = 0{,}137$ gewählt. Die Simulation im Bild 9 zeigt aber, daß dieser Regler doch ein gutes Ergebnis liefert. Er ist sogar noch etwas besser als der schnelle T_Σ-Regler. Der *Chien-Hrones-Reswick*-Regler ist hier sehr schnell. Dies liegt daran, daß er die langsame Zeitkonstante, die sich wegen der Nullstelle erst nach dem Wendepunkt bemerkbar macht, nicht kennt. Er zeigt wieder sein typisches Unterschwingen. Nach *Ziegler-Nichols* konnte hier kein Regler eingestellt werden, weil diese Strecke mit einem P-Regler nicht instabil wird.

Die Beispiele zeigen, daß die neue T_Σ-Regel gute Ergebnisse liefert. Bei P-T_1- und P-T_2-ähnlichen Strecken wird mit der schnellen Variante ein rasches Einschwingen erzielt, bei Strecken höherer Ordnung muß man die normale Variante anwenden.

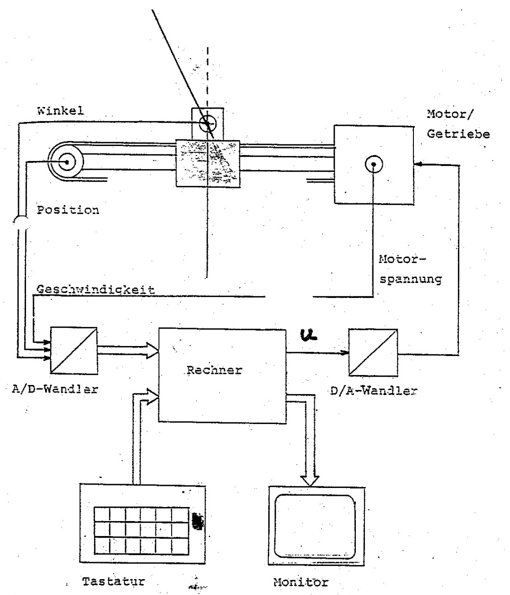
Die Einstellregel von *Latzel* liefert bei allen Beispielen sehr gute Ergebnisse, das Entwurfsziel, etwa 5% Überschwingen bei raschem Einschwingvorgang, wird immer erreicht. Allerdings erfordert dieses Verfahren für seine Anwendung eine Tabelle zum Nachschlagen. Da es drei spezielle Werte der Sprungantwort benötigt ($t_{10\%}$, $t_{50\%}$, $t_{90\%}$), ist es empfindlicher bezüglich Meßstörungen als die T_Σ-Regel, bei der T_Σ bei gestörtem Meßsignal durch Integration der Sprungantwort gewonnen werden kann.

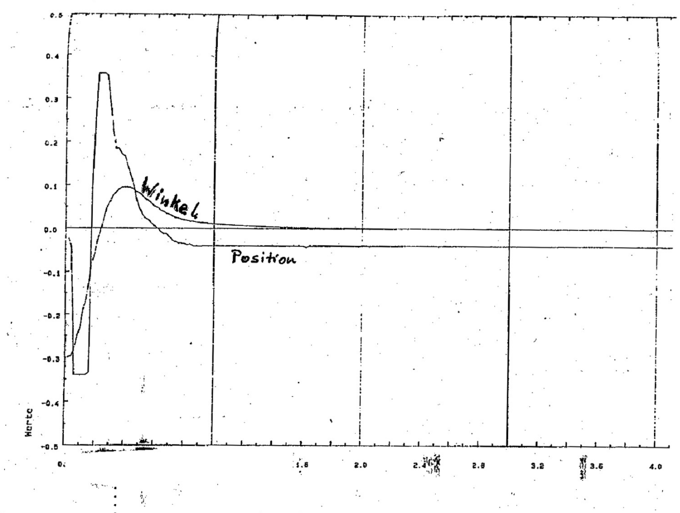
Die Regeln von *Ziegler-Nichols* und *Chien-Hrones-Reswick* zeigen zumeist unbefriedigendes Ein-

Chapter 5

Digital feedback control systems

Plant for control of an inverted pendelum





```
      CALL MBDATO (15,2,KOUT(1),0)

      CALL MBSTOU (3,17,0)                       ! DA-Steuercode 17
      CALL MBSTOU (5,130,0)                      ! AD-Steuercode 130

      CALL TIMRES (0,211,0)                      ! Interrupt aktiviert

      S C H L E I F E

      IF ( IABT .EQ. 0) GOTO 399                 !

      GOTO (355,360,370,380,390) LREG!           !

      CALL HNDABT                                !
      GOTO 395                                   !

      CALL PIDABT                                !
      GOTO 395                                   !

      CALL KAPABT                                !
      GOTO 395                                   !

      CALL PHIABT                                !
      GOTO 395                                   !

      CALL ZUPABT                                !

5     IABT = 0                                   !

9     CALL GETBUC (ICHAR)
      IF (ICHAR .GT. 0) GOTO 1000                !
      GOTO 350

00    CALL TIMRES (1,211,0)                      !
      CALL MBDATO (3,1,0,0)                      !

      Ende des Regelvorgangs,Abspeichern
      in die Datei MESS.DAT

10    CALL MBDATO (3,1,0,0)
      WRITE (20,1020) ANZ,WEG,PHI0,VSOLL
20     FORMAT (1H ,F5.1,4F10.5)
      CLOSE (UNIT = 20)

      IF (ANZ .GT. 800) ANZ = 800
      IANZ = IFIX (ANZ)

      RETURN

      END
```

```
C-----------------------------------------------------------------------
C     Quelle: PIDABT.FOR     Autor: Si                              Ver
C-----------------------------------------------------------------------
C -----------------------------------------------------------------------
C -
C -                       P I D A B T
C -
C -
C -
C -                 Abtastregelung mit PID-Regler
C -
C -
C -----------------------------------------------------------------------
C
      SUBROUTINE PIDABT (ITO)
      COMMON/ABT/ XSOLL,VGR, XR,XP,XI,XD, IOFF,CXV
      COMMON/PID/ A0,A1,A2,B0,B1,B2,XDI1,XDI2,VSOLL1,VSOLL2
      COMMON/ANZ/ IANZ
      DIMENSION IN(3)
C
C-----------------------------------------------------------------------
C
C  Einlesen der Messwerte, Endabfrage und
C  Berechnung der Positionsabweichung
C
30    IANZ = IANZ + 1
C
      CALL MBDATI (5,3,IN(1),0)
      V     = 7.5964E-4*IN(1)              ! (1.1027 mm/s)/1451.6
      WEG   = 3.1738E-4*IN(2)              ! (0.650 m)/2048
      PHI   = 1.704E-4*(IN(3) - IOFF)      ! (20/57.3 rad)/2048
C
      XD0 = XSOLL - WEG
C
      TYPE *,WEG,PHI
C-----------------------------------------------------------------------
C  Berechnung der Stellgroesse VSOLL0 mit einem PIDT1-Algorithmus
C  fuer den Weg
C
      VSOLL0 = (B0*XD0 + B1*XDI1 + B2*XDI2
     *  - A1*VSOLL1 - A2*VSOLL2)
C
C-----------------------------------------------------------------------
C  Geschwindigkeitsbegrenzung von VSOLL
C  Software-Endschalter, Berechnung der INTEGER-Steuergroesse
C
      VSOLL = VSOLL0
      IF (VSOLL .GT. VGR) VSOLL = VGR
50    IF (VSOLL .LT. -VGR) VSOLL = -VGR
C
60    IF (WEG .LT. 0.5) GOTO 70                 !
      IF (VSOLL .GT. 0) VSOLL = 0.0             ! Begrenzung der
       GOTO 80                                  ! groesse in Rich
70    IF (WEG .GT. -0.5) GOTO 80                ! Bahnmitte
      IF (VSOLL .LT. 0) VSOLL = 0.0             !
C
80    IVSOLL = IFIX (-1857.3*VSOLL)             ! - 2048/(1.1027
C
C-----------------------------------------------------------------------
C  Ausgabe der Stellgroesse IVSOLL und
C  Abspeichern der Messwerte in die Datei MESS.DAT
C
      CALL MBDATO (3,1,IVSOLL,0)
C
      IF (IANZ .GT. 800) GOTO 95
      WRITE (20,90) IANZ,WEG,PHI,VSOLL0
90     FORMAT (1H ,I4,3F10.5)
C
      GOTO 100
C
95    DO 96 I = 1,42                  ! Warteschleife
       X = I                          ! als Ersatz
       Y = X*X                        ! fuer die
96    CONTINUE                        ! Datenspeicherung
C
C-----------------------------------------------------------------------
C  Verschiebung der Differenzengleichungsvariablen
C  in die Vergangenheit
C
100   VSOLL2 = VSOLL1
      VSOLL1 = VSOLL0
      XDI2   = XDI1
      XDI1   = XD0
```

Exercise: Design of a digital controller

system. This rule gives quite short sampling periods. The Nyquist frequency will be about 5–20 times larger than the crossover frequency. $u \to \boxed{\frac{1}{s+1}} \to \boxed{\frac{1}{s}} \to x$ Position

Example 8.2—Digital redesign of lead compensator

Consider the system in Example A.2, which is a normalized model of a motor. The closed-loop transfer function

$$G_c(s) = \frac{4}{s^2 + 2s + 4} \tag{8.9}$$

is obtained with the lead compensator

$$G_k(s) = 4\frac{s+1}{s+2} \tag{8.10}$$

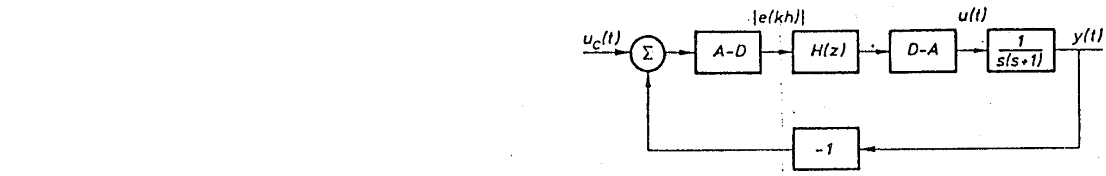


Figure 8.3 Digital control of the motor example.

The closed-loop system has a damping of $\zeta = 0.5$ and a natural frequency of $\omega_0 = 2$ rad/s. The objective is now to find $H(z)$ in Fig. 8.3, which approximates (8.10).

Euler's method gives the approximation

$$H_E(z) = 4\frac{z - 1 + h}{z - 1 + 2h} = 4\frac{z - (1 - h)}{z - (1 - 2h)} \tag{8.11}$$

while Tustin's approximation gives

$$H_T(z) = 4\frac{(2+h)z - 2 + h}{(2+2h)z - 2 + 2h} = 4\frac{2+h}{2+2h} \cdot \frac{z - (2-h)/(2+h)}{z - (1-h)/(1+h)}$$

Finally, zero-order hold sampling of (8.10) gives

$$H_{ZOH}(z) = \frac{4z - 2(1 + e^{-2h})}{z - e^{-2h}} = 4\frac{z - 0.5(1 + e^{-2h})}{z - e^{-2h}}$$

All approximations have the form

$$H(z) = \frac{b_0 z + b_1}{z + a_1}$$

The crossover frequency of the continuous-time process in cascade with the compensator (8.10) is $\omega_c = 1.6$ rad/s. The rule of thumb above gives a sampling period of about 0.1–0.3s.

Figure 8.4 shows the control signal and the process output when Euler's approxi-

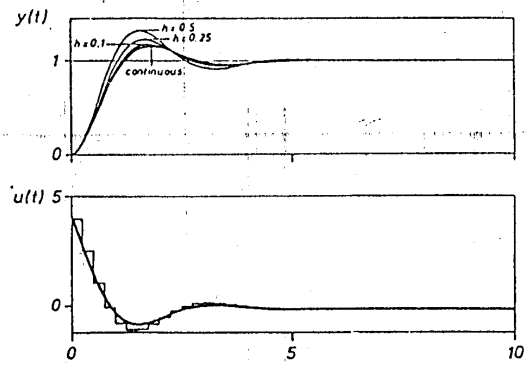


Figure 8.4 Process output, $y(t)$, when the motor is controlled using the compensator of (8.11) when $h = 0.1$, 0.25, and 0.5. The control signal is shown for $h = 0.25$. For comparison, the continuous-time signals are also shown.

mation has been used for different sampling times. The other approximations give similar results. The closed-loop system has a satisfactory behavior for all compensators when the sampling time is short. The rule of thumb also gives reasonable values for the sampling period. The overshoot when $h = 0.5$ is about twice as large as for the continuous-time compensator. In the example, the change in u_c occurs at a sampling instant. This is not true in practice, and there may be a delay in the response of at most one sampling period.

Chapter 6

Fuzzy feedback Control

In the beginning of the 80s fuzzy controllers came into scene to control plants without mathematical-physical modelling. Idea: implementing the human expertise of an plant-operator who controls the plant manually.
We have to work in that manner, always if modelling is not possible (complex, non-linear behaviour), for example: a biological-chemical reactor with control value: ph-value (for example).

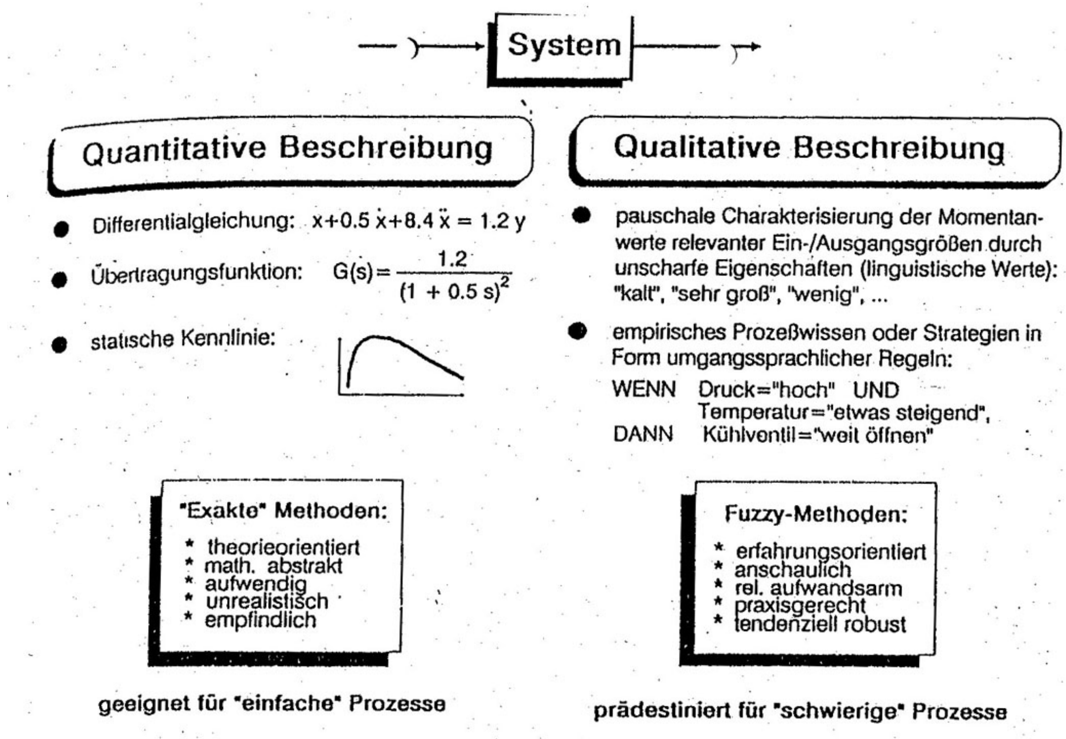
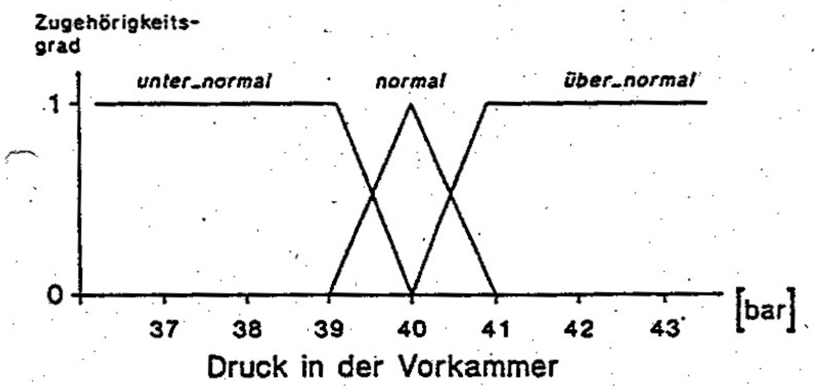
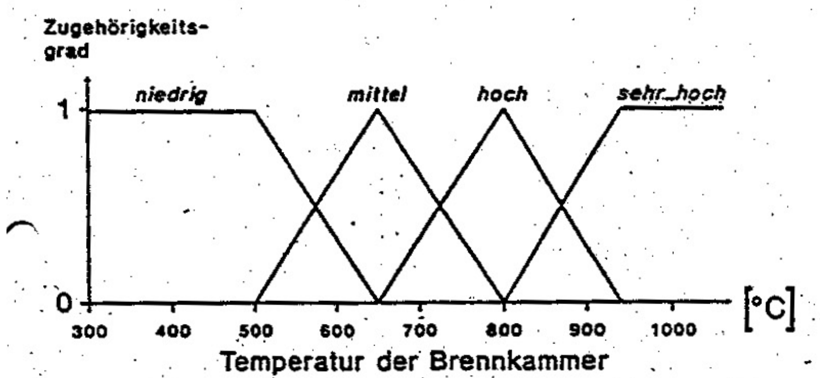


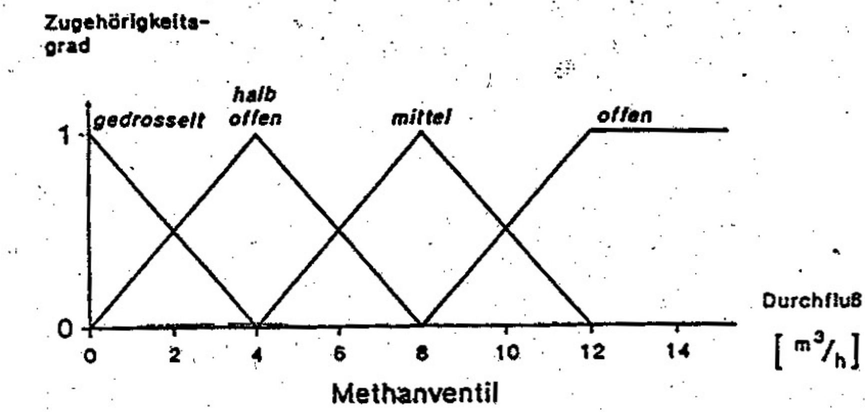
Bild 1. Quantitative und qualitative Methoden der Systembeschreibung.

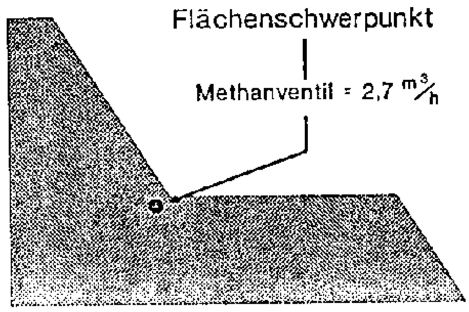
Regel (1):
WENN Temperatur=sehr_hoch
ODER
Vorkammerdruck=über_normal
DANN Methanventil=gedrosselt

Regel (2):
WENN Temperatur=hoch
UND Vorkammerdruck=normal
DANN Methanventil=halb_offen









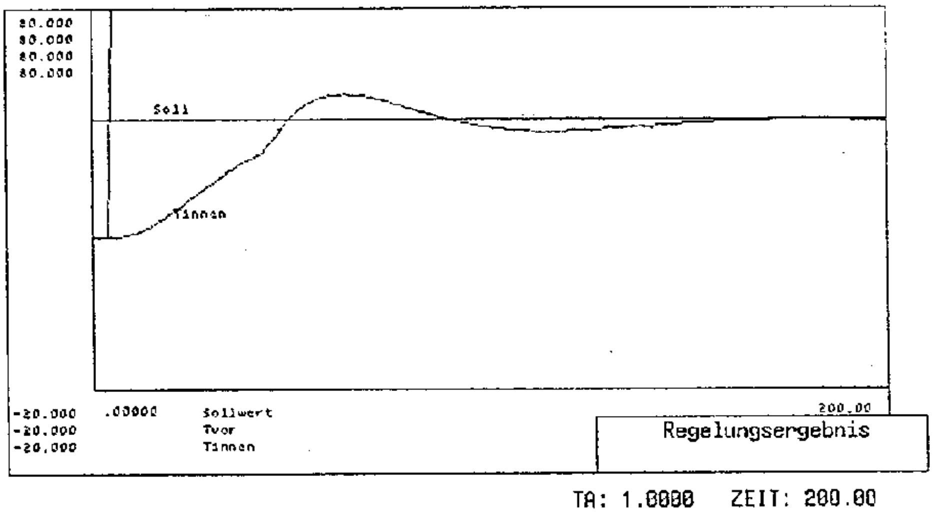


Bild 20. Überschwingen des PID-geregelten Reaktors bei exothermer Reaktion.

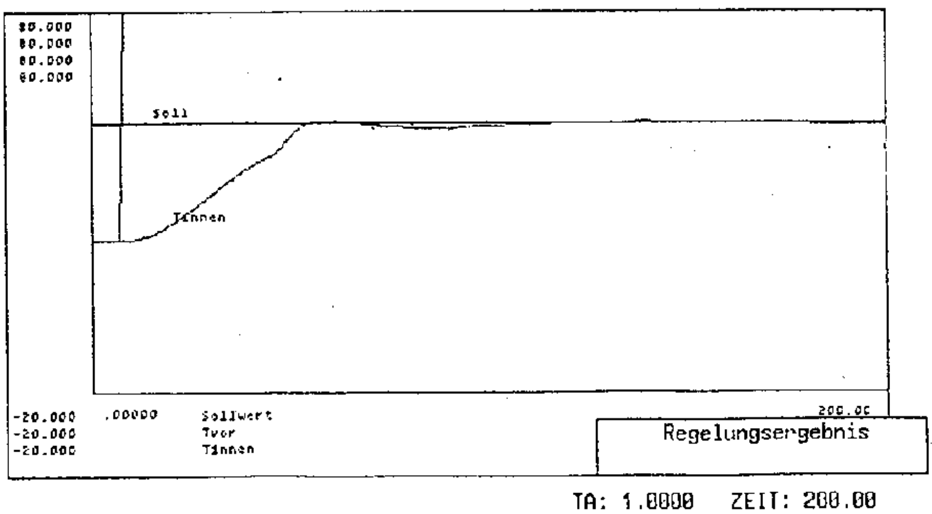


Bild 21. Überschwingungsvermeidung durch Fuzzy-Regelung.