Task 1.1.1

In a random experiment three LED lamps are turned on simultaneously. It must be assured that every single LED lamp might be faulty. (1) Define the simplest possible sample space for this random experiment contains the events.

A_1 = { exactly one LED lamps is on }

and

A_2 = { maximum of two LED lamps are on }

Suppose that the probabilities of the events A_1 and A_2 are given as: $P(A_1) = \frac{1}{4}$ and $P(A_2) = \frac{1}{2}$

(1) Three LED lamps turned on. Any can be faulty.

Maximum lamps faulty = 3

Sample Space for the number of lamps that turn on $\Rightarrow E = \{0, 1, 2, 3\}$

E = { no lamps turn on, only one lamp turn on, two lamps turn on, all three lamps turn on }

(2) Possible events in the collective sample space

A_1 = { exactly one lamp is on } = $\{1\} \Rightarrow \overline{A_1} = \{0, 2, 3\}$

A_2 = { maximum two lamps are on } = $\{0, 1, 2\} \Rightarrow$

$\overline{A_2}$ = { all lamps are on } = $\{3\}$

A_3 = { exactly on lamp is on OR all lamps are on }

$= A_1 + \overline{A_2} = \{1, 3\} \Rightarrow \overline{A_3} = \{0, 2\}$

~~are on~~

$E = \{0, 1, 2, 3\}$

$\emptyset = \{\emptyset\} = \{\}$

(3) Probabilities of the events possible: compliment probabilities

$P(A_1) = \frac{1}{4}$, $\boxed{P(\bar{A}_1) = 1 - \frac{1}{4}}$

$P(A_2) = \frac{1}{2}$ given. , $\boxed{P(\bar{A}_2) = 1 - \frac{1}{2}}$

$P(A_3) = P(A_1) + P(\bar{A}_2) \Rightarrow \frac{1}{4} + \left(1 - \frac{1}{2}\right)$

$\boxed{P(A_3) \Rightarrow \frac{3}{4}}$

$\boxed{P(\bar{A}_3) = \frac{1}{4}}$

$\boxed{P(E) = 1}$ Whole sample space is an event that is certain to occur.

$\boxed{P(\emptyset) = 0}$

Task 1.1.2

The union of two disjoint events A and B is the certain event. The conditional probability of an event X and events A and B are

$$P(X|A) = \frac{1}{4}, \quad P(X|B) = \frac{1}{3}, \quad P(A|X) = \frac{1}{2}$$

Determine $P(A)$ and $P(B)$

Conditional Probability: Probability of a ~~certain~~ an event when another event has already occured

Example : $P(X|A) = \frac{P(X \cap A)}{P(A)}$

Possible for dependent events as for X.

$$P(X \cap A) = P(X|A)\, P(A)$$

Probability of X and A is probability of A times probability of X when A already occured.

We know $P(X|A) = \frac{1}{4}$

$P(A|X) = \frac{1}{2}$

$$P(X \cap A) = P(X/A)\,P(A) \quad \text{——— (1)}$$

$$P(A \cap X) = P(A/X)\,P(X) \quad \text{——— (2)}$$

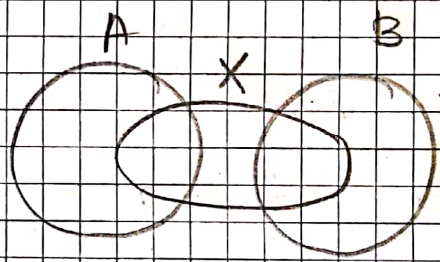
$$\Rightarrow \quad P(X \cap A) = P(A \cap X) \quad \text{——— (3)}$$

→ Using (1) and (2) in (3)

$$P(X|A)\,P(A) = P(A|X)\,P(X)$$

$$\frac{P(A)}{4} = \frac{P(X)}{2}$$

$$P(X) = \frac{P(A)}{2} \quad \text{——— (4)}$$



We know X is union of two disjoint events

$$P(X) = P(A \cap X) + P(B \cap X)$$

$$= P(X \cap A) + P(X \cap B)$$

$$= P(X|A)\,P(A) + P(X|B)\,P(B)$$

$$P(X) = \frac{P(A)}{4} + \frac{P(B)}{3} \quad \text{——— (5)}$$

As A and B are disjoint events

$$P(B) = 1 - P(A) \quad \text{——— (6)}$$

→ Using (6) in (5)

$$P(X) = \frac{P(A)}{4} + \frac{1}{3} - \frac{P(A)}{3} \Rightarrow P(X) = -\frac{P(A)}{12} + \frac{1}{3}$$

→ Using (4)

$$\frac{P(A)}{2} = \frac{-P(A)}{12} + \frac{1}{3}$$

$$\boxed{P(A) = \frac{4}{7}}$$

→ Using (5)

$$\boxed{P(B) = \frac{3}{7}}$$

Task 1.1.3

Total resistors 10,000

5000 resistors from A_1 (1% do not meet specs.)

3000 resistors from A_2 (2% do not meet specs.)

2000 resistors from A_3 (5% do not meet specs)

Probability of event B that randomly picked resistor is out of specification.

B : Randomly selected resistor is out of specification

$$P(B) = \frac{\text{Total number of out of specification resistors}}{\text{Total number of resistors}}$$

$$\rightarrow P(B) = \frac{N(B)}{N(S)}$$

$$\Rightarrow N(B) = (5000 \times 1\%) + (3000 \times 2\%) + (2000 \times 5\%)$$

$$= 210$$

$$\Rightarrow N(S) = 10{,}000$$

$$\rightarrow P(B) = \frac{210}{10{,}000} = 0.021$$

$$\boxed{P(B) = 0.021}$$

Task 1.1.4

Probability that no interruption occurs:

The AND condition of probabilities:

$= P(\bar{A}_1) \times P(\bar{A}_2) \times P(\bar{A}_3) \times P(\bar{A}_4) \times P(\bar{A}_5) \times P(\bar{A}_6) \times P(\bar{A}$

$= (1-0.4) \times (1-0.5) \times (1-0.6) \times (1-0.02) \times (1-0.7) \times (1-0.5) \times (1-0.4$

$= 0.010584$

Exersice

1.2.1

$$f_x(x) = \begin{cases} \frac{K}{8} e^{-\frac{x}{K}+2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) Calculate constant K?

we know that $\int_{-\infty}^{\infty} f_x(x) = 1$ (PDF)

$$\Rightarrow \int_{-\infty}^{\infty} \frac{K}{8} e^{-\frac{x}{K}+2} = 1$$

$$\Rightarrow \frac{K}{8} \int_{-\infty}^{\infty} e^{-\frac{x}{K}+2} = 1$$

$$\frac{K}{8} \int_{0}^{\infty} e^{-\frac{x}{K}+2} = 1$$

$$\frac{K}{8} \left. \frac{e^{-\frac{x}{K}+2}}{-\frac{1}{K}} \right|_{0}^{\infty} = 1$$

$$\frac{K}{8} \left[-K e^{-\frac{x}{K}+2} \right]_{0}^{\infty} = 1$$

$$\frac{K}{8} \left[-K e^{-\infty} - (-K e^{2}) \right] = 1$$

$$\frac{K}{8} \left[K e^{2} \right] = 1 \quad , \quad \frac{K^2 e^2}{8} = 1$$

$$\left(K = \pm \frac{2\sqrt{2}}{e} \right) \text{. ANS}$$

b) mean → $m_x^{(1)}$

we know that $m_x^{(1)} = \int_{-\infty}^{\infty} x f_x(x)\, dx$

$\Rightarrow \int_0^{\infty} x \cdot \frac{K}{8} e^{-\frac{x}{K}+2} dx$

$\Rightarrow \frac{K}{8} \int_0^{\infty} \left\{ x \cdot e^{-\frac{x}{K}+2} \right\} dx$

$\Rightarrow$ Using $\int u \cdot v\, dx = u \int v\, dx - \int u' \int v\, dx$

$\frac{K}{8} \left[x \cdot \int_0^{\infty} e^{-\frac{x}{K}+2} dx - \int_0^{\infty} 1 \cdot \int e^{-\frac{x}{K}+2} dx \right]$

$\frac{K}{8} \left\{ \left[x \cdot \frac{e^{-\frac{x}{K}+2}}{-\frac{1}{K}} \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-x/K+2}}{-1/K} dx \right\}$

$\frac{K}{8} \left\{ \text{N.D} + K \left[\frac{e^{-x/K+2}}{-1/K} \right]_0^{\infty} \right\}$

$\frac{K}{8} \left\{ \text{N.D} - K^2 (-e^2) \right\} \rightarrow$ (eq a)

$\frac{K}{8} \left[e^2 \cdot K^2 \right] = \frac{K^3 e^2}{8}$ ANS

Substitute values of K i.e $\left(\frac{2\sqrt{2}}{e}, -\frac{2\sqrt{2}}{e} \right)$

$\frac{\left(\frac{2\sqrt{2}}{e}\right)^3 e^2}{8} = \frac{2\sqrt{2}}{e}$ & for negative K we have mean

equals $-\frac{2\sqrt{2}}{e}$

Variance $= m_x^{(2)} - (m_x^{(1)})^2$

we know $m_x^{(1)}$, we need to find $m_x^{(2)}$.. (Pg 19 eq (3.3))

$$m_x^{(2)} = \int_{-\infty}^{\infty} x^2 f_x(x)\, dx$$

$$\Rightarrow \int_0^\infty x^2 \frac{k}{8} e^{-\frac{x}{k}+2}\, dx$$

$$\Rightarrow \frac{k}{8}\int_0^\infty x^2 e^{-\frac{x}{k}+2}\, dx$$

Again applying $\int u \cdot v\, dx$

$$\Rightarrow \frac{k}{8}\left\{\left[x^2 \frac{e^{-\frac{x}{k}+2}}{-\frac{1}{k}}\right]_0^\infty - \int_0^\infty 2x \frac{e^{-x/k+2}}{-1/k}\, dx\right\}$$

$$\frac{k^2}{8}\left\{\underbrace{x^2 e^{-x/k+2}\Big|_0^\infty}_{0} + \int_0^\infty 2x e^{-x/k+2}\, dx\right\}$$

$$\frac{k^2}{8}\left\{+2\int_0^\infty x e^{-x/k+2}\, dx\right\} \Rightarrow +\frac{k^2}{4}\underbrace{\int_0^\infty x e^{-x/k+2}\, dx}_{\text{using eq a'}}$$

$\frac{k^2}{4} \cdot e^2 k^2 \Rightarrow \frac{k^4 e^2}{4}$, Putting value of k we get $\frac{16}{e^2}$

variance $= m_x^{(2)} - (m_x^{(1)})^2$

$\Rightarrow \frac{16}{e^2} - \left(\frac{2\sqrt{2}}{e}\right)^2 = \frac{8}{e^2}$ ANS

c) Probability $\{-1 \le f(x) < 2\}$

$\int_{-1}^{2} f_x(x) \Rightarrow \int_{0}^{2} f_x(x)$

$\Rightarrow \int_0^2 \frac{k}{8} e^{-x/k+2} \Rightarrow -\frac{k^2}{8}\left[e^{-x/k+2}\right]_0^2$

$\Rightarrow -\frac{k^2}{8}\left[e^{-2/k+2} - e^2\right]$

Substituting value of k i.e $\pm \frac{2\sqrt{2}}{e}$

$\Rightarrow -\frac{\left(\frac{2\sqrt{2}}{e}\right)^2}{8}\left[e^{-2/\frac{2\sqrt{2}}{e}+2} - e^2\right]$ Taking (+)ve value

$\Rightarrow -\frac{1}{e^2}\left[e^{-e/\sqrt{2}+2} - e^2\right] = 0.853$ ✓

& for $k = -\frac{2\sqrt{2}}{e}$ we will get

$-\frac{1}{e^2}\left[e^{e/\sqrt{2}+2} - e^2\right] =$ (−)ve answer -5.83

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Task 1 Class Questions

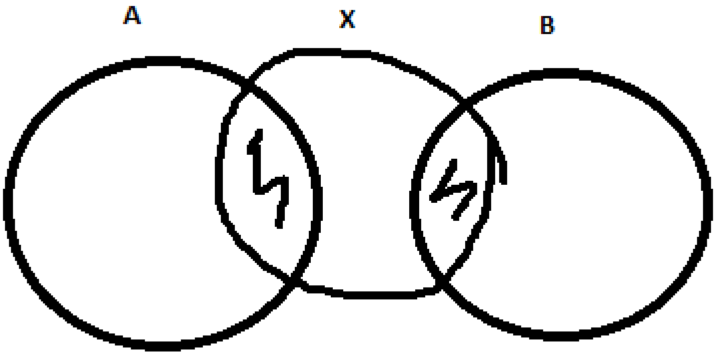
Task 1.1.1

A3= 2 elemental sample space

{0,1} or {0, 2} or {0, 3} or {1,2} or {1, 3} so on until 12 combinations

A3 = {1,3} to make from events A1 and A2' which is information known

Task 1.1.2



Venn diagram, A and B Venns should join. Not necessarily, need only to show A and B are disjointed. The can NOT have any value, presence of gap between A and B doesn't ensure presence of elements which is Venn configurations.

Task 1.1.3 Or probability

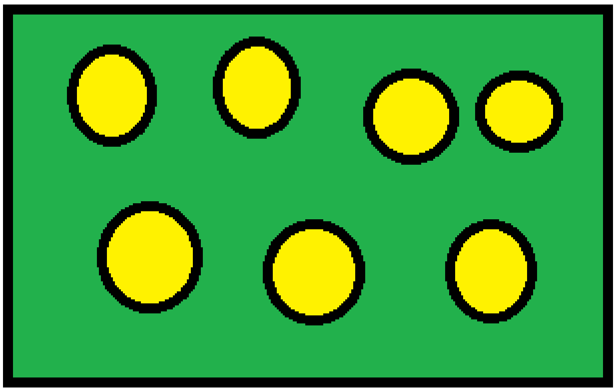
Task 1.1.4 AND probability

Question = Could multiply all events together and compliment the product:

P(A)= P(A1)xP(A2)x…xP(A7)

P(no interrupts) = 1 – P(A)

This approach is incorrect because:



P(A) (yellow) is all interrupts happening

1-P(A) (green) any other event happening other than the events of P(A), which is all other events including the event of no interruptions

Exersice

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$$\Rightarrow \frac{k}{8} \int_0^{\infty} \{x \cdot e^{-\frac{x}{k}+2}\} dx$$

⇒ Using $\int u \cdot v\, dx = u \int v\, dx - \int u' \int v\, dx$

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Substitute values of k i.e $\left(\frac{2\sqrt{2}}{e}, -\frac{2\sqrt{2}}{e}\right)$

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equals $-\frac{2\sqrt{2}}{e}$

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we know $m_x^{(1)}$, we need to find $m_x^{(2)}$..

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$\Rightarrow \frac{16}{e^2} - \left(\frac{2\sqrt{2}}{e}\right)^2 = \frac{8}{e^2}$ ANS

c) Probability $\{-1 \le f(x) < 2\}$

$\cancel{\frac{e^2}{8}} \int_{-1}^{2} f_x(x) \Rightarrow \int_{0}^{2} f_x(x)$

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& for $k = -\frac{2\sqrt{2}}{e}$ we will get

$-\frac{1}{e^2}\left[e^{e/\sqrt{2}+2} - e^2\right] = (-)$ve answer $\underline{-5.83}$