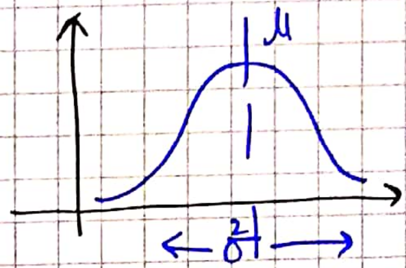
KALMAN FILTER

State of the process

$$x = \begin{bmatrix} P \\ V \end{bmatrix}$$

All real world applications are employable by Gaussian curves, the process thus has a mean (μ) and variance (σ^2)



The two variables governing the state have a correlation Σ which makes a covariance matrix P_K. So a state is defined by when k-1 is current and k is state

$$\hat{x}_k = \begin{bmatrix} \text{position} \\ \text{velocity} \end{bmatrix}$$

$$P_k = \begin{bmatrix} \Sigma_{pp} & \Sigma_{pv} \\ \Sigma_{vp} & \Sigma_{vv} \end{bmatrix}$$

For next state or future state prediction

$$p_k = p_{k-1} + \Delta t\, v_{k-1}$$

$$v_k = v_{k-1}$$

OR

$$\hat{x}_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{k-1} \\ v_{k-1} \end{bmatrix}$$

$$\hat{x}_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \hat{x}_{k-1}$$

$$\hat{x}_k = F_k \hat{x}_{k-1}$$

prediction matrix for next state

For covariance, its identity is used

$Cov(x) = \Sigma$

$\Rightarrow$ $Cov(Ax) = A \Sigma A^T$

how to update covariance for next state

$$P_k = F_k P_{k-1} F_k^T$$

Mathematical Prediction OR Measurement

$$\boxed{\begin{aligned} \hat{x}_k &= F_k \hat{x}_{k-1} \\ P_k &= F_k P_{k-1} F_k^T \end{aligned}}$$

new best estimate

new uncertainty

previous best estimate

old uncertainty

when k is future state

$k-1$ is current

External Influence

To compensate any external influences further steps are taken.

For example, influence due to changing acceleration

$$p_k = p_{k-1} + \Delta t \, v_{k-1} + \frac{1}{2} a \Delta t^2$$

$$v_k = v_{k-1} + a \Delta t^2$$

In matrix:

$$\hat{x}_k = F_k \hat{x}_{k-1} + \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix} a$$

OR

$$\hat{x}_k = F_k \hat{x}_{k-1} + \underbrace{B_k}_{\text{control matrix}} \underbrace{\vec{u}_k}_{\text{control vector}}$$

External Uncertainty

External influences that can not be factored in the process. Uncertainty true to the real world example.

Treat the untracked influences as noise with covariance Q_k

Finally,

State by measurements

$$\hat{x}_k = F_k \hat{x}_{k-1} + B_k \vec{u}_k$$
$$P_k = F_k P_{k-1} F_k^T + Q_k$$

additional uncertainty from environment

The new best estimate $\hat{x}_k$ is a prediction F_k made from previous best estimate $\hat{x}_{k-1}$ plus a correction for known external influences $\vec{u}_k$

The new uncertainty P_k is predicted F_k from the old uncertainty P_{k-1} with some additional uncertainty from the environment Q_k

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Sensor Readings / Update step

The sensor reading are modelled in a similar manner with an expected mean value and covariance

$$\vec{\mu}_{expected} = H_k \hat{x}_k$$

H_k measurement matrix

$$\Sigma_{expected} = H_k P_k H_k^T$$

modeled or predicted states

For uncertainty or range or reliability surrounding sensor reading, we observe and note R_k and z_k to be the sensor measured readings where R_k is the covariance of uncertainty.

In process these are two separate estimates. Factoring these estimates together, We take multiplication or intersection of their Gaussian curves.

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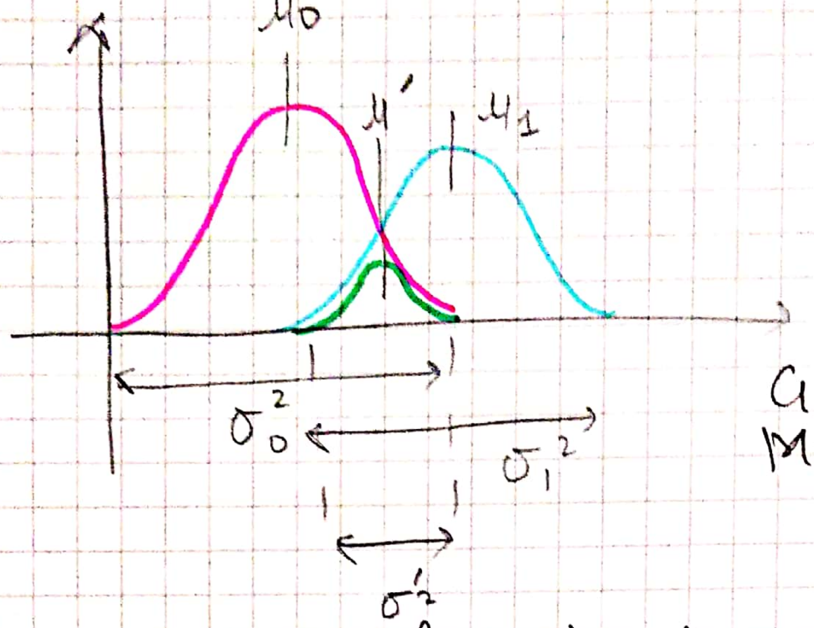

Gaussian
Multiplication

Intersection of best estimates lowers the uncertainty involved in process

Normal eq of bell curve

$$N(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The multiplication of normals of both bell curves provides

$$\mu' = \mu_0 + \frac{\sigma_0^2(\mu_1 - \mu_0)}{\sigma_0^2 + \sigma_1^2}$$

$$\sigma'^2 = \sigma_0^2 - \frac{\sigma_0^4}{\sigma_0^2 + \sigma_1^2}$$

We observe a common,

$$k = \frac{\sigma_0^2}{\sigma_0^2 + \sigma_1^2}$$

$$\mu' = \mu_0 + k(\mu_1 - \mu_0)$$

$$\sigma'^2 = \sigma_0^2 - k\sigma_0^2$$

In matrix form $\sigma_0^2 \rightarrow \Sigma$

Kalmain Gain : $K = \Sigma_0(\Sigma_0 + \Sigma_1)^{-1}$ — ①

$$\vec{\mu}_1' = \vec{\mu}_0 + K(\vec{\mu}_1 - \vec{\mu}_0) \text{ — ②}$$

$$\Sigma' = \Sigma_0 - K\Sigma_0 \text{ — ③}$$

We know,

→ predicted measurement

$$(\mu_0, \Sigma_0) = (H_k \hat{x}_k, H_k P_k H_k^T)$$

→ observed measurement/sensor

$$(\mu_1, \Sigma_1) = (\vec{z}_k, R_k)$$

for ② and ③

by 2 $H_k \hat{x}'_k = H_k \hat{x}_k - K(\vec{z}_k - H_k \hat{x}_k)$ — ④

by 3 $H_k P_k' H_k^T = H_k P_k H_k^T - K(H_k P_k H_k^T)$ — ⑤

for ①

$$K = H_k P_k H_k^T (H_k P_k H_k^T + \cancel{H_k P_k} R_k)^{-1}$$

→ ⑥

Dividing ④ ⑤ ⑥ by $H_k \; H_k^T$

$$\hat{x}_k' = \hat{x}_k + K'(\vec{z}_k - H_k \hat{x}_k)$$

$$P_k' = P_k - K' H_k P_k$$

$$K' = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1}$$

Update step