**Department of Mathematics**
**Indian Institute of Technology Bhilai**
**IC104: Linear Algebra-I**
**Tutorial Sheet 1: Systems of Equations**

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1. Solve the following system of equations using Gauss Elimination method

$$(a)\ \begin{cases} x+3y=1 \\ 2x+y=-3 \\ 2x+2y=-2 \end{cases} \qquad (b)\ \begin{cases} x+2y=4 \\ y-z=0 \\ x+2z=4 \end{cases}$$

2. For what values of $\alpha$ are there no solutions, many solutions, or unique solution to this system?

$$\begin{aligned} x+y&=1 \\ 3x+3y&=\alpha \end{aligned}$$

3. Pick the elementary matrices out of the following collection

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

4. Let $A \in M_{m\times n}(\mathbb{R})$ be a any given matrix and the elementary operation $e$ is $R_i \to R_i + 2R_j$, then for what matrix $E$, the equation $e(A) = EA$ holds good? Find the inverse of $E$ also.

5. Are the following matrices row equivalent to each other?

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 10 \\ 2 & 0 & 4 \end{bmatrix}$$

6. Find the RRE forms of the following matrices. If they are invertible, find the inverse.

$$A = \begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 0 & 5 & 6 \\ 1 & 5 & 1 & 5 \\ 2 & 3 & 7 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 3 & 2 \\ 2 & 4 & 7 \end{bmatrix}$$

7. What is the rank of the following matrices?

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 4 \\ 4 & 2 & 3 \\ 2 & 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 4 \\ 4 & 4 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 4 & 2 & 3 & 2 \end{bmatrix},$$

8. Solve the following system of equations using Gauss Jordan Elimination method

$$(a)\ \begin{cases} x+y=0 \\ 2x+y+3z=3 \\ x+2y+z=3 \end{cases} \qquad (b)\ \begin{cases} x+2y=1 \\ 2x+z=2 \\ 3x+2y+z-w=4 \end{cases} \qquad (c)\ \begin{cases} x+y-2z=-2 \\ y+3z=7 \\ x-z=-1 \end{cases}$$

9. Find the coefficients $a, b$ and $c$ so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2), (-1, 6)$, and $(2, 3)$.

10. Prove that, the following linear equations have the same solution set

$$ax + by = c$$

and

$$ax + dy = e$$

where $a, b, c, d, e \in \mathbb{R}$. Also determine the solution set.

11. Let $A \in M_{m\times n}(\mathbb{R})$ then is it possible for the system $Ax = b$ to have only a finitely many (greater than 1) solutions for any choice of $m$ and $n$? Give reasons for your answer.

12. Let $Ax = b$ be a linear system of $m$ equations in 2 variables. What are the possible choices for $RRE([A\ b])$, if $m \geq 1$?

13. Let $A \in M_n(\mathbb{C})$ such that $A \neq \alpha I$ for any $\alpha \in \mathbb{C}$ then prove that there exists a non-singular matrix $S$ such that $SAS^{-1} = B$ with $B = (b_{ij})$ having $b_{11} = 0$

14. Justify your answer

    (a) True or false: a system with more unknowns than equations has at least one solution or never inconsistent.

    (b) True or false: a system with more equations than unknown can not have a unique solution solutions.

    (c) True or false: a system with more equations than unknown is always consistent.