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IC 152 Linear Algebra **Instructor: Dr. Avijit Pal**

1. Let A be a real $n \times n$ orthogonal matrix, that is , $A^tA = AA^t = I_n$ the $n \times n$ identity matrix. Which of the following statements are true? Justify your answer.

   (a) $\langle Ax, Ay \rangle = \langle x, y \rangle, \ \forall x, y \in \mathbb{R}^n$.

   (b) All eigenvalues are either $+1$ or -1.

   (c) The rows of A form an orthonormal basis for $\mathbb{R}^n$.

2. Let $W = \text{span}\{(i, 0, 1)\}$ in $\mathbb{C}^3$. Find the orthonormal basis for W and $W^{\perp}$.

3. Determine whether the following is a IPS over the given vector space.

   (a) $\langle (a, b), (c, d) \rangle = ac - bd$, on $\mathbb{R}^2$.

   (b) $\langle A, B \rangle = \text{tr}(A + B)$, on $M_{2\times2}(\mathbb{R})$.

   (c) $\langle f(x), g(x) \rangle = \int_0^1 f'(t)g(t)dt$, on P(R), where $'$ denote the differentiation.

4. Let $V = C([-1, 1])$. Suppose W_e and W_o denote the subspace of V consisting of even and odd functions respectively. Prove that $W_e^{\perp} = W_o$, where the inner product is defined as

$$\langle f, g \rangle = \int_{-1}^{1} f(t)g(t)dt.$$

5. Let $\{(1, -1, 1, 1), (1, 0, 1, 0), (0, 1, 0, 1)\}$ be a linearly independent set in $\mathbb{R}^4(\mathbb{R})$. Find an orthonormal set v_1, v_2, v_3 such that

$$\text{span}\{(1, -1, 1, 1), (1, 0, 1, 0), (0, 1, 0, 1)\} = \text{span}\{v_1, v_2, v_3\}.$$