Tutorial 6: Probability and Statistics (IC105)

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1. The joint pdf of X and Y is given as

$$f_{X,Y}(x,y) = \begin{cases} k(1-x-y), & x>0, y>0, x+y<1 \\ 0, & \text{Otherwise} \end{cases}$$

   Find the $E(X|Y=1/2)$ and $E(Y|X=1/2)$

2. A two dimensional discrete random vector (X,Y) having pmf as

$$f_{X,Y}(x,y) = P(x=x, Y=y) = \begin{cases} c(3x+4y), & x=0,1,2,3, y=1,2,3,4 \\ 0, & \text{Otherwise} \end{cases}$$

   Find the $E(X|Y=1)$ and $E(Y|X=0)$

3. The joint pdf of X and Y is given as

$$f_{X,Y}(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0<x<2,\ 2<y<4, \\ 0, & \text{Otherwise} \end{cases}$$

   Find the $E(X|Y=3)$ and $E(Y|X=1)$

4. Let X and Y be two independent $U(0,1)$ random variables. Then find the distributions of (a) XY, (b) $\frac{X}{Y}$.

5. Let X be Poisson $\mathcal{P}(2)$ and Y be $Bin(10,3/4)$ random variables. X and Y are independent, then find the value of $P(XY=0)$.

6. The pmf of a two dimension discrete random vector (X,Y) is given as

Y/X	-1	0	1
-2	1/6	1/12	1/6
1	1/6	1/12	1/6
2	1/12	0	1/12

   Find the joint distribution of $|X|$ and Y^2.

7. Let X and Y be independent and identically distributed uniform random variable over the interval $(0,1)$ and let $S = X+Y$. Find the probability that the quadratic equation $9x^2+9Sx+1=0$ has no real root.

8. Let the random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = \begin{cases} ce^{-(x+y)}, & y>x>0, \\ 0, & \text{Otherwise} \end{cases}$$

(a) Find the value of c.

(b) Find the value of $E(Y|X=2)$.

9. Let X_1 and X_2 be independent random variables, each having exponential distribution with parameter λ. Then find the conditional distribution of X_1 given that $X_1 + X_2 = 1$.

10. The conditional probability density function of X given $Y = y(> 0)$ is

$$f_{X|Y}(x|y) = \begin{cases} ye^{-yx}, & x > 0, \\ 0, & \text{Otherwise} \end{cases}$$

and the marginal pdf of Y is

$$f_Y(y) = \begin{cases} \alpha e^{-\alpha y}, & y > 0, \\ 0, & \text{Otherwise} \end{cases}$$

Find the conditional probability density function of Y given $X = x$.

11. X_1, X_2, X_3, X_4, X_5 independent random variables with $X_1 \sim N(200, 8)$, $X_2 \sim N(104, 8)$, $X_3 \sim N(108, 15)$, $X_4 \sim N(120, 15)$ and $X_5 \sim N(210, 15)$. Let $U = \frac{X_1+X_2}{2}$ and $V = \frac{X_3+X_4+X_5}{3}$. Then find the value of $P(U > V)$.

12. (a) Six fair dice are thrown independently. Let S denote the number of dice showing even numbers on their upper faces. Then the find mean the variance of S.

(b) Let X_1 and X_2 be independent random variables with respective moment generating function $M_1(t) = \left(\frac{3}{4} + \frac{1}{4}e^t\right)^3$ and $M_2(t) = e^{2(e^t-1)}$. Find the value of $P(X_1 + X_2 = 1)$.