Tutorial 6

Linear Algebra-(IC152)
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31/03/2023

Question 1

Reduce the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ to the normal form and show that it is positive definite.

Question 2

Show that the quadratic form $x^2 + 2y^2 + 3z^2 - 2xy + 4yz$ is indefinite.

Question 3

Let B is a real $n \times n$ matrix. Show that the symmetric matrix $B^t B$ is either positive definite or positive semidefinite and it is positive definite or positive semidefinite according as B is non-singular or singular.

Question 4

Prove that eigenvalues of a real symmetric matrix are all real.

Question 5

Let A be any $n \times n$ invertible symmetric matrix. Show that if the quadratic form $x^T A x$ is positive definite, then so is the quadratic form $x^T A^{-1} x$.

Question 6

Let A and B be symmetric $n \times n$ matrices whose eigenvalues are all positive. Show that the eigenvalues of $A + B$ are all positive.

Question 7

Reduce the equation $x^2 - 6xy + y^2 - 4x - 4y + 12 = 0$ into canonical form and determine the nature of the conic.

Question 8

Write down the following Quadratic form into matrix form

1. $2x^2 + 3y^2 + 6xy$.

2. $2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx$.