Tutorial 3: Calculus I (IC153)

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1. Discuss the convergence/ divergence of the following series

   (i) $\sum_{n=1}^{\infty} \frac{n^2+1}{(n+3)(n+4)}$ (ii) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$ (iii) $\sum_{n=1}^{\infty} \frac{\sin n+1}{n^2+1}$ (iv) $\sum_{n=1}^{\infty} \frac{1}{2^n+n}$ (v) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$ (vi) $\sum_{n=1}^{\infty} \frac{n}{4n^3-2}$

   (vii) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ (viii) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{n^2}}$ (ix) $\sum_{n=1}^{\infty} \frac{5^n}{3^n+4^n}$ (x) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p},\ p > 0$ (xi) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$

2. Test convergence/ divergence of the following series

   (i) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+5n-1}$ (ii) $\sum_{n=1}^{\infty} \frac{n^2}{10n^2+n-13}$ (iii) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}+\sqrt{n}}{3n+n^2+\sqrt{n}}$ (iv) $\sum_{n=1}^{\infty} \frac{3^n+1}{7^n+4}$ (v) $\sum_{n=1}^{\infty} \frac{3^n+1}{2^n+400}$ (vi) $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$

   (vii) $\sum_{n=1}^{\infty} \sin^2 \frac{1}{n}$ (viii) $\sum_{n=1}^{\infty} n \tan \frac{1}{n^2}$

3. Test convergence/ divergence of the following series

   (i) $\sum_{n=1}^{\infty} \frac{5^{n+1} 7^{n-1}}{n!}$ (ii) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ (iii) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ (iv) $\sum_{n=1}^{\infty} \frac{a^n n!}{n^n}$ (v) $\sum_{n=1}^{\infty} \frac{n!}{2^{2n}}$ (vi) $\sum_{n=1}^{\infty} \frac{2\cdot 4 \cdots 2n}{5\cdot 8 \cdots (3n+2)}$ (vii) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

4. Test convergence/ divergence of the following series

   (i) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+3}\right)^n$ (ii) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$ (iii) $\sum_{n=1}^{\infty} \frac{5^n}{n^{n+1}}$

5. (i) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ using Cauchy condensation test

   (ii) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$.

   (iii) Show that the series $\sum_{n=3}^{\infty} \frac{1}{n(\log n)(\log \log n)}$ is divergent.

   (iv) Prove that the series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n(\log n)^{1/3}}$ is conditionally convergent.