Indian Institute of Technology, Bhilai
Department of Mathematics

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Tutorial -1, Instructor: Dr. Avijit Pal, Linear Algebra (IC152) Semester: Winter

1. Let $V = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : a, b > 0 \right\}$ is a subspace over the field $\mathbb{R}$, where the vector addition and the scalar multiplication is defined by $\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ bd \end{bmatrix}$ and $t \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a^t \\ b^t \end{bmatrix}$, for $t \in \mathbb{R}$. Now define a map $T : V \to \mathbb{R}^2$ such that

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \log_2 y \\ \log_2 x \end{pmatrix}.$$

Then $\longrightarrow$

   (a) Find the basis β for V.
   (b) Prove that T is linear.
   (c) Calculate the kernel of T.
   (d) Find the matrix of T related to the basis β and the standard basis for $\mathbb{R}^2$.

2. Give an example of a linear map, which has no eigen values.

3. Let $D : P_4 \to P_4$ be a linear transformation such that $D(p(x)) = \frac{d}{dx} p(x)$, where $p(x) \in P_4$ (the set of polynomial with degree less than or equal to 4 over $\mathbb{R}$). Then relative to the basis $\{1, x, x^2, x^3\}$ of P_4, determine the matrix of D. Hence find the eigen value and eigen vector of D.

4. Let T be the linear operator on $M_{n \times n}(\mathbb{R})$, defined by $T(A) = A^t$. Show that ± 1 are the eigenvalues of T.

5. (a) Let λ be an eigen value of an $n \times n$ matrix A, prove that λ^m is an eigen value of the matrix A^m, where m is a positive integer.
   (b) $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the eigen values of a non-singular matrix A of order n. Then find the eigen values of the matrix
      i. A^{-1},
      ii. adj$A$.

6. If every non-zero vector $\mathbb{R}^n$ be an eigen vector of a real $n \times n$ matrix A corresponding to a real eigen value λ, prove that A is the scalar matrix λI_n.