Tutorial 3: Probability and Statistics (IC105)

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1. Let X be a random variable with distribution function F. Then find the distribution function fo $|X|,\ aX+b,$ where $a \neq 0, b \in \mathbb{R},\ \max\{X,0\}$ and $\min\{X,0\}$.

2. Let X be a discrete random variable with p.m.f. $P(X=-2)=\frac{1}{5}$, $P(X=-1)=\frac{1}{6}$, $P(X=0)=\frac{1}{5}$, $P(X=1)=\frac{1}{15}$ and $P(X=2)=\frac{11}{30}$. Find the p.m.f. and d.f. of $Y=X^2$.

3. Let X be a random variable with pdf
$$f_X(x) = \begin{cases} \frac{1}{2}, & -1<x<1 \\ 0, & \text{Otherwise.} \end{cases}$$
Find the distribution function of $Y=\max\{X,0\}$.

4. The random variable X has pdf $f_X(x)=\frac{1}{2}e^{-|x|},\ -\infty<x<\infty$. Find the distribution of $Y=X^2$.

5. Suppose X have the density function
$$f_X(x) = \begin{cases} c(x+1), & -1<x<2 \\ 0, & \text{Otherwise.} \end{cases}$$
Find the value of c. Hence calculate the pdf and cdf of $Y=X^2$.

6. Suppose X have the density function
$$f_X(x) = \begin{cases} 3x^2, & 0<x<1 \\ 0, & \text{Otherwise.} \end{cases}$$
Find the density function of $Y=40(1-X)$.

7. Among the 15 applicants for a job, 9 are women and 6 are men. 5 applicants are randomly selected from the applicant pool for final interviews. Let X is the number of female applicants among the final 5. (i) Give the probability mass function for X . (ii) Define Y, the number of male applicants among the final 5, as a function of X . Find the probability mass function for Y.

8. If X is a random variable such that $E(X)=3$ and $E(X^2)=13$, then determine a lower bound for $P(-2<X<8)$.

9. Let the random variable X have the m.g.f.
$$M(t)=\frac{e^{-2t}}{8}+\frac{e^{-t}}{4}+\frac{e^{2t}}{8}+\frac{e^{3t}}{2}.$$
Find the distribution function of X and find $P(X^2=4)$.

10. Let X be a random variable with m.g.f. $M(t), \ -h < t < h$

    (a) Prove that $P(X \geq a) \leq e^{-at}M(t), \ 0 < t < h$;

    (b) Prove that $P(X \leq a) \leq e^{-at}M(t), \ -h < t < 0$;

11. Suppose 15% of items produced at a manufacturing facility are defective. What is the probability that a lot of randomly selected 10 items contains more than 3 defective items?

12. The average number of trains either arriving at or departing from a railway station is one every 5 minutes. What is the probability that at least 10 trains arrive/depart during a selected hour? What is the probability that fewer than 4 such train will take place in an hour?

13. A electronic system consists of n parts each of which function independently with probability p. The entire system will be able to operate effectively, if at least one-half of its components function. For what values of p, a 5-component system more likely to operate effectively than a 3-component system?

14. The DVD produced by a certain company are defective with probability 0.01, independently of each other. The company sells the DVDs in packs of size 10 and offers a money-back guarantee if more than one of the 10 DVDs in the pack is found to be defective. If you buy 3 packs, what is the probability that at most one pack will be returned.

15. The number of times that an individual contracts cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose that a new drug has been just marketed that reduces the parameter λ to 2 for 75% of the population. For the other 25% of the population the drug has no appreciable effect on the cold. If an individual tries the drug for a year and has no cold in that time, how likely is it that the drug is beneficial for him?