Tutorial 5: Probability and Statistics (IC105)
Indian Institute of Technology Bhilai

1. Suppose X denotes the time (in hours) needed to locate and rectify a problem in the software that governs the timing of lights in a large city. Let X follow $N(10, 9)$. Find the probability that the next problem will require at most 15 hours to locate and rectify.

2. In a town adult population have equally divided opinion on the implementation of a municipality project. A random sample of 100 adults is asked to give their opinion. Using binomial approximation to normal determine the probability that at least 60% of adults in the sample are in favour of the project.

3. In an industrial process the diameter of a ball bearing is an important component. The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The diameter has a normal distribution with mean 3 cm. and s.d. 0.005 cm. On the average what percentage manufactured balls will be scrapped?

4. The height a university high jumper will clear, each time he jumps, is a normal r.v. with mean 2 meters and s.d. 10 cm. What is the greatest height that he will jump with probability 0.95? What is the height that he will clear only 10% of the time?

5. If a set of marks on a Statistics exam is approximately $N(74, 62.41)$, find

   (a) the lowest passing grade if the lowest 10% of the students are given F's;

   (b) the highest B if the top 5% of the students are given A's

6. A box contain 5 white balls numbered 0, 1, 2, 3, 4; 4 red balls numbered 0,1,2,3; 3 black balls numbered 0, 1, 2 and 2 blue balls numbered 0,1. A ball was drawn at random from the bag. Let the random variables X and Y defined as follows: X takes values 0, 1, 2, 3 respectively for white, red, black and blue balls. Y denotes the numbered of the ball. Find the joint distribution of X and Y . Also find the marginal distribution of X and Y.

7. A boy and a girl plan to meet a coffee shop between 4 pm and 5 pm each of agreing not to wait more than 20 minutes for the other. They arrived at the coffee shop independently between 4pm to 5 pm. What is the probability that they will meet.

8. The joint pdf of X and Y is given as

$$f_{X,Y}(x, y) = \begin{cases} k(1 - x - y), & x > 0, y > 0, x + y < 1 \\ 0, & \text{Otherwise} \end{cases}$$

   (a) Find the value of k

   (b) Find the marginal pdf of X and Y.

   (c) Find $E(X)$, $E(Y)$, $E(XY)$ and $Cov(X, Y)$ and $\rho_{X,Y}$.

9. A two dimensional discrete random vector (X, y) having pmf as

$$f_{X,Y}(x, y) = P(x = x, Y = y) = \begin{cases} c(3x + 4y), & x = 0, 1, 2, 3, y = 1, 2, 3, 4 \\ 0, & \text{Otherwise} \end{cases}$$

   (a) Find the value of c
   (b) Find the marginal distribution of X and Y.
   (c) Find $P(X \geq 2|Y \leq 3)$ and $P(Y = 2|X = 3)$.

10. The joint pmf of a discrete random vector (X, Y) given as

X/Y	−1	0	1
0	0	1/3	0
1	1/3	0	1/3

   (a) Find the marginal distribution of X and Y.
   (b) Find $E(X), E(Y), E(XY)$ and $\rho_{X,Y}$.
   (c) Are X and Y independent?

11. Let (X, Y) with the joint pdf

$$f_{X,Y}(x, y) = \begin{cases} 6xy(2 - x - y), & 0 < x < 1, \ 0 < y < 1, \\ 0, & \text{Otherwise} \end{cases}$$

   (a) Find the marginal distribution of X and Y
   (b) Find $E(X), E(Y), E(XY)$ and $\rho_{X,Y}$.
   (c) Are X and Y independent?

12. Let (X, Y) with the joint pdf

$$f_{X,Y}(x, y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2, \ 2 < y < 4, \\ 0, & \text{Otherwise} \end{cases}$$

   (a) Find the marginal distribution of X and Y
   (b) $P(X < 1, Y < 3), P(X + Y < 3), P(X < 1|Y = 3)$ and $P(X < 1|Y < 3)$.