Solu^n- tutorial 4

(i) Let X denote the length of AP.

$PB = (2a - X)$. $\quad X \sim U(0, 2a)$

$$f_X(x) = \begin{cases} \frac{1}{2a}, & 0 < x < 2a \\ 0, & o/w \end{cases}$$

$$E(AP \cdot PB) = \int_0^{2a} x(2a-x) f_X(x)\, dx = \frac{2a^2}{3}$$

(ii) $|AP - PB| = 2|X - a|$

$$E|AP - PB| = 2\int_0^{2a} |x-a| f_X(x)\, dx$$

$$= 2\int_0^{a} (a-x) f_X(x)\, dx + 2\int_a^{2a} (x-a) f_X(x)\, dx$$

$$= a.$$

(iii) $\max\{AP, PB\} = \max\{X, 2a-X\} = \begin{cases} 2a - X & \text{if } X < a \\ x & \text{if } X \geq a. \end{cases}$

$$E(\max\{AP, PB\}) = \int_0^{a} (2a-x) f_X(x)\, dx + \int_a^{2a} x f_X(x)\, dx$$

$$= \frac{3}{2} a.$$

(2) Suppose n bombs are dropped and let X denote the number of direct hits. Then $X \sim Bin(n, \frac{1}{2})$

we want n s.t.

$$P(X \geq 2) \geq 0.99$$

$$\text{or } 1 - P(X=0) - P(X=1) \geq 0.99$$

$$\text{or } P(X=0) + P(X=1) \leq 0.01$$

$$\text{or } \left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n \leq 0.01$$

$$\text{or } 2^n \geq 100(n+1) \quad \text{—— } (*)$$

The smallest value of n for which $(*)$ is satisfied is $n = 11$.

(3) – Same as 2 do yourself.

(4) Let $X \rightarrow$ time in minutes past 7.00 a.m. that the passenger arrive at the bus stop. Then $X \sim U(0,30)$. The passenger will have to wait less than 5 min, if he/she arrives between 7:10 & 7:15 or between 7:25 & 7:30 am. (a) Hence required prob $P(10 < X < 15) + P(25 < X < 30) = \frac{5}{30} + 5/30 = 1/3$

(b) Similarly the P(wait at least 12 min) $= P(0 < X < 3) + P(15 < X < 18)$
$= 1/5$.

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Let p be the cut point.

Let X be the length of AP.

$X \sim U(0,2)$

Then the required prob is

$$P(\max\{x, 2-x\} \geq 2\min\{x, 2-x\})$$

There are two possibilities. if x is bigger than $(2-x)$

Then $X > 2(2-x) \Rightarrow x > 3/4$. ——(I)

If x is smaller than $(2-x)$ Then

$$(2-X) \geq 2X \Rightarrow X < 3/2 \quad \text{——(II)}$$

Now $\max\{X, 2-X\} \geq 2 \min\{X, 2-X\}$ will hold if any one of (I) & (II) holds and $X < 3/2$ & $X > 3/4$ are mutually disjoint

$$P(\max\{X, 2-X\} \geq 2\min\{X, 2-X\}) = P(X < 3/2) + P(3/4 < X < 2) = \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\left(\frac{2}{3}\right) = 2/3$$

(6) Let X denote the r.v. daily consumption of oil in city in excess of 30,000 gallons.

$X \sim \text{Gamma}\left(2, \frac{1}{10000}\right)$

$$f_X(x) = \begin{cases} \left(\frac{1}{10000}\right)^2 x e^{-x/10000} & , x > 0 \\ 0 & \text{o/w} \end{cases}$$

$E(X) = 20000$

The required prob is $P(X > 10000)$

$$= \int_{10000}^{\infty} \frac{x}{(10000)^2} e^{-x/10000} \quad , \text{let } y = \frac{x}{10000}$$

$$= \int_{1}^{\infty} y e^{-y} dy = 2e^{-1}.$$

(7) X denote the the time to failure (in years)

. 1 the density of X is

$$f_X(x) = \begin{cases} \frac{1}{8} e^{-x/8} & , x > 0 \\ 0 & \text{o/w}. \end{cases}$$

Then the percentage of TV's will fail with warranty period is $= P(X<1) \times 100\% = 0.1175 \times 100\%$

$= 11.75\%$

$E(\text{Profit on sell of one TV}) = 10000\, P(X>1) - 15000\, P(X \le 1) = \alpha$ (say)

(Find α)

$E(\text{Profit on } 1000 \text{ TV}) = 1000 \times \alpha$.

(8) Given that the lead time of oders of diodes from a certain manufacturer follow a Gamm (r, λ) where $\frac{r}{\lambda} = 20$, $\frac{r}{\lambda^2} = 100$

$\Rightarrow r = 4$, $\lambda = 1/5$. Let X denote the lead time $X \sim \text{Gamma}(4, 1/5)$

Required prob $P(X \le 15) = \int_0^{15} \frac{1}{5^4} \frac{x^3 e^{x/5}}{\Gamma(4)} dx.$

(9) Let X denote the no. of defects in a 2% area of the total surface.

$X \sim \mathcal{P}(\lambda)$, $\lambda = 300 \times 0.02 = 6$.

Required prob

$$P(X \leq 4) = \sum_{x=0}^{4} \frac{e^{-6} 6^x}{x!} \simeq 0.285.$$

(10) Let X denote the life of a bulb in hours

Given that

$X \sim Exp(\lambda)$, with $E(X) = \frac{1}{\lambda} = 50$

$$\Rightarrow \lambda = \frac{1}{50}.$$

$$f_X(x) = \begin{cases} \frac{1}{50} e^{-x/50}, & x > 0 \\ 0, & o/w. \end{cases}$$

Now P(A bulb working after 100 hrs) $= P(X > 100)$

$= e^{-2}$

Let Y denote the no of bulb working after 100 hrs. Then $Y \sim Bin(10, e^{-2})$, $[p = e^{-2}]$

$$P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$$
$$= 1 - (1-e^{-2})^{10} - 10(e^{-2})(1-e^{-2})^9$$