Indian Institute of Technology, Bhilai
Department of Mathematics

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Tutorial-4, Instructor: Dr. Avijit Pal, Linear Algebra (IC152) Semester: Winter

1. Show that if $v_1, v_2, \ldots, v_n$ are pairwise orthogonal in a real inner product space V, then

$$||v_1 + v_2 + \cdots + v_n||^2 = ||v_1||^2 + ||v_2||^2 + \cdots + ||v_n||^2.$$

   If $v_1, v_2, \ldots, v_n$ are orthonormal, then prove that

$$||v_1 + v_2 + \ldots v_n|| = \sqrt{n}.$$

2. Show that $\langle u, v\rangle = \frac{1}{4}\|u+v\|^2 - \frac{1}{4}\|u-v\|^2$ for any u and v in a real inner product space.

3. Find a polynomial $f(t) = a + bt$, where a and b is real scalar, such that $f(t)$ is perpendicular to the polynomial $g(t) = 1 - t$ with respect to the inner product

$$\langle f, g\rangle = \int_0^1 f(t)g(t)dt.$$

4. Prove or disprove: There is an inner product on $\mathbb{R}^2$ such that the associated norm is given by

$$||(x_1, x_2)|| = |x_1| + |x_2|,\ \forall (x_1, x_2) \in \mathbb{R}^2.$$

5. Let A be an $n \times n$ Hermitian matrix. Check that the linear transformation $T_A : \mathbb{C}^n \to \mathbb{C}^n$ defined by

$$T_A(z) = Az$$

   for every $z^t \in \mathbb{C}^n$ is a self-adjoint operator.

6. Consider the real vector space $\mathbb{R}^2$. In this example, we define three products that satisfy two conditions out of the three conditions for an inner product. Hence the three products are not inner products. Define $\langle \mathbf{x}, \mathbf{y} \rangle = \langle (x_1, x_2), (y_1, y_2) \rangle = x_1 y_1$. Then it is easy to verify that the third condition is not valid whereas the first two conditions are valid. Define $\langle \mathbf{x}, \mathbf{y} \rangle = \langle (x_1, x_2), (y_1, y_2) \rangle = x_1^2 + y_1^2 + x_2^2 + y_2^2$. Then it is easy to verify that the first condition is not valid whereas the second and third conditions are valid. Define $\langle \mathbf{x}, \mathbf{y} \rangle = \langle (x_1, x_2), (y_1, y_2) \rangle = x_1 y_1^3 + x_2 y_2^3$. Then it is easy to verify that the second condition is not valid whereas the first and third conditions are valid.