Tutorial 2: Probability and Statistics (IC105)

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1. Check the following functions are distribution functions are not

(i) $F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \le x \le \frac{3}{2} \\ 1, & x > \frac{3}{2} \end{cases}$ (ii) $F(x) = \begin{cases} 0, & x \le 1 \\ 1 - \frac{1}{x^2}, & x > 1 \end{cases}$

(iii) $F(x) = \begin{cases} 0, & x \le 0 \\ \frac{1}{2} + \frac{e^{-x}}{2}, & x > 0 \end{cases}$ (iv) $F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{8}, & 0 \le x < 1 \\ \frac{x+1}{8}, & 1 \le x < 2 \\ \frac{2x+1}{8}, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$

2. Consider a function F as

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-x}, & x \ge 0 \end{cases}$$

   (a) Show that F is a distribution function.

   (b) Find the value of $P(2 < X \le 3),\ P(-2 < X \le 3),\ P(1 \le X < 4),\ P(5 \le X < 8)$.

3. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{2}{3}, & 2 \le x < 5 \\ \frac{7-6k}{6}, & 5 \le x < 9 \\ \frac{3k^2-6k+7}{6}, & 9 \le x < 14 \\ \frac{16k^2-16k+19}{16}, & 14 \le x \le 20 \\ 1, & x > 20 \end{cases}$$

   (a) Find the value of constant k;

   (b) Show that the r.v. X is of discrete type and find its support;

   (c) Find the p.m.f. of X.

4. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{4}, & 0 \le x < 1 \\ \frac{x}{3}, & 1 \le x < 2 \\ \frac{3x}{8}, & 2 \le x < 5/2 \\ 1, & x \ge 5/2 \end{cases}$$

   (a) Prove that X is neither continuous nor discrete random variable.

(b) Find the value of $P\left(1 < X \leq \frac{5}{2}\right)$, $P\left(1 < X < \frac{5}{2}\right)$, $P\left(1 \leq X < \frac{5}{2}\right)$, $P(-2 \leq X < 1)$, $P(X \geq 2)$, $P(X > 2)$.

5. Let X be a continuous type random variable with p.d.f.

$$f(x) = \begin{cases} k - |x|, & |x| < \frac{1}{2} \\ 0, & \text{Otherwise,} \end{cases}$$

where $k \in \mathbb{R}$.

(a) Find the value of constant k;

(b) Evaluate $P(X < 0)$, $P(X \leq 0)$, $P(0 < X \leq 1/4)$ $P(-1/8 \leq X \leq 1/4)$

(c) Find the distribution function of X.

6. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1 \\ 0, & \text{Otherwise.} \end{cases}$$

(a) What is the value of c ?

(b) Find the cumulative distribution function of X.

(c) Find $E(X)$ and $Var(X)$.

7. Consider a function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{1}{\beta}\left(1 - \frac{|x-\alpha|}{\beta}\right), & \alpha - \beta < x < \alpha + \beta \\ 0, & \text{Otherwise,} \end{cases}$$

where $\alpha \in \mathbb{R}$, $\beta > 0$.

(a) Show that $f(x)$ is a probability density function.

(b) Find the distribution function.

(c) Find $E(X)$ and $Var(X)$.