

## Tutorial 4: Probability and Statistics (IC105)

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1. A point  $P$  is chosen at random on a line segment  $AB$  of length  $2a$ . Find the expected values of (i)  $AP \cdot PB$  (ii)  $|AP - PB|$  (iii)  $\max\{AP, PB\}$ .
2. In a precision bombing attack there is a 50% chance that a bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least 99% chance of completely destroying the target?
3. Suppose 5% of chips manufactured at a plant are defective. How many should he/she buy in order that there is more than 99% chance of having at most one defective chip?
4. Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits (a) less than 5 minutes for a bus; (b) more than 12 minutes for a bus.
5. If a string of 2 meter is cut into two pieces at a random point along its length, what is the probability that the length of the longer piece is at least twice the length of the shorter piece?
6. The daily consumption of oil in a city, in excess of 30,000 gallons, is approximately distributed as a gamma distributed with parameters  $\alpha = 2$  (two consumptions in a day) and  $\beta = 1/10000$  (time taken in days per consumption of one gallon of oil). The city has a daily stock of 40,000 gallons. What is the probability that the stock is insufficient on a particular day?
7. TV- manufacturer offers a one year warranty. If a TV fails during this warranty period, it is replaced free of cost. The time to failure (in years)  $X$  has a shifted exponential distribution with location parameter 0 and scale parameter 8. What percentage of TVs will fail within the warranty period? The manufacturing cost of a TV is Rs 25,000 and the net profit per unit scale is Rs 10,000. If in the first week of January 2010, thousand TVs are sold, what is the expected profit after a year?
8. The lead time for orders of diodes from a certain manufacturer is known to have a gamma distribution with a mean of 20 days and a standard deviation of 10 days. Determine the probability of receiving an order within 15 days of placement date.
9. A large microprocessor chip contains multiple copies of circuits. If a circuit fails, the chip knows it and knows how to select the proper logic to repair itself. The average number of defects per chip is 300. What is the probability that no more than 4 defects will be found in a randomly selected area that comprises 2% of the total surface area?
10. A small industrial unit has 10 bulbs whose lifetimes are independent exponentially distributed with mean 50 hours. If all the bulbs are used at a time, find the probability that even after 100 hours there are at least two bulbs working.