# Week-3:Generating and Solving Uniform Random 3-SAT

Chetankumar kamani(20251603005), Sakariya Devraj (20251603006), Divyesh Dodiya (20251603007),
M.Tech Sem-1 IIIT Vadodara

## I. Problem Statement

Write programs to generate uniform random $k$-SAT instances and, for $k$=3, solve a set of random 3-SAT problems for different $(m, n)$ combinations. Compare **Hill-Climbing**, **Beam-Search** (beam widths 3 and 4), and **Variable-Neighborhood Descent** (three neighborhoods). Use two heuristic functions and compare them with respect to *penetrance* (fraction of runs solved).

### A. Uniform Random $k$-SAT (Fixed Clause Length Model)

Given $k, m, n$, generate $m$ clauses. Each clause picks $k$ *distinct* variables from $\{x_1, \ldots, x_n\}$ uniformly at random; each literal is negated with probability $1/2$. For $k$=3 we obtain uniform random 3-SAT.

## II. Python Code

### A. A. Simple $k$-SAT Generator

```
import random

def generate_kSAT(k, m, n):
    clauses = []
    for _ in range(m):
        # pick k distinct variables
        vars_chosen = random.sample(range(1, n+1),
             k)
        clause = []
        for v in vars_chosen:
            if random.choice([True, False]):
                clause.append(f"x{v}")
            else:
                clause.append(f" x {v}")
        clauses.append(clause)
    return clauses

# Example usage
k = 3   # length of each clause
m = 5   # number of clauses
n = 4   # number of variables

formula = generate_kSAT(k, m, n)

print("Random k-SAT formula:")
for i, clause in enumerate(formula, 1):
    print(f"C{i}: ({'     '.join(clause)})")

%% Example function signature:
%%   generate_kSAT(k, m, n) -> list[list[str]]
```

### B. B. 3-SAT (HC, Beam=3/4, VND) with Two Heuristics

```
import random

#3-SAT
def generate_3sat(m, n):
    """Generate random 3-SAT instance with m
        clauses, n variables"""
    clauses = []
    for _ in range(m):
        vars3 = random.sample(range(1, n+1), 3)
        clause = []
        for v in vars3:
            if random.choice([True, False]):
                clause.append(v)
            else:
                clause.append(-v)
        clauses.append(clause)
    return clauses

def evaluate(clauses, assignment):
    """Number of satisfied clauses"""
    count = 0
    for c in clauses:
        ok = False
        for lit in c:
            v = abs(lit)
            val = assignment[v]
            if (lit > 0 and val) or (lit < 0 and
                not val):
                ok = True
        if ok: count += 1
    return count

# two heuristics
def h1(clauses, A): return evaluate(clauses, A)
def h2(clauses, A): return 2*evaluate(clauses, A)
    - len(clauses)

# hill climbing
def hill_climbing(clauses, n, heuristic, steps
    =1000):
    A = [None] + [random.choice([False, True]) for
         _ in range(n)]
    for _ in range(steps):
        if evaluate(clauses, A) == len(clauses):
            return True

        best_score, best_var = heuristic(clauses,
            A), None
        for v in range(1, n+1):
            A[v] = not A[v]
            score = heuristic(clauses, A)
            A[v] = not A[v]
            if score > best_score:
                best_score, best_var = score, v
        if best_var is None:
            v = random.randint(1, n)
            A[v] = not A[v]
        else:
            A[best_var] = not A[best_var]
    return False
```

```
def beam_search(clauses, n, heuristic, beam_width
    =3, steps=200):
    beam = []
    for _ in range(beam_width):
        A = [None] + [random.choice([False, True])
             for _ in range(n)]
        beam.append(A)
    for _ in range(steps):
        new_beam = []
        for A in beam:
            if evaluate(clauses, A) == len(clauses
                ): return True
            for v in range(1, n+1):
                B = A.copy()
                B[v] = not B[v]
                new_beam.append(B)
        new_beam.sort(key=lambda x: heuristic(
            clauses, x), reverse=True)
        beam = new_beam[:beam_width]
    return False

def vnd(clauses, n, heuristic, steps=1000):
    A = [None] + [random.choice([False, True]) for
         _ in range(n)]
    for _ in range(steps):
        if evaluate(clauses, A) == len(clauses):
            return True
        improved = False
        for v in range(1, n+1):
            A[v] = not A[v]
            if heuristic(clauses, A) > heuristic(
                clauses, A):
                improved = True
            A[v] = not A[v]
        if not improved:
            v = random.randint(1, n)
            A[v] = not A[v]
    return False


n = 6   # variables
m = 15  # clauses
clauses = generate_3sat(m, n)
print("Generated 3-SAT clauses:", clauses)

for hname, hfun in [("h1", h1), ("h2", h2)]:
    print(f"\nUsing heuristic {hname}:")
    print(" Hill-Climbing:", hill_climbing(clauses
        , n, hfun))
    print(" Beam Search (width=3):", beam_search(
        clauses, n, hfun, beam_width=3))
    print(" Beam Search (width=4):", beam_search(
        clauses, n, hfun, beam_width=4))
    print(" VND:", vnd(clauses, n, hfun))

%%   - instance generator generate_3sat(m, n)
%%   - evaluate(), heuristics h1 and h2
%%   - hill_climbing(), beam_search(beam_width
    =3/4), vnd()
%%   - a small main that prints solved/unsolved
    for both heuristics
```

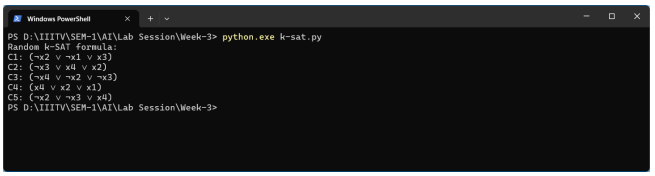
## III. How to Run

- Generate $k$-SAT or run 3-SAT solvers from terminal:

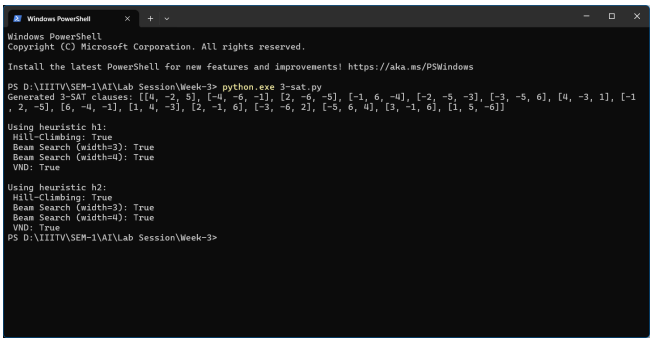
```
python k-sat.py
python 3-sat.py
```

## IV. Screenshots of Results

### A. *k-sat.py*




### B. *3-sat.py*




## V. Code Availability

- The complete source code is available at: GitHub Repository (CS659 – AI Laboratory).
- **GitHub Repository:** https://github.com/ChetanKamani/CS659-LAB-TASK