UDAAN

2026

REAL NUMBERS

MATHS LECTURE-5

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Topics *to be covered*

- **A** Real Numbers (Basic of Rational and Irrational Numbers)
- **B** Proof of Irrationality


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Real Numbers

$\frac{p}{q}$ ← Integers, $q \neq 0$

Rational

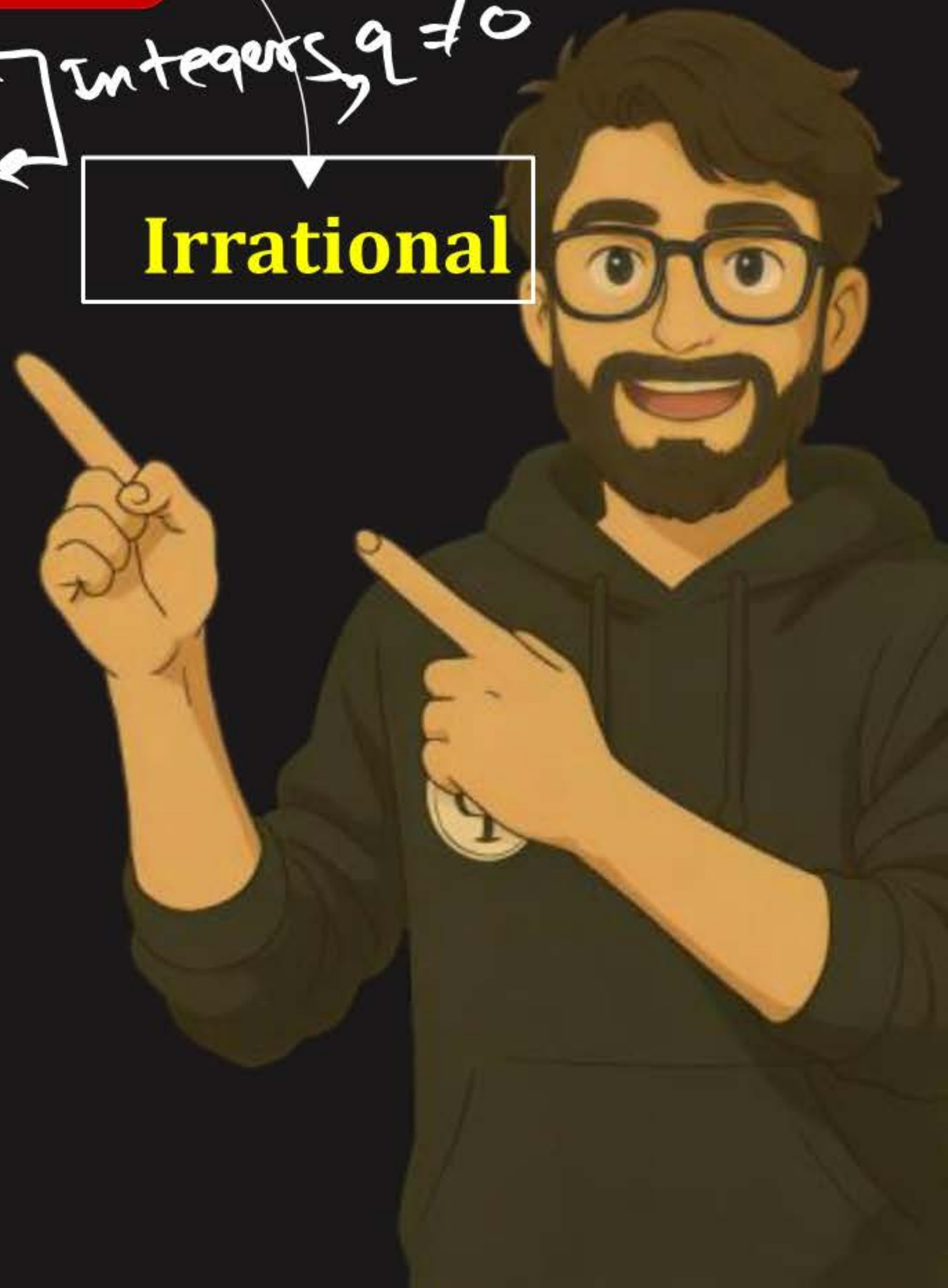
$\neq \frac{p}{q}$ ← Integers, $q \neq 0$

Irrational

Integers	Natural Numbers	Whole Numbers
{−∞...−2,−1,0,1,2...∞}	{1,2,3,4,5,....∞}	{0,1,2,3,... ∞}

naahi -ve
naahi +ve

$\frac{3}{-5}$ ✓



Real no.

T

N.T.R

N.T.N.R

2.5

3.52

2.142857

$(2.\overline{15})$ 2.15151515....

$(3.92\overline{5})$ 3.925555555.....

2.813813813...
$(2.\overline{813})$

6.9236578
945625...

Rational nos.

Irrational no.

Irrational no.

$\sqrt{3} = 3^{1/2}$

1. N.T.N.R
2. $\sqrt{\text{not a perfect Square}}$
3. $\sqrt{\text{Prime no.}}$

Perfect square no.s

1, 4, 9, 16, 25, 36

$(1)^2$ $(2)^2$ $(3)^2$ .... $(6)^2$

Q $\sqrt{8}$ = Irr.

Q $\sqrt{5}, \sqrt{7}, \sqrt{11}, \sqrt{17}$

Q $\sqrt{49}$ = rational.

$49^{1/2} = (7^2)^{1/2} = 7^{2\times\frac{1}{2}}$

$= 7$

1. R + Irr = Irr
2. R − Irr = Irr
3. R × Irr = Irr
4. R ÷ Irr = Irr

non-zero

$$\text{Irr} \; {}^{+}_{\times} \; {}^{-}_{\div} \; \text{Irr} = \text{R} / \text{Irr}$$

Q. $2+\sqrt{3} = $ Irr.

Q. $\sqrt{3} \times \sqrt{7} = \sqrt{21} = $ Irr.

Q. $\dfrac{3}{\sqrt{2}} = $ Irr.

Q. $(3+\sqrt{2}) + (5-\sqrt{2}) = 8$

Irr Irr Rational.

Concept #1

Rational $= \dfrac{p}{q}$ coprime integers.

$\dfrac{5}{1}$ $\cancel{\dfrac{25}{5}}$

$\dfrac{\cancel{44}\ \cancel{22}\ 11}{\cancel{28}\ \cancel{14}\ 7} = 11/7$

Theorem 2

Let p be a prime number and a be a positive integer.
If p divides a^2, then p divides a.

$2 \to 4^2$, $2 \to 4$

$3 \to 9^2$, $3 \to 9$

$5 \to p^2$, $5 \to p$

Concept 3

$21 = \underline{3} \times 7$

$\downarrow$

$\boxed{3 \text{ divides } 21}$

$q = 7a$

$\downarrow$

$\boxed{7 \text{ divides } q}$

$P = 3c$

$\downarrow$

$\boxed{3 \text{ divides } p}$

$p^2 = 5q^2$

$\downarrow$

$\boxed{5 \text{ divides } p^2}$ +ve integer.

$\boxed{5 \text{ divides } p \text{ also}}$

2 divides q.

$\downarrow$

$q = 2 \times c$

$q = 2a$

$q = 2b$

$q = 2d$

#Q. Prove that $\sqrt{3}$ is an irrational number.

NCERT, CBSE 2009, 10, 19, 23

p or q ka 1 key alawa koi or common facta nahi hoga.

Let $\sqrt{3}$ be rational.

$\therefore \sqrt{3} = \frac{p}{q}$ ['p' and 'q' are coprime integers]

Squaring both sides.

$(\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$

$3 = \frac{p^2}{q^2}$

$3q^2 = p^2$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ $\boxed{\text{3 divides p also}}$ — (1)

let, $\boxed{p = 3c}$

$3q^2 = (3c)^2$

$3q^2 = 9c^2$

$\Rightarrow q^2 = \frac{\cancel{9}c^2}{\cancel{3}}$

$q^2 = 3c^2$

$\Rightarrow$ 3 divides q^2 (2)

$\Rightarrow$ $\boxed{\text{3 divides q also}}$

From (1) and (2)

3 is a common factor of p and 'q'.

this makes our assumption wrong.

$\therefore$ $\sqrt{3}$ is irrational.

H.P

#Q. Prove $\boxed{\text{that } \sqrt{2} \text{ is an irrational number.}}$

NCERT, CBSE 2010, 23

Let, $\sqrt{2}$ be rational.

$\therefore \sqrt{2} = \frac{p}{q}$ ['p' and 'q' are coprime integers]

Squaring both sides,

$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$

$2 = \frac{p^2}{q^2}$

$2q^2 = p^2$

$\Rightarrow$ 2 divides p^2

$\Rightarrow$ $\boxed{\text{2 divides } p \text{ also}}$ (1)

Let, $p = 2c$

$2q^2 = (2c)^2$

$2q^2 = 4c^2$

$q^2 = 2c^2$

$\Rightarrow$ 2 divides q^2

$\Rightarrow$ $\boxed{\text{2 divides } q \text{ also}}$ (2)

From (1) and (2)

2 is a common factor of p and q.

this makes our assumption wrong.

$\therefore \sqrt{2}$ is irrational.

H.P

#Q. Given that $\boxed{\sqrt{2}}$ is $\boxed{\text{irrational}}$, prove that $\boxed{(5 + 3\sqrt{2}) \text{ is an irrational number.}}$

CBSE 2018

Let, $5+3\sqrt{2}$ be rational.

$\therefore\ 5+3\sqrt{2} = \frac{p}{q}$ ['p' and 'q' are coprime integers]

$3\sqrt{2} = \frac{p}{q} - \frac{5}{1}$

$3\sqrt{2} = \frac{p-5q}{q}$

$\boxed{\sqrt{2} = \frac{p-5q}{3q}}$ → Rational.

Irr.

this is not possible.

$\therefore\ 5+3\sqrt{2}$ is irrational.

$I \times I = I \rightarrow R$

$I + I = I \rightarrow R$

$I - I = I \rightarrow R$

$\frac{I}{I}$ = Rational.

$\frac{4}{2} = 2$ I, R. $\frac{2}{4} = \frac{1}{2}$ R

#Q. Prove that $3 + 2\sqrt{5}$ is an irrational.

NCERT

#GPk

#Q. Prove that $\boxed{\frac{2+\sqrt{3}}{5}}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

CBSE 2019

let, $\frac{2+\sqrt{3}}{5}$ be rational.

$\therefore \frac{2+\sqrt{3}}{5} = \frac{p}{q}$ ['p' and 'q' are integers]

$2+\sqrt{3} = \frac{5p}{q}$

$\sqrt{3} = \frac{5p}{q} - \frac{2}{1}$

$\boxed{\sqrt{3} = \frac{5p-2q}{q}}$ → R

Irr

this is not possible.

$\therefore \frac{2+\sqrt{3}}{5}$ is irrational.

$(a-b)^2 = a^2 + b^2 - 2ab$

#Q. Prove that $\boxed{\sqrt{2}+\sqrt{3}}$ is irrational.

NCERT Exemplar

Let, $\sqrt{2}+\sqrt{3}$ is rational.

$\therefore \sqrt{2}+\sqrt{3} = \frac{p}{q}$ ['p' and 'q' are integers]

$\sqrt{3} = \frac{p}{q} - \sqrt{2}$

squaring both sides.

$(\sqrt{3})^2 = \left(\frac{p}{q} - \sqrt{2}\right)^2$

$3 = \left(\frac{p}{q}\right)^2 + (\sqrt{2})^2 - 2 \cdot \frac{p}{q} \cdot \sqrt{2}$

$3 = \frac{p^2}{q^2} + 2 - \frac{2\sqrt{2}p}{q}$

$\frac{2\sqrt{2}p}{q} = \frac{p^2}{q^2} + 2 - 3$

$\frac{2\sqrt{2}p}{q} = \frac{p^2}{q^2} - 1$

$$\frac{2\sqrt{2}p}{q} = \frac{p^2 - q^2}{q^2}$$

$$\sqrt{2} = \frac{(p^2 - q^2)\cancel{q}}{2p \cdot q^{\cancel{2}}}$$

I

$$\sqrt{2} = \frac{p^2 - q^2}{2pq}$$

R

this is not possible.

$\Rightarrow$ Our assumption was wrong.

$\therefore \sqrt{2} + \sqrt{3}$ is irrational.

#Q. If p, q are prime positive integers, prove that $\sqrt{p} + \sqrt{q}$ is an irrational number.

NCERT Exemplar

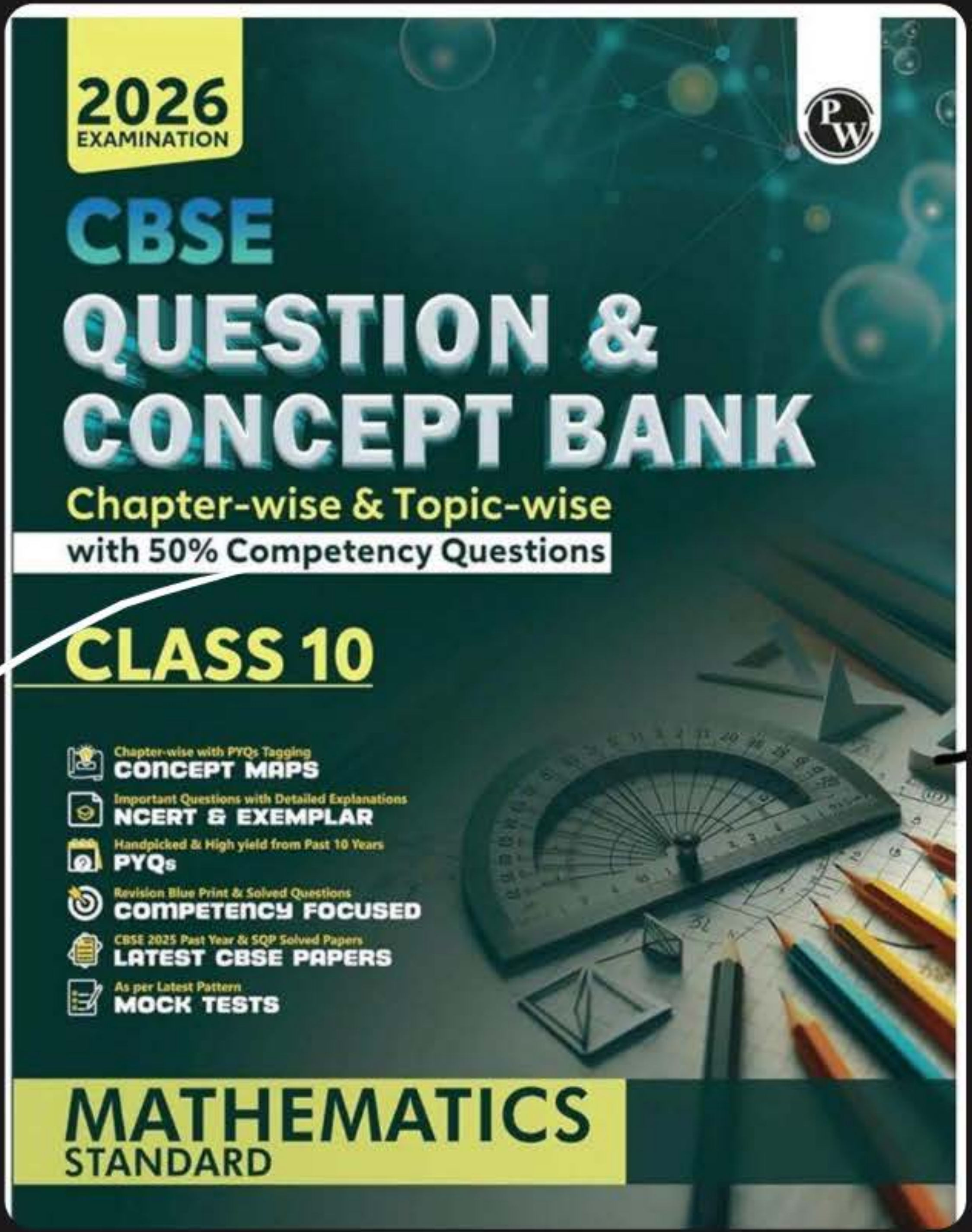
#GPK

Let, $\sqrt{p}+\sqrt{q} \rightarrow$ Rational

$\therefore \sqrt{p}+\sqrt{q} = \frac{a}{b}$ ['a' and 'b' integers]

$(\sqrt{q})^2 = \left(\frac{a}{b} - \sqrt{p}\right)^2$

#DPP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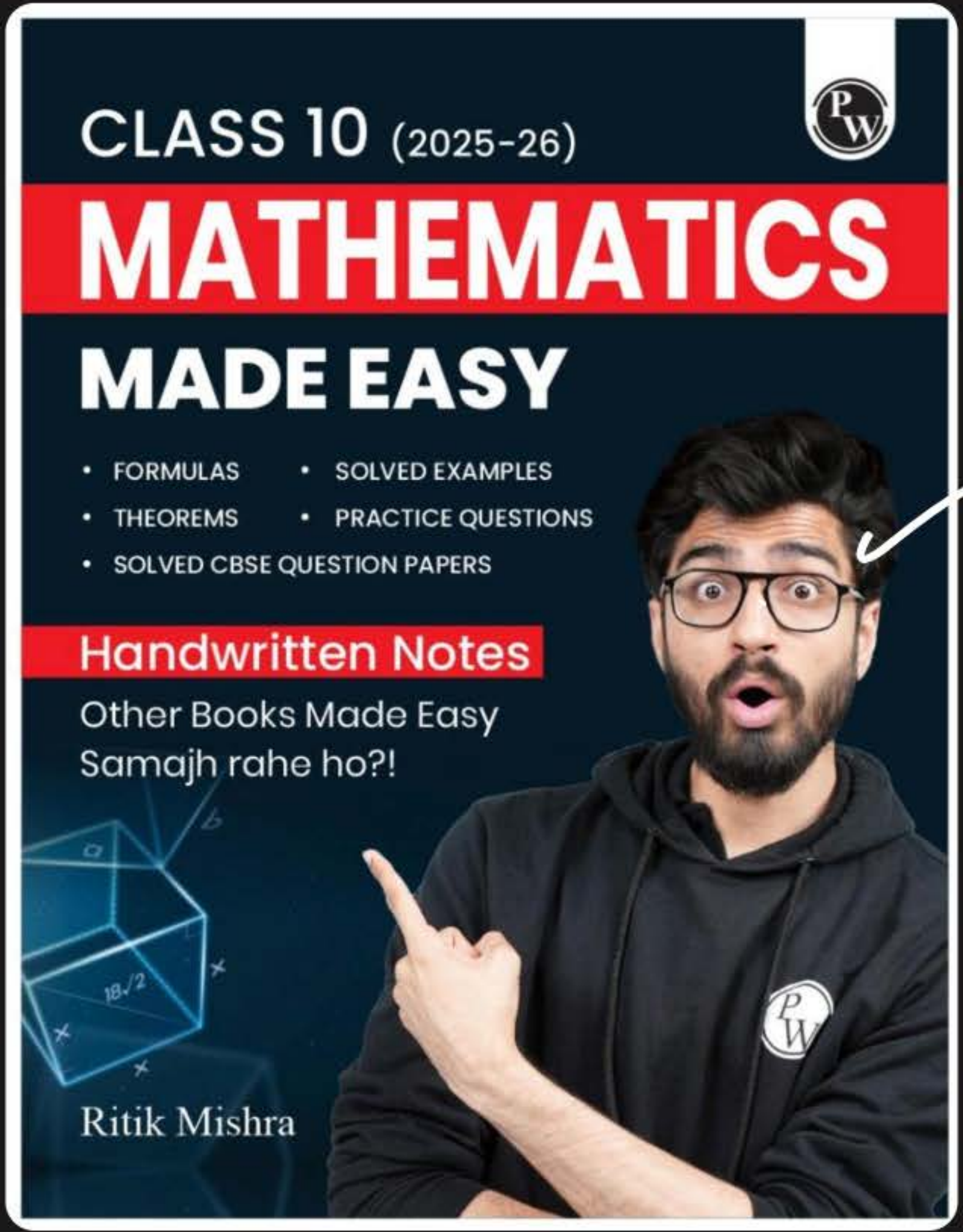
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