UDAAN

2026

Quadratic Equations

MATHS LECTURE- 05

BY-RITIK SIR

Topics to be covered

A Questions on nature of roots.

B ~~Quadratic Formula~~ Completing the Square. **Method**

C Quadratic Formula **(Proof)**

$$\boxed{x = \frac{-b \pm \sqrt{D}}{2a}}$$

$D = b^2 - 4ac$

(I) $D > 0$ ($D = +ve$)	Real & distinct
(II) $D = 0$	Real and equal
(III) $D < 0$ ($D = -ve$)	no real roots (imaginary roots)

#Gpk

#Q. Using quadratic formula solve the following quadratic equations :

CBSE 2004

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

$ax^2 + bx + c = 0$

$$\boxed{\begin{aligned} a &= p^2 \\ b &= p^2 - q^2 \\ c &= -q^2 \end{aligned}}$$

$D = b^2 - 4ac$

$= (p^2 - q^2)^2 - 4(p^2)(-q^2)$

$= (p^2)^2 + (q^2)^2 - 2p^2q^2 + 4p^2q^2$

$D = (p^2)^2 + (q^2)^2 + 2p^2q^2$

$\boxed{D = (p^2 + q^2)^2}$ //

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$x = \dfrac{-(p^2 - q^2) \pm \sqrt{(p^2 + q^2)^{\cancel{2}}}}{2p^2}$

$x = \dfrac{-p^2 + q^2 \pm (p^2 + q^2)}{2p^2}$

$x = \dfrac{-\cancel{p^2} + q^2 + \cancel{p^2} + q^2}{2p^2}$

$x = \dfrac{2q^2}{2p^2} = \boxed{q^2/p^2}$

$x = \dfrac{-p^2 + \cancel{q^2} - p^2 - \cancel{q^2}}{2p^2}$

$x = \dfrac{-\cancel{2p^2}}{\cancel{2p^2}} = \boxed{-1}$

#Q. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$, has equal roots, find the value of k.

CBSE 2002, 09, 14

-5 root

$2x^2+px-15=0$

$2(-5)^2+p(-5)-15=0$

$50-5p-15=0$

$35-5p=0$

$35=5p$

$\boxed{7=p}$

$p(x^2+x)+k=0$ → equal roots.

$px^2+px+k=0$

$D=0$

$b^2-4ac=0$

$(p)^2-4(p)(k)=0$

$(7)^2-4(7)(k)=0$

$49-28k=0$

$49=28k$

$\frac{49}{28}=k$

$\boxed{\frac{7}{4}=k}$

#Q. Find the values of k for which the given equation has real and equal roots:

CBSE 2015

(i) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

$a = k+1,\ b = -2(k-1),\ c = 1$

$D = 0$

$b^2 - 4ac = 0$

$[-2(k-1)]^2 - 4(k+1)(1) = 0$

$4(k-1)^2 - 4(k+1) = 0$

$4(k^2+1-2k) - 4k - 4 = 0$

$4k^2 + \cancel{4} - 8k - 4k - \cancel{4} = 0$

$4k^2 - 12k = 0$

$[4k][k-3] = 0$

$4k = 0,\ k-3 = 0$

$\boxed{k=0},\ \boxed{k=3}$

Verify karo.

#Q. Find the values of k for which the given equation has real and equal roots:

CBSE 2013, 17

(ii) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$

#GPK

$\cancel{12}$ xxx

#Q. Find the value of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$, $p \neq -1$ has equal roots. Hence, find the roots of the equation.

CBSE 2015

$(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$

$D = 0$

$b^2 - 4ac = 0$

$[-6(p+1)]^2 - 4(p+1)3(p+9) = 0$

$36(p+1)^2 - 12(p+1)(p+9) = 0$

$36(p^2+1+2p) - 12(p^2+9p+p+9) = 0$

$36p^2 + 36 + 72p - 12p^2 - 120p - 108 = 0$

$24p^2 - 48p - 72 = 0$

$24[p^2 - 2p - 3] = 0$

$\boxed{p^2 - 2p - 3 = 0}$

$p^2 - 2p - 3 = 0$

$S = -2,\ P = -3$

$-3, 1$

$p^2 - 3p + 1p - 3 = 0$

$p(p-3) + 1(p-3) = 0$

$(p-3)(p+1) = 0$

$p = 3,\ \cancel{p = -1}$

$p = 3$

$(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$

$p = 3$

$4x^2 - 24x + 36 = 0$

$4(x^2 - 6x + 9) = 0$

$x^2 - 6x + 9 = 0$

$S = -6,\ P = 9$

$-3, -3$

$x^2 - 3x - 3x + 9 = 0$

$x(x-3) - 3(x-3) = 0$

$(x-3)(x-3) = 0$

$x = 3, 3$

HOT

#Q. Prove that the equation $x^2(a^2+b^2)+2x(ac+bd)+(c^2+d^2)=0$ has no real root, if $ad \neq bc$.

$$x^2\underbrace{(a^2+b^2)}_{a} + \underbrace{2(ac+bd)}_{b}x + \underbrace{c^2+d^2}_{c} = 0$$

$$D = b^2 - 4ac$$

$$D = [2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2)$$

$$= 4(ac+bd)^2 - 4(a^2c^2+a^2d^2+b^2c^2+b^2d^2)$$

$$= 4(a^2c^2+b^2d^2+2acbd) - 4(a^2c^2+a^2d^2+b^2c^2+b^2d^2)$$

$$= \cancel{4a^2c^2} + \cancel{4b^2d^2} + 8acbd - \cancel{4a^2c^2} - 4a^2d^2 - 4b^2c^2 - \cancel{4b^2d^2}$$

$D = -4a^2d^2 - 4b^2c^2 + 8acbd$

$D = -4\{a^2d^2 + b^2c^2 - 2acbd\}$

$D = -4[ad - bc]^2$

$ad - bc = 0$ ✗ $\rightarrow \because ad \neq bc$

$ad - bc = -ve$

$ad - bc = +ve$

$\therefore (ad - bc)^2$ will always be +ve

$D = -4 \times +ve$

$D = -$

Hence no real roots.

#Q. If the roots of the equation $\boxed{(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0}$ are $\boxed{\text{equal,}}$ then prove that $\frac{a}{b} = \frac{c}{d}$.

From previous Question.

CBSE 2017

$D = -4(ad - bc)^2$

$\because$ Roots are equal

$\therefore \boxed{D = 0}$

$-4(ad - bc)^2 = 0$

$(ad - bc)^2 = 0$

$ad - bc = \pm\sqrt{0}$

$ad - bc = 0$

$ad - bc = 0$

$ad = bc$

$\boxed{\frac{a}{b} = \frac{c}{d}}$

#Q. If the equation $(1 + m^2) x^2 + 2 mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

CBSE 2007

$(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$

$a = 1+m^2$
$b = 2mc$
$c = c^2 - a^2$

$D = 0$

$b^2 - 4ac = 0$

$(2mc)^2 - 4(1+m^2)(c^2-a^2) = 0$

$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$\cancel{4m^2c^2} - 4c^2 + 4a^2 - \cancel{4m^2c^2} + 4m^2a^2 = 0$

$-4c^2 + 4a^2 + 4m^2a^2 = 0$

$4a^2 + 4m^2a^2 = 4c^2$

$\frac{4a^2 + 4m^2a^2}{4} = c^2$

$\frac{\cancel{4}a^2[1+m^2]}{\cancel{4}} = c^2$

$\boxed{a^2(1+m^2) = c^2}$

H.P

#Q. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

$D=0$

CBSE 2002

#GPL

$$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca}$$

Solution of a Quadratic Equation by Completing the Square

#Q. Solve the quadratic equation $9x^2 - 15x - 6 = 0$ by the method of completing the square.

NCERT

$$\frac{9x^2}{9} - \frac{15x}{9} - \frac{6}{9} = \frac{0}{9}$$

$$x^2 - \frac{5}{3}x - \frac{2}{3} = 0$$

$$x^2 - \frac{5x}{3} = \frac{2}{3}$$

$$x^2 - \frac{5}{3}x + \frac{25}{36} = \frac{2}{3} + \frac{25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{24+25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{49}{36}$$

$$x - \frac{5}{6} = \pm\sqrt{\frac{49}{36}}$$

$$\boxed{x - \frac{5}{6} = \pm\frac{7}{6}}$$

$$x - \frac{5}{6} = \frac{7}{6}, \quad x - \frac{5}{6} = -\frac{7}{6}$$

$$x = \frac{7}{6} + \frac{5}{6}, \quad x = -\frac{7}{6} + \frac{5}{6}$$

$$x = 2, -\frac{1}{3}$$

#Q. Solve the equation $\boxed{2x^2 - 5x + 3 = 0}$ by the method $\boxed{\text{of completing square}}$.

NCERT

$$\frac{\cancel{2}x^2}{\cancel{2}} - \frac{5x}{2} + \frac{3}{2} = \frac{0}{2}$$

$$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$$

$$x^2 - \frac{5}{2}x = \frac{-3}{2}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{-3}{2} + \frac{25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{-24 + 25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{1}{16}}$$

$$\boxed{x - \frac{5}{4} = \pm\frac{1}{4}}$$

$$x - \frac{5}{4} = \frac{1}{4}, \quad x - \frac{5}{4} = -\frac{1}{4}$$

$$x = \frac{1}{4} + \frac{5}{4}, \quad x = -\frac{1}{4} + \frac{5}{4}$$

$$x = \frac{6}{4}, \frac{4}{4}$$

$$x = \frac{3}{2}, 1$$

#Q. By using the method of completing the square, show that the equation $4x^2 + 3x + 5 = 0$ has no real roots.

NCERT

$$\frac{\cancel{4}x^2}{\cancel{4}} + \frac{3x}{4} + \frac{5}{4} = \frac{0}{4}$$

$$x^2 + \frac{3}{4}x + \frac{5}{4} = 0$$

$$x^2 + \frac{3}{4}x = -\frac{5}{4}$$

$$x^2 + \frac{3}{4}x + \frac{9}{64} = -\frac{5}{4} + \frac{9}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{-80+9}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$$

$$x + \frac{3}{8} = \pm\sqrt{-\frac{71}{64}}$$

Clearly, no real roots

Proof of Sridharacharya Formula

$$\frac{\cancel{a}x^2}{\cancel{a}}+\frac{bx}{a}+\frac{c}{a}=\frac{0}{a}$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{-4ac+b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

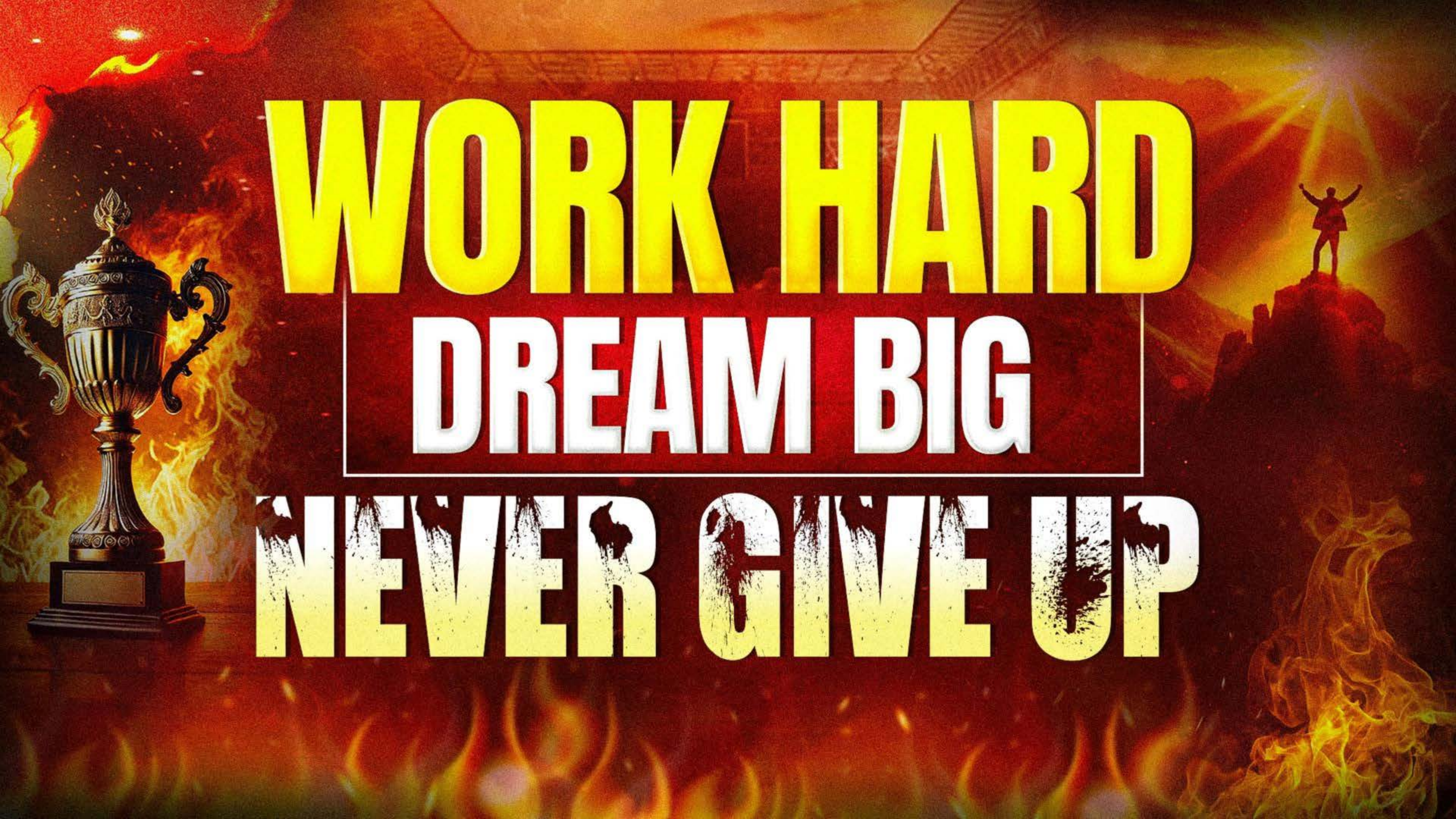
$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

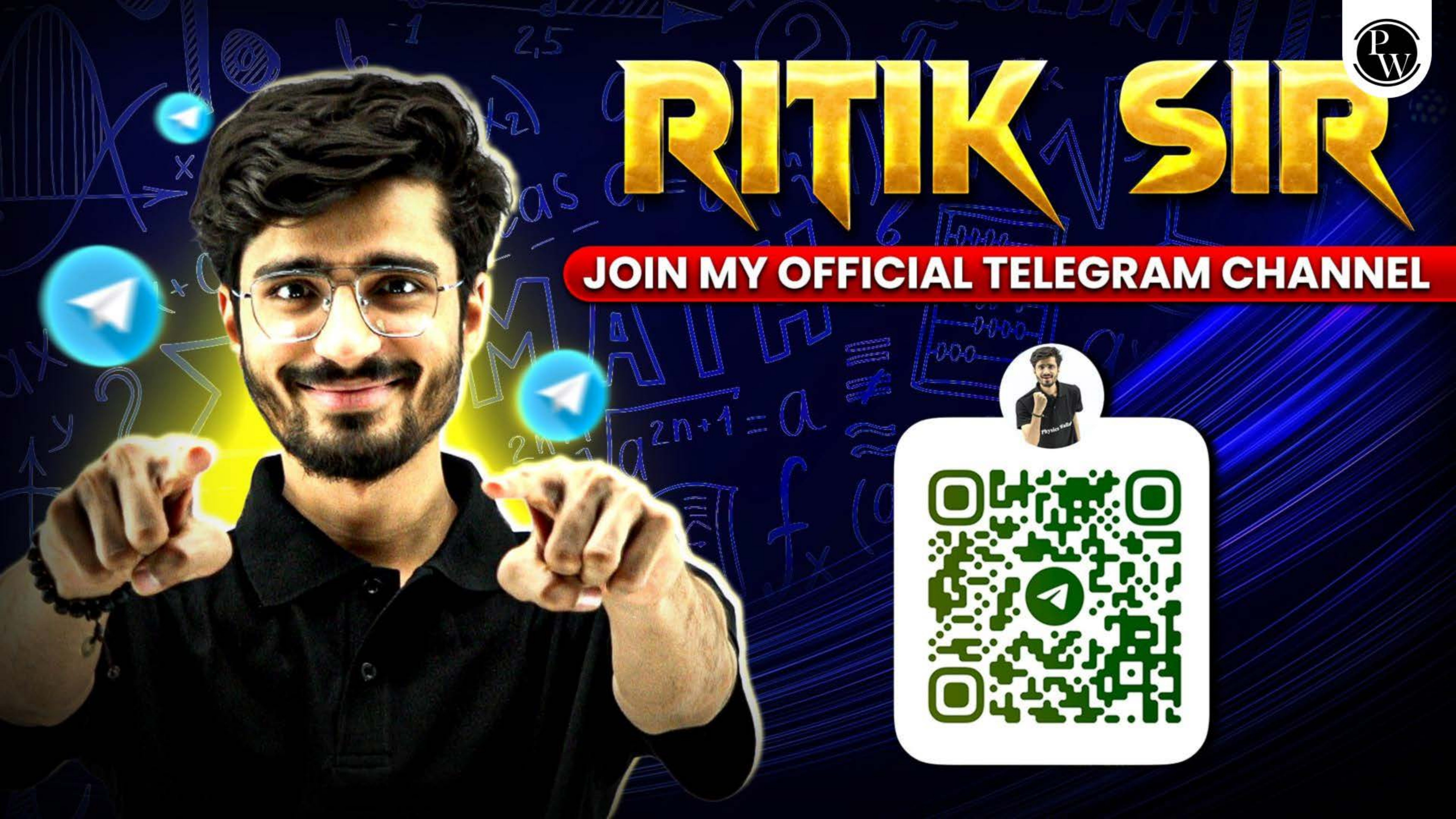
$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

#Q. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square?

#GPL


WORK HARD
DREAM BIG
NEVER GIVE UP


RITIK SIR
JOIN MY OFFICIAL TELEGRAM CHANNEL

Thank

You