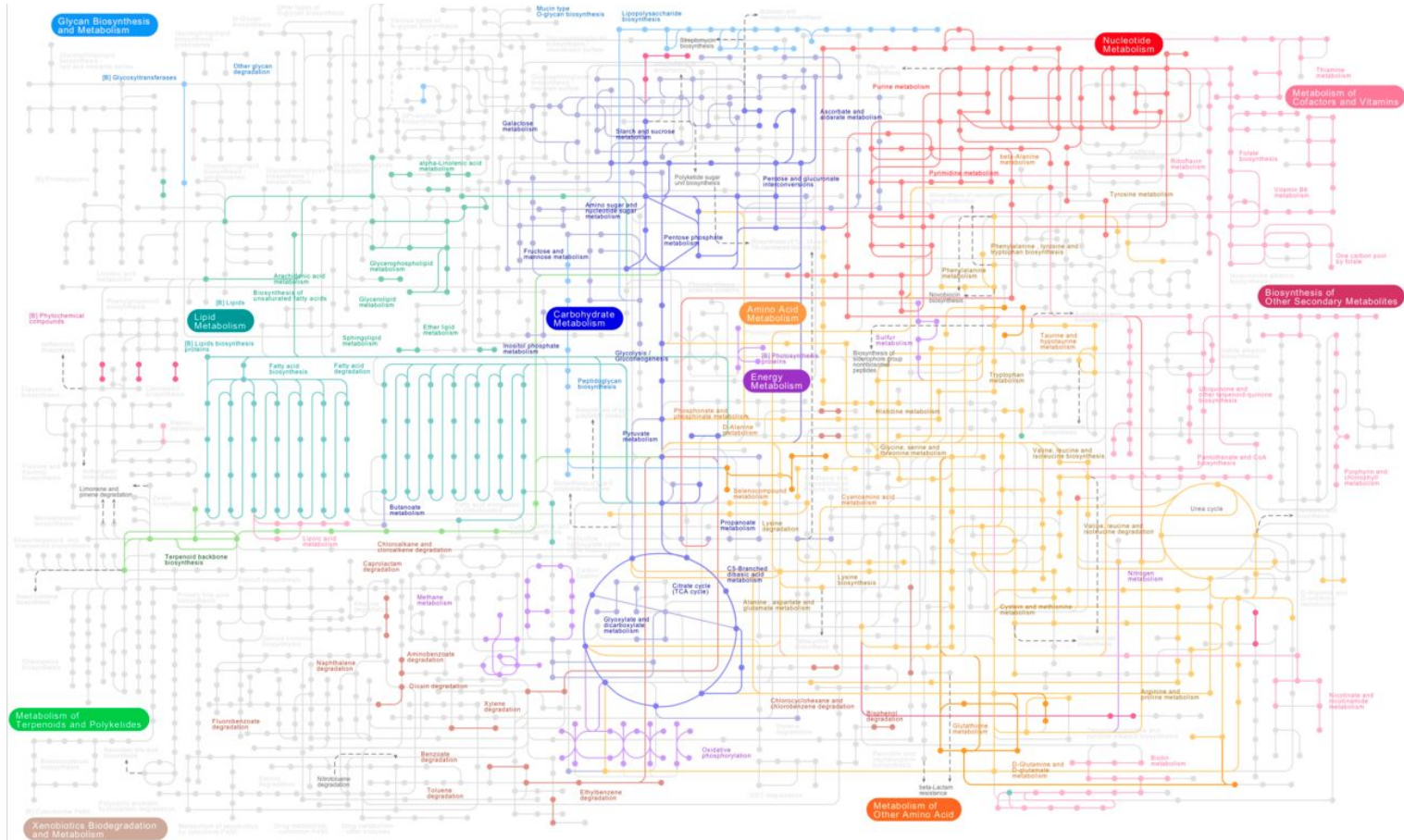


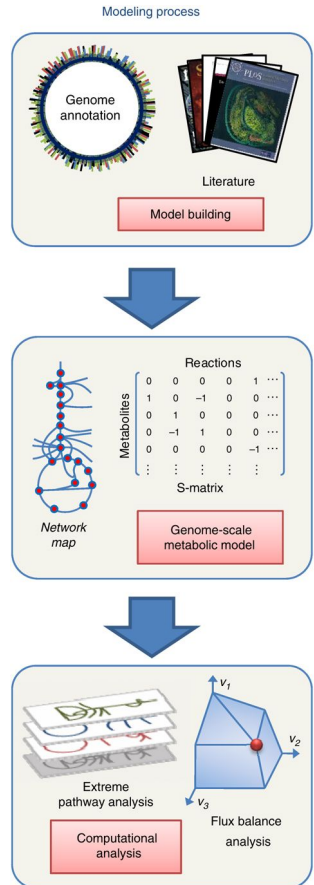
Lecture 13: Metabolomics

- Genome-scale metabolic models
- Flux balance analysis

Metabolic networks



Genome-scale metabolic network reconstruction & model



Genome-scale metabolic network reconstruction:

- A collection of biochemical transformation derived from the genome annotation and the literature of a particular organism.
- Formed based on an organism-specific knowledge base.
- A network reconstruction is unique to an organism.

Genome-scale metabolic network model:

- Derived from a *reconstruction* by converting it into a mathematical form (i.e., an in silico model) and by assessing its phenotypic properties computationally.

Genome-scale metabolic network reconstruction & model

1. Contextualization of HT data

Several studies have overlaid gene microarray data on a metabolic GENRE to determine condition-dependent cell phenotypes. Metabolic GENREs have also been used to interpret metabolomic data, ^{13}C flux data, and to link multiple data types.



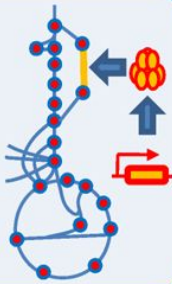
2. Guidance of metabolic engineering

Metabolic GENREs guided efforts to engineer malate and succinate producing strains of *S. cerevisiae* and *M. succiniciproducens*. GENREs have also helped determine ways to increase the respiration rate of *G. sulfurreducens* and scale-up vaccine production against *N. meningitidis*.



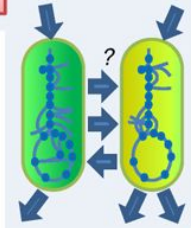
3. Directing hypothesis-driven discovery

A Metabolic GENRE aided in determining pathway usage and discovering a novel citramalate synthase gene in *G. sulfurreducens*. GENREs have also helped study the effects of transposons on downstream genes, and identify transcriptional timing patterns in *S. cerevisiae*.



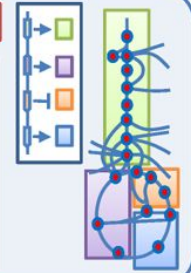
4. Interrogation of multi-species relationships

A dual-species metabolic model was built to study interactions between the syntrophic bacteria, *D. vulgaris* and *M. maripaludis*. Metabolic models have also been used in comparisons of multiple species, such as an analysis of pathway differences between four halophilic bacteria.



5. Network property discovery

Metabolic GENREs have been used to study metabolite connectivity, and pathway redundancy *in silico*. Pathway-analysis tools have also spawned techniques such as flux coupling analysis, which has helped identify novel drug targets in *M. tuberculosis*.



Constructing a genome-scale metabolic model

Draft construction

Genome databases

| | |
|--|---|
| Comprehensive Microbial Resource (CMR) | http://cmr.jcvi.org/cgi-bin/CMR/CmrHomePage.cgi |
| Genomes OnLine Database (GOLD) | http://www.genomesonline.org/ |
| TIGR | http://www.tigr.org/db.shtml |
| NCBI Entrez Gene | http://www.ncbi.nlm.nih.gov/sites/entrez |
| SEED database ³² | http://theseed.uchicago.edu/FIG/index.cgi |

Biochemical databases

| | |
|--|---|
| KEGG ⁴¹ | http://www.genome.jp/kegg/ |
| BRENDA ⁴² | http://www.brenda-enzymes.info/ |
| Transport DB ⁸⁹ | http://www.membranetransport.org/ |
| PubChem ⁸⁶ | http://pubchem.ncbi.nlm.nih.gov/ |
| Transport Classification Database (TCDB) | http://www.tcdb.org/ |
| pK _a Plugin | http://www.chemaxon.com/product/pka.html |
| pK _a DB | http://www.acdlabs.com/products/phys_chem_lab/pka/ |

1. Draft reconstruction

- 1| Obtain genome annotation.
- 2| Identify candidate metabolic functions.
- 3| Obtain candidate metabolic reactions.
- 4| Assemble draft reconstruction.
- 5| Collect experimental data.

Organism-specific databases

| | |
|--------------------------|---|
| Ecocyc ⁴³ | http://ecocyc.org/ |
| PyloriGene ³⁷ | http://genolist.pasteur.fr/PyloriGene |
| Gene Cards | http://www.genecards.org/ |

Protein localization databases

| | |
|----------------------|---|
| PSORT ⁴⁷ | http://www.psort.org/psortb/ |
| PA-SUB ⁴⁸ | http://www.cs.ualberta.ca/~bioinfo/PA/Sub/ |

Bio-numbers

| | |
|---|---|
| CyberCell Database (CCDB) ⁸⁸ | http://redpoll.pharmacy.ualberta.ca/CCDB/cgi-bin/STAT_NEW.cgi |
| B10NUMB3R5 | http://bionumbers.hms.harvard.edu/ |

Constructing a genome-scale metabolic model

Refinement of draft construction

2. Refinement of reconstruction

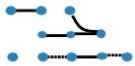
- 6| Determine and verify substrate and cofactor usage.
- 7| Obtain neutral formula for each metabolite.
- 8| Determine the charged formula.
- 9| Calculate reaction stoichiometry.
- 10| Determine reaction directionality.
- 11| Add information for gene and reaction localization.
- 12| Add subsystems information.
- 13| Verify gene–protein–reaction association.
- 14| Add metabolite identifier.
- 15| Determine and add confidence score.
- 16| Add references and notes.
- 17| Flag information from other organisms.
- 18| Repeat Steps 6 to 17 for all genes.
- 19| Add spontaneous reactions to the reconstruction.
- 20| Add extracellular and periplasmic transport reactions.
- 21| Add exchange reactions.
- 22| Add intracellular transport reactions.
- 23| Draw metabolic map (optional).
- 24–32| Determine biomass composition.
- 33| Add biomass reaction.
- 34| Add ATP-maintenance reaction (ATPM).
- 35| Add demand reactions.
- 36| Add sink reactions.
- 37| Determine growth medium requirements.

Constructing a genome-scale metabolic model

Refinement of draft construction

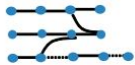
Mass & charge balancing; Filling-in H^+ & water; adjusting metabolites to a particular pH

Draft reconstruction



| Gene alias | Locus name | EntrezGene function | EcoCyc function | EC number | Reaction |
|------------|------------|---------------------|-----------------|-----------|---|
| glk | b2388 | Glucokinase | Glucokinase | 2.7.1.2 | β -D-glucose + ATP \rightarrow β -D-Glucose-6-Phosphate + ADP |

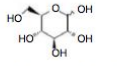
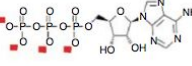
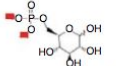
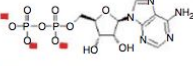
Curated reconstruction



| Rxn Abb Name | Rxn | Reaction | Pathway | GPR | EC | CSNotes | References |
|--------------|-------------|--|------------|-------|---------|---|--|
| GLK | Glucokinase | $[c]: 1 \text{ Glc} + 1 \text{ ATP} \rightarrow 1 \text{ G6p} + 1 \text{ ADP} + 1 \text{ H}^+$ | Glycolysis | b2388 | 2.7.1.2 | 4 Protein structure has been crystallized[1]; Cloned and sequenced[2]; biochemical activity measured [2]; [c] = cytosol | [1] Lunin <i>et al.</i> 2004. J. Bacteriol. 186(20):691 5-27; [2] Meyer <i>et al.</i> 1997. J. Bacteriol. 179(4):1298 -306 |

| | Gene | Peptide | Protein |
|--|-------|---------|---------|
| | b2388 | glk | Glk |

| | Locus | Gene | Protein | Reaction |
|-----------------|-------------------|------|---------|----------|
| Genomics | ORF annotation | | | |
| Transcriptomics | mRNA levels | | | |
| Proteomics | protein levels | | | |
| Fluxomics | flux measurements | | | |

| | Substrates | Glc | ATP | G6p | ADP |
|----------------|------------|---|---|---|---|
| Neutral | | $C_6H_{12}O_6^0$ | $C_{10}H_{16}N_5O_{13}P_3^0$ | $C_6H_{13}O_9P^0$ | $C_{10}H_{15}N_5O_{10}P_2^0$ |
| Charged | | $C_6H_{12}O_6^0$ | $C_{10}H_{12}N_5O_{13}P_3^{4-}$ | $C_6H_{11}O_9P^{2-}$ | $C_{10}H_{12}N_5O_{10}P_2^{3-}$ |
| | |  |  |  |  |
| Stoichiometry | | $C_{16}H_{24}O_{19}P_3, 4e^-$ | $1 \text{ Glc} + 1 \text{ ATP}$ | $C_{16}H_{23}O_{18}P_3, 5e^-$ | $1 \text{ G6p} + 1 \text{ ADP} + 1 \text{ H}^+$ |
| Directionality | | $1 \text{ Glc} + 1 \text{ ATP}$ | \rightarrow | $1 \text{ G6p} + 1 \text{ ADP} + 1 \text{ H}^+$ | |
| Location | | $\text{cytosol}: 1 \text{ GLc} + 1 \text{ ATP}$ | \rightarrow | $1 \text{ G6p} + 1 \text{ ADP} + 1 \text{ H}^+$ | |

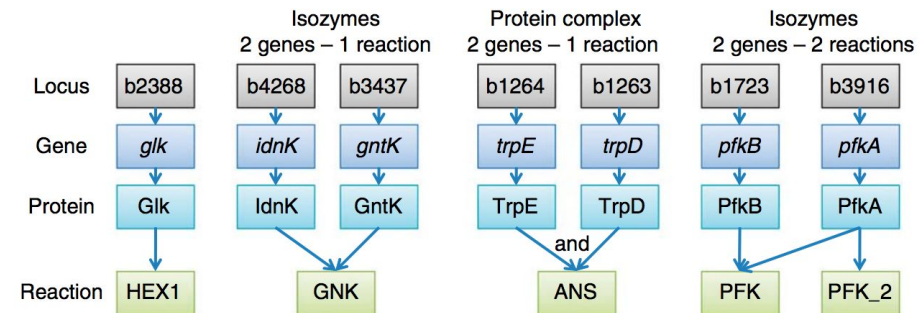
Constructing a genome-scale metabolic model

Refinement of draft construction

Subcellular localization

| Compartment | Commonly used symbol [#] | Achaea | Bacteria | Eukaryotic pathogens ^a | Fungi ^b | Photosynthetic eukarya ^c | Baker's yeast | Human |
|-----------------------|-----------------------------------|--------|----------|-----------------------------------|--------------------|-------------------------------------|---------------|-------|
| Extracellular space | [e] | | | | | | | |
| Periplasm | [p] | | | | | | | |
| Cytoplasm | [c] | | | | | | | |
| Nucleus | [n] | | | | | | | |
| Mitochondrion | [m] | | | | | | | |
| Chloroplast | [h] | | | | | | | |
| Lysosome [*] | [l] | | | | | | | |
| Vacuole | [v] | | | | | | | |
| Golgi apparatus | [g] | | | | | | | |
| Endoplasmatic | [r] | | | | | | | |

Gene-protein-reaction associations



| Reaction abbreviation | Reaction name | E. C.number | GPR |
|-----------------------|----------------------------|-------------|---------------------|
| HEX1 | Hexokinase (D-glucose:ATP) | 2.7.1.1 | (b2388) |
| GNK | Gluconokinase | 2.7.1.12 | (b3437) or (b4268) |
| ANS | Anthranilate synthase | 4.1.3.27 | (b1264) and (b1263) |
| PFK | Phosphofructokinase | 2.7.1.11 | (b1723) or (b3916) |
| PFK_2 | Phosphofructokinase (2) | 2.7.1.11 | (b3916) |

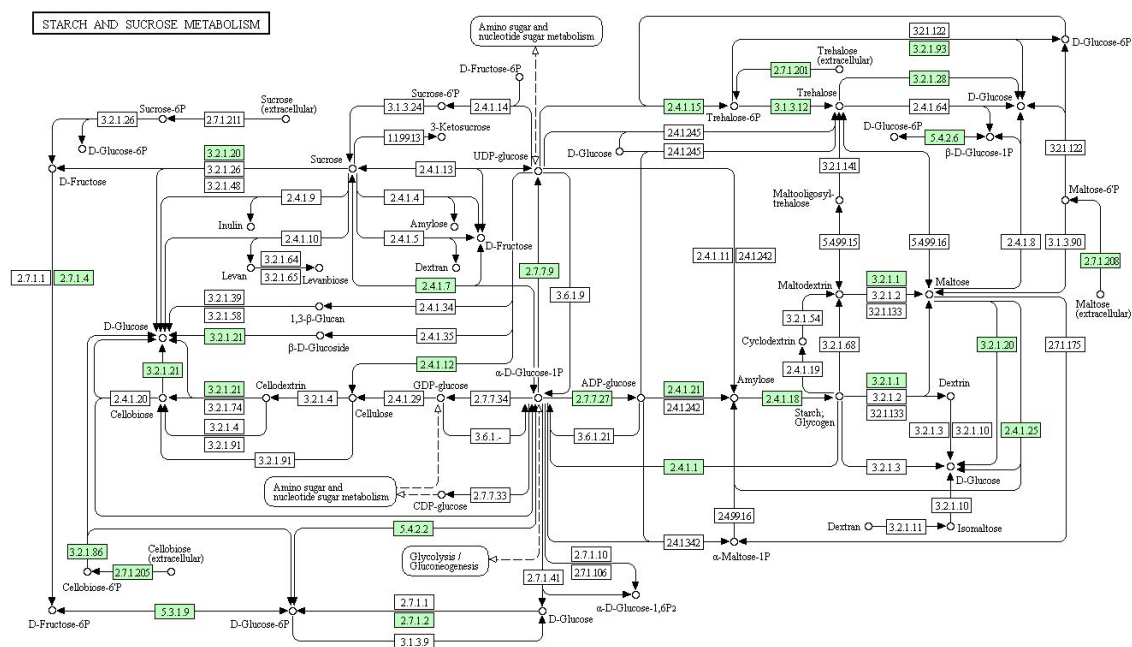
Constructing a genome-scale metabolic model

Refinement of draft construction

Chemical composition of a cell

| Cellular component | Cellular content % (wt/wt) |
|---------------------|----------------------------|
| Protein | 55 |
| RNA | 20.5 |
| DNA | 3.1 |
| Lipids | 9.1 |
| Lipopolysaccharides | 3.4 |
| Peptidoglycan | 2.5 |
| Glycogen | 2.5 |
| Polyamines | 0.4 |
| Other | 3.5 |
| Total | 100.00 |

Identification of missing functions



Constructing a genome-scale metabolic model

Refinement of draft construction

| Evidence type | Confidence score | Examples |
|--------------------|------------------|---|
| Biochemical data | 4 | Direct evidence for gene product function and biochemical reaction: protein purification, biochemical assays, experimentally solved protein structures and comparative gene-expression studies (e.g., Chhabra <i>et al.</i> ⁹⁵) |
| Genetic data | 3 | Direct and indirect evidence for gene function: knockout characterization, knock-in characterization and overexpression |
| Physiological data | 2 | Indirect evidence for biochemical reactions based on physiological data: secretion products or defined medium components serve as evidence for transport and metabolic reactions |
| Sequence data | 2 | Evidence for gene function: genome annotation and SEED annotation ³² |
| Modeling data | 1 | No evidence is available, but reaction is required for modeling. The included function is a hypothesis and needs experimental verification. The reaction mechanism may be different from the included reaction(s) |
| Not evaluated | 0 | |

Constructing a genome-scale metabolic model

Conversion of reconstruction into a model

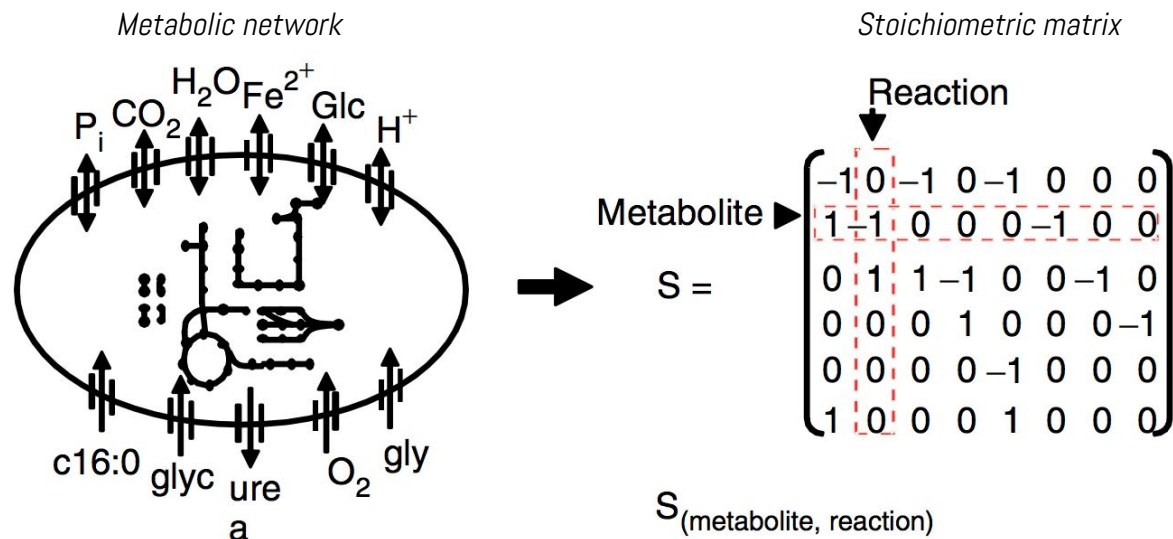
3. Conversion of reconstruction into computable format

- 38| Initialize the COBRA toolbox.
- 39| Load reconstruction into Matlab.
- 40| Verify S matrix.
- 41| Set objective function.
- 42| Set simulation constraints.

Constructing a genome-scale metabolic model

Conversion of reconstruction into a model

Mathematical representation



By definition:

- Substrates have negative coefficients (i.e., they are consumed)
- Products have positive coefficients (i.e., they are produced)

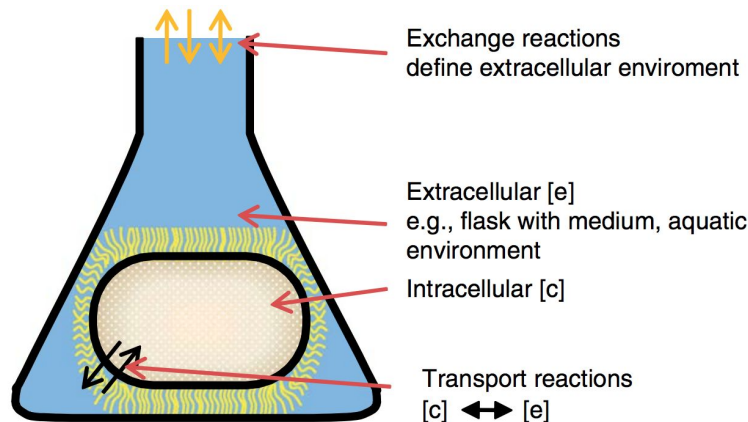
\mathbf{v} is a vector of reaction fluxes

Conservation of mass: All steady states can be described by $\mathbf{S}\mathbf{v} = 0$

Constructing a genome-scale metabolic model

Conversion of reconstruction into a model

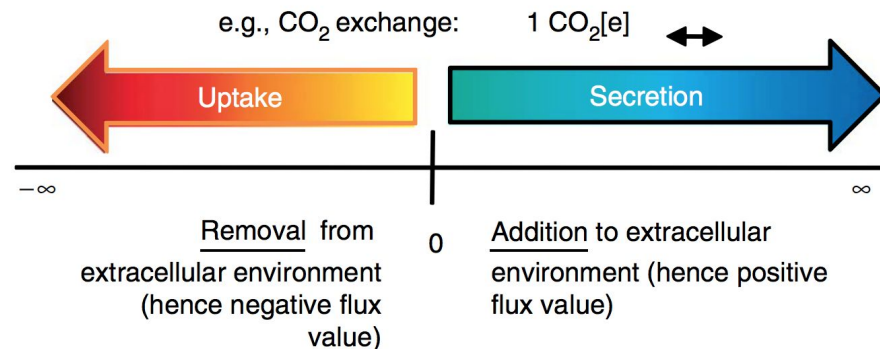
Definition of system boundaries



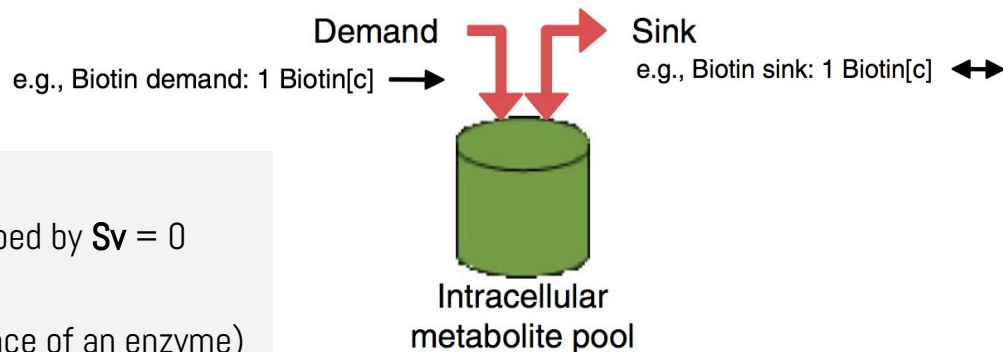
Constraints:

- Mass conservation: all steady states can be described by $Sv = 0$
- Thermodynamics (reaction directionality)
- Enzyme capacity or regulation (i.e., presence/absence of an enzyme)

Exchange reactions



Demand/sink reactions



Constructing a genome-scale metabolic model

Network evaluation \asymp Debugging

4. Network evaluation

43–44| Test if network is mass-and charge balanced.

45| Identify metabolic dead-ends.

46–48| Perform gap analysis.

49| Add missing exchange reactions to model.

50| Set exchange constraints for a simulation condition.

51–58| Test for stoichiometrically balanced cycles.

59| Re-compute gap list.

60–65| Test if biomass precursors can be produced in standard medium.

66| Test if biomass precursors can be produced in other growth media.

67–75| Test if the model can produce known secretion products.

76–78| Check for blocked reactions.

79–80| Compute single gene deletion phenotypes.

81–82| Test for known incapacibilities of the organism.

83| Compare predicted physiological properties with known properties.

84–87| Test if the model can grow fast enough.

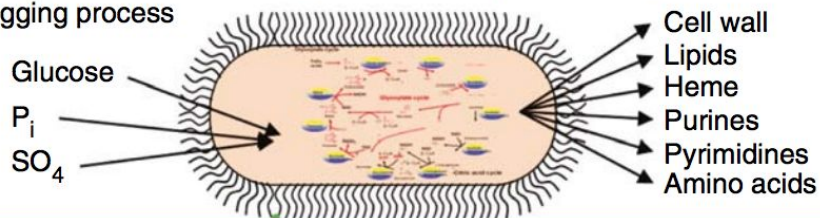
88–94| Test if the model grows too fast.

Constructing a genome-scale metabolic model

Network evaluation \approx Debugging

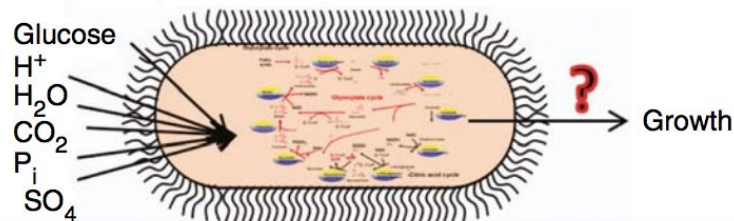
Analysis of biomass precursors synthesis

- Biomass precursors = cellular growth requirements
- Pathways to synthesize precursors must be complete (i.e., functional) for the network to simulate growth
- Testing synthesis of each separate biomass precursor is part of the debugging process



Analysis of growth in minimal medium

- Minimal medium is defined for many organisms and can be found in primary literature
- Contains at least 1 C-, N-, S- and P-source
- Auxotrophs may require the presence of addition metabolites



Test for growth on known carbon sources

- Exchange reactions define medium and environment
- Transport reactions allow network to consume carbon sources
- Biodegradative pathways that are required for carbon utilization

Secretion capability

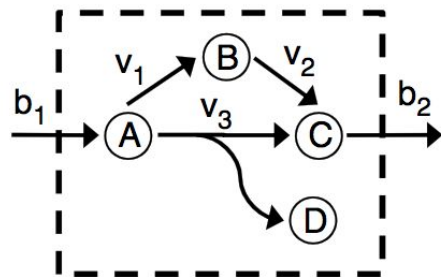
- Transport and exchange reactions are required in reconstruction to enable secretion
- Secretion may only occur under certain circumstances (e.g., D-lactic acid formation under anoxic conditions)
- Comparison with known secretion pattern of multiple metabolites (e.g., secretion of a certain ratio of CO₂ and acetate)

Constructing a genome-scale metabolic model

Network evaluation \approx Debugging

Identifying gaps

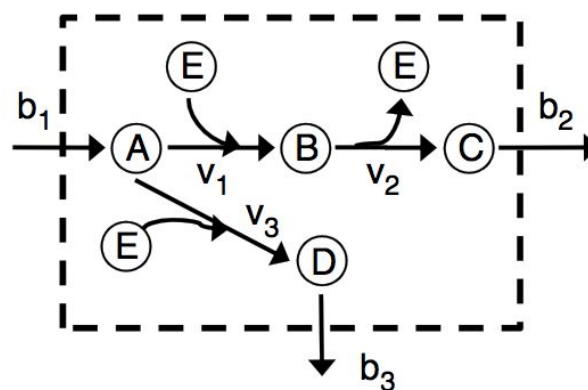
Connectivity based (topology):



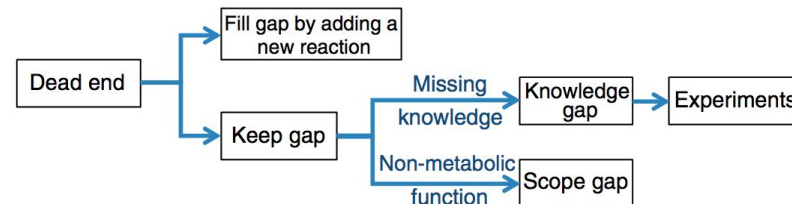
| | v_1 | v_2 | v_3 | b_1 | b_2 |
|---|-------|-------|-------|-------|-------|
| A | -1 | 0 | -1 | 1 | 0 |
| B | 1 | -1 | 0 | 0 | 0 |
| C | 0 | 1 | 1 | 0 | -1 |
| D | 0 | 0 | 1 | 0 | 0 |

Dead end

Functionality based (computation)



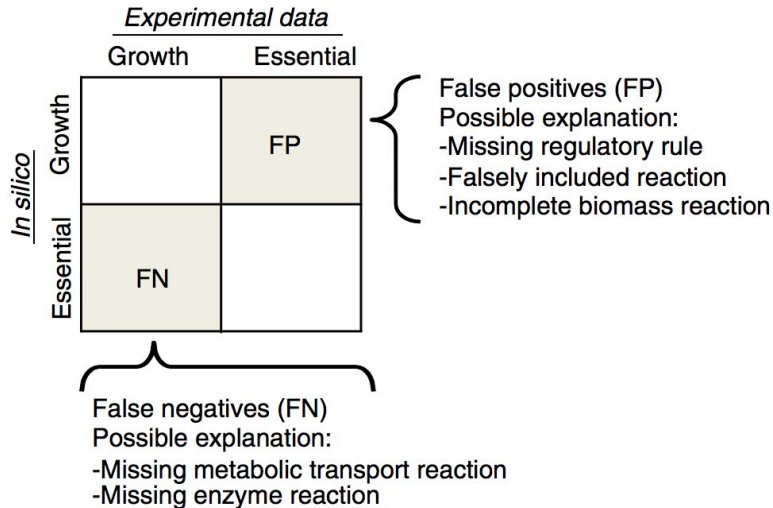
| | v_1 | v_2 | v_3 | b_1 | b_2 | b_3 |
|---|-------|-------|-------|-------|-------|-------|
| A | -1 | 0 | -1 | 1 | 0 | 0 |
| B | 1 | -1 | 0 | 0 | 0 | 0 |
| C | 0 | 1 | 0 | 0 | -1 | 0 |
| D | 0 | 0 | 1 | 0 | 0 | -1 |
| E | -1 | 1 | -1 | 0 | 0 | 0 |



Constructing a genome-scale metabolic model

Network evaluation \asymp Debugging

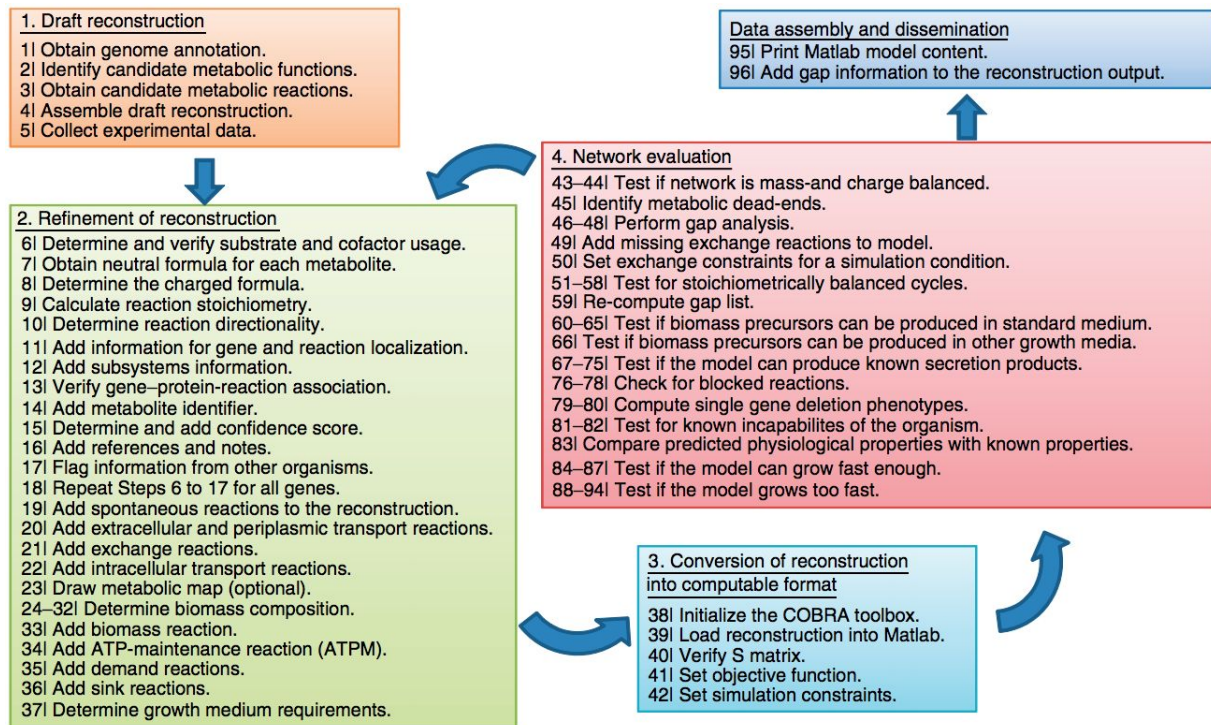
Gene essentiality



Constructing a genome-scale metabolic model

Procedure to iteratively
reconstruct metabolic networks.

- Iterate stages 2–4 are continuously...
- ...until model predictions are similar to the phenotypic characteristics of the target organism and/or all experimental data for comparison are exhausted.



Constructing a genome-scale metabolic model

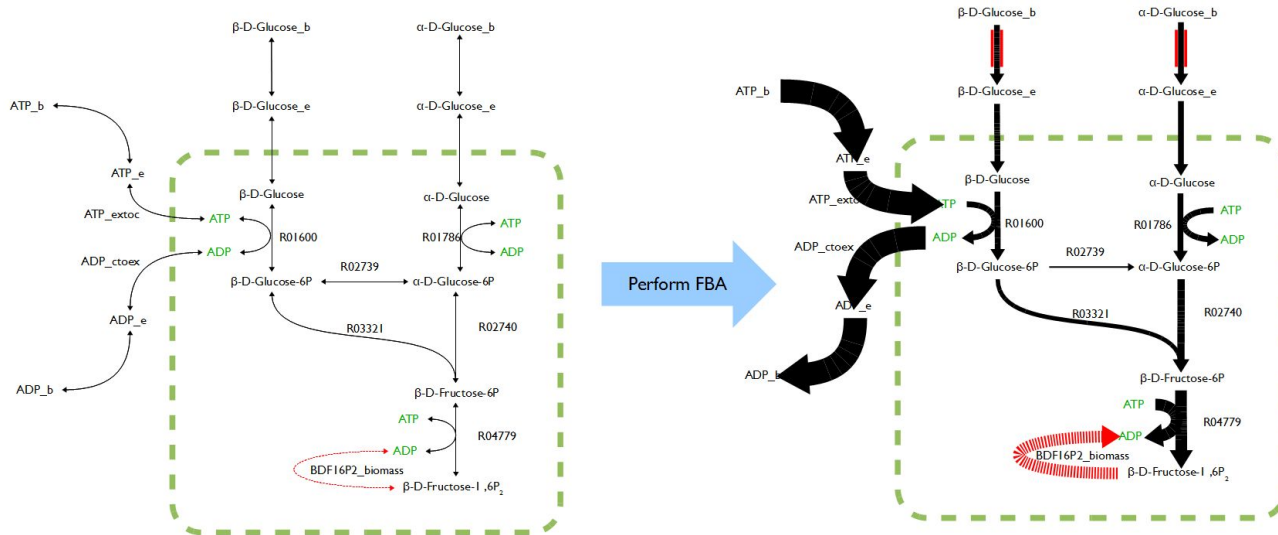
| Organism | Strain | Genes | Version | GR | Mets | Rxns | Comp |
|---------------------------------|------------|--------|----------|-------|-------|-------|---------------------|
| <i>Bacillus subtilis</i> | | 4,225 | model_v3 | 844 | 988 | 1,020 | 2 (c,e) |
| <i>Escherichia coli</i> | K12 MG1655 | 4,405 | iAF1260 | 1,260 | 1,039 | 2,077 | 3 (c,e,p) |
| <i>Helicobacter pylori</i> | 26695 | 1,632 | iIT341 | 341 | 485 | 476 | 2 (c,e) |
| <i>Pseudomonas putida</i> | KT2440 | 5,350 | iNJ746 | 746 | 911 | 950 | 3 (c,p,e) |
| <i>Pseudomonas putida</i> | KT2440 | 5,350 | iJP815 | 815 | 886 | 877 | 2 (c,e) |
| <i>Pseudomonas aeruginosa</i> | PA01 | 5,640 | iM01056 | 1,056 | 760 | 883 | 2 (c,e) |
| <i>Mycoplasma genitalium</i> | G-37 | 521 | iPS189 | 189 | 274 | 262 | 2 (c,e) |
| <i>Lactobacillus plantarum</i> | WCFS1 | 3,009 | | 721 | 531 | 643 | 2 (c,e) |
| <i>Streptomyces coelicolor</i> | A3(2) | 8,042 | | 700 | 500 | 700 | 2 (c,e) |
| <i>Leishmania major</i> | Friedlin | 8,370 | iAC560 | 560 | 1,101 | 1,112 | 8 (a,f,y,c,e,m,r,n) |
| <i>Saccharomyces cerevisiae</i> | Sc288 | 6,183 | iMM904 | 904 | 713 | 1,412 | 8 (c,e,m,x,n,r,v,g) |
| <i>Homo sapiens</i> | | 28,783 | Recon 1 | 1,496 | 2,766 | 3,311 | 8 (c,e,m,x,n,r,v,g) |

Flux balance analysis (FBA)

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

The results of FBA on a metabolic network of the top six reactions of glycolysis.

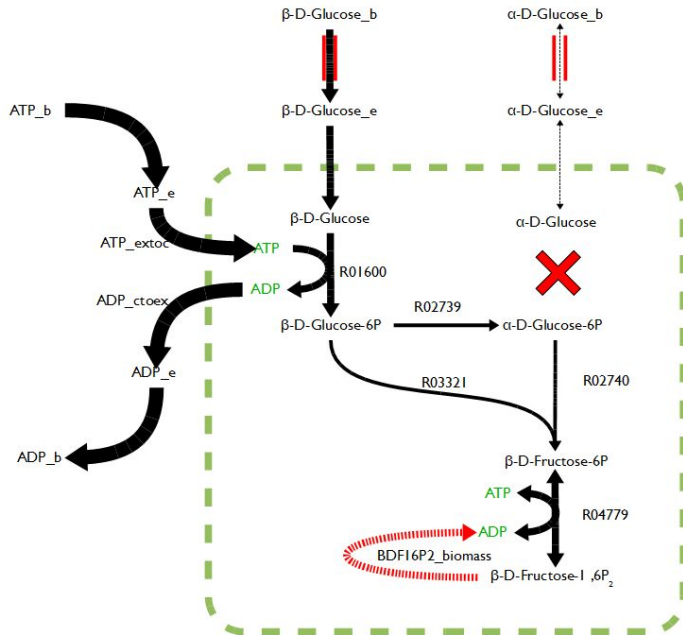
- The predicted flux through each reaction is proportional to the width of the line.
- **Red springy arrow**: Objective function; **Red bars**: Constraints on α -D-glucose and β -D-glucose import.



Flux balance analysis

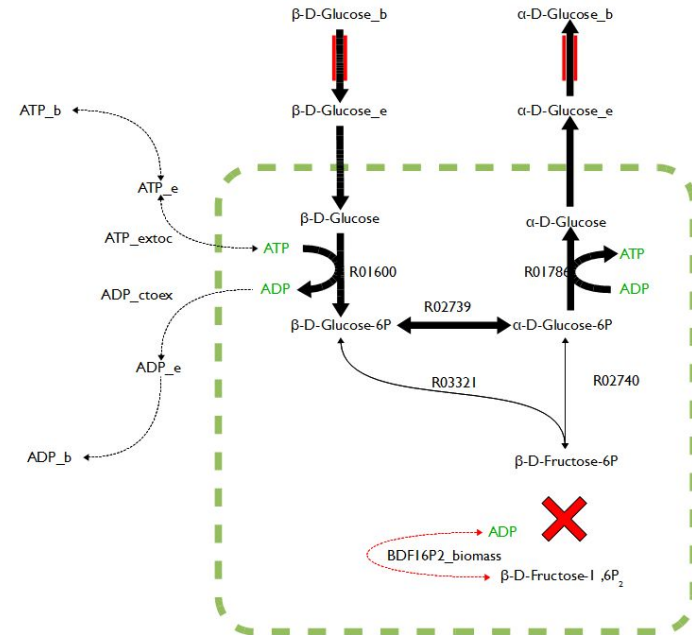
Non-lethal gene deletion in a metabolic network.

- Flux through the objective function is halved but is still present.



Lethal gene deletion in a metabolic network.

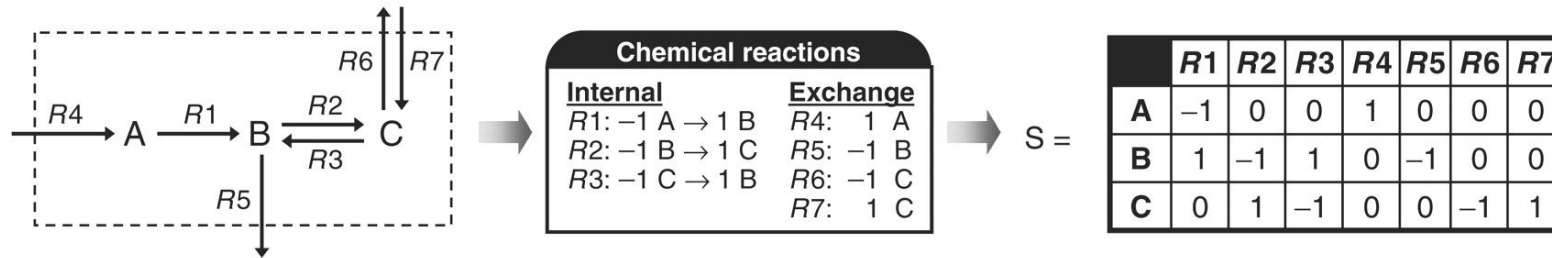
- No flux through the objective function \rightarrow pathway is no longer functional.



Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

I. Reaction network formalism



Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

II. FBA formulation

Dynamic mass balance

$$\frac{dC}{dt} = Sv$$

C : Concentration
 t : Time
 S : Stoichiometric matrix
 v : Flux vector

Steady-state assumption

$$Sv = 0$$

LP formulation

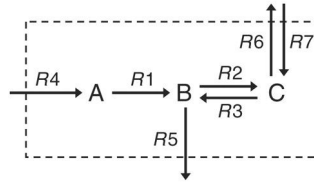
Objective: $\max Z = v_5$

$$Z = c \cdot v$$

Constraints:

| | $R1$ | $R2$ | $R3$ | $R4$ | $R5$ | $R6$ | $R7$ |
|---|------|------|------|------|------|------|------|
| A | -1 | 0 | 0 | 1 | 0 | 0 | 0 |
| B | 1 | -1 | 1 | 0 | -1 | 0 | 0 |
| C | 0 | 1 | -1 | 0 | 0 | -1 | 1 |

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_7 \end{bmatrix} = 0$$



- Principal constraint: mass balance.
- Additional constraints: e.g. reversibility of a reaction, energy requirement for cell maintenance.
- All constraints represent a set of linear equations.
 - No. of equations (one per reactant) << no. of unknown variables (reaction fluxes).
 - An under-determined set of linear equations.
- Therefore, optimize fluxes given cellular objective.

$$0 \leq v_1, \dots, v_7 \leq 10$$

Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

II. FBA formulation

Dynamic mass balance

$$\frac{dC}{dt} = Sv$$

C : Concentration
 t : Time
 S : Stoichiometric matrix
 v : Flux vector

Steady-state assumption

$$Sv = 0$$

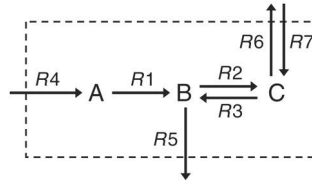
LP formulation

Objective: $\max Z = v_5$

$$Z = c \cdot v$$

Constraints:

$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} R1 & R2 & R3 & R4 & R5 & R6 & R7 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_7 \end{bmatrix} = 0$$



Objective functions: physiologically meaningful objectives OR design objectives for the interrogation or exploitation of a given system.

- Maximizing biomass or cell growth, maximizing ATP production or maximizing the rate of synthesis of a particular product.
- Minimizing ATP production in order to determine conditions of optimal metabolic energy efficiency
- Minimizing nutrient uptake in order to evaluate the conditions under which a cell will perform its metabolic functions while consuming the minimum amount of nutrients.

$$0 \leq v_1, \dots, v_7 \leq 10$$

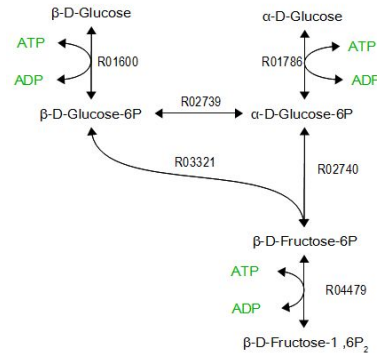
Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

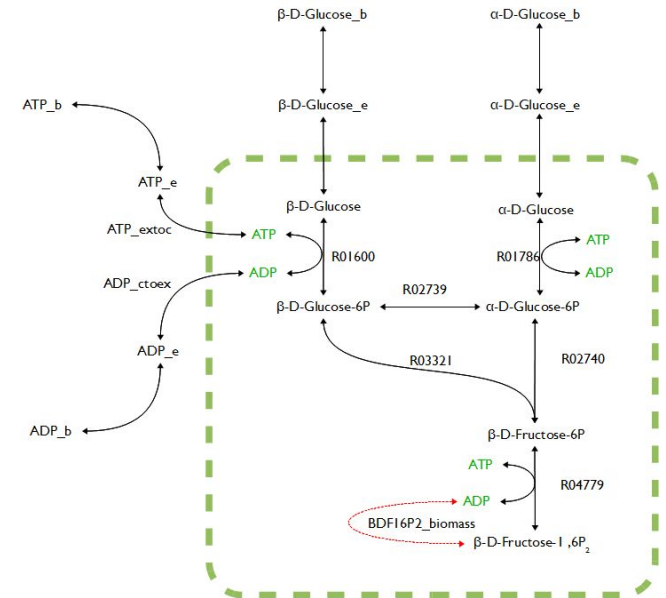
II. FBA formulation

Preparing the first six reactions in glycolysis for FBA:

- Addition of an objective function (red).
- Import & export of nutrients (ATP, ADP, BDG, ADG) across the system boundary (dashed green line).



Prepare for FBA

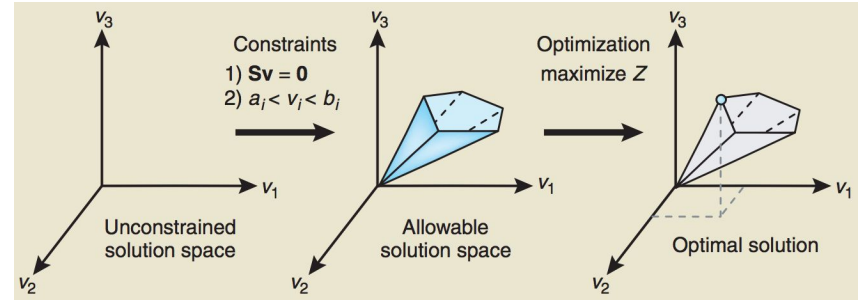


Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

Constraint-based modeling:

- No constraints: flux may lie at any point in solution space.
- Mass balance constraints (imposed by the stoichiometry) and capacity constraints (imposed by the lower and upper bounds (a_i & b_i)): defines allowable solution space.
 - Any flux distribution within this space is allowable
 - Points outside this space are denied
- Optimization of an objective function: A single optimal flux distribution that lies on the edge of the allowable solution space.

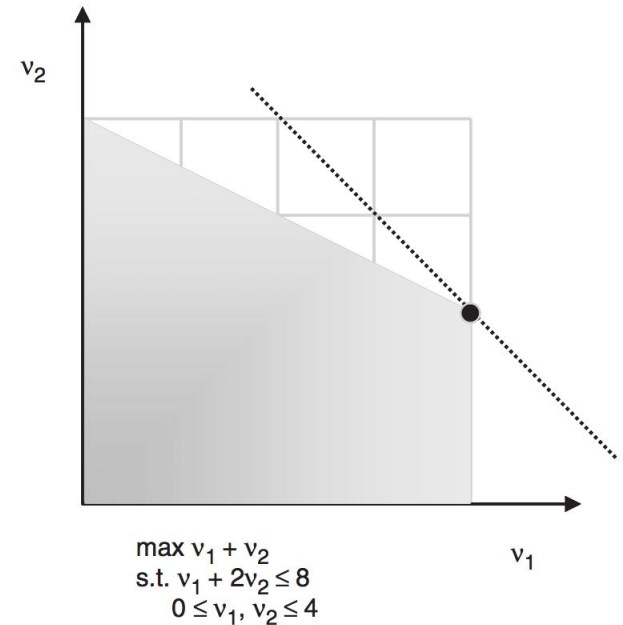


Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

Linear programming

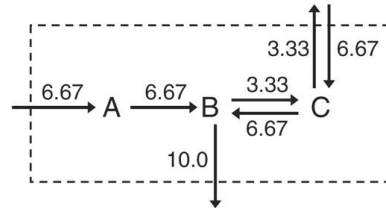
- Fluxes: v_1 and v_2
- Feasible solution space: shaded area and solid lines, defined by flux capacities, stoichiometric relationships, and design specification (e.g. gene deletions).
- Objective function: dotted line
- Optimal solution: circular dot



III. Hypothetical flux distribution at steady-state

$$Z = 10$$

$$\mathbf{v} = [6.67 \ 3.33 \ 6.67 \ 6.67 \ 10.0 \ 3.33 \ 6.67]^T$$



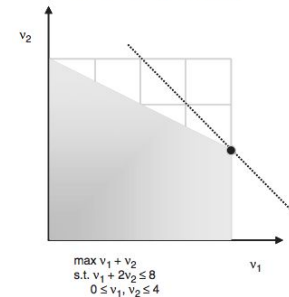
Flux balance analysis

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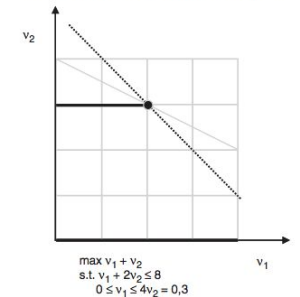
Linear programming

- Fluxes: v_1 and v_2
- Feasible solution space: shaded area and solid lines, defined by flux capacities, stoichiometric relationships, and design specification (e.g. gene deletions).
- Objective function: dotted line
- Optimal solution: circular dot

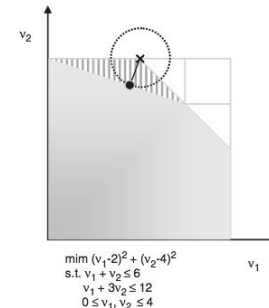
(a) Linear programming (LP)



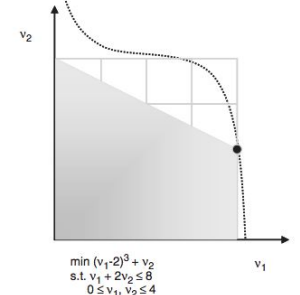
(b) Mixed integer linear programming (MILP)



(c) Quadratic Programming (QP)



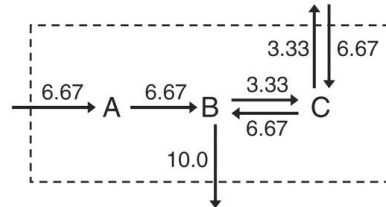
(d) Non-linear Programming (NLP)



III. Hypothetical flux distribution at steady-state

$$Z = 10$$

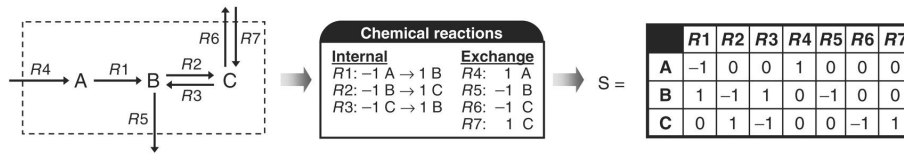
$$\mathbf{v} = [6.67 \ 3.33 \ 6.67 \ 6.67 \ 10.0 \ 3.33 \ 6.67]^T$$



Flux balance analysis

FBA is a formalism that defines the metabolic network as a linear programming optimization problem. The main constraints in FBA are imposed by the steady-state mass conservation of metabolites.

I. Reaction network formalism



II. FBA formulation

Dynamic mass balance

$$\frac{dC}{dt} = Sv$$

C : Concentration
t : Time
S : Stoichiometric matrix
v : Flux vector

Steady-state assumption

$$Sv = 0$$

LP formulation

Objective: max $Z = v_5$

Constraints:

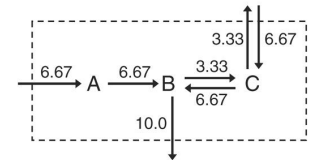
$$\begin{matrix} A \\ B \\ C \end{matrix}
 \begin{bmatrix} R1 & R2 & R3 & R4 & R5 & R6 & R7 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 & 1 \end{bmatrix}
 \begin{bmatrix} v_1 \\ \vdots \\ v_7 \end{bmatrix} = 0$$

$0 \leq v_1, \dots, v_7 \leq 10$

III. Hypothetical flux distribution at steady-state

$$Z = 10$$

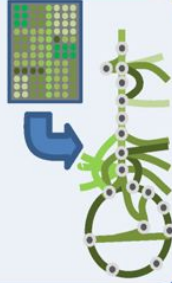
$$v = [6.67 \ 3.33 \ 6.67 \ 6.67 \ 10.0 \ 3.33 \ 6.67]^T$$



Genome-scale metabolic network reconstruction & model

1. Contextualization of HT data

Several studies have overlaid gene microarray data on a metabolic GENRE to determine condition-dependent cell phenotypes. Metabolic GENREs have also been used to interpret metabolomic data, ^{13}C flux data, and to link multiple data types.



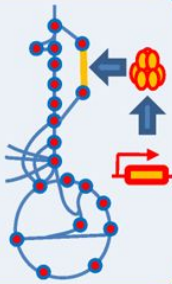
2. Guidance of metabolic engineering

Metabolic GENREs guided efforts to engineer malate and succinate producing strains of *S. cerevisiae* and *M. succiniciproducens*. GENREs have also helped determine ways to increase the respiration rate of *G. sulfurreducens* and scale-up vaccine production against *N. meningitidis*.



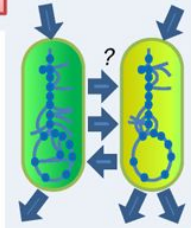
3. Directing hypothesis-driven discovery

A Metabolic GENRE aided in determining pathway usage and discovering a novel citramalate synthase gene in *G. sulfurreducens*. GENREs have also helped study the effects of transposons on downstream genes, and identify transcriptional timing patterns in *S. cerevisiae*.



4. Interrogation of multi-species relationships

A dual-species metabolic model was built to study interactions between the syntrophic bacteria, *D. vulgaris* and *M. maripaludis*. Metabolic models have also been used in comparisons of multiple species, such as an analysis of pathway differences between four halophilic bacteria.



5. Network property discovery

Metabolic GENREs have been used to study metabolite connectivity, and pathway redundancy *in silico*. Pathway-analysis tools have also spawned techniques such as flux coupling analysis, which has helped identify novel drug targets in *M. tuberculosis*.

