PoSW from Skip List

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Abstract

1 Introduction

1.1 PoSW

1. PoSW vs PoW vs Time-lock puzzles
2. Original construction from depth-robust graphs

1.2 Related Work

1. Time-release crypto [RSW00, BGJ$^+$15, May93, MMV11]
2. Original construction [MMV13] using depth-robust graphs [MMV13, EGS75]
3. [CP18] construction doesn't require depth-robustness

1.3 Our Contribution

1. Intuitive construction of POSW based on skip lists [Pug90]
2. Larger gap in proof generation and verification using PRPs and sloth hash function [LW17]
3. PRP used instead of a hash function and the input to it is squared before

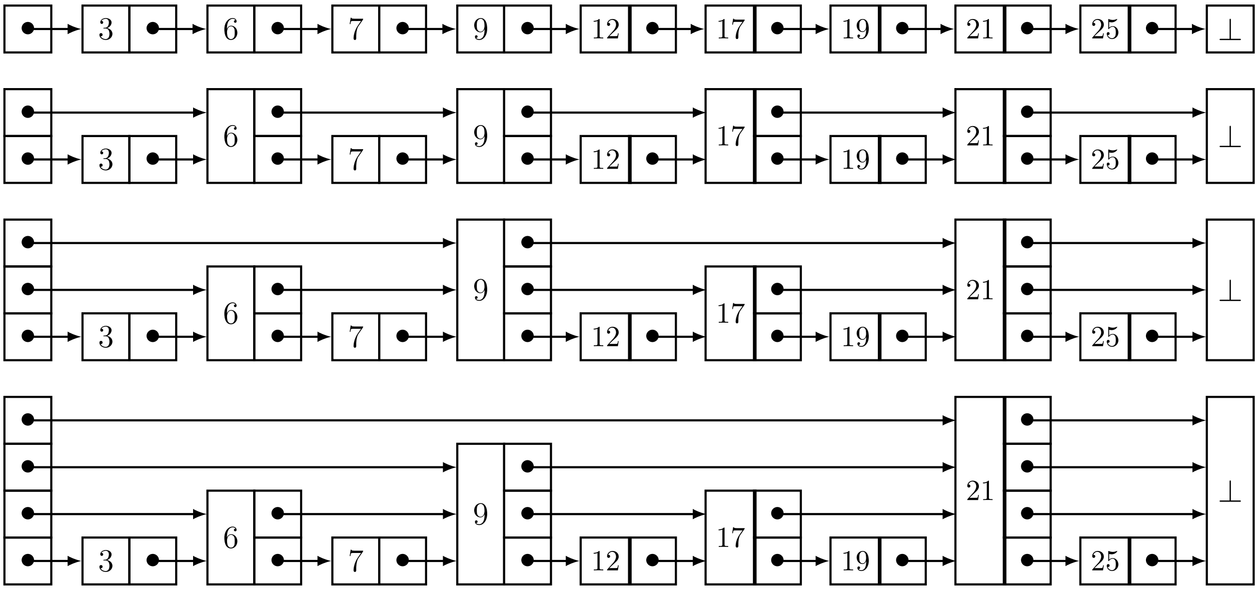


Figure 1: Linked list vs. skip lists

2 Preliminaries

2.1 Formal Definitions

1. PoSW

2.

2.2 Skip list

1. [Pug90]

2. randomised vs. deterministic (binary)

3. Figure

2.3 [CP18] Construction

2.4 The Sloth Hash Function

1. [LW17]

2. Assumptions: computing square-root requires logarithmically many squarings

3 Warm-up: PoSW from Skip Lists

We start with basic construction that uses an ensemble of PRPs and the skip list, and then show that it is a proof of sequential work.

3.1 Construction

The construction takes as input a time parameter $N = 2^n$ for $n \in \mathbb{N}$ and two statistical parameters $w, t \in \mathbb{N}$. We assume an ensemble of random permutations $P := P_0, \ldots, P_n$ with $P_i : \{0,1\}^{(i+1)\cdot w} \to \{0,1\}^{(i+1)\cdot w}$ sampled uniformly at random from $\mathcal{P}_{(i+1)\cdot w}$, the set of permutations on $(i+1)\cdot w$-bit-long strings in $\{0,1\}^*$. Let P^{-1} denote the "inverse" oracle of P.

1. The verifier sends the statement $\chi \in \{0,1\}^{(n+1)\cdot w}$ to the prover
2. The prover computes the sequence of states $\sigma_0, \ldots, \sigma_N$ and sets $\phi = \sigma_N$ and $\phi_p = \sigma_0, \ldots, \sigma_N$. (We will see later how one can trade-off space for time just like in [CP18].)
3. The verifier, on receipt of ϕ, challengers the prover on t random leaf nodes $\gamma_1, \ldots, \gamma_t$, where $\gamma_i \in [N]$.

Definition 1. An adversary A, with oracle access to P and P^{-1}, on an input $x \in \{0,1\}^w$ outputs a P-sequence $x_0, \ldots, x_s$ of length s if

1. $x \subset x_0$, where $\subset$ denotes that x is a continuous substring of x_0 (i.e., x_0 is of the form $a \circ x \circ b$ for some $a, b \in \{0,1\}^*$).
2. For all $j \in [0, s-1]$, there exists some $i \in [0, n]$ such that $P_i(x_j) \subset x_{j+1}$, where $\subset$ denotes that some continuous substring of $P_i(x_j)$ of length w is present as a continuous subsequence of x_{j+1}.

Claim 0.1. *The probability that an adversary outputs a P-sequence of length s making (strictly) less than s sequential queries is*

$$2q \cdot \frac{Q + \sum_{i=0}^{s} |x_i^2|}{2^{(t-1)w}},$$

where q denotes the total number of queries that the adversary is allowed to make to the random permutations and Q their total length.

"Proof". The three ways that A can output a P-sequence $x_0, \ldots, x_s$ making less than s queries are given below.

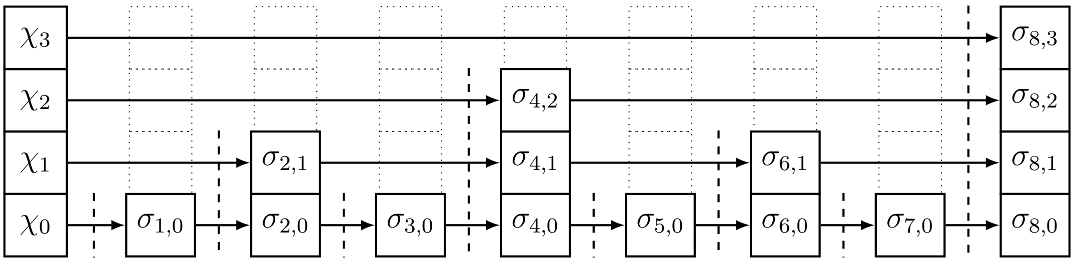


Figure 2: Schematic diagram for $N = 8$. The statement is of the form $\chi = \chi_0 \circ \cdots \circ \chi_3$ and is the initial state σ_0 in the construction. The proof, ϕ, is the final state $\sigma_8 = \sigma_0 \circ \cdots \circ \sigma_3$. The vertical lines represent an application of the permutation P with the length indicative of the exact permutation: e.g., the rightmost vertical line denotes an application of P_4 to the input $\chi_3 \circ \sigma_{4,2} \circ \sigma_{6,1} \circ \sigma_{7,0}$.

1. Lucky guess of a value of P: for some $i \in [s], j \in [n]$ it holds that $P(x_i) \subset x_{i+1}$ and the adversary *did not* make the query $P(x_i)$. As P is random, the probability of this event can be upper-bounded by
$$q \cdot \frac{\sum_{i=0}^{s} |x_i^2|}{2^{(t-1)w}}?$$
2. Collision: The x_js were not computed sequentially. That is it holds that for some $0 \leq j < k \leq s-1$ a query x_j is made in round j and a query x_k is made in round k where $P(x_j) \subset x_k$. Again, since P is uniformly random, the probability is
$$q \cdot \frac{Q^2}{2^{(t-1)w}}?$$
3. 1. or 2. occurs with P^{-1}: similar bounds hold.

The original bound follows by a union bound. □

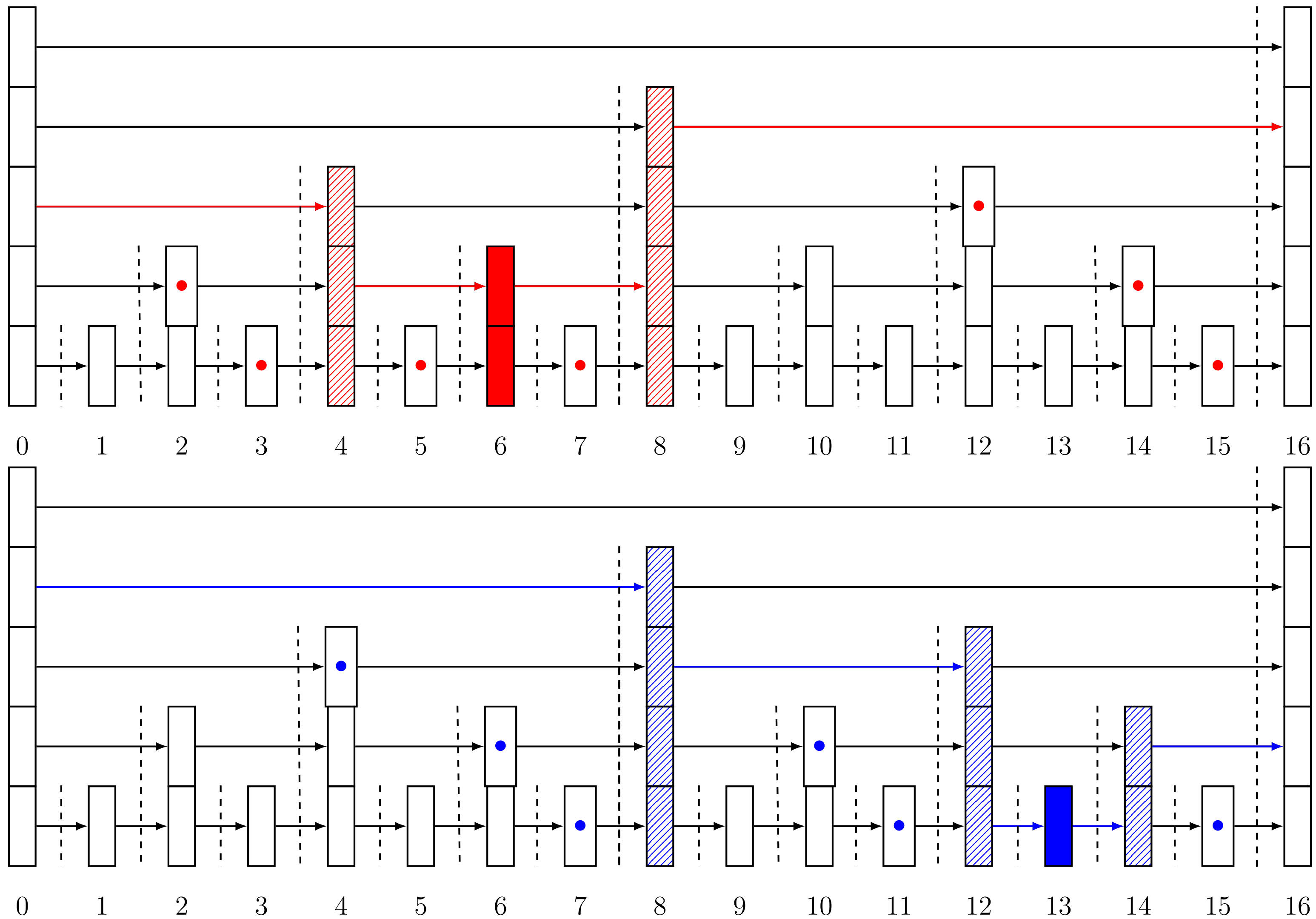
3.2 Trade-off

4 Main Construction

References

[BGJ+15] Nir Bitansky, Shafi Goldwasser, Abhishek Jain, Omer Paneth, Vinod Vaikuntanathan, and Brent Waters. Time-lock puzzles

Figure 3: The red and blue box represent two (independent) challenges $c_1 = 6$ and $c_2 = 13$. The red (resp., blue) path is the shortest path from 0 to 16 that goes through 6 (resp., 13). The (sub)states with red (resp., blue) bullets are required for the verification of the red (resp., blue) path. The shaded boxes are recomputed during the verification.



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