Coresets for k-median/means Clustering

TOP: Data Clustering 076/091

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Outline

1. Coresets for k-median/means
2. Coreset Construction for k-median

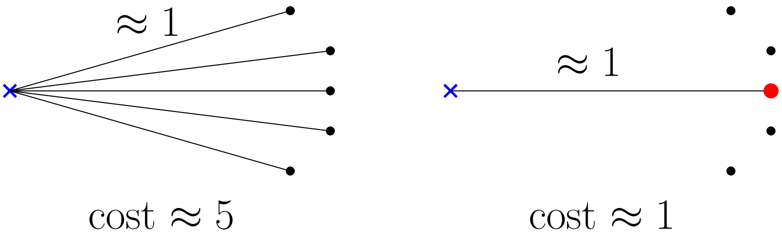
Euclidean k-median

Given a set X of n points in $\mathbb{R}^d$

- Find a set C of k points (cluster centers) in $\mathbb{R}^d$ that minimizes,

$$\text{cost}(X, C) = \sum_{p \in X} ||p - NearestCenter(p)||$$

Coresets



Cost comparison of original dataset and coreset

Coresets

Given a set S of points with weight w_p for each $p \in S$, the k-median cost of S w.r.t a set of centers C

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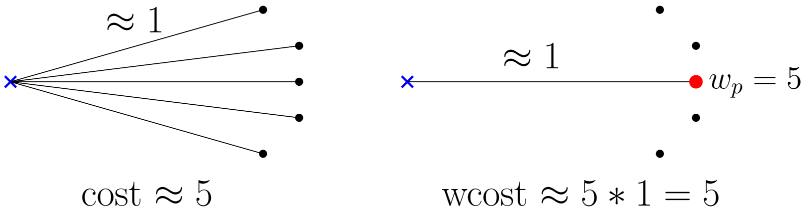
(ϵ-coreset.) A weighted subset $S \subseteq X$ is an ϵ-coreset if for any set of k centers C,

$$(1 - \epsilon) \cdot \text{cost}(X, C) \leq \text{wcost}(S, C) \leq (1 + \epsilon) \cdot \text{cost}(X, C)$$

Back to Our Example



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- We will take multiple balls per cluster

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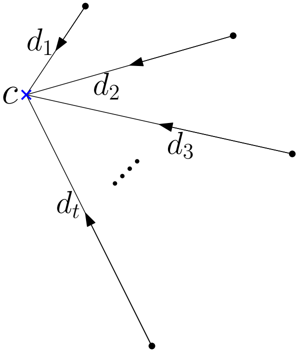
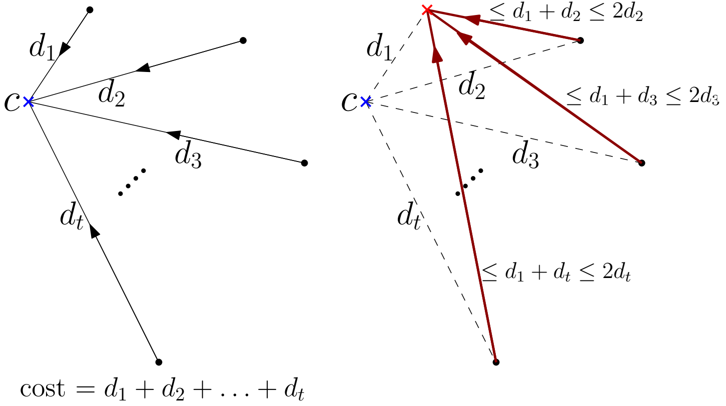
A Cluster with a Center not from X



$\text{cost} = d_1 + d_2 + \ldots + d_t$

Reassign Points to the One Closest to c



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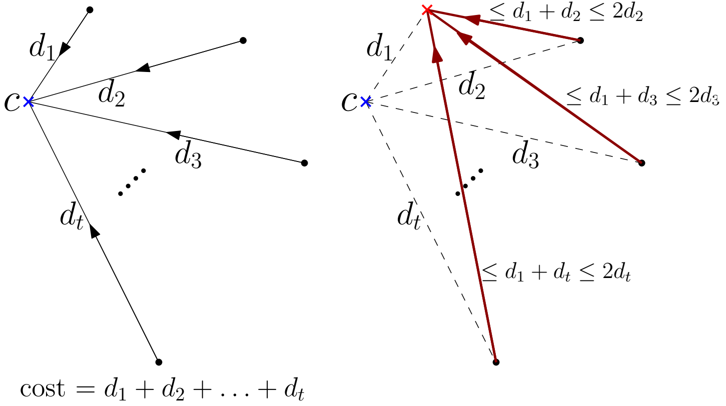

There is a subset of k centers of X that has cost $\leq 2 \cdot$ OPT

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Local search yields a 3-factor approximation to the best solution $\Longrightarrow$ we obtain a clustering of cost $\leq 6\cdot$ OPT

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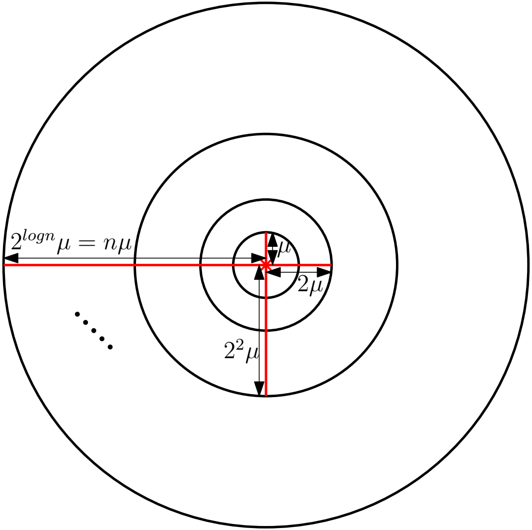
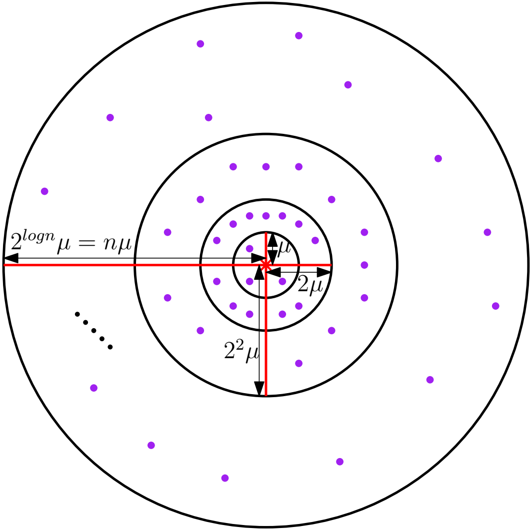
- For each cluster, do the following

Division of a Cluster into Rings



All Points in the Cluster are in the Rings



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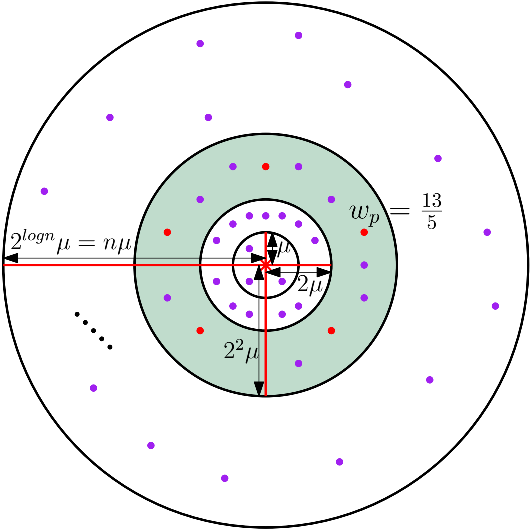
- Pick a uniformly random sample of size s from each ring R and add them to coreset
- Distribute the total weight $|R|$ equally to the chosen coreset points from R, i.e., $w_p = |R|/s$

A Random Sample is Picked from each Ring



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- Total coreset size: $O(k^2 d \log^2 n/\epsilon^3)$