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1) a) We define a random variable 'x' that marks the number of students that take the same seat in both classes. If we denote with S_j that jth student has the same seat, we have the following :-

$$P(x=0) = 1 - P(x \geq 1) = 1 - P\left(\bigcup_j S_I\right)$$

Using the inclusion-exclusion principle (formula) and the symmetry, we have

$$P\left(\bigcup_j S_I\right) = \sum_j (-1)^{j-1} \binom{100}{j} P\left(\bigcap_{k=1}^{j} S_k\right)$$

Evening

The probability that first j students sit on their seats is simply $\frac{(100-j)!}{100!}$

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∴ We have

$$P\left(\bigcup_j S_j\right) = \sum_j (-1)^{j-1} \binom{100}{j} \frac{(100-j)!}{100!} = \sum_{j=1}^{100} \frac{(-1)^{j-1}}{j!}$$

Finally, $P(x=0) = 1 - \sum_{j=1}^{100} \frac{(-1)^{j-1}}{j!} = \sum_{j=0}^{100} \frac{(-1)^j}{j!}$

b) We define an indicator random variable I_j that indicates if S_j has occurred, we have

$$x = \sum_{j=1}^{100} I_j$$

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We know that $P(I_j) = \frac{1}{100}$ and that we can approximate :-

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$$P(I_j = 1) \cap P(I_k \cap 1) = \frac{1}{100} \times \frac{1}{99} \approx \left(\frac{1}{100}\right)^2$$

$$= P(I_j = 1) \cdot P(I_k = 1)$$

So we can consider them as independent random variables. Next, we can approximate x with Poisson distribution with parameter $\lambda = E(x) = 100\, E(I_j) = 1$

So, we have

$$P(x = 0) \approx \frac{1^0}{0!} e^{-1} \approx 0.37$$

c) Using Poisson distribution, we finally obtain that,

$$P(X \geq 2) = 1 - P(x = 0) - P(x = 1) \approx 1 - e^{-1} - e^{-1}$$

$$= 1 - 2e^{-1}$$

$$\approx 0.26$$

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2) Let us consider that there are only 2 people, Alice and Bob, and 2 seats on the airplane. Alice picks a seat at random. The only way Bob gets to pick his designated seat if Alice correctly picks hers. Since there are only 2 choices for Alice, she picks her own seat correctly with probability $\frac{1}{2} \times \frac{1}{2}$

$\therefore P(\text{Bob gets his seat}) = P(\text{Alice chooses her own seat}) = \frac{1}{2}$

Now let us consider one more case where there are now 3 people, Alice, Bob and Carlos. Alice picks a random seat first. In this case, Carlos gets to sit on his designated seat

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in one of the following possibilities—

(1) Alice picks her own seat.

(2) Alice picks Bob's seat and Bob picks Alice's seat.

In all other cases, probability of Carlos picking his own seat is 0.

Hence, P(Carlos gets his seat) = P(Alice picks her seat) + P(Alice picks Bob's seat) × P(Bob picks Alice's seat)

∴ P(Carlos gets his seat) = $\frac{1}{3} + \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

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