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1) The only events that are independent of themselves are those with probability either 0 or 1. That follows from the fact that a number is its own square iff its either 0 or 1. The only way a random variable X can be independent of itself is y for every measurable set A, either $P(X \in A) = 1$ or $P(X \in A) = 0$

2) We know that,
$$P(A \cap B) = P(A) P(B)$$
$$P(A^c) = 1 - P(A)$$
and $P(B^c) = 1 - P(B)$

Assuming A and B are independent,
$$P(A^c \cap B^c) = 1 - P(A \cup B)$$
$$= 1 - P(A) - P(B) + P(A \cap B)$$
$$= 1 - P(A) - P(B) + P(A) P(B)$$

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$$= 1 - P(A) - P(B)[1 - P(A)]$$
$$= [1 - P(A)][1 - P(B)]$$
$$= P(A^c) P(B^c)$$