Simple Book Example

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Chapter 1

Common concepts

1.1 Propositions

Definition 1 (Proposition). *A proposition is a statement that is either true or false (and usuallly has a variable).*

Definition 2 (Compound Proposition). *Compound propositions: Combine multiple propositions using and/or/not/implies*

Definition 3 (Predicate). *A predicate is a proposition whose truth value dedpends on one or more variables.*

For e.g. "n is a perfect square" P(n) ::= "n is a perfect square" P(4) is true, P(5) is false

Definition 4 (Quantifier). *A quantifier is an operator that specifies how many individuals in the domain of discourse satisfy an open formula. ($\forall$ or $\exists$)*

For instance, the universal quantifier $\forall$ in the first order formula $\forall x P(x)$ expresses that everything in the domain satisfies the property denoted by P. On the other hand, the existential quantifier $\exists$ in the formula $\exists x P(x)$ expresses that there is something in the domain which satisfies that property

Lemma 1 (Negating Quantifiers). *Universal negation:*

$$\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x).$$

Existential negation:

$$\neg(\exists x \in X, P(x)) \equiv \forall x \in D, \neg P(x).$$

Lemma 2 (Order of mixed quantifiers). *When multiple quantifiers are of the same type, e.g. all $\forall$ or all $\exists$, then it is ok.*

$$\forall x \forall y.Likes(x, y) \equiv \forall y \forall x.Likes(x, y)$$

But if quantifiers are of different type, then they do not mean the same

$$\forall x \exists y.Likes(x, y) \neq \exists y \forall x.Likes(x, y)$$

Definition 5 (Validity). *A propositional formula is called valid when it evaluates to T no matter what truth values are assigned to the individual propositional variables.*

Definition 6 (Satisfiability). *A proposition is satisfiable if some setting of the variables makes the proposition true*

E.g. e, P $\wedge$ ¬Q is satisfiable because the expression is true if P is true or Q is false.

1.2 Induction

Definition 7 (Well Ordering principle). *Every nonempty set of nonnegative integers has a smallest element.*

Definition 8 (Induction). *Let P(n) be a predicate. If*

- *P (0) is true, and*
- *P (n) IMPLIES P(n + 1) for all nonnegative integers, n,*

then

- *P (m) is true for all nonnegative integers, m.*

In other words: $$\frac{P(0) \quad \forall n \in \mathbb{N}.P(n) \implies P(n+1)}{\forall m \in \mathbb{N}.P(m)}$$

Definition 9 (Invariant). *A property that is preserved through a series of operations or steps is known as an invariant*

Lemma 3 (Invariant method). *If you would like to prove that some property NICE holds for every step of a process, then it is often helpful to use the following method:*

- *Define $P(t)$ to be the predicate that NICE holds immediately after step t.*

- *Show that* $P(0)$ *is true, namely that NICE holds for the start state.*
- *Show that* $\forall t \in \mathbb{N}. P(t) \implies P(t+1)$*; namely, that for any* $t \geq 0$*, if NICE holds immediately after step* t*, it must also hold after the following s*

Definition 10 (Strong Induction). *Let P(n) be a predicate*

- $P(0)$ *is true and*
- $P(1), P(2), ...P(n)$ *together imply* $P(n+1)$

then $P(n)$ *is true for all* $n \in \mathbb{N}$

Chapter 2

The Second Chapter