Number Theory Notes

TeXstudio Team

January 2021

Contents

1 Fundamental theorem on Numbers **1**
1.1 Division of numbers 1
1.2 Floor & Ceiling functions 1
1.3 Floor and Ceiling inequalities 1

2 The Second Chapter **3**

Chapter 1

Fundamental theorem on Numbers

1.1 Division of numbers

Given any two integers $a, d \in \mathbb{N}$,

$$a = d * \left\lfloor \frac{a}{d} \right\rfloor + a \bmod d \tag{1.1}$$

Here the quotient is $q = \left\lfloor \frac{a}{d} \right\rfloor$ and remainder is $r = a \bmod d$

1.2 Floor & Ceiling functions

$$\begin{aligned} \lfloor x \rfloor &= \textit{greatest integer less than or equal to } x \\ \lceil x \rceil &= \textit{smallest integer greater than or equal to } x \end{aligned} \tag{1.2}$$

1.3 Floor and Ceiling inequalities

$$\boxed{x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1} \tag{1.3}$$

When $x \in \mathbb{N}$, following equation holds:

$$\boxed{\lceil x \rceil = \lfloor x \rfloor = x} \tag{1.4}$$

Integers can move in and out of floors and ceilings, i.e. $n \in \mathbb{N}$

$$\begin{aligned} \lfloor x + n \rfloor &= \lfloor x \rfloor + n \\ \lceil x + n \rceil &= \lceil x \rceil + n \end{aligned} \tag{1.5}$$

Chapter 2

The Second Chapter