# 1 Calculus

**Definition 1.1. Continuity at a point $x = a$:**

A function $f$ is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$

There are three subtle points that should not be missed:

- Two sided limit $\lim_{x \to a} f(x)$ exists and is finite
- The function is defined at $x = a$, i.e. $f(a)$ exists
- The above two quantities are equal i.e $\lim_{x \to a} f(x) = f(a)$

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Some of the cases the continuity fails is, e.g. two sided limit exists but $f(a)$ has a different value, or two sided limit exists and $f(a)$ is not defined.

In essence, for $f$ to be continuous at a point $x = a$, two sided limit should exist and be same as $f(a)$ at $x = a$.

**Definition 1.2. Continuity on an interval**
$f$ is continuous on the interval $(a, b)$ if it is continuous at every point in the interval.

**Definition 1.3. Continuity and Intermediate Value Theorem(IVT)**
If $f$ is continuous on $[a, b]$, and $f(a) < 0$ and $f(b) > 0$,
then there is at least one number $c$ in the interval $(a, b)$ such that $f(c) = 0$.

The same is true if instead $f(a) > 0$ and $f(b) < 0$.

**Definition 1.4. Continuity and Max-Min Theorem**
If $f$ is continuous on $[a, b]$,
then f has atleast one maximum and one minimum on $[a, b]$.