1 Basic definitions

Definition 1.1. Support The set of values a random variable can take (set of all the numerical realizations of outcomes) e.g. for a binary random variable X, it's support is $R_X = \{0, 1\}$

Definition 1.2. Sample Point In an experiment, a sample point is one of the possible outcomes of experiment denoted by ω

Definition 1.3. Independent Events Two events A and B are independent if and only if

$$P(A \cap B) = P(A) * P(B) \tag{1}$$

Definition 1.4. Jointly/Mutually Independent Events Let $E_1, ..., E_n$ be n events.
$E_1, ..., E_n$ are jointly independent(or mutually independent) if and only if for any sub-collection of k events $(k \leq n) E_{i1}, ..., E_{ik}$:

$$P(\bigcap_{j=1}^{k} E_{ij}) = \prod_{j=1}^{k} P(E_{ij}) \tag{2}$$

Definition 1.5. Random Variable A random Variable X is a function from sample space Ω to set of real numbers $\mathbb{R}$, i.e $X : \Omega- > \mathbb{R}$.

The real number $X(\omega)$ associated with sample point $\omega \in \Omega$ is called a realization of the random variable. The set of all possible realizations is called the support and denoted by R_X

Definition 1.6. Probability Mass Function The PMF of a discrete random variable X is a function $p_X : \mathbb{R}- > [0, 1]$ such that

$$p_X = P(X = x) \quad \forall x \in \mathbb{R} \tag{3}$$

where $P(X = x)$ is the probability of realization of random variable X will be equal to x. Basically PMF is numerical realizations $- >$ respective Probabilities.

Definition 1.7. Distribution Function/Cumulative Distribution Function If X is a random variable, its distribution/cdf is a function $F_X : \mathbb{R}- > [0, 1]$ such that

$$F_X(x) = P(X \leq x) \quad \forall x \in \mathbb{R} \tag{4}$$

Definition 1.8. Expected Value of a Random Variable The expected value of random variable X is the weighted average of the values that X can take on where each possible realization value is weighted by its respective probability i.e.

$$\mathbf{E}[X] = \sum_{x \in R_X} x p_X(x) \tag{5}$$

In other words, it is the weighted average of all possible numerical outcomes(in the support) widghted with respect to their respective probabilities

Definition 1.9. Expected value of function of Random Variable Expected value of function $g(Y)$ say $g(Y) = Y^2$:

$$E[g(Y)] = \sum_{y \in R_Y} g(y)p(y) \tag{6}$$

Definition 1.10. Linearity of Expectation If X is a random variable and $a \in R$ is a constant, then

$$E[aX] = aE[X] \tag{7}$$

In a more general setting:
If Y is a random variable such that $Y = a + bX$, where X is a random variable, and a and b are constants,

$$\begin{aligned} E[Y] &= E[a] + E[bX] \\ &= a + bE[X] \\ E[Y] &= a + bE[X] \end{aligned} \tag{8}$$

Definition 1.11. Conditional expectation of Random Variable Given random variables X and Y, The conditional expectation of X given $Y = y$ is the weighted average of values that X can take on, where each possible value is weighted by its respective conditional probability (conditinal on the information Y = y), denoted by $E[X|Y = y]$

$$E[X|Y = y] = \sum_{x \in R_x} x p_{X|Y=y}(x) \tag{9}$$

Another fact worth noting is that $E[X|Y = y]$ when y is known is a number, but when y is unknown, $E[X|Y]$ acts like a random variable that is a function of random variable Y.i.e. $E[X|Y] = g(Y)$

Definition 1.12. Total Expectation/Iterated Expectation The law of total expectation/law of iterated expectation/tower rule/smoothing theorem/adam's law among other names states that if, X is a random variable whose expected value $E[X]$ is defined, and Y is any random variable on same probability space then

$$E[X] = E[E[X|Y]] \tag{10}$$

This comes from the fact that if there are A_i is a finite countable partition of the sample space, then

$$E[X] = \sum_i E[X|A_i]P[A_i] \tag{11}$$

Definition 1.13. Quantile The $p-$Quantile of a Random Variable X is given $p \in (0, 1)$ and X having cumulative distribution function(cdf) to be $F_X(x)$, the $p-$Quantile is:

$$Q_X(p) = min\{x \in R : F_X(x) \geq p\} \tag{12}$$

This translates roughtly equivalently to the definition of percentiles. e.g 0.5-quantile is a median. 0.25-Quantiles are quartiles etc.

Definition 1.14. Deviation of a Random Variable The Deviation of a Random Variable is its difference from its mean value/Expected value. It is denoted by

$$\overline{X} = X - \mathbf{E}[X] \tag{13}$$

Deviation of Random Variable is also a random variable in it's own respect.

Definition 1.15. Variance It is a measure of dispersion of a random variable. Let X be a random variable. The variance of X, denoted by $Var[X]$ is defined as follows:

$$Var[X] = \mathbf{E}[(X - \mathbf{E}[X])^2] = \mathbf{E}[\overline{X}^2] = \mathbf{E}[X^2] - \mathbf{E}[X]^2 \tag{14}$$

You can also think of it as mean of the square of the deviations of original random variable.

Definition 1.16. Standard Deviation It is a measure of dispersion of a random variable. Let X be a random variable. The standard deviation of X, denoted by $stdev[X]$ or $std[X]$ is defined as follows:

$$stdev[X] = \sqrt{Var[X]} \tag{15}$$

You can also think of it as rms deviation.

Definition 1.17. Covariance It is a measure of association between two random variables. Covariance of two random variables X and Y is , provided Expected values are well defined,

$$Cov[X, Y] = \mathbf{E}[(X - \mathbf{E}[X]) * (Y - \mathbf{E}[Y])] = \mathbf{E}[\overline{X} * \overline{Y}] \tag{16}$$

where we say deviation of X is $\overline{X} = X - \mathbf{E}[X]$ and
deviation of Y is $\overline{Y} = Y - \mathbf{E}[Y]$
or in other words, covariance is expectation of product of deviations.

Definition 1.18. Random Vector It is a multidimensional generalisation of the concept of Random Variable. Associated probability functions have the word "joint" in front of them e.g. Joint PMF, Joint cdf etc correspond to a Random Vector.

2 Logistic

A logistic function or logistic curve is a common 'S' shaped curve with equation:

$$f(x) = \frac{L}{1 + e^{-k(x - x_0)}} \tag{17}$$

where

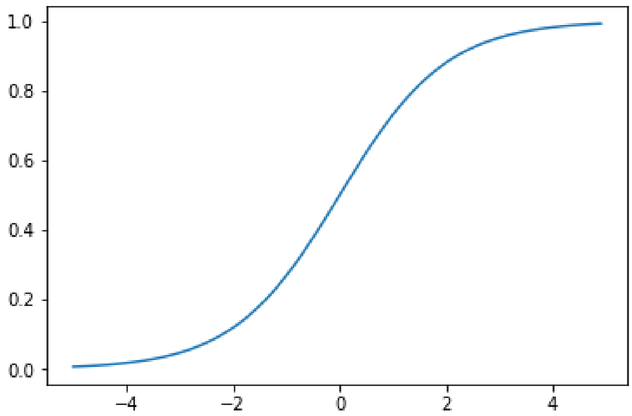
- e = natural log
- x_0 = x-value of sigmoid's midpoint
- L = the curve's maximum value
- k = the steepness of the curve

2.1 Standard Logistic

The standard logistic function is the logistic function with parameters given $(k = 1, x_0 = 0, L = 1)$ i.e.

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{18}$$

which when plotted looks like



Why is logistic function so important? Because it can take any real input $x, (x \in R)$, whereas the output always takes values between 0 and 1, and hence is interpretable as probability.