Fluid Statics

Introduction

Fluid statics, or hydrostatics, is the study of fluids at rest. It forms the foundation for understanding pressure distribution in liquids, buoyancy, and measurement devices like manometers. In chemical engineering, knowing how to calculate hydrostatic pressure is crucial for designing tanks, piping systems, and instrumentation.

Key Concepts

- Hydrostatic Pressure
- Pressure Variation with Depth
- Manometers
- Pressure Measurement
- Applications in Chemical Engineering

Definitions

- **Hydrostatic Pressure:** The pressure exerted by a fluid at rest due to gravity.
- **Manometer:** A device used to measure pressure or pressure differences using a column of fluid.
- **Gauge Pressure:** Pressure relative to atmospheric pressure.
- **Absolute Pressure:** Total pressure including atmospheric pressure.

Hydrostatic Pressure

Hydrostatic pressure is the pressure exerted by a fluid at rest due to the force of gravity. It increases with depth and depends on fluid density.

The general equation is:

$$P = P_0 + \rho g h$$

where:

- P = pressure at depth (Pa)
- P_0 = pressure at the surface (Pa)
- ρ = fluid density ($\mathrm{kg/m^3}$)
- g = acceleration due to gravity (9.81 $\mathrm{m/s^2}$)
- h = depth below the surface (m)

Example: What is the pressure 2 meters below the surface of water ($\rho = 1000$ $\mathrm{kg/m^3}$) open to the atmosphere ($P_0 = 101.3$ kPa)?

$$P = 101.3\,\mathrm{kPa} + 1000 \cdot 9.81 \cdot 2 = 101.3 + 19.62 \approx 120.92\,\mathrm{kPa}$$

Pressure Variation with Depth

For fluids of varying density (e.g., seawater), hydrostatic pressure is determined using integration:

$$P = P_0 + \int_0^h \rho(z) g \, dz$$

Manometers

Manometers are devices used to measure pressure using a column of liquid. Common types include U-tube and inclined manometers.

U-tube manometer:

- Measures the difference between two pressures by the height difference of a liquid column.
- Pressure difference:

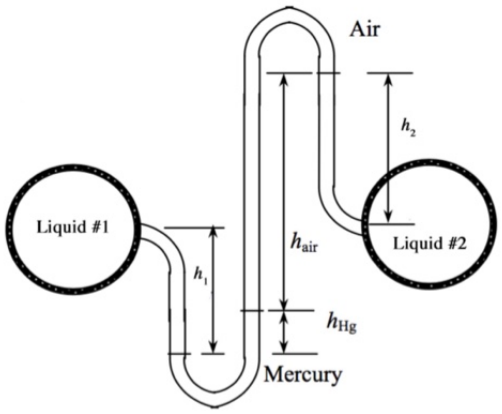
$$\Delta P = \rho g h$$

- You want to choose a reference (in our example our reference is the line above Mercury and its counterpart on the adjacent left tube). You need to make sure that your reference consists of the same material on each line. You then form a balance, as those reference pressures will be equal. You need to make sure that you include pressures in tanks (ie. the pressures in Liquid 1 and 2)

U-Tube:

Problem 5: Manometers

For the double U-tube manometer shown below find the relationship between the pressures of Liquid #1 and Liquid #2 flowing through their respective pipes.



$$P_a = P_1 + \rho_{Hg} g h_1$$
$$P_b = P_2 + \rho_{air} g (h_{air} - h_2)$$
$$P_a = P_b$$
$$P_1 + \rho_{Hg} g h_1 = P_2 + \rho_{air} g (h_{air} - h_2)$$
$$\Delta P = \rho_{Hg} g h_1 - \rho_{air} g (h_{air} - h_2)$$

Inclined Manometer:

- Works like a U-tube, but the fluid column is inclined to increase measurement sensitivity.
- Vertical height is related to the inclined length by

$$h = L \sin \theta$$

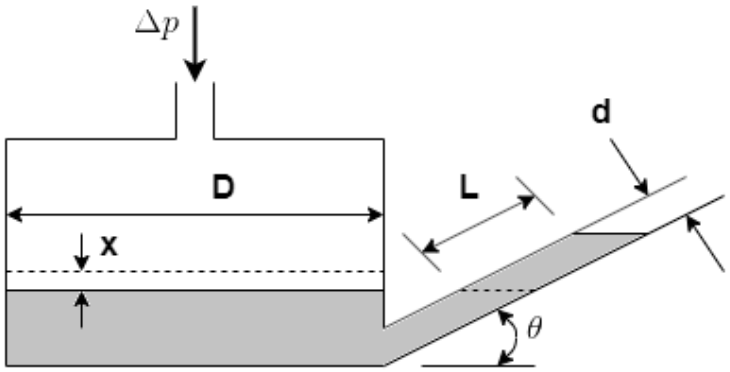
where L is the length of the fluid column along the incline and θ is the angle of inclination.

- Pressure difference:

$$\Delta P = \rho g h = \rho g L \sin\theta$$

- Same approach: pick a reference, then apply a pressure balance.

Inclined Manometer Diagram:



Let Δp be the atmospheric pressure P_{atm}

Let the pressure at the outlet of the tube be P_2

$$P_a = P_{atm}$$

$$P_b = P_2 + \rho g L sin\theta$$

$$P_a = P_b$$

$$P_2 + \rho g L \sin\theta = P_{atm}$$

Applications in Chemical Engineering

- Measuring liquid levels in tanks
- Determining pressure drop in piping systems
- Calibrating pressure gauges and sensors
- Designing dams, vessels, and hydraulic equipment

Recap

Fluid statics is the study of fluids at rest and the pressures they exert. Hydrostatic pressure depends on depth and fluid density, and devices like manometers are practical tools for measuring pressure differences. Mastery of these concepts is essential for designing and operating chemical engineering systems safely and effectively.