Econometric Analysis

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Ch. 11 More Topics in Linear Unobserved Effects Models

11.1 Unobserved Effects Models without the Strict Exogeneity Assumption

Models in Ch. 10 assume strict exogeneity $E(u_{it} \mid \mathbf{x}_{i1},\cdots,\mathbf{x}_{iT},c_i)=0$**. However, this is not satisfied if** y affects future x (ex. y: output, x: capital)

$y_{it}=\mathbf{x}_{it}\boldsymbol{\beta}+c_i+u_{it}$, $t=1,\cdots,T$

Sequential Moment Restrictions: $\mathbf{x}_{it}$ are **sequentially exogenous conditional on the unobserved effect** if $E(u_{it} \mid \mathbf{x}_{i1},\cdots,\mathbf{x}_{it},c_i)=0$, $t=1,\cdots,T$

u_{it} **can correlate to future** $\mathbf{x}_{it}$**, but uncorrelated to past** $\mathbf{x}_{it}$

Ex. u_{it}: technology shock. Future capital x_{it+1} depends on past GDP y_{it} and thus past technology shocks u_{it} through investment. But u_{it} does not depend on past capital

Example 11.2 lung cancer and sales of cigarette

$y_{it}=\mathbf{z}_{it}\boldsymbol{\gamma}+\delta w_{it}+c_i+u_{it}$, $w_{it}=\mathbf{z}_{it}\boldsymbol{\xi}+\rho_1 y_{i,t-1}+\psi c_i+r_{it}$

$\mathbf{z}_{it}$: strictly exogenous, w_{it}: sequentially exogenous

w_{it}: number of **lung cancer patients per 1000**, y_{it}:**cigarette sales**

If $\rho_1\neq 0$**, strict exogeneity is not satisfied (**$E(w_{i,t+1}u_{it})=\rho_1 E(u_{it}^2)\neq 0$**)**

Even with sequential moment restrictions, the FE or FD estimator of the model with feedback is not consistent without strict exogeneity

⇨ **Estimate FD transformation model with strictly exogenous instruments**

<u>How to estimate the model with Sequentially exogenous regressors (Pooled 2SLS)</u>

1. FD transformation: $\Delta y_{it}=\Delta\mathbf{x}_{it}\boldsymbol{\beta}+\Delta u_{it}$, $t=2,\cdots,T$ $\Delta y_{it}=y_{it}-y_{i,t-1}, \Delta\mathbf{x}_{it}=\mathbf{x}_{it}-\mathbf{x}_{i,t-1}, \Delta u_{it}=u_{it}-u_{i,t-1}$

2. Since $E(\mathbf{x}_{is}'\Delta u_{it})=0$ **for** $s=1,\cdots,t-1$**, estimate the model by POLS using** $\Delta\mathbf{x}_{i,t-1}$ **(or variants of** $\mathbf{x}_{i,t-1}^o\equiv(\mathbf{x}_{i1},\cdots,\mathbf{x}_{i,t-1})$**) as instruments for** $\Delta\mathbf{x}_{it}$

Note that $E(\Delta\mathbf{x}_{it}'\Delta\mathbf{x}_{i,t-1})\neq 0$**. For the IV to be valid, this correlation must be high enough**

Example 11.3

11.1.2 Models with Strictly and Sequentially Exogenous Explanatory Variables

$y_{it} = \mathbf{z}_{it}\boldsymbol{\gamma} + \mathbf{w}_{it}\boldsymbol{\delta} + c_i + u_{it}$, $t = 1,\cdots,T$

$\mathbf{z}_{it}$: strictly exogenous, $\mathbf{w}_{it}$: sequentially exogenous

<u>How to estimate the model with sequentially exogenous (Pooled 2SLS)</u>

1. FD transformation: $\Delta y_{it} = \Delta\mathbf{z}_{it}\boldsymbol{\gamma} + \Delta\mathbf{w}_{it}\boldsymbol{\delta} + \Delta u_{it}$, $t = 2,\cdots,T$

2. P2SLS estimation using for example $(\Delta\mathbf{z}_{it}, w_{i,t-1}, w_{i,t-2})$ **as instruments for** $(\Delta\mathbf{z}_{it}, \Delta\mathbf{w}_{it})$

(Note that any of ($\mathbf{z}_{it}, \mathbf{w}_{it-1}, \ldots, \mathbf{w}_{i1}$) can be instruments)

<u>Estimation Procedure</u>

1. Remove unobservable effect by either FD or FE

$\Delta\log(wage_{it}) = \Delta\mathbf{z}_{it}\boldsymbol{\gamma} + \delta_1\Delta cigs_{it} + \Delta u_{it}$

2. 2SLS using **strictly exogenous** $\mathbf{z}_i$ **or （under the assumption that u is not serially correlated） 2 lagged w or y, or other exogenous variable as IV**

P2SLS or GMM estimate the model using cigarette price or cigarette tax (in the U.S., there is a variation across shops /states) as instruments for $\Delta cigs_{it}$

It is possible to use FE transformation to remove unobserved effect, but with FEtransformation, we cannot use lagged w as IV (FD is better)

11.1.3 Models with Contemporaneous Correlation between Some Explanatory Variables and the Idiosyncratic Error

$y_{it} = \mathbf{z}_{it}\boldsymbol{\gamma} + \mathbf{w}_{it}\boldsymbol{\delta} + c_i + u_{it}$, $t = 1,\cdots,T$

$\mathbf{z}_{it}$: strictly exogenous, $\mathbf{w}_{it}$: contemporaneously correlated with u_{it} (due to **omitted variables, measurement errors, simultaneity**)

Example 11.5 Effect of smoking on wage

$\log(wage_{it}) = \mathbf{z}_{it}\boldsymbol{\gamma} + \delta_1 cigs_{it} + c_i + u_{it}$, $cigs_{it}$:smokes per day

If cigarette is a normal good, high income (wage) leads to high consumption of cigarette (δ_1 is upward biased)

⇨ **Correlation between regressors and u (OLS is inconsistent, and IVs neede)**

<u>With measurement error (attenuation bias)</u>

$y_{it} = \beta x_{it}^* + c_i + u_{it}$, $E(u_{it} \mid \mathbf{x}_i^*, \mathbf{x}_i, c_i) = 0$, $t = 1,\cdots,T$,

x_{it}^*:true value, x_{it}:observed $x_{it} = x_{it}^* + r_{it}$

$$p\lim_{N\to\infty}\hat{\beta}_{POLS} = \beta + \frac{Cov(x_{it}, c_i) - \beta\sigma_r^2}{Var(x_{it})}, \quad \sigma_r^2 = Var(r_{it}) = Cov(x_{it}, r_{it})$$

$$p\lim_{N\to\infty}\hat{\beta}_{FD} = \beta + \left(1 - \frac{\sigma_r^2(1-\rho_r)}{\sigma_{x^*}^2(1-\rho_{x^*}) + \sigma_r^2(1-\rho_r)}\right), \quad \rho_{x^*} = Corr(x_{it}^*, x_{i,t-1}^*), \rho_r = Corr(r_{it}, r_{i,t-1})$$

=> do not know which generates smaller bias, **POLS or FD**

FD + IV for the consistent estimation

11.1.4 Summary of Models without Strictly Exogenous Explanatory Variables

In general

１） FD or FE transformation to remove unobserved effect

２） IV estimation

11.2 Models with Individual-Specific Slopes

11.2.1 A Random Trend Model

$y_{it} = c_i + g_i t + \mathbf{x}_{it}\boldsymbol{\beta} + u_{it}$, $t = 1,\cdots,T$, $E(u_{it} \mid \mathbf{x}_i, c_i, g_i) = 0$:strict exogeneity

<u>**Estimation Method**</u> **If taking FD, the model is just one with** unobserved effect model!

$\Delta y_{it} = g_i + \Delta\mathbf{x}_{it}\boldsymbol{\beta} + \Delta u_{it}$

FD or FE estimation（need Ｔ＞２）

Same estimation method can be used if the coefficients on explanatory variables varies across time

11.2.2 General Models with Individual-Specific Slopes

$y_{it} = \mathbf{z}_{it}\mathbf{a}_i + \mathbf{x}_{it}\boldsymbol{\beta} + u_{it}$, $t = 1,\cdots,T$, a_i: Jx1 (individual specific **slope**)

<u>**Assumption FE.1'**</u> **(strict exogeneity)**: $E(u_{it} \mid \mathbf{x}_i, \mathbf{z}_i, \mathbf{a}_i) = 0$

Stack across time and write with the vector $\mathbf{y}_i = \mathbf{Z}_i\mathbf{a}_i + \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i$,

$\mathbf{Z}_i = (\mathbf{z}_{i1},\cdots,\mathbf{z}_{iT})'$, $\mathbf{X}_i = (\mathbf{x}_{i1},\cdots,\mathbf{x}_{iT})'$ here $\mathbf{M}_i = \mathbf{I}_T - \mathbf{Z}_i(\mathbf{Z}_i'\mathbf{Z}_i)^{-1}\mathbf{Z}_i'$ and $\ddot{\mathbf{X}}_i = \mathbf{M}_i\mathbf{X}_i$, $\ddot{\mathbf{y}}_i = \mathbf{M}_i\mathbf{y}_i$, $\ddot{\mathbf{u}}_i = \mathbf{M}_i\mathbf{u}_i$, then $\ddot{\mathbf{y}}_i = \ddot{\mathbf{X}}_i\boldsymbol{\beta} + \ddot{\mathbf{u}}_i$

<u>**Assumption FE.2'**</u> **(rank condition)**: $rankE(\ddot{\mathbf{X}}_i'\ddot{\mathbf{X}}_i) = K$, where $\ddot{\mathbf{X}}_i = \mathbf{M}_i\mathbf{X}_i$,

Under AFE1' and AFE2', SOLS is consistent.

<u>**Assumption FE.3'**</u> **(homoskedasticity)**: $E(\mathbf{u}_i\mathbf{u}_i' \mid \mathbf{z}_i, \mathbf{x}_i, \mathbf{a}_i) = \sigma_u^2\mathbf{I}_T$

Asymptotic variance $A\hat{\text{var}}(\hat{\beta}_{FE}) = \hat{\sigma}_u^2\left(\sum_{i=1}^{N}\ddot{\mathbf{X}}_i'\ddot{\mathbf{X}}_i\right)^{-1}$

$\hat{\sigma}_u^2 = [N(T-J)-K]^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{u}_{it}^2 = SSR/[N(T-J)-K]$, $\hat{u}_{it} = \ddot{y}_{it} - \ddot{\mathbf{x}}_{it}\hat{\boldsymbol{\beta}}_{FE}$

F-test is valid. If AFE3' is not satisfied, use robust variance matrix

<u>**How to obtain $\mathbf{a} = E(\mathbf{a}_i)$**</u>

$\mathbf{a} = E[(\mathbf{Z}_i'\mathbf{Z}_i)^{-1}\mathbf{Z}_i'(\mathbf{y}_i - \mathbf{X}_i\boldsymbol{\beta})]$, $\hat{\mathbf{a}} = N^{-1}\sum_{i=1}^{N}(\mathbf{Z}_i'\mathbf{Z}_i)^{-1}\mathbf{Z}_i'(\mathbf{y}_i - \mathbf{X}_i\hat{\boldsymbol{\beta}}_{FE})$

Expanding this equation and using asymptotic normality of beta, we can calculate asymptotic variance of $\hat{\mathbf{a}}$

11.3 GMM Approaches to Linear Unobserved Effects Models

11.3.1 Equivalence between 3SLS and Standard Panel Data Estimation

11.3.2 Chamberlain's Approach to Unobserved Effects Models

11.4 Hausman and Taylor-Type Models

PS 9

I. Example 11.3

II. Example 11.6

III. 11.15

11.5 Applying Panel Data Methods to Matched Pairs and Cluster Samples

Matched pairs sample

Ex. sibling $y_{i1} = \mathbf{x}_{i1}\boldsymbol{\beta} + f_i + u_{i1}$, $y_{i2} = \mathbf{x}_{i2}\boldsymbol{\beta} + f_i + u_{i2}$

=> estimate by **RE, FE**

Cluster sample

Ex. Cluster by school $y_{is} = \mathbf{x}_{is}\boldsymbol{\beta} + c_i + u_{is}$

Even if the cluster size is not same, demean and FE estimation

Peer effects

Ex. School average traits $y_{is} = \mathbf{x}_{is}\boldsymbol{\beta} + \overline{\mathbf{w}}_{i(s)}\boldsymbol{\delta} + u_{is}$

$\overline{\mathbf{w}}_{i(s)}$: cluster average of $\mathbf{x}_{is}$ **except for s**

Under strict exogeneity, POLS is consistent