

<b>This paper is not to be removed from the Examination Halls</b>
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**UNIVERSITY OF LONDON**

**MT105A ZB**

**BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences**

**Mathematics 1**

Thursday, 5 May 2016 : 10:00 to 12:00

Candidates should answer all **EIGHT** questions: all **SIX** questions of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

Calculators may not be used for this paper.

**PLEASE TURN OVER**

## SECTION A

Answer all **six** questions from this section (60 marks in total).

1. A monopoly has fixed costs of 10 and marginal cost function  $6q^2 + 8$ . The demand equation for its product is  $p + 2q = 40$ .

Determine the profit function in terms of  $q$ .

Hence find the production level that maximises the profit.

2. Express the following system of equations in matrix form, and solve it **using row operations**.

$$\begin{aligned}4x - y - z &= 8, \\2x - 3y - z &= 8, \\2x - 5y + z &= 16.\end{aligned}$$

3. Use the method of Lagrange multipliers to find the positive values of  $x$  and  $y$  which maximise

$$\frac{2x}{x+2} + \frac{y}{y+1}$$

subject to the constraint  $x + y = 120$ .

4. Determine the following integrals:

$$\int \frac{x}{x^2 + 7x + 12} dx,$$

$$\int \frac{\ln(x+1)}{\sqrt{x+1}} dx.$$

5. Suppose that  $a$  is a positive number and that the function  $f$  is given by

$$f(x, y) = y^4 - 8a^2y^2 - x^2 + 1.$$

Find the critical points of  $f$ . For each critical point of  $f$ , determine whether it is a local minimum, local maximum, or a saddle point.

6. Suppose the function  $f$  is given by

$$f(x, y) = \cos(2y - x) + e^x \ln(y^2 + 1).$$

Show that

$$\frac{\partial^2 f}{\partial x \partial y} + 2 \frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial y} - 2 \frac{\partial f}{\partial x} = 0.$$

## SECTION B

Answer **both** questions from this section (20 marks each).

7. (a) A firm is the only producer of two goods,  $X$  and  $Y$ . The demand quantities  $x$  and  $y$  for  $X$  and  $Y$  (respectively) and the corresponding prices  $p_X$  and  $p_Y$  are related by the equations

$$\begin{aligned} -p_X + p_Y + 13 - 3x &= 0, \\ p_X - 4p_Y + 26 - 3y &= 0. \end{aligned}$$

The firm's joint total cost function (that is, the cost of producing  $x$  of  $X$  and  $y$  of  $Y$ ) is  $14x + 7y$ .

Find an expression in terms of  $x$  and  $y$  for the profit function.

Hence determine the quantities  $x$  and  $y$  that maximise the profit.

- (b) Suppose the function  $f$  is given by  $f(x) = e^x - x$ . Prove that, for  $x \geq 0$ ,  $f'(x) \geq 0$ . Hence show that, for all  $x \geq 0$ ,

$$e^x \geq 1 + x.$$

Now suppose the function  $g$  is given by

$$g(x) = e^x - x - \frac{x^2}{2}.$$

Show that  $g'(x) \geq 0$  for all  $x \geq 0$ . Hence show that, for all  $x \geq 0$ ,

$$e^x \geq 1 + x + \frac{x^2}{2}.$$

Now use a similar method (considering an appropriate function  $h$  and its derivative) to show that, for all  $x \geq 0$ ,

$$e^x \geq 1 + x + \frac{x^2}{2} + \frac{x^3}{6}.$$

8. (a) On the first day of 2016 there are 1000 on-line book retailers in a particular country. During each subsequent year, the number of new such retailers grows by 30, but by the end of the year 2% of all the on-line book retailers that were in business at the start of the year will have closed down. Find an expression, in terms of  $N$  (and in as simple a form as possible) for the number of on-line book retailers  $N$  years after the first of January 2016. What happens to the number of such retailers in the long run?

(b) Suppose the quantity  $y$  is defined as a function of  $x$  through the equation

$$x^2y^3 - 6x^3y^2 + 2xy = 1.$$

Find the value of  $y$  when  $x = 1/2$ . [You might find it useful to note that  $y^3 - 3y^2 + 4y - 4 = (y - 2)(y^2 - y + 2)$ .]

Find a general expression for the derivative  $y'(x) = \frac{dy}{dx}$ .

Hence determine the value of  $y'(1/2)$ .

END OF PAPER