This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON **MT105A ZA**

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences

Mathematics 1

Friday, 5 May 2017: 14:30 to 16:30

Candidates should answer all **EIGHT** questions: all **SIX** questions of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

Calculators may not be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **six** questions from this section (60 marks in total).

1. The demand equation for a good is $q(p+2) = 6$ and the supply equation is $q - p + 3 = 0$, where p is the price and q is the quantity demanded.

Determine the equilibrium price and quantity.

2. Determine the following integrals:

$$\int x^2 e^x \, \mathrm{d}x,$$

$$\int x^2 \sqrt{x-2} \, \mathrm{d}x.$$

3. The function f is defined, for $x > 0$, by

$$f(x) = a \ln x - b(\ln x)^2,$$

where a and b are positive numbers. Show that f has one critical point. Determine whether it is a local maximum, a local minimum or a point of inflexion.

4. Use the Lagrange multiplier method to find the values of x, y which maximise $x^2\sqrt{y}$ subject to the constraint $x + y = 100$.

5. Find the critical points of the function

$$f(x, y) = y^3 - x^3 - 2xy + 5.$$

For each critical point of f, determine whether it is a local minimum, local maximum, or a saddle point.

6. A sequence of numbers $x_0, x_1, x_2, \ldots$ is such that $x_0 = 3$ and, for each $n \geq 1$,

$$x_n = \frac{1}{2}x_{n-1} + 2.$$

Show that x_n can be written in the following form, for some numbers a and r (which you should find):

$$x_n = ar^n + 2\left(1 + r + r^2 + \cdots + r^{n-1}\right).$$

Hence, or otherwise, find an explicit expression for x_n in terms of n, simplifying your answer as far as possible. Hence describe the behaviour of x_n as $n \to \infty$.

SECTION B

Answer **both** questions from this section (20 marks each).

7. **(a)** The function f is given, for $x > 0$, by

$$f(x) = \frac{a}{x} + bx + cx^2$$

for some constants a, b and c. The following facts about f and its derivative f' are known:

$$f(1) = 4, \ f(2) = -3, \ f'(1) = -9.$$

Show that a, b and c satisfy the following equations:

$$\begin{aligned} a + b + c &= 4 \\ a + 4b + 8c &= -6 \\ a - b - 2c &= 9. \end{aligned}$$

By solving this system of equations **using row operations**, find the values of a, b and c.

(b) Determine the integral $\displaystyle\int \frac{1}{1 + e^{2x}}\, \mathrm{d}x$.

8. **(a)** The function f is given, for $x, y > 0$, by

$$f(x,y) = y^3 \ln\left(1 + \frac{y}{x}\right).$$

Find the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ and verify that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f(x,y).$$

(b) Consider the problem of maximising the function

$$q(k,l) = \left(\frac{1}{k} + \frac{2}{l}\right)^{-1}$$

subject to the constraint $vk + wl = M$, where v, w, M are positive constants. By using the Lagrange multiplier method, show that the optimal values of k and l satisfy

$$l = \left(\frac{2v}{w}\right)^{1/2} k.$$

Suppose now that $v = 2w$. Find the optimal values of k and l in terms of M and v. Then find the maximum value of $q(k,l)$, also in terms of M and v.

END OF PAPER