

<b>This paper is not to be removed from the Examination Hall</b>
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**UNIVERSITY OF LONDON**

**MT105A ZB**

**BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences**

**Mathematics 1**

Friday, 4 May 2018: 14:30 to 16:30

Candidates should answer all **EIGHT** questions: all **SIX** questions of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

Calculators may not be used for this paper.

PLEASE TURN OVER

## SECTION A

Answer all **six** questions from this section (60 marks in total).

1. Find the points of intersection of the curves

$$y = 1 - 3x \quad \text{and} \quad y = 3 - 2x - x^2.$$

Sketch these curves on the same set of axes.

2. A monopoly has a marginal cost function given by

$$MC(q) = e^q - 4q,$$

and fixed costs of 12. Find its total cost function.

The demand for its product is given by the equation

$$p + 2q = 14.$$

Find the profit function for this monopoly as a function of  $q$ .

What value of  $q$  will maximise this profit?

3. Use row operations to solve the system of equations

$$\begin{aligned}x + 3y + z &= 4, \\4x + 2y - z &= 1, \\2x - 4y + 3z &= 23.\end{aligned}$$

4. Find the integrals

$$(i) \quad \int (x+2)^2 e^x \, dx \quad \text{and} \quad (ii) \quad \int \frac{e^x \, dx}{(3 - e^x)(3 + e^x)}.$$

5. Use the method of Lagrange multipliers to find the values of  $x$  and  $y$  that minimise  $3x + 2y$  subject to the constraint  $x^{1/3}y^{1/2} = 48$  with  $x, y > 0$ .

6. At the beginning of 2018, Martin takes out a loan of \$1,000 at an interest rate of 2% per month. He has to make monthly repayments of \$ $D$  at the beginning of each subsequent month for the next 25 months.

Explain why he owes  $1000(1.02) - D$  after his first repayment.

How much does he owe after  $n$  repayments? (Simplify your answer as far as possible.)

Given that  $1.02^{25}$  is approximately 1.64, show that  $D$  is approximately 51.25.

## SECTION B

Answer **both** questions from this section (20 marks each).

7. (a) A company is the only producer of two goods, X and Y. When the prices for X and Y are  $p_X$  and  $p_Y$  per unit (respectively), consumers will purchase quantities  $x$  and  $y$  (respectively) according to the demand equations

$$p_X + p_Y = 54 - 4x - 4y \quad \text{and} \quad p_X - p_Y = 22 - 2x + 2y.$$

If the company's joint total cost function (that is, the cost of producing quantities  $x$  of X and  $y$  of Y) is

$$TC(x, y) = 10 + 2x^2 - 3xy + 2y^2,$$

find their profit function,  $\pi(x, y)$ .

Hence determine the quantities  $x$  and  $y$  that maximise this profit.

- (b) Show that for all values of the number  $k$ , the function

$$f(x, y) = x^2 + kxy + y^2,$$

has a critical (or stationary) point at  $(0, 0)$ .

For each value of  $k$ , determine whether this critical point is a local minimum, a local maximum or a saddle point.

8. A firm has a production function given by

$$q(k, l) = k^{1/4}l^{1/12}$$

where  $k$  and  $l$  are the amounts of capital and labour used.

- (a) If the prices of capital and labour are  $u$  and  $v$  per unit respectively, use the method of Lagrange multipliers to find the values of  $k$  and  $l$  that will minimise the firm's costs when they produce a quantity  $Q$ .

The firm sells its product at a fixed price of  $w$  per unit.

- (b) What is the firm's profit function?  
(c) Determine the firm's breakeven quantity.  
(d) Find the quantity that maximises the firm's profit.

END OF PAPER