This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON **MT105A ZA**

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences

Mathematics 1

Thursday, 5 May 2016 : 10:00 to 12:00

Candidates should answer all **EIGHT** questions: all **SIX** questions of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

Calculators may not be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **six** questions from this section (60 marks in total).

1. A monopoly has fixed costs of 20 and marginal cost function $3q^2 + 4$. The demand equation for its product is $p + q = 20$.

Determine the profit function in terms of q.

Hence find the production level that maximises the profit.

2. Express the following system of equations in matrix form, and solve it **using row operations**.

$$\begin{aligned} 4x - y - z &= 4, \\ 2x - 3y - z &= 4, \\ 2x - 5y + z &= 8. \end{aligned}$$

3. Use the method of Lagrange multipliers to find the positive values of x and y which maximise

$$\frac{2y}{y+2} + \frac{x}{x+1}$$

subject to the constraint $x + y = 120$.

4. Determine the following integrals:

$$\int \frac{x+1}{x^2+7x+10}\,\mathrm{d}x,$$

$$\int \frac{\ln(2x+1)}{\sqrt{2x+1}}\,\mathrm{d}x.$$

5. Suppose that a is a positive number and that the function f is given by

$$f(x,y) = x^4 - 2a^2x^2 - y^2 + 1.$$

Find the critical points of f. For each critical point of f, determine whether it is a local minimum, local maximum, or a saddle point.

6. Suppose the function f is given by

$$f(x,y) = \sin(2x-y) + e^y \ln(x^2+1).$$

Show that

$$\frac{\partial^2 f}{\partial x \partial y} + 2\frac{\partial^2 f}{\partial y^2} - \frac{\partial f}{\partial x} - 2\frac{\partial f}{\partial y} = 0.$$

SECTION B

Answer **both** questions from this section (20 marks each).

7. (a) A firm is the only producer of two goods, X and Y. The demand equations for X and Y are given by

$$\begin{aligned} x &= 600 - p_X - p_Y, \\ y &= 450 - \frac{1}{2}p_X - p_Y, \end{aligned}$$

where x and y are the quantities of X and Y demanded (respectively) and p_X, p_Y are (respectively) the prices of X and Y. The firm's joint total cost function (that is, the cost of producing x of X and y of Y) is

$$x^2 + 3xy + y^2 + 100.$$

Find an expression in terms of x and y for the profit function.

Hence determine the quantities x and y that maximise the profit, and find the corresponding prices p_X, p_Y.

(b) Suppose the function f is given by $f(x) = \mathrm{e}^x - x$. Prove that, for $x \geq 0$, $f'(x) \geq 0$. Hence show that, for all $x \geq 0$,

$$\mathrm{e}^x \geq 1 + x.$$

Now suppose the function g is given by

$$g(x) = \mathrm{e}^x - x - \frac{x^2}{2}.$$

Show that $g'(x) \geq 0$ for all $x \geq 0$. Hence show that, for all $x \geq 0$,

$$\mathrm{e}^x \geq 1 + x + \frac{x^2}{2}.$$

Now use a similar method (considering an appropriate function h and its derivative) to show that, for all $x \geq 0$,

$$\mathrm{e}^x \geq 1 + x + \frac{x^2}{2} + \frac{x^3}{6}.$$

8. (a) The number of fish in a lake is 10000 at the start of 2016. Each year, 5% of the fish who were alive at the start of the year die and 1000 new fish are born. Find an expression, in as simple a form as possible, for the number of fish in the lake N years after the start of 2016. Describe what happens to the number of fish in the lake in the long run.

(b) Suppose the quantity y is defined as a function of x through the equation

$$x^2y^3 - 6x^3y^2 + 2xy = 1.$$

Find the value of y when $x = 1/2$. [You might find it useful to note that $y^3 - 3y^2 + 4y - 4 = (y-2)(y^2 - y + 2)$.]

Find a general expression for the derivative $y'(x) = \dfrac{dy}{dx}$.

Hence determine the value of $y'(1/2)$.

END OF PAPER