SIM GLOBAL EDUCATION

SINGAPORE INSTITUTE OF MANAGEMENT

PRELIMINARY EXAM 2014

PROGRAMME(S)	:	University of London Degree and Diploma Programmes (Lead College: London School of Economics & Political Science)
SUBJECT	:	**MT105A MATHEMATICS 1 (HALF UNIT)**
DATE	:	Friday, 28 February 2014
DURATION	:	2 Hours

INSTRUCTIONS :-

DO NOT TURN OVER THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Candidates should answer all **EIGHT** questions: all **SIX** questions of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Calculators may **not** be used for this paper.

Total number of pages: 3 (including this page)

SECTION A

Answer all **six** questions from this section (60 marks in total).

1. Functions f and g are given by

$$f(x) = x^4 + 2x^3 + 2x^2 + 2 \quad \text{and} \quad g(x) = -x^4 + 2x^3 + 18x^2 + 20.$$

Show that the curves $y = f(x)$ and $y = g(x)$ intersect for exactly two values of x. Find these values of x. (Do not attempt to sketch the curves.)

2. Write the equations

$$2x - 3y - z = 0, \quad 2x + 3y + z = 8 \quad \text{and} \quad x - 2y + 3z = 3$$

in matrix form. Hence, **use a matrix method** to find the numbers x, y and z that satisfy them.

3. The function f is given, for $x > 0$, by

$$f(x) = x^x.$$

Show that f has one critical (or stationary) point and determine its nature.

4. Find the critical (or stationary) points of the function

$$f(x, y) = 10 - x^2 + 2y^2 - 2y^4 + 2xy^2.$$

Determine whether each critical point is a local maximum, local minimum or saddle point.

5. Determine the following integrals.

(a) $\displaystyle\int \frac{\ln(x+1)}{x^2}\, \mathrm{d}x.$

(b) $\displaystyle\int \frac{\cos x}{(1 - \sin x)(2 + \sin x)}\, \mathrm{d}x.$

(c) $\displaystyle\int e^{\sqrt{x}}\, \mathrm{d}x.$

6. Find the first term and common ratio of the geometric series that has a second term of 5 and a sum to infinity of 20.

SECTION B

Answer **both** questions from this section (20 marks each).

7. (a) A travel company is the only provider of holidays (of one week's duration) to two private island resorts, X and Y. The demand equations for such holidays are given by

$$x = 200 - p_X \quad \text{and} \quad y = 100 - p_Y,$$

where x and y are the numbers of week-long holidays at X and Y demanded (respectively) and p_X and p_Y are (respectively) the prices of these holidays. The company's joint total cost function (that is, the cost of providing x holidays in X and y holidays in Y) is $2x^2 + y^2 + 2xy$.

Find an expression in terms of x and y for the profit the company obtains from selling these holidays. Determine the numbers x and y of holidays in resorts X and Y that will maximise the company's profit.

(b) James opens a savings account with a bank. The interest rate is guaranteed to be fixed at a rate of 5% per annum for K years after he opens the account. (Here, K is an integer greater than 1.) He opens the savings account with a payment of \$200 on 1 January 2014, and makes further deposits of \$200 yearly, on 1 January each year, starting 1 January 2015. Find an expression (involving K, and in as simple a form as possible) for the value of his savings K years after he opens the account (that is, K years from 1 January 2014, just after he has made a new deposit of \$200).

Suppose now that he continues to make the same deposits after this time, but that the interest rate falls to the fixed rate of 4% for the next N years (where N is an integer greater than 1). Find an expression (involving K and N, and in as simple a form as possible) for the value of his savings $K + N$ years after he opened the account (that is, $K + N$ years from 1 January 2014, just after he has made a new deposit of \$200).

8. A consumer has a utility function

$$u(x, y) = \left((x+1)^3 + y^3\right)^{1/3}$$

when consuming amounts x and y of two goods, X and Y, respectively. Each unit of X costs the consumer p dollars and each unit of Y costs him q dollars. If the consumer has a budget for X and Y of M dollars, use the Lagrange multiplier method to determine the quantities, x^* and y^*, of these two goods that he should consume if he wants to maximise his utility subject to his budget constraint. (You should simplify your answers as far as possible.)

Suppose that, for given values of p, q and M, the consumer's maximum utility subject to his budget constraint is $V(p, q, M)$. Find, in as simple a form as possible, an expression for V.

Use your values of x^* and y^* to determine, in as simple a form as possible, the corresponding value of the Lagrange multiplier, λ^*.

Hence verify that $\dfrac{\partial V}{\partial M} = \lambda^*$.

 END OF PAPER