Mathematics 1 mock examination paper (2016)

UNIVERSITY OF LONDON   **MT105a Mock**

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diplomas in Economics and Social Sciences and Access Route

Mathematics 1

Time allowed : 2 hours

Candidates should answer all **EIGHT** questions: all **SIX** of Section A (60 marks in total) and **BOTH** questions from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Calculators may not be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **six** questions from this section (60 marks in total).

1. A firm is the only producer of a particular good. The firm's marginal revenue function is $MR = 9 - q$, where q denotes the quantity of the good produced by the firm. The firm's fixed costs are 12 and its average variable cost function is $1 + q/2$. Find an expression for the firm's profit function, $\Pi(q)$. Find the value of the production, q, which maximises the firm's profit, and hence calculate the firm's maximum profit.

2. Using the method of row operations, solve the following system of linear equations to find x, y and z.

$$3x + y + 3z + 22 = x + 2y - 2z + 150,$$

$$x + y + 73 = 2x - 2y + 2z + 44,$$

$$x - 2y + z + 102 = x - y + 130.$$

3. The function f is defined for positive y and all x by

$$f(x, y) = x^2 \ln y - y \ln y.$$

Find the critical (or stationary) points of f and determine whether each critical point is a local maximum, local minimum or saddle point.

4. Two functions $W(x, y)$ and $U(x, y)$ are connected by the equation

$$W(x, y) = e^{x-4y}U(x, y).$$

Find the partial derivatives

$$\frac{\partial W}{\partial x}, \quad \frac{\partial W}{\partial y} \quad \text{and} \quad \frac{\partial^2 W}{\partial x^2},$$

in terms of U and its partial derivatives.
If W satisfies

$$\frac{\partial W}{\partial y} = \frac{\partial^2 W}{\partial x^2} - 2\frac{\partial W}{\partial x} - 3W,$$

show that the function U then satisfies the equation

$$\frac{\partial U}{\partial y} = \frac{\partial^2 U}{\partial x^2}.$$

5. (a) Determine the integral $\displaystyle\int \frac{x^{-1}dx}{(1+\ln x)\ln x}$.

   (b) An arithmetic progression is such that its second term is 7 and its thirteenth term is ten times its first term. Determine the first term and the common difference.

6. An investor saves money in a bank account paying interest at a fixed rate of $100r\%$, where the interest is paid once per year, at the end of the year. She deposits an amount D at the beginning of each of the next N years. Show that she will then have saved an amount equal to

$$\frac{D}{r}\Big((1+r)^N - 1\Big)$$

just after the last of these deposits.

SECTION B

Answer **both** questions from this section (20 marks each).

7. (a) Use the Lagrange multiplier method to find the minimum value of

$$\left(\frac{1}{x^2} + \frac{1}{y^2}\right)^{1/2}$$

   among all positive x, y satisfying $x + y = \sqrt{2}$.

   (b) A firm's marginal cost function is given by

$$MC = qe^q + \frac{q}{q^2 + 4q + 3},$$

   where q denotes the quantity produced. If its production is increased from $q = 1$ to $q = 2$, use an integral to find the increase in total cost.

8. A firm has a weekly production function

$$q(k,l) = k^A l^A,$$

where A satisfies $0 < A < 1/2$. Here, k denotes the amount of capital and l the amount of labour used. Each unit of capital costs V and each unit of labour costs W. Show that the minimum amount that the firm can spend in a week on capital and labour if it manufactures a total amount q is

$$2\sqrt{VW}q^{\frac{1}{2A}}.$$

The product manufactured by the firm sells at a fixed price P. The raw materials required to produce each unit cost an amount C, where $C < P$. Suppose the firm acts so as to minimise its capital and labour costs. Using the result just obtained, determine the production level, q, that will maximise the firm's weekly profit.