20181018f Integration by substitution

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### Integration by substitution

Find Int (3*x+5)^12 dx:*  
*Let u = 3*x+5;  
du / dx = 3 => dx = du / 3;  
then it became Int u^12 / 3 du => 1/(13*3)*  u^13 = > 1/(13*3)*  (3\*x+5)^13

### Verified by Ryacas

library(Ryacas)  
library(mosaic)

# LHS  
x = Sym('x')  
LHS = Integrate((3\*x+5)^12,x)  
LHS

## expression(531441 \* x^13/13 + 885735 \* x^12 + 8857350 \* x^11 +   
## 54128250 \* x^10 + 225534375 \* x^9 + 676603125 \* x^8 + 1503562500 \*   
## x^7 + 2505937500 \* x^6 + 3132421875 \* x^5 + 2900390625 \*   
## x^4 + 1933593750 \* x^3 + 878906250 \* x^2 + 244140625 \* x)

# RHS  
u = 3\*x+5  
deriv(u,x)

## expression(3)

dx = Sym('dx'); du = Sym('du')  
Solve(du/dx==3,dx)

## expression(list(dx == du/3))

u = Sym('u')  
Integrate(u^12,u) / 3

## expression(u^13/39)

intfdu = makeFun(u^13/39~u)  
intfdx = intfdu(u=3\*x+5)  
Expand(intfdx)

## expression(531441 \* x^13/13 + 885735 \* x^12 + 8857350 \* x^11 +   
## 54128250 \* x^10 + 225534375 \* x^9 + 676603125 \* x^8 + 1503562500 \*   
## x^7 + 2505937500 \* x^6 + 3132421875 \* x^5 + 2900390625 \*   
## x^4 + 1933593750 \* x^3 + 878906250 \* x^2 + 244140625 \* x +   
## 1220703125/39)

### More difficult examples

Find Int x^3 \* (2\*x^2 + 7)^8 dx:

# LHS  
x = Sym('x')  
LHS = Integrate(x^3 \* (2\*x^2 + 7)^8, x)  
LHS

## expression(64 \* x^20/5 + 3584 \* x^18/9 + 5488 \* x^16 + 43904 \*   
## x^14 + 672280 \* x^12/3 + 3764768 \* x^10/5 + 1647086 \* x^8 +   
## 6588344 \* x^6/3 + 5764801 \* x^4/4)

# RHS  
# Let u = 2\*x^2 + 7  
u = 2\*x^2 + 7  
deriv(u,x)

## expression(4 \* x)

# So that dx = du / (4\*x) =>   
# Int x^2 / 4 \* u^8 du  
# Consider u = 2\*x^2 + 7 => x^2 = (u-7)/2  
# Therefore our integral became (u-7)/8 \* u^8  
u = Sym('u')  
Integrate((u-7)/8 \* u^8, u)

## expression(u^10/80 + -7 \* u^9/72)

intfdu = makeFun(u^10/80 + -7 \* u^9/72 ~ u)  
intfdx = intfdu(u=2\*x^2 + 7)  
Expand(intfdx)

## expression(64 \* x^20/5 + 3584 \* x^18/9 + 5488 \* x^16 + 43904 \*   
## x^14 + 672280 \* x^12/3 + 3764768 \* x^10/5 + 1647086 \* x^8 +   
## 6588344 \* x^6/3 + 5764801 \* x^4/4 - 282475249/720)