20181019a Integration by parts

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### Principles

From differentiation, we can get d/dx f(x)g(x) == f’(x)g(x) + g’(x)f(x)  
Therefore, Int f’(x)g(x) dx + Int g’(x)f(x) dx == f(x)g(x)  
=> Int f’(x)g(x) dx == f(x)g(x) - Int g’(x)f(x) dx

### Example

Find Int x ln(x) dx

library(Ryacas)  
library(mosaic)

# LHS  
x = Sym('x')  
LHS = Integrate(x\*logb(x),x)  
LHS

## expression(log(x) \* x^2/2 - x^2/4)

# RHS  
# Int x\*ln(x)dx; f'(x)=x => f(x) = 1/2\*x^2; g(x)=ln(x) => g'(x)=1/x  
# Therefore Int x\*ln(x) dx can be expressed by:  
# 1/2\*x^2 \* logb(x) - Int 1/x \* 1/2\*x^2 dx  
# => 1/2\*x^2 \* logb(x) - Int x/2 dx  
# => x^2/2 \* logb(x) - x^2/4

### Finding Int ln(x) using integration by parts

LHS = Integrate(logb(x),x)  
LHS

## expression(x \* log(x) - x)

# RHS  
# Consider logb(x) = 1\*logb(x)  
# Therefore f(x) = x; g(x) = logb(x); g'(x) = 1/x  
# Int logb(x) dx == f(x)g(x) - Int f(x)g'(x) dx  
# => x \* logb(x) - Int x (1/x) dx  
# => x \* logb(x) - x