20181019b partial fractions

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### Partial fractions demonstrated by examples

This is done when the denominator could be factorized

Find Int x / (x^2 - x - 2) dx

(Actually the nominator can be many other things other than x)

library(Ryacas)  
library(mosaic)

# LHS  
x = Sym('x')  
LHS = Integrate(x / (x^2 - x - 2), x)  
LHS

## expression((2 \* log(3 \* (x - 2)) + log(3 \* (x + 1)))/3)

# RHS  
# Let f(x) == x and g(x) == (x^2 - x - 2)  
# Factorize: x^2 - x - 2 == (x+1)\*(x-2)  
# It then becomes x / (x+1)\*(x-2)  
# Consider: A1 / (x+1) + A2 / (x-2) == f(x) / g(x);  
# f(x) == A1(x-2) + A2(x+1)  
# In this case, f(x) == x => x == A1(x-2) + A2(x+1)  
# The cover-up rule: let x = 2 and x = -1  
# For x = 2, 2 == A2\*3 => A2 = 2/3  
# For x = -1, -1 == A1\*-3 => A1 = 1/3

# Consider: A1 / (x+1) + A2 / (x-2) == f(x) / g(x)  
# f(x) / g(x) == 1/(3\*(x+1)) + 2/(3\*(x-2))  
Integrate(1/(3\*(x+1)) + 2/(3\*(x-2)),x)

## expression((2 \* log(3 \* (x - 2)) + log(3 \* (x + 1)))/3)

### Summary of the Prinicple

For Int f(x) / g(x) dx;  
While g(x) factorizable into (x-a1)*(x-a2)*  
*f(x) / g(x) == A1/(x-a1)*  A2/(x-a2)  
And f(x) == A1(x-a2) + A2(x-a1)  
A1 and A2 can thus be found by the ‘cover-up’ rule  
And the A1/(x-a1) \* A2/(x-a2) can thus be integrated by substitution