20181019e implicit partial differentiation

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### Principles

For g(x,y) == constant; theorectically we can express y in terms of x, i.e. finding dy/dx  
However, most of the time it is difficult to do so  
In this case, we need to do this implicit partial differentiation:

dy/dx = - dgdx / dgdy

library(Ryacas)  
library(mosaic)

##### Defined: x2*y^3 - 6*x3*y^2 + 2*x\*y == 1

# LHS  
x = Sym('x')  
y = Sym('y')  
g\_LHS = Solve(x^2\*y^3 - 6\*x^3\*y^2 + 2\*x\*y == 1,y)  
g\_LHS

## expression(list(y == ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \*   
## x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6) + root(((3 \* (x^2 \*   
## (2 \* x)) - (6 \* x^3)^2)/(9 \* x^4))^3 + ((-9 \* (x^2 \* (6 \*   
## x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6))^2,   
## 2))^(1/3) + ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 -   
## 2 \* (-(6 \* x^3))^3)/(54 \* x^6) - root(((3 \* (x^2 \* (2 \* x)) -   
## (6 \* x^3)^2)/(9 \* x^4))^3 + ((-9 \* (x^2 \* (6 \* x^3) \* (2 \*   
## x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6))^2, 2))^(1/3) +   
## 2 \* x^3/x^2, y == complex\_cartesian(2 \* x^3/x^2 - (((-9 \*   
## (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \*   
## x^6) + root(((3 \* (x^2 \* (2 \* x)) - (6 \* x^3)^2)/(9 \* x^4))^3 +   
## ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \*   
## x^3))^3)/(54 \* x^6))^2, 2))^(1/3) + ((-9 \* (x^2 \* (6 \*   
## x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6) -   
## root(((3 \* (x^2 \* (2 \* x)) - (6 \* x^3)^2)/(9 \* x^4))^3 +   
## ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \*   
## (-(6 \* x^3))^3)/(54 \* x^6))^2, 2))^(1/3))/2, root(3/4,   
## 2) \* (((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \*   
## (-(6 \* x^3))^3)/(54 \* x^6) + root(((3 \* (x^2 \* (2 \* x)) -   
## (6 \* x^3)^2)/(9 \* x^4))^3 + ((-9 \* (x^2 \* (6 \* x^3) \* (2 \*   
## x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6))^2, 2))^(1/3) -   
## ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \*   
## x^3))^3)/(54 \* x^6) - root(((3 \* (x^2 \* (2 \* x)) - (6 \*   
## x^3)^2)/(9 \* x^4))^3 + ((-9 \* (x^2 \* (6 \* x^3) \* (2 \*   
## x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6))^2,   
## 2))^(1/3))), y == complex\_cartesian(2 \* x^3/x^2 - (((-9 \*   
## (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \*   
## x^6) + root(((3 \* (x^2 \* (2 \* x)) - (6 \* x^3)^2)/(9 \* x^4))^3 +   
## ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \*   
## x^3))^3)/(54 \* x^6))^2, 2))^(1/3) + ((-9 \* (x^2 \* (6 \*   
## x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6) -   
## root(((3 \* (x^2 \* (2 \* x)) - (6 \* x^3)^2)/(9 \* x^4))^3 +   
## ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \*   
## (-(6 \* x^3))^3)/(54 \* x^6))^2, 2))^(1/3))/2, -(root(3/4,   
## 2) \* (((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \*   
## (-(6 \* x^3))^3)/(54 \* x^6) + root(((3 \* (x^2 \* (2 \* x)) -   
## (6 \* x^3)^2)/(9 \* x^4))^3 + ((-9 \* (x^2 \* (6 \* x^3) \* (2 \*   
## x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6))^2, 2))^(1/3) -   
## ((-9 \* (x^2 \* (6 \* x^3) \* (2 \* x)) - -27 \* x^4 - 2 \* (-(6 \*   
## x^3))^3)/(54 \* x^6) - root(((3 \* (x^2 \* (2 \* x)) - (6 \*   
## x^3)^2)/(9 \* x^4))^3 + ((-9 \* (x^2 \* (6 \* x^3) \* (2 \*   
## x)) - -27 \* x^4 - 2 \* (-(6 \* x^3))^3)/(54 \* x^6))^2,   
## 2))^(1/3))))))

# OMG

# RHS  
g = x^2\*y^3 - 6\*x^3\*y^2 + 2\*x\*y - 1  
dgdx = deriv(g,x)  
dgdy = deriv(g,y)  
RHS = - dgdx / dgdy  
PrettyForm(RHS)

##   
## / 3 2 2 \  
## -\ 2 \* x \* y - 18 \* x \* y + 2 \* y /  
## --------------------------------------  
## 2 2 3   
## x \* 3 \* y - 6 \* x \* 2 \* y + 2 \* x

### Extension of the theory

Suppose g(x,y,z) == c  
dz/dx = -dg/dx / dg/dz;  
dz/dy = -dg/dy / dg/dz