20181019f optimization in partial differentiation

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### Principles

Consider F = f(x,y)  
For critical points, dF/dx == 0 and dF/dy ==0

For the properties of critical points:  
Maximum: Fxx*Fyy - Fxy^2 > 0 & Fxx < 0*  
*Minimum: Fxx*Fyy - Fxy^2 > 0 & Fxx > 0  
Saddle: Fxx\*Fyy - Fxy^2 < 0

library(Ryacas)  
library(mosaic)  
library(manipulate)

##### Consider F = f(x,y) = x^4 + 2*x2y + 2y2 + 2*y

x = Sym('x')  
y = Sym('y')  
F = x^4 + 2\*x^2\*y + 2\*y^2 + 2\*y  
dFx = deriv(F,x)  
dFy = deriv(F,y)  
dFx; dFy

## expression(4 \* x^3 + 4 \* (x \* y))

## expression(2 \* x^2 + 4 \* y + 2)

# Need to express y in terms of x by using:  
# dFx-dFy == 0; dFx == 0; dFy == 0  
# (We need 3 equations to solve 2 var)  
Solve(Simplify(dFx-dFy)==0,y)

## expression(list(y == -((2 \* x^3 - x^2 - 1)/(2 \* x - 2))))

Solve(dFy==0,y)

## expression(list(y == -((2 \* x^2 + 2)/4)))

Solve(dFx==0,y)

## expression(list(y == -(x^3/x)))

# Then need to solve x using the 3 equations  
Solve(-((2 \* x^2 + 2)/4)==-(x^3/x),x)

## expression(list(x == 1, x == -1))

Solve(-((2 \* x^3 - x^2 - 1)/(2 \* x - 2))==-((2 \* x^2 + 2)/4),x)

## expression(list(x == 0, x == -1))

Solve(-((2 \* x^3 - x^2 - 1)/(2 \* x - 2))==-(x^3/x),x)

## expression(list(x == -1))

# solve y using substitution  
Eval(-((2 \* x^2 + 2)/4),list(x=c(0,1,-1)))

## [1] -0.5 -1.0 -1.0

So that the critical points would be: (0,-0.5), (1,-1), (-1,-1)

ddFxx = deriv(dFx,x)  
ddFyy = deriv(dFy,y)  
ddFxy = deriv(dFx,y)  
Eval(ddFxx\*ddFyy - ddFxy^2, list(x=c(0,1,-1),y=c(-0.5,-1,-1)))

## [1] -8 16 16

Eval(ddFxx, list(x=c(0,1,-1),y=c(-0.5,-1,-1)))

## [1] -2 8 8

(0,-0.5) being a saddle point  
(1,-1) and (-1,-1) being minima

plotFun(x^4 + 2\*x^2\*y + 2\*y^2 + 2\*y ~ x & y, surface = T,  
 xlim = c(-3,3), ylim = c(-3,3))

