20181019i Constrained optimization

cheungngo

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### The Lagrange multiplier

Suppose f(x,y) is to be optimized subject to a constraint g(x,y) == 0  
Then we we have to find the critical points in  
L (x,y,lambda) == f(x,y) - lambda\*g(x,y)  
L is known as the Lagrangean; lambda is known as the Lagrange multiplier

### Example

library(Ryacas)  
library(mosaic)  
library(manipulate)

f(x) == x^2 + y^2; subject to the constraint x + y == 1;  
i.e. L == x^2 + y^2 - lambda\*(x+y-1)

x = Sym('x')  
y = Sym('y')  
lambda = Sym('lambda')  
L = x^2 + y^2 - lambda\*(x+y-1)  
dLx = deriv(L,x); dLx

## expression(2 \* x - lambda)

dLy = deriv(L,y); dLy

## expression(2 \* y - lambda)

dLlambda = deriv(L,lambda); dLlambda

## expression(1 - (x + y))

Solve(List(dLx==0,dLy==0,dLlambda==0),List(x,y,lambda))

## expression(list(list(x == 1/2, y == 1/2, lambda == 1)))