20181019n systems of linear equations

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library(Ryacas)

### Solving systems of linear equations

x1 + x2 + x3 == 6  
2x1 + 4x2 + x3 == 5  
2x1 + 3x2 + x3 == 6

The system can be re-written in matrix form: AX = B

A = matrix(c(1,2,2,1,4,3,1,1,1),ncol = 3); A

## [,1] [,2] [,3]  
## [1,] 1 1 1  
## [2,] 2 4 1  
## [3,] 2 3 1

x1 = Sym('x1'); x2 = Sym('x2'); x3 = Sym('x3')  
X = matrix(c(x1,x2,x3),ncol = 1); X

## [,1]  
## [1,] "x1"  
## [2,] "x2"  
## [3,] "x3"

B = matrix(c(6,5,6),ncol = 1); B

## [,1]  
## [1,] 6  
## [2,] 5  
## [3,] 6

To solve X, we can have A-1B

solve(A)%\*%B

## [,1]  
## [1,] 2  
## [2,] -1  
## [3,] 5

x1 = 2; x2 = -1; x3 = 5

### Elementary row operations

Without computer, we can also try to solve such linear equations by hand

Consider again:  
x1 + x2 + x3 == 6  
2x1 + 4x2 + x3 == 5  
2x1 + 3x2 + x3 == 6

Aug = matrix(c(1,2,2,1,4,3,1,1,1,6,5,6),ncol = 4); Aug

## [,1] [,2] [,3] [,4]  
## [1,] 1 1 1 6  
## [2,] 2 4 1 5  
## [3,] 2 3 1 6

We aim at creating a echelon form, i.e. 1 on the diag and 0 on the lower tri

# To create 0 in the second row:  
# R2 = R2 - 2\*R1  
Aug[2,] = Aug[2,]-2\*Aug[1,]; Aug

## [,1] [,2] [,3] [,4]  
## [1,] 1 1 1 6  
## [2,] 0 2 -1 -7  
## [3,] 2 3 1 6

# To create 1 in the second row:  
# R2 = R2 / 2  
Aug[2,] = Aug[2,] / 2; Aug

## [,1] [,2] [,3] [,4]  
## [1,] 1 1 1.0 6.0  
## [2,] 0 1 -0.5 -3.5  
## [3,] 2 3 1.0 6.0

# To create 0 in the third row:  
# R3 = R3 - 2\*R1  
Aug[3,] = Aug[3,] - 2\*Aug[1,]; Aug

## [,1] [,2] [,3] [,4]  
## [1,] 1 1 1.0 6.0  
## [2,] 0 1 -0.5 -3.5  
## [3,] 0 1 -1.0 -6.0

# To create another 0 in the third row:  
# R3 = R3 - R2  
Aug[3,] = Aug[3,] - Aug[2,]; Aug

## [,1] [,2] [,3] [,4]  
## [1,] 1 1 1.0 6.0  
## [2,] 0 1 -0.5 -3.5  
## [3,] 0 0 -0.5 -2.5

# To create 1 in the third row:  
# R3 = R3 / R2  
Aug[3,] = Aug[3,] \* -2; Aug

## [,1] [,2] [,3] [,4]  
## [1,] 1 1 1.0 6.0  
## [2,] 0 1 -0.5 -3.5  
## [3,] 0 0 1.0 5.0

Hence, we can get:  
x3 = 5;  
x2 - 0.5(5) = -3.5 => x2 = -1  
x1 -1 +5 = 6 => x1 = 2

### Note that throughout the operation, we only use + / - between rows; if we have to use multiply we just use scaler multiplications