

Lecture 7 (I) Recursion

GNBF5010

Instructor: Jessie Y. Huang

What is recursion?

- Algorithmically: a way to design solutions to problems by **divide-and-conquer** or **decrease-and-conquer**
 - reduce a problem to simpler versions of the same problem
- Semantically: a programming technique where a **function calls itself**
 - In programming, the goal is to NOT have infinite recursion
 - must have **1 or more base cases** that are easy to solve
 - must solve the same problem on **some other input** with the goal of simplifying the larger problem input

Multiplication – iterative solution

- “multiply $a * b$ ” is equivalent to “add a to itself b times”

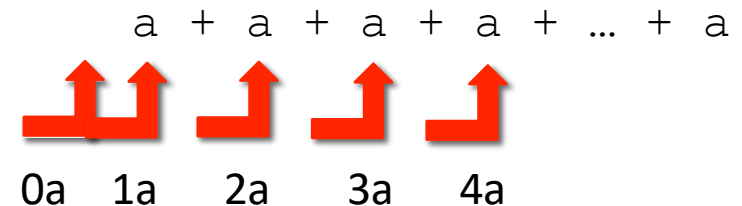
- capture **state** by

- an **iteration** number (i) starts at b

$i \leftarrow i - 1$ and stop when 0

- a current **value of computation** ($result$)

$result \leftarrow result + a$



```
def mult_iter(a, b):  
    result = 0  
    while b > 0:  
        result += a  
        b -= 1  
    return result
```

Multiplication – recursive solution

■ recursive step

- think how to reduce problem to a **simpler/smaller version** of same problem

■ base case

- keep reducing problem until reach a simple case that can be **solved directly**
- when $b = 1$, $a * b = a$

$$\begin{aligned} a * b &= \underbrace{a + a + a + a + \dots + a}_{b \text{ times}} \\ &= a + \underbrace{a + a + a + \dots + a}_{b-1 \text{ times}} \\ &= a + \boxed{a * (b-1)} \end{aligned}$$

recursive reduction

```
def mult(a, b):
```

```
    if b == 1:
        return a
```

```
    else:
        return a + mult(a, b-1)
```

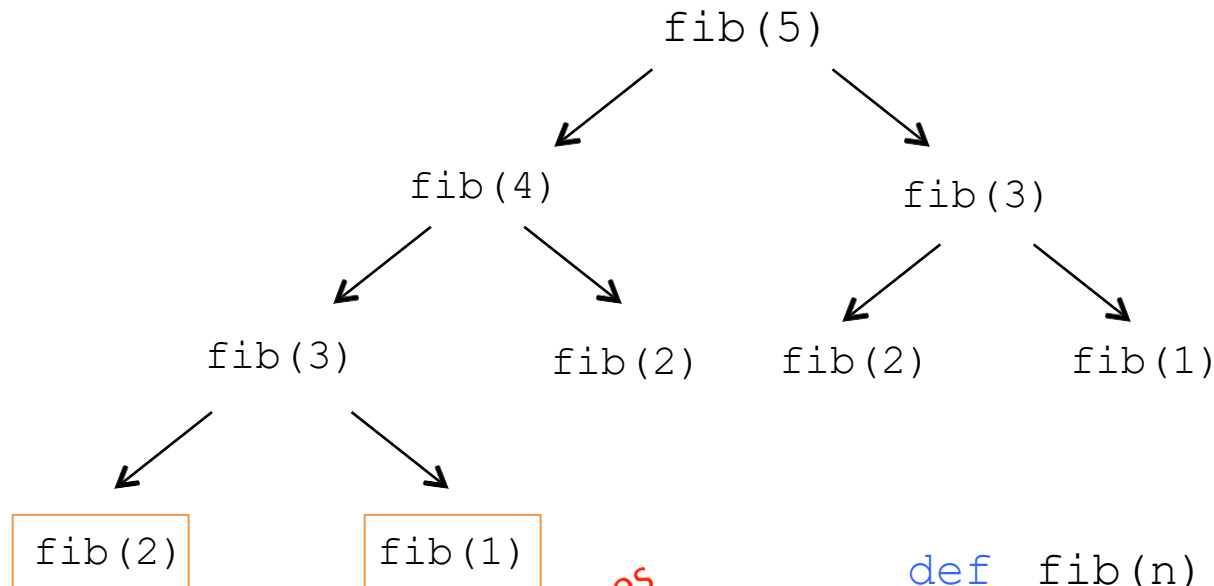
Inductive reasoning

- How do we know that our recursive code will work?
- `mult()` called with $b > 1$ makes a recursive call with a smaller version of b
- It must eventually reach the call with $b=1$

```
def mult(a, b):  
    if b == 1:  
        return a  
    else:  
        return a + mult(a, b-1)
```

Example 1: Fibonacci numbers

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$



base cases

```
def fib(n):  
    # assume n >= 1  
    if n == 1 or n == 2:  
        return 1  
    else:  
        return fib(n-1) + fib(n-2)
```

Example 2: Factorial

$$n! = n * (n-1) * (n-2) * (n-3) * \dots * 1$$

- for what n do we know the factorial?

```
n = 1      →      if n == 1:
                    return 1
```

base case

- how to reduce problem? Rewrite in terms of something simpler to reach base case

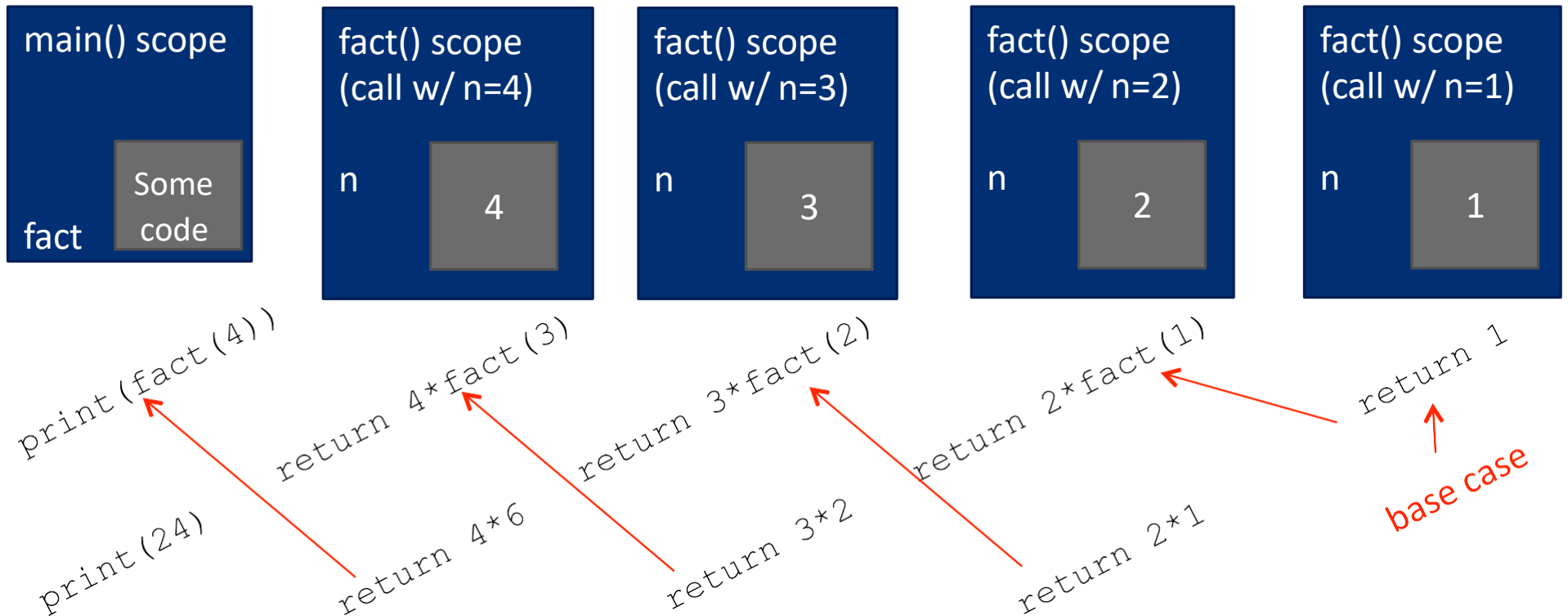
```
n*(n-1)!      →      else:
                        return n*factorial(n-1)
```

recursive step

Recursive function scope

```
def fact(n):  
    if n == 1:  
        return 1  
    else:  
        return n*fact(n-1)  
  
def main():  
    print(fact(4))
```

main()



Iteration vs. Recursion

```
def factorial_iter(n):  
    prod = 1  
    for i in range(1, n+1):  
        prod *= i  
  
    return prod
```

```
def factorial(n):  
    if n == 1:  
        return 1  
    else:  
        return n*factorial(n-1)
```

- Every **recursion** can be implemented with **iteration**
- **Recursion** is usually **slower**
 - as function calls are stored in a stack to allow return to the caller
- Infinite **recursion** can lead to **system crash**
 - whereas infinite iteration consumes CPU cycles
- Reasons to use recursion
 - Some problems are more easily solved with recursion than with a loop.
 - For example, computing **factorials** and **Fibonacci numbers**, where the mathematical definition lends itself to the recursion, and searching binary trees.

Reading

- Chapter 12 of Starting Out with Python, 4th Edition