# Well Testing

- Part of reservoir engineering
- Determination of well/reservoir behavior based on, e.g., flow and transient pressure responses in reservoirs
- If properly designed, executed, and analyzed provides information about, e.g., formation permeability, initial reservoir pressure, average pressure, well damage and stimulation (skin effects)
- **Records well pressure as function of time**
- Analysis of data is based on analysis of solutions of radial diffusivity equation



# Radial flow equation in porous medium

Diffusivity equation for radial flow:

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial p}{\partial r}\right)=\frac{c_{eff}\cdot\varphi\cdot\mu}{k}\cdot\frac{\partial p}{\partial t}$$

***Transient regime:***

Initial condition: $p\left(r,t=0\right)=p_i$

Boundary conditions: $\left(r\frac{\partial p}{\partial r}\right)_{r=r_w}=-\frac{qB_0\mu}{2\pi kh}$

$$\left(\frac{\partial p}{\partial r}\right)_{r\to\infty}=0$$



# Radial flow equation in porous medium

Dimensionless form: $\frac{\partial p_D}{\partial t_D}=\frac{1}{r_D}\frac{\partial}{\partial r_D}(r_D\frac{\partial p_D}{\partial r_D})$

With: $t_D=\frac{k\,t}{\varphi\,c_{eff}\,\mu\,r_w^2}$ $\quad r_D=\frac{r}{r_w}$ $\quad p_D=\frac{\left(p-p_i\right)2\pi\,k\,h}{q\,B_0\,\mu}$

***Transient regime:***

Initial condition: $p_D(r_{D,}t_D=0)=0$

Boundary conditions:

$$\left(r_D\frac{\partial p_D}{\partial r_D}\right)_{R_D=1}=1$$

$$\left(\frac{\partial p_D}{\partial r_D}\right)_{r_D=R/r_w}=0$$



# Well Testing

- **Drawdown test**:
  - based on pressure data **after well is put** on production
  - data usually scattering and cannot easily be analyzed
  - for new wells data influenced by cleanup process of several days

- **Pressure buildup test:**
  - follows drawdown
  - based on bottom-hole pressure data **after producing well is shut in**
  - easy experimental determination

# Well Testing

- **Multi-rate drawdown test:**
  - applied if difficult to maintain constant flow rate for complete drawdown test
  - either one test with variable flow rates
  - or series of test with various constant flow rates
  - exact flow rate determination crucial for results



# Superposition theorem

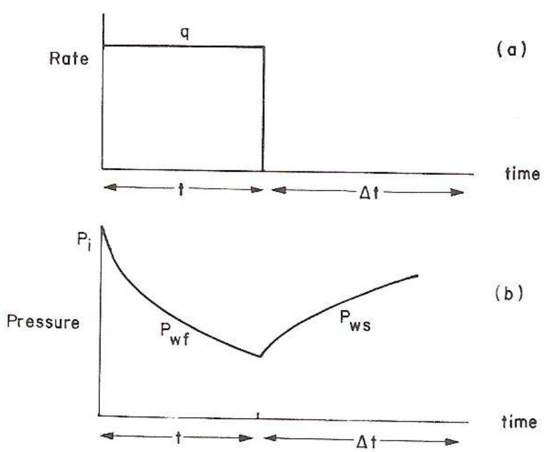
- Superposition theorem states that any sum of individual solutions of a 2nd order **linear** PDE is also a solution of the equation.

- All the solutions should satisfy boundary conditions.

- Powerful tool for engineers to solve complex flow problems.



# Well testing
## Pressure drawdown-buildup test



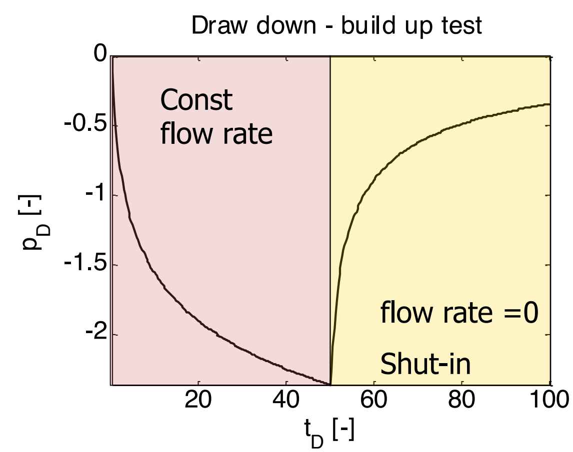

Most common form of well testing

Rate schedule and corresponding pressure response given in graphs

Well at constant flow rate q for total time t; then well is shut in; pressure in latter period experimentally determined



# Well testing
## Pressure drawdown-buildup test



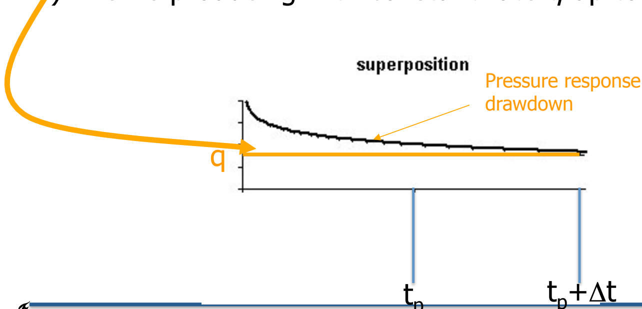
# Well testing
## Pressure drawdown-buildup test

Suppose reservoir produces with constant $q$ up to time $t_A$.

From $t_A$ to $t_B$ the well is shut in (no flow rate) and pressure builds up.

This can be described by superposition of two processes:

1) Well is producing with constant rate $q$ up to $t_P + \Delta t$





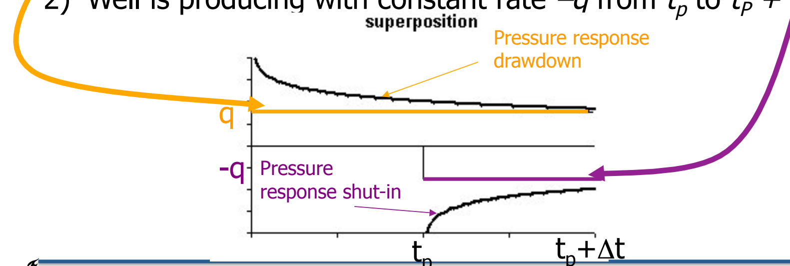

# Well testing
## Pressure drawdown-buildup test

Suppose reservoir produces with constant $q$ up to time $t_A$.

From $t_A$ to $t_B$ the well is shut in (no flow rate) and pressure builds up.

This can be described by superposition of two processes:

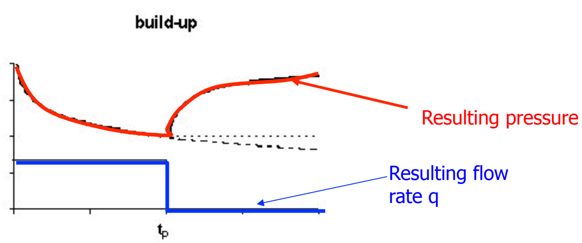
1) Well is producing with constant rate $q$ up to $t_P + \Delta t$
2) Well is producing with constant rate $-q$ from $t_p$ to $t_P + \Delta t$






# Well testing
## Pressure drawdown-buildup test

Super positioning the two solutions gives (graphically):




Mathematical solution is given by:

$$p_D^s = p_D\left(t_{pD} + \Delta t_D\right) - p_D\left(\Delta t_D\right)$$



# Well testing
## Pressure drawdown-buildup test

Dimensionless pressure at well bore is given by (self-similarity solution):

At t= $\Delta t_{pD}$: $p_D\left(r_D = 1, \Delta t_{pD}\right) \approx -\frac{1}{2}\cdot \ln\left(\frac{4\cdot \Delta t_{pD}}{\exp(\gamma)}\right)$

And at t=$t_{pD}+\Delta t_D$ $p_D\left(r_D = 1, t_{pD} + \Delta t_D\right) \approx -\frac{1}{2}\cdot \ln\left(\frac{4\cdot\left(t_{pD} + \Delta t_D\right)}{\exp(\gamma)}\right)$

Superposition gives the dimensionless shut-in pressure:

$$p_D^s = p_D\left(t_{pD} + \Delta t_D\right) - p_D\left(\Delta t_D\right) = -\frac{1}{2}\ln\frac{t_{pD} + \Delta t_D}{\Delta t_D}$$

# Well testing
## Pressure drawdown-buildup test

In full dimensional formulation obtained by introducing

$$p = p_R p_D + p_i \qquad\qquad p_R = \frac{qB_o\mu}{2\pi kH}$$

We obtain
$$p^s = p_i - \frac{qB_o\mu}{4\pi kH}\ln\frac{t_p+\Delta t}{\Delta t}$$
$$p^s = p_i - \frac{qB_o\mu\ln(10)}{4\pi kH}\log\frac{t_p+\Delta t}{\Delta t}$$

The latter form was used in the past to graphically determine the product: $k{\cdot}h$

*Plotting measured buildup response as function of Δt/(t+Δt) on semi-logarithmic paper ⇒ slope of curve gives* $\frac{qB_o\mu\ln(10)}{4\pi kH}$



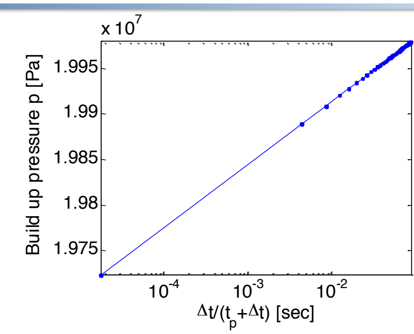
# Well testing - Pressure buildup test
## Determination of *kh and S*

Recall build up pressure

$$p_{well} = +\frac{q\mu}{4\pi kH}\left(\ln\left(\frac{\Delta t}{t_p+\Delta t}\right)\right)$$

In $\log_{10}$ notation

$$p_{well} = +\frac{q\mu\ln(10)}{4\pi kH}\left(\log_{10}\left(\frac{\Delta t}{t_p+\Delta t}\right)\right)$$



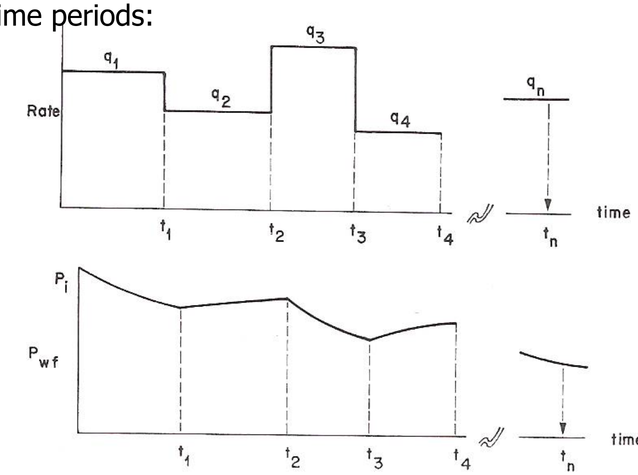

Drawdown pressure can then be used to determine skin *S* with the value for *kh* determined from the build-up pressure

$$p_{well}(t) = p_i - \frac{q\cdot\mu}{2\cdot\pi\cdot k\cdot h}\cdot\left(\frac{1}{2}\cdot\ln\left(\frac{4\cdot k\cdot t}{\varphi\cdot c_{eff}\cdot\mu\cdot r_w^2\cdot\exp(\gamma)}\right)+S\right)$$



# Well testing
## Multi-rate drawdown test

Suppose a well produces at series of constant rates for different time periods:






# Well testing
## Multi-rate drawdown test

Superposition theorem is used to obtain the wellbore pressure after time $t_n$ when current flow rate is $q_n$:

| | | |
|---|---|---|
| $q_1$ | Acting for time | $t_n$ |
| $+\ (q_2-q_1)$ | " " " | $(t_n-t_1)$ |
| $+\ (q_3-q_2)$ | " " " | $(t_n-t_2)$ |
| . | | |
| . | | |
| $+\ (q_j-q_{j-1})$ | " " " | $(t_n-t_{j-1})$ |
| . | | |
| . | | |
| . $(q_n-q_{n-1})$ | " " " | $(t_n-t_{n-1})$ |

# Well testing
# Multi-rate drawdown test

Solution of radial diffusivity equation (lhs) has to be same as the sum of the different solutions in different time periods (rhs):

$$\frac{2\pi kh}{\mu}(p_i - p_{wf_n}) = (q_1 - 0)(p_D(t_{D_n} - 0) + S)$$
$$+ (q_2 - q_1)(p_D(t_{D_n} - t_{D_1}) + S)$$
$$+ (q_3 - q_2)(p_D(t_{D_n} - t_{D_2}) + S)$$
$$\vdots$$
$$+ (q_j - q_{j-1})(p_D(t_{D_n} - t_{D_{j-1}}) + S)$$
$$\vdots$$
$$+ (q_n - q_{n-1})(p_D(t_{D_n} - t_{D_{n-1}}) + S)$$

$P_{wfn}$: specific value of bottom hole flowing pressure at time $t_n$



# Well testing
# Multi-rate drawdown test

Rewriting the solution gives

$$\frac{2\pi kh}{\mu}(p_i - p_{wf_n}) = \sum_{j=1}^{n} \Delta q_j \, p_D(t_{D_n} - t_{D_{j-1}}) + q_n S$$

in which $\Delta q_j = q_j - q_{j-1}$

The basic solution to analyze pressure-time data collected during well tests.

Applicable for oils and with minor modifications also for gas.



# Well testing
# Single-rate drawdown test

Single well with constant rate for extended period of time:

$$q_1 = q\,;\ \Delta q_1 = q \quad \text{and} \quad t_{D_n} = t_D$$

With this the general solution:

$$\frac{2\pi kh}{\mu}(p_i - p_{wf_n}) = \sum_{j=1}^{n} \Delta q_j \, p_D(t_{D_n} - t_{D_{j-1}}) + q_n S$$

Reduces to:

$$\frac{2\pi kh}{q\mu}(p_i - p_{wf}) = p_D(t_D) + S$$

$P_{wf}$ recorded during experiment; analyzed as function of t $\Rightarrow$ k is determined from slope, S from the cross-section with y-axes



# Well testing
# Pressure drawdown-buildup test

In this case we have:

$$q_1 = q\,;\ \Delta q_1 = q\,;\ t_{D_n} = t_D + \Delta t_D$$
$$q_2 = 0\,;\ \Delta q_2 = (0 - q)\,;\ t_{D_n} - t_{D_1} = \Delta t_D$$

And with the solution from the superposition theorem:

$$\frac{2\pi kh}{\mu}(p_i - p_{wf_n}) = \sum_{j=1}^{n} \Delta q_j \, p_D(t_{D_n} - t_{D_{j-1}}) + q_n S$$

We get: $$\frac{2\pi kh}{q\mu}(p_i - p_{ws}) = p_D(t_D + \Delta t_D) - p_D(\Delta t_D)$$

This is equation for analysis of pressure buildup curves;

Most common interpretation: plotting of shut-in pressure $p_{ws}$ as function of $log(t+\Delta t)/\Delta t \Rightarrow p_i$, $\overline{p}$ , $k{\cdot}h$, and $S$

# Well testing
# Multi-rate drawdown test

Series of different rates for different periods of time

Equation $\frac{2\pi kh}{\mu}(p_i - p_{wf_n}) = \sum_{j=1}^{n} \Delta q_j \, p_D(t_{D_n} - t_{D_{j-1}}) + q_n S$

Is used for analysis of data.

Sequence is arbitrary but often plotted in series of increasing or decreasing rate

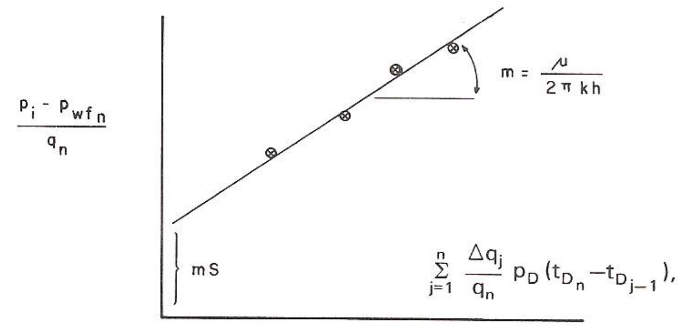
If $q_n \neq 0$ Odeh-Jones method can be used (dividing solution by $q_n$):

$$\frac{2\pi kh}{\mu}\frac{(p_i - p_{wf_n})}{q_n} = \sum_{j=1}^{n} \frac{\Delta q_j}{q_n} p_D\,(t_{D_n} - t_{D_{j-1}}) + S$$

$\Rightarrow$ *Kh* from slope and *S* from intercept assuming that $p_i$ is determined prior flowing well at first rate



# Well testing
# Multi-rate drawdown test






**Non-Darcy Flow**



# Non-Darcy Flow

- flow of gas and other fast flowing liquids
- accounting for inertia effects

Darcy's law replaced by Forchheimer equation:

$$-\left(\frac{\mu\vec{u}}{k} + \beta\rho|\vec{u}|\vec{u}\right) = \nabla\left(p + \rho gz + \frac{1}{2}\rho v^2\right)$$

*Note: density and viscosity are assumed to be pressure dependent*