

Ollscoil na hÉireann, Corcaigh
National University of Ireland, Cork



Quantum Reservoir Computing

Thesis presented by

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for the degree of

Bachelor's of Science

**University College Cork
Department of Physics**

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March 14, 2023

Declaration of Authorship

This is to certify that the work I am submitting is my own and has not been submitted for another degree, either at University College Cork or elsewhere. All external references and sources are clearly acknowledged and identified within the contents. I have read and understood the regulations of University College Cork concerning plagiarism and intellectual property.

Signature

Date

Abstract

This thesis is a test and i am a fking idiot this about qrc and i am most defn going to talk about qrc so watr is qrc and where can i go from there also 600 word limit? perhaps

Acknowledgements

I want to thank myself

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Chapter 1

Introduction

Analogous to classical computers which is based on the smallest unit of transferable information called bits, a quantum computer is based on quantum bits, otherwise known as qubits. Small quantum objects, such as electrons or photons, whose spins represent the quantum pure state, can define physical qubits. Quantum computing is based on two fundamental laws of quantum mechanics: superposition and entanglement of states [1]. In short, quantum superposition of a state, is the linear combination of pure states (for example, particles with spin up and down, can represent such state), whose complex coefficients represent the probability distribution of the corresponding pure states. On the other hand, quantum entanglement, a special case of superposition but the properties of two or more particles are correlated and can instantaneously affect each other, as a result of their shared quantum state. Even if the particles are separated by large distances, the state of one particle can not be described independently of the other particle. Quantum computation is both more efficient and complex by using a superposition of states, where all possible states have an equal probabilistic outcome of existence. This is realised into a series of steps using qubits and quantum gates, producing a probability distribution. The final output is a single pure combination of the qubits

with the highest frequency, determined by repeating measurements [2]. For quantum computing to be better than classical computing, the results of the quantum algorithm must be correct and be less complex.

A biological brain is flexible and can learn from example, their resilience allows them to have versatility and adaptivity in practical situations. Computers based on Artificial Neural Networks (ANN) are in principle analogous to that of a biological brain. For instance, ANN's are utilized in various fields for a variety of tasks [3–8] and have the ability to identify patterns in noisy [9,10] or inadequate data [11], as well as in disturbed systems [12]. This chapter will provide an introduction to the theory of how ANN's are intergrated to Reservoir Computing (RC) and in turn to Quantum Reservoir Computers (QRC). Quantum gates will be realised and simulated and the data will be presented in this chapter for both comparisons and accuracy testing. "The experimental objective of this thesis paper will be stated then after."

- talk about brief history - applications in industry - potential futures - be sure to reference papers - "this chapter will provide an introduction to the theory and experimental..." - "This chapter concludes with a summary of the objectives of the work presented in this thesis, followed by a brief outline of the remaining chapters in this thesis"

1.1 Recurrent Neural Networks

ANN's are a system of interconnected non linear nodes, divided into input, one or more hidden, and output layers (see fig. 1.1). Each node connects to another and has an associated weight and threshold. If output of such node is above this

threshold quantity, then that node is activated, sending information to the next layer of the network [13]. One can visualise that complexity increases rapidly as more nodes are introduced, making the engineering of the nodes that much more difficult, one of the major drawbacks of such architecture.

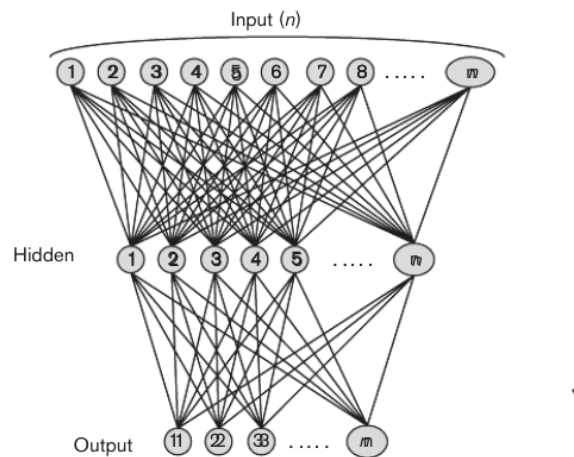


Figure 1.1: A representation of ANN's, an excerpt from [13]. Generally there are 3 layers, an input, hidden and an output layer. One can engineer the weights and threshold of each connection between nodes, via a training procedure which can ultimately after many iterations provide the most optimum outcome.

Recurrent Neural Networks (RNN) are an alternative neural network architecture wherein the connections inside the network are now fixed but random [14]. Due to the randomness, the engineering of such nodes is much easier, where the connections and thresholds are random within the input and hidden layers. The other main advantage is that in such architecture, one need only to engineer or focus on the output layer to get the optimum output. This enables one to essentially work with a large number of nodes/connections. RNN's are unique in that they have a "memory" feature, which means they use past input data to influence the processing of current input(s) and output(s). Hence, unlike conventional deep

neural networks that treat inputs and outputs as separate entities, the output of a RNN's is based on previous elements within the sequence.

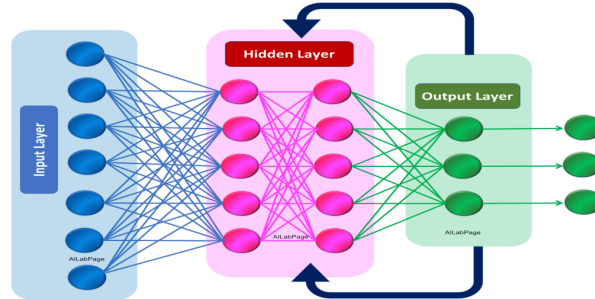


Figure 1.2: A representation of RNN's, an excerpt from [15]. Similar to Figure 1.1, the RNN possess a unique trait in that prior inputs influence any future inputs and outputs.

1.1.1 Applications?

perhaps add some examples? idk

1.2 Reservoir Computing

Reservoir computing (RC) is a computational method based on RNN's. It uses a fixed, high-dimensional system, known as a reservoir, to perform tasks such as image recognition, time series prediction, and classification [14]. The reservoir acts as a preprocessor that maps inputs into high-dimensional states and the output is determined by a simple combination of these states, which is then trained by an output layer.

One of the major benefits of RC is that it requires significantly less training data compared to traditional RNN's [16,17]. This is because the reservoir's internal dynamics can already capture (puts in memory) the relationships between inputs over time. Additionally, the reservoir can be designed to work in parallel and can be implemented on hardware optimized for parallel computation [18], making it ideal for real-time applications [].

Examples of RC include Echo State Networks and Liquid State Machines [].

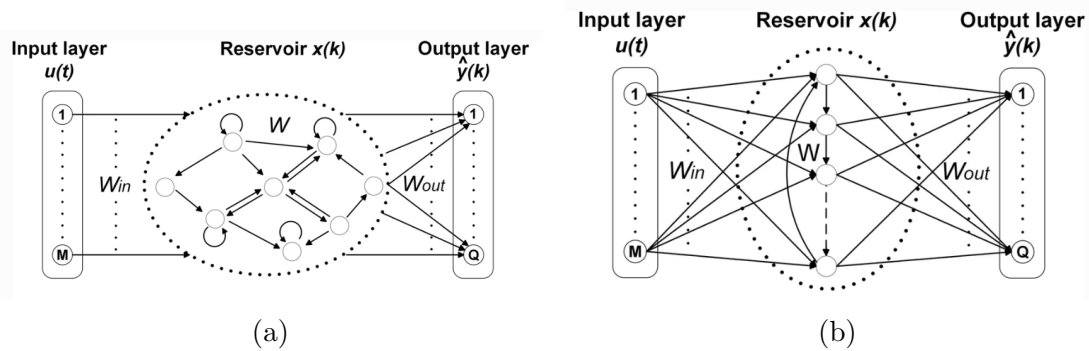


Figure 1.3: (a) shows an example architecture where all connections, W & W_{in} , are fixed but random, except for those whose connections are with the output layer W_{out} , which can be trained for a specific task. (b) shows an alternative architecture with cyclic properties. Figure adapted from [18]

1.2.1 Applications?

getting layout atleast

1.3 Quantum Reservoir Computing

Quantum Reservoir Computing (QRC) is based on RC, where now the reservoir is in the quantum domain and consequently so are the nodes, but the input and output nodes can be classical or quantum in nature. QRC utilizes a quantum system as a computational resource to tackle challenges that classical computers cannot resolve or have difficulty in doing so. The core concept of QRC lies in utilizing the inborn quantum mechanical properties of the system to conduct computation in a manner distinct from classical computing. A common architecture, the quantum circuit model is to be realised in order to go forward in this paper, such that, a quantum operation is decomposed with fundamental quantum gates (31). Quantum computers with a limited number of qubits have been realised thus far (33-34) due to the need of precise engineering of these gates (32).

Before understanding more about the quantum circuit model architecture, which is to be introduced next, let us understand more about the QRC setup introduced by S. Ghosh et al. [19]. For the main network, called the Quantum Network (QN), a set of quantum nodes is coupled via uncontrolled and random quantum tunneling [20]. An important quality of this QN is that the connection weights between these quantum nodes are not engineered. Examples (39-41)

1.3.1 Applications?

note that the titles are ofc temp

1.3.2 Rabi Oscillations (this could be in like result section - initial observations?)

A simple simulation of a single network, that is, a one computational and one reservoir qubit system (Henceforth I will start calling such systems as C+R systems, where C and R are placeholders for the number of computational and reservoir qubits in system respectively)(**perhaps add this as footnote?**) is made. Where in such a system, there is no hopping terms, \mathbf{K} , as there is only a single reservoir qubit (refer to page blabla where I explain the hamiltonian stuff). The total hamiltonian of this $\mathbf{1+1}$ system is found from the expansion of equation(refer) and some sample states were chosen for this two level system (**perhaps add examples of states, perhaps in section explaining hamiltonian**) such that the QN starts in a vacuum state (that is, the single reservoir qubit starts in such a state). The time evolution operator is found by using $U = \exp(-iH\tau/\hbar)$, where the units are scaled with respect to some value to keep units dimensionless (more information can be found in supplementary). With this we can evolve the initial states and see the evolution with respect to time. Figure fig. 1.4 represents a simulation with the initial state of the computational and reservoir qubits both in the ground state. According to fig. 1.4b, we see that there is a anti-symmetrical, reducing in amplitude, oscillation between the two represented basis states as time evolves. This is to be expected as this scenario is an example of a two level quantum system in the presence of a oscillatory driving field, the so called Rabi cycle or oscillation [].

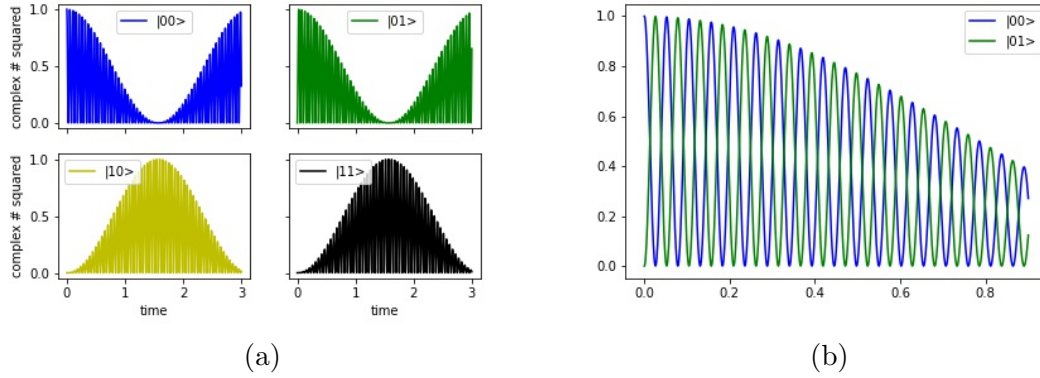


Figure 1.4: **(a)** The result of simulation of time evolution of a 1+1 system. An initial state represented by $|\psi_{initial}\rangle = (1, 0, 0, 0)$ is chosen for testing purposes, where the reservoir qubit is in the vacuum state. The probability distribution of the final state is represented in the y axis. I have chosen scaling values of $E_0 = 1$, $dt = (1/100)E_0$ and $P, J = (60/E_0, 2/E_0)$ with the onsite potentials for the QN being randomly chosen in the intervals of $\pm E_0/2$. **(b)** shows the comparison between the two basis states using the same parameters, one can note the reduction in amplitude as well as the symmetry as time evolves

1.3.3 Quantum State Representation

1.3.4 Density Matrix

According to one of the fundamental postulates of quantum mechanics, the complete information about any suitable system is contained in its so called wavefunction. But such a wavefunction can only describe a pure system. As a result, one can utilise the density operator formalism, to quantitatively describe physical systems with mixed as well as pure states. Consider a quantum system with a n -dimensional Hilbert space \mathcal{H} and an arbitrary state $|\psi\rangle \in \mathcal{H}$. Then the expectation

value of some observable:

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \quad (1.1)$$

$$= \sum_k \langle \psi | \hat{A} | \phi_k \rangle \langle \phi_k | \psi \rangle \quad (1.2)$$

where $|\phi_k\rangle$ are the basis state representation of $|\psi\rangle$. Then, rearranging, we get:

$$\langle \hat{A} \rangle = \sum_k \langle \phi_k | (|\psi\rangle \langle \psi|) \hat{A} | \phi_k \rangle \quad (1.3)$$

And the density operator is defined as:

$$\rho = |\psi\rangle \langle \psi| \quad (1.4)$$

and the other terms define the matrix element wise sum across the diagonal, which simply put is the trace of the matrix, thus we define:

$$\langle \hat{A} \rangle = Tr(\rho \hat{A}) \quad (1.5)$$

An important property of a trace of a matrix, is that it is independent of the basis representation of the system one is taking the trace of. Also holding the properties:

$$Tr(\rho^2) = 1 \quad (1.6)$$

For a pure ensemble, and

$$Tr(\rho^2) < 1 \quad (1.7)$$

for a mixed system. In fact, one can test the purity of operations on states by comparing 1.6. Now consider, a composite system consisting of two subsystems \mathbf{A} and \mathbf{B} with Hilbert spaces \mathcal{H}_A and \mathcal{H}_B , where the combined space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. Now focusing on the physical behavior of \mathbf{B} , that is, the outcome probabilities and expectation values of any measurements performed on \mathbf{B} . This can be calculated from $|\psi\rangle$, the state of the combined system; however, $|\psi\rangle$ also carries information about subsystem \mathbf{A} , which based on the stated focus, is not relevant. Thus, to isolate a subsystem \mathbf{B} , we can define the reduced density operator as:

$$\hat{\rho}_B = \text{Tr}_A[\rho] \quad (1.8)$$

$$= \sum_j (\langle j|_A \otimes \mathbf{1}_B) \rho (|j\rangle_A \otimes \mathbf{1}_B) \quad (1.9)$$

Where $|j\rangle$ is the basis vectors of which the corresponding subsystem is represented in. This results in tracing over the \mathcal{H}_A of space \mathcal{H} , which yields an operator acting on \mathcal{H}_B . To best illustrate this, consider the simplest and maximal examples of quantum entanglement of two qubits, one of the Bell's states (in total, four maximally entangled states).

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \quad (1.10)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (1.11)$$

With the basis representation $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The density matrix of our system is:

$$\rho_{AB} = |\phi^+\rangle\langle\phi^+| \quad (1.12)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \quad (1.13)$$

$$\rho_{AB} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad (1.14)$$

$$(1.15)$$

We can trace out subsystem **B** using [1.8](#), for example, and get:

$$\rho_A = (\mathbb{1}_A \otimes \langle 0|_B) \rho_{AB} (\mathbb{1}_A \otimes |0\rangle_B) + (\mathbb{1}_A \otimes \langle 1|_B) \rho_{AB} (\mathbb{1}_A \otimes |1\rangle_B) \quad (1.16)$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1.17)$$

$$(1.18)$$

That is, for qubit representing subsystem **A**, the classical probabilities of state of qubit to be in either $|0\rangle$ or $|1\rangle$ is $\frac{1}{2}$.

A pure state is such a system where a single wavefunction is enough to describe such system.

Chapter 2

Quantum Algorithm

2.1 Quiskit

This thesis is based on the paper by Sanjit Ghosh [19].

The laser excites the quantum nodes such that you suck¹. I can use² to make a placeholder and not define it until later².

2.1.1 Python

perhaps something about packages and reference it here?

2.2 Why is it Important?

2.3 Purpose of this paper?

¹a footnote with no socks lel!

²kek this is how this works

Chapter 3

thignstodo

1) Create chapter solely for explaining states and formalism? perhaps. Maybe too simple so perhaps not 2) Need to represent recent literature findings and such to QRC and the benefits/advantages 3) Can explain more on about qiskit perhaps, nice examples on how it works check out <https://qiskit.org/textbook/ch-quantum-hardware/density-matrix.html> 4) Need to talk about decoherence in density matrix section and the importance of mixed state representation when dealing with non ideal or noisy systems (i.e reality :D) 5)

Chapter 4

Conclusion

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4.1 Figures

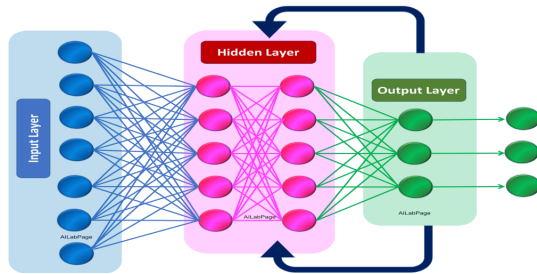


Figure 4.1: Caption

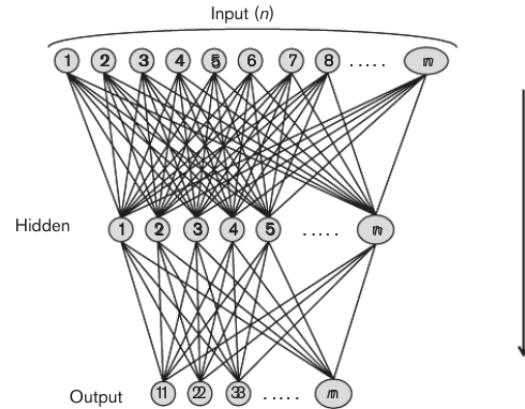


Figure 4.2: Caption

the graphics are gay fig. 4.1 and you are shit fig. 4.2 ok so i am understanding fig. 4.3a and not so much fig. 4.3b which are in fig. 4.3 the difference 4.3 is showing 4.3a and 4.3b Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Nunc ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam.



(a) Caption



(b) Caption

Figure 4.3: for subfigure representation

Appendix A

Extra Notes

this is not appendix

Appendix B

Bibliography

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