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Quantum Reservoir Computing

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Declaration of Authorship

This is to certify that the work I am submitting is my own and has not been submitted for another degree, either at University College Cork or elsewhere. All external references and sources are clearly acknowledged and identified within the contents. I have read and understood the regulations of University College Cork concerning plagiarism and intellectual property.

Signature

Date

Abstract

This thesis is a test and i am a fking idiot this about qrc and i am most defn going to talk about qrc so whatr is qrc and where can i go from there also 600 word limit? perhaps

Acknowledgements

I want to thank myself

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Chapter 1

Introduction

Analogous to classical computers which is based on the smallest unit of transferable information called bits, a quantum computer is based on quantum bits, otherwise known as qubits. Small quantum objects, such as electrons or photons, whose spins represent the quantum pure state, can define physical qubits. Quantum computing is based on two fundamental laws of quantum mechanics: superposition and entanglement of states [1]. In short, quantum superposition of a state, is the linear combination of pure states (for example, particles with spin up and down, can represent such state), whose complex coefficients represent the probability distribution of the corresponding pure states. On the other hand, quantum entanglement, a special case of superposition but the properties of two or more particles are correlated and can instantaneously affect each other, as a result of their shared quantum state. Even if the particles are separated by large distances, the state of one particle can not be described independently of the other particle. Quantum computation is both more efficient and complex by using a superposition of states, where all possible states have an equal probabilistic outcome of existence. This is realised into a series of steps using qubits and quantum gates, producing a probability distribution. The final output is a single pure combination of the qubits.

with the highest frequency, determined by repeating measurements [2]. For quantum computing to be better than classical computing, the results of the quantum algorithm must be correct and be less complex.

1.1 Basics of Quantum mechanics and computing

1.1.1 Quantum Superposition

One can represent a qubit in any state vector in a two-dimensional Hilbert space (a vector space), and in general one can write such a state as a linear combination of two basis vectors. The computational basis is used in this paper, where the two basis vectors are $|0\rangle$ and $|1\rangle$. Any coherent superposition of the two basis vectors can represent a pure qubit of the form $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex numbers in which the modulus squared describes the probability and the normalization condition $|\alpha|^2 + |\beta|^2$ holds. In fact, we can use any orthonormal basis vectors $|x\rangle, |y\rangle$ and represent our state as $|\psi\rangle = \mu|x\rangle + \zeta|y\rangle$ and the same quantum properties hold. By using quantum gates, qubits can be manipulated, where these gates are unitary operators that preserve the norm of qubits, and the state vectors describing the qubits essentially undergo rotations around the vector space.

1.1.2 Quantum Gates

Just like in classical computers, information is sent via bits and manipulated via classical logic gates, quantum computers send information via qubits and manip-

ulated via quantum gates. Quantum gates are distinct from their classical counterparts due to the unitary property, which ensures that every unitary operator is invertible, making every quantum gate inherently reversible. This distinguishes them from classical logic gates. The reason for this lies in the fact that a gate's physical implementation is an experimental arrangement that involves a Hamiltonian acting on a qubit for a specific duration. According to the Schrödinger equation, the Hamiltonian generates the time evolution operator, which is always unitary by definition.

Single qubit quantum gates, like the X gate (like the classical NOT logic gate), can be represented by matrix such that the state vector representing the qubit undergoes a rotation:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \hat{=} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (1.1)$$

Then applying the X gate (Pauli X) operation to such a qubit then:

$$X|\psi\rangle = \beta|0\rangle + \alpha|1\rangle \hat{=} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \quad (1.2)$$

$$X \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (1.3)$$

Other examples of quantum gates are:

$$Z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad Y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad CX \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad (1.4)$$

The Pauli Z, Y gates and the powerful Hadamard gate along with the CNOT gate, shown respectively. The Hadamard gate essentially puts a qubit say in state $|0\rangle$ into a superposition of states $H|qubit\rangle \hat{=} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. While the CNOT or control not gate takes a control qubit and a target qubit and changes the state of the target qubit if and only if the control qubit is in a state of $|1\rangle$. These are but a few of the many qubits that make quantum circuits really powerful and robust, requiring smart quantum algorithms to use effectively.

Now when measuring (the act of inferring or accessing information of a quantum state) say a single qubit in the computational basis $\{|0\rangle, |1\rangle\}$, the quantum properties associated with the quantum state is reduced such that the superposition collapses and as a consequence the state of the qubit is either $|0\rangle$ or $|1\rangle$ depending on $|\alpha|^2$ or $|\beta|^2$, the associated probability respectively.

1.1.3 Bloch Sphere

In the computational basis $\{|0\rangle, |1\rangle\}$, the state vector representing the qubit is:

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where $\alpha, \beta \in \mathbb{C}$ and in Euler representation:

$$\alpha = r_1 e^{i\phi_1} \quad (1.5)$$

$$\beta = r_2 e^{i\phi_2} \quad (1.6)$$

where $r_1, r_2, \phi_{1,2} \in \mathbb{R}$. Rewriting, one can get:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1.7)$$

$$= e^{i\phi_1}(r_1|0\rangle + r_2 e^{i(\phi_2 - \phi_1)}|1\rangle) \quad (1.8)$$

eq. (1.8) represents (insert bloch spehre)

1.1.4 Quantum Entanglement

Quantum entanglement can be exploited when there are many qubit interactions. Consider, a two qubit system with qubits, A and B with Hilbert spaces \mathcal{H}_A and \mathcal{H}_B , where the combined tensor product space (in the computational basis represented above) $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. If the computational basis for the two qubits can be represented $\{|i\rangle_A, |j\rangle_B\}$, then the tensor space is $|i\rangle_A \otimes |j\rangle_B = |ij\rangle$. For an N qubit system there are 2^N combinations our quantum state can be in which also corresponds to the number of complex coefficients needed to describe the quantum state fully.

For the two qubit case, the quantum state is (in computational basis of $\{|0\rangle, |1\rangle\}$):

$$|\psi\rangle = \alpha|0\rangle \otimes |0\rangle + \beta|0\rangle \otimes |1\rangle + \gamma|1\rangle \otimes |0\rangle + \delta|1\rangle \otimes |1\rangle \quad (1.9)$$

Now if the qubits were entangled, then they can not be described independently from each other and in quantum mathematical representation, they can not be factored.

Entanglement promises to dramatically improve computation speed and security (encryption) via quantum teleportation and super dense coding [3]. There are many different types of quantum computers from photonic [4] based to neutral atom based quantum computers [5]. This paper will focus on the approach of quantum computing, the foundation or framework and not so much the actual build of the quantum computer.

”The experimental objective of this thesis paper will be stated then after.”

- talk about brief history - applications in industry - potential futures - be sure to reference papers - ”this chapter will provide an introduction to the theory and experimental...” - ”This chapter concludes with a summary of the objectives of the work presented in this thesis, followed by a brief outline of the remaining chapters in this thesis”

1.2 Quantum State Representation

1.2.1 Density Matrix

According to one of the fundamental postulates of quantum mechanics, the complete information about any suitable system is contained in its so called wavefunc-

tion. But such a wavefunction can only describe a pure system (pure states were assumed in previous definitions). As a result, one can utilise the density operator formalism, to quantitatively describe physical systems with mixed as well as pure states. Consider a quantum system with a N-dimensional Hilbert space \mathcal{H} and an arbitrary state $|\psi\rangle \in \mathcal{H}$. Then the expectation value of some observable:

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \quad (1.10)$$

$$= \sum_k \langle \psi | \hat{A} | \phi_k \rangle \langle \phi_k | \psi \rangle \quad (1.11)$$

where $|\phi_k\rangle$ are the basis state representation of $|\psi\rangle$. Then, rearranging, we get:

$$\langle \hat{A} \rangle = \sum_k \langle \phi_k | (|\psi\rangle \langle \psi|) \hat{A} | \phi_k \rangle \quad (1.12)$$

And the density operator is defined as:

$$\rho = |\psi\rangle \langle \psi| \quad (1.13)$$

and the other terms define the matrix element wise sum across the diagonal, which simply put is the trace of the matrix, thus we define:

$$\langle \hat{A} \rangle = Tr(\rho \hat{A}) \quad (1.14)$$

An important property of a trace of a matrix, is that it is independent of the basis representation of the system one is taking the trace of. Also holding the properties:

$$Tr(\rho^2) = 1 \quad (1.15)$$

For a pure ensemble, and

$$Tr(\rho^2) < 1 \quad (1.16)$$

for a mixed system. In fact, one can test the purity of operations on states by comparing 1.15. Now consider, a composite system consisting of two subsystems **A** and **B** with Hilbert spaces \mathcal{H}_A and \mathcal{H}_B , where the combined space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. Now focusing on the physical behavior of **A**, that is, the outcome probabilities and expectation values of any measurements performed on **A**. This can be calculated from $|\psi\rangle$, the state of the combined system; however, $|\psi\rangle$ also carries information about subsystem **B**, which based on the stated focus, is not relevant. Thus, to isolate a subsystem **A**, we can define the reduced density operator as:

$$\hat{\rho}_A = Tr_B[\rho] \quad (1.17)$$

$$= \sum_j (\mathbb{1}_A \otimes \langle j|_B) \rho (\mathbb{1}_A \otimes |j\rangle_B) \quad (1.18)$$

Where $|j\rangle$ is the basis vectors of which the corresponding subsystem is represented in. This results in tracing over the \mathcal{H}_B of space \mathcal{H} , which yields an operator acting on \mathcal{H}_A . To best illustrate this, consider the simplest and maximal examples of quantum entanglement of two qubits, one of the Bell's states (in total, four

maximally entangled states).

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \quad (1.19)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (1.20)$$

With the basis representation $|0\rangle \hat{=} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle \hat{=} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The density matrix of our system is:

$$\rho_{AB} = |\phi^+\rangle \langle \phi^+| \quad (1.21)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \quad (1.22)$$

$$\rho_{AB} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad (1.23)$$

We can trace out subsystem \mathbf{B} using 1.17, for example, and get:

$$\rho_A = (\mathbb{1}_A \otimes \langle 0|_B) \rho_{AB} (\mathbb{1}_A \otimes |0\rangle_B) + (\mathbb{1}_A \otimes \langle 1|_B) \rho_{AB} (\mathbb{1}_A \otimes |1\rangle_B) \quad (1.24)$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1.25)$$

That is, for qubit representing subsystem \mathbf{A} , the classical probabilities of state of qubit to be in either $|0\rangle$ or $|1\rangle$ is $\frac{1}{2}$.

To best see the difference between using the wavefunction and density matrix representation consider the following illustration fig. 1.1:

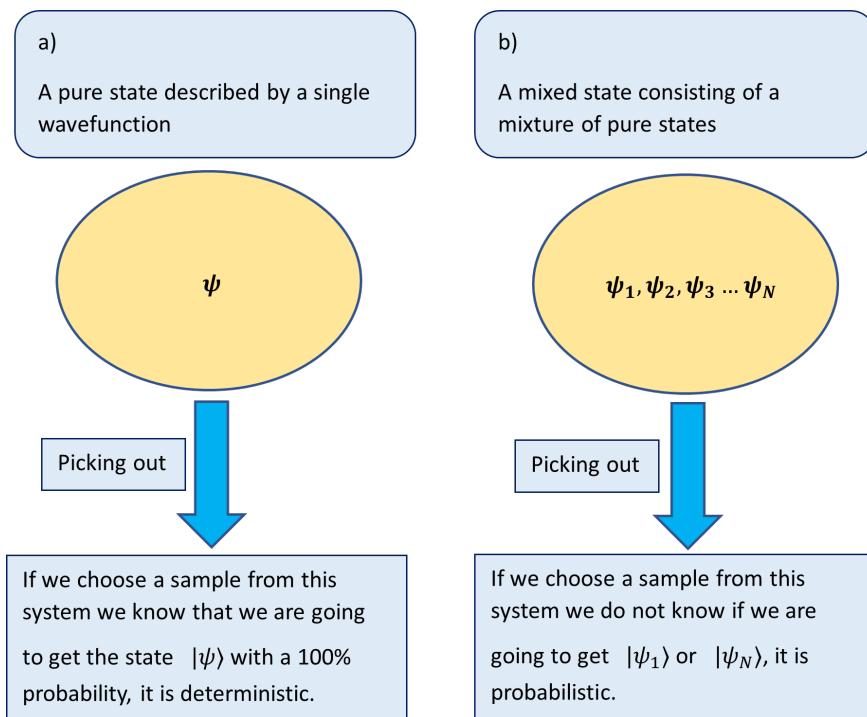


Figure 1.1: An illustration of choosing states from some sample shown in yellow. a) shows that if we were to choose from this sample then we know with a 100% certainty that we are going to get a state $|\psi\rangle$, in other words the choosing process is deterministic. But in b) we do not know which state we will get, or the choosing process is probabilistic.

So we use the density matrix formulation when we are dealing with quantum systems where we do not have enough information about the system itself, and this is the main premise of Quantum Reservoir Computing, where we do not know much about the actual interactions going on in the system (reservoir), more about this in a later section.

The structure of this thesis hereafter is as follows: I will be talking about Quantum Reservoir Computing, which is inspired by Classical Reservoir Computing, and talk about my research in general and potential goals. Then I will show my current results and discuss about such and finally give future research endeavours based on those results along with the conclusion. All materials used or referenced can be found in the supplementary section.

Chapter 2

Recurrent Neural Networks

A biological brain is flexible and can learn from example, their resilience allows them to have versatility and adaptivity in practical situations. Computers based on Artificial Neural Networks (ANN) are in principle analogous to that of a biological brain. For instance, ANN's are utilized in various fields for a variety of tasks [6–11] and have the ability to identify patterns in noisy [12, 13] or inadequate data [14], as well as in disturbed systems [15]. This chapter will provide an introduction to the theory of how ANN's are intergrated to Reservoir Computing (RC) and in turn to Quantum Reservoir Computers (QRC). Quantum gates will be realised and simulated and the data will be presented in this chapter for both comparisons and accuracy testing.

2.1 Understanding Recurrent Neural Networks

ANN's are a system of interconnected non linear nodes, divided into input, one or more hidden, and output layers (see fig. 2.1). Each node connects to another

and has an associated weight and threshold. If output of such node is above this threshold quantity, then that node is activated, sending information to the next layer of the network [16]. One can visualise that complexity increases rapidly as more nodes are introduced, making the engineering of the nodes that much more difficult, one of the major drawbacks of such architecture.

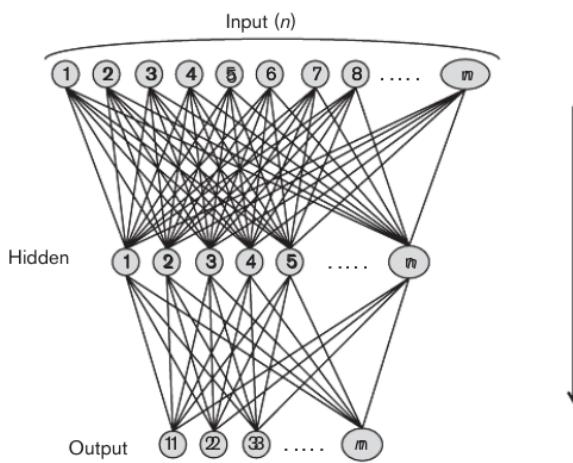


Figure 2.1: A representation of ANN's, an excerpt from [16]. Generally there are 3 layers, an input, hidden and an output layer. One can engineer the weights and threshold of each connection between nodes, via a training procedure which can ultimately after many iterations provide the most optimum outcome.

Recurrent Neural Networks (RNN) are an alternative neural network architecture wherein the connections inside the network are now fixed but random [17]. Due to the randomness, the engineering of such nodes is much easier, where the connections and thresholds are random within the input and hidden layers. The other main advantage is that in such architecture, one need only to engineer or focus on the output layer to get the optimum output. This enables one to essentially work with a large number of nodes/connections. RNN's are unique in that they have a "memory" feature, which means they use past input data to influence

the processing of current input(s) and output(s). Hence, unlike conventional deep neural networks that treat inputs and outputs as separate entities, the output of a RNN's is based on previous elements within the sequence.

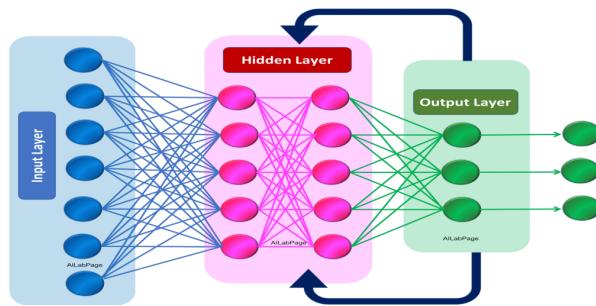


Figure 2.2: A representation of RNN's, an excerpt from [18]. Similar to Figure 1.1, the RNN possess a unique trait in that prior inputs influence any future inputs and outputs.

2.1.1 Applications?

perhaps add some examples? idk

Chapter 3

Reservoir Computing

Reservoir computing (RC) is a computational method based on RNN's. It uses a fixed, high-dimensional system, known as a reservoir, to perform tasks such as image recognition, time series prediction, and classification [17]. The reservoir acts as a preprocessor that maps inputs into high-dimensional states and the output is determined by a simple combination of these states, which is then trained by an output layer.

One of the major benefits of RC is that it requires significantly less training data compared to traditional RNN's [19,20]. This is because the reservoir's internal dynamics can already capture (puts in memory) the relationships between inputs over time. Additionally, the reservoir can be designed to work in parallel and can be implemented on hardware optimized for parallel computation [21], making it ideal for real-time applications [].

Examples of RC include Echo State Networks and Liquid State Machines [].

3.0.1 Applications?

getting layout atleast

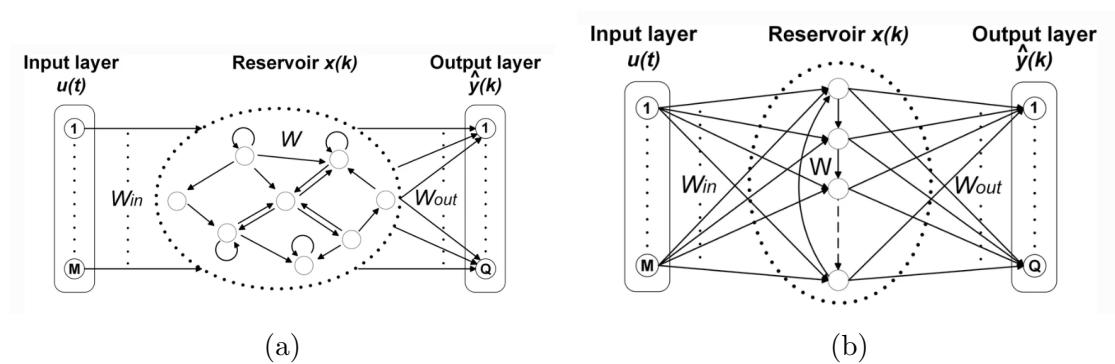


Figure 3.1: (a) shows an example architecture where all connections, W & W_{in} , are fixed but random, except for those whose connections are with the output layer W_{out} , which can be trained for a specific task. (b) shows an alternative architecture with cyclic properties. Figure adapted from [21]

Chapter 4

Quantum Reservoir Computing

Quantum Reservoir Computing (QRC) is based on RC, where now the reservoir is in the quantum domain and consequently so are the nodes, but the input and output nodes can be classical or quantum in nature. QRC utilizes a quantum system as a computational resource to tackle challenges that classical computers cannot resolve or have difficulty in doing so. The core concept of QRC lies in utilizing the inborn quantum mechanical properties of the system to conduct computation in a manner distinct from classical computing. A common architecture, the quantum circuit model is to be realised in order to go forward in this paper, such that, a quantum operation is decomposed with fundamental quantum gates (31). Quantum computers with a limited number of qubits have been realised thus far (33-34) due to the need of precise engineering of these gates (32).

Before understanding more about the quantum circuit model architecture, which is to be introduced next, let us understand more about the QRC setup introduced by S. Ghosh et al. [22]. For the main network, called the Quantum Network (QN), a set of quantum nodes is coupled via uncontrolled and random

quantum tunneling [23]. An important quality of this QN is that the connection weights between these quantum nodes are not engineered. Examples (39-41)

4.0.1 Applications?

note that the titles are ofc temp

4.0.2 General System

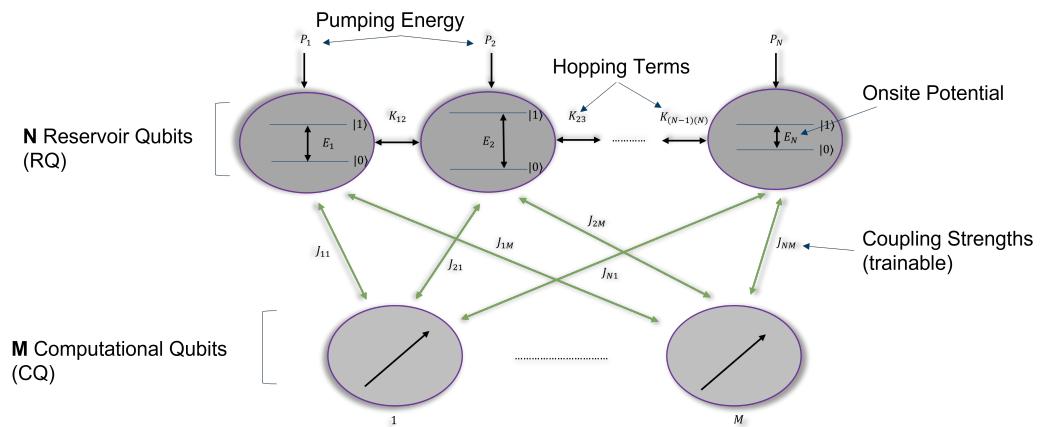


Figure 4.1: general system test

The general structure of the QRC is seen in 4.1. There is a reservoir layer which contains reservoir qubits and a computational layer which contains computational qubits. The qubits can either be in states of $|0\rangle$ or $|1\rangle$, analogous to classical bits, but these qubits can also be both states at the same time, a case of superposition (see page XX about superposition). The way it is represented here is that the state $|1\rangle$ is like an excited state and $|0\rangle$ like a ground state, this is done for visual clarity. Each reservoir qubit has onsite potentials labelled \mathbf{E} , nearest neighbour hopping terms labelled \mathbf{K} and pumping terms labelled \mathbf{P} . The

only form of interaction within the system is between the reservoir and computational qubits via quantum tunnelling (see XX for more on quantum tunneling). The coupling strength is controlled by tuning the \mathbf{J} terms, all the connections are random and arbitrary. Dr. Ghosh and his team showed that by using machine learning techniques (like genetic based algorithms) to tune the coupling strengths, they were able to make the reservoir layer induce specific quantum gate operations on the computational qubits. They showed also that by having a set of these quantum gates they can essentially simulate quantum circuits. The power of the reservoir layer to induce this effect is that by tuning these terms that describe the system, setting the reservoir layer to these quantum gate operations, we can effectively make the reservoir layer act like a computer and put it into any many qubit system, without having to know what interactions have been made in the reservoir system. This is different from the conventional method of using quantum computers where we have to interact with the quantum system in order to make specific operations. Quantum circuits require a successive application of quantum gates but with QRC, the whole reservoir layer can act like these quantum circuits and do not require these successive applications and also a single network or layer is enough to simulate any quantum circuits, effectively compressing the amount of information.

The hermitian Hamiltonian of this general system can be condensed down to the following form:

$$\hat{H} = \sum_l E_l \hat{a}_l^\dagger \hat{a}_l + \sum_{ll'} K_{ll'} (\hat{a}_l^\dagger \hat{a}_{l'} + \hat{a}_{l'}^\dagger \hat{a}_l) + \sum_l (P_l^* \hat{a}_l + P_l \hat{a}_l^\dagger) + \sum_{kl} (J_{kl}^* \hat{\sigma}_k^+ \hat{a}_l + J_{kl} \hat{a}_l^\dagger \hat{\sigma}_k^-) \quad (4.1)$$

The \hat{a}^\dagger is the creation and \hat{a} the destruction operator. The Pauli matrices are

given as follows:

$$\sigma_k^\pm = \frac{1}{2}(\sigma_k^x \mp i\sigma_k^y) \quad (4.2)$$

$$\sigma_k^\pm = \frac{1}{2} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mp i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right) \quad (4.3)$$

where

$$\sigma_k^- = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \& \sigma_k^+ = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad (4.4)$$

and, the basis is taken as:

$$|g\rangle \hat{=} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \& |e\rangle \hat{=} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (4.5)$$

The full quantum state (total Hamiltonian follows same property) of a CQ+RQ system is:

$$|\psi\rangle = |CQ_1\rangle \otimes |CQ_2\rangle \otimes |CQ_3\rangle \otimes \dots |CQ_C\rangle \otimes |RQ_1\rangle \otimes |RQ_2\rangle \otimes |RQ_3\rangle \otimes \dots |RQ_R\rangle \quad (4.6)$$

Where CQ and RQ are the computational and reservoir qubits respectively.

Chapter 5

Results

5.1 Simple Analysis: 1+1 System

Focusing on a more simple system, a one reservoir and computational qubit system, the aim is to analyze the dynamics and behaviours of the system and from such, map it onto more complex systems such as the many qubit system. For such a

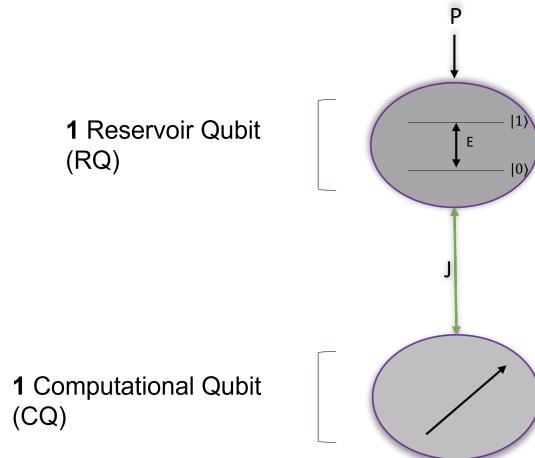


Figure 5.1: Simple System

system the Hamiltonian 4.1 can be simplified to a 4 by 4 matrix:

$$\hat{H} = \begin{pmatrix} 0 & P^* & 0 & 0 \\ P & E & J & 0 \\ 0 & J^* & 0 & P^* \\ 0 & 0 & P & E \end{pmatrix} \quad (5.1)$$

Typical states represented in the same basis as \hat{H} is:

$$|\psi\rangle = \alpha|0\rangle \otimes |0\rangle + \beta|0\rangle \otimes |1\rangle + \gamma|1\rangle \otimes |0\rangle + \delta|1\rangle \otimes |1\rangle \quad (5.2)$$

That is, for our 2 qubit system, the possible combinations of our system is represented as a superposition of product states with the complex coefficients (α, β, \dots) squared representing the probability that our system will be in that particular product state if a measurement is made on the system. Recall that the left most digit represents the state of the computational qubit and the right most, the state of the reservoir qubit.

If you want to find the evolution of such state, we need to define the unitary time evolution operator:

$$\hat{U}(t) = \exp\left(\frac{-i}{\hbar}\hat{H}\right) \quad (5.3)$$

$$|\psi(t)\rangle = \hat{U}(t)|\psi_{initial}\rangle \quad (5.4)$$

5.2 Results: Rabii Oscillations for 1+1 system

Evolving the 1+1 system, where we started off with the $|00\rangle$ product state and let it evolve wherein the quantum state will become a superposition of product states (5.2), we find trivial oscillations between the probability amplitudes of our product states.

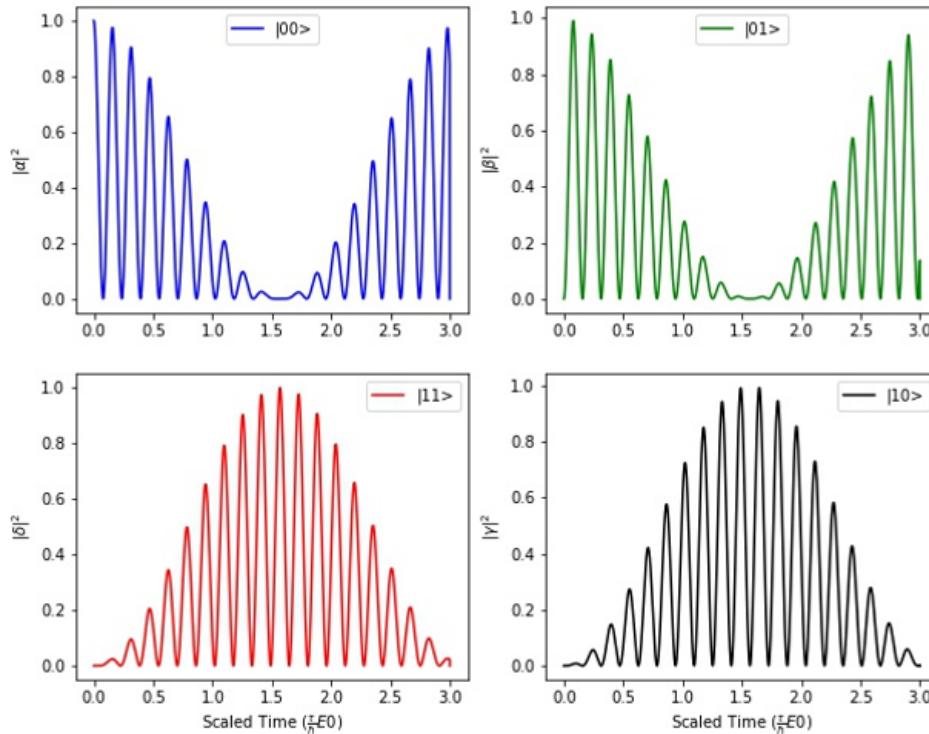


Figure 5.2: For 1 RQ and CQ, the evolution of this 2 qubit system exhibits particular probability oscillations with respect to each other. Starting with an initial state of $|00\rangle$ and with $P,J,E = (20/E0, 2/E0, 1/E0)$, where $E0 = 1$

Taking a closer look at the oscillations with respect to each product state:

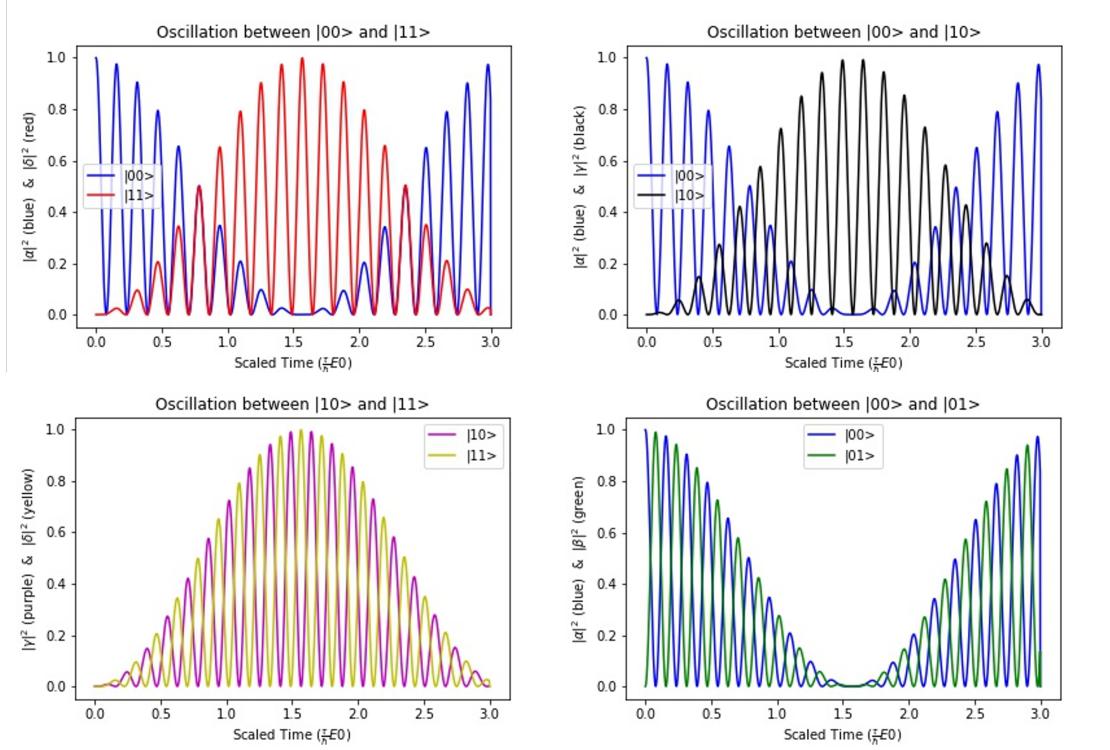


Figure 5.3: The oscillations with respect to each product state for the same simulation seen in fig. 5.2

The x axis represents the scaled time in units of $\frac{\tau}{\hbar}E_0$, where E_0 is the variance (a number interval were parameters are scaled on), the y axis represents the probability amplitude of the corresponding product state that our quantum state (5.2) can be in.

Taking a look at fig. 5.3, we see that for the oscillations between the $|00\rangle$ and the $|11\rangle$, at initial time, our state is in this $|00\rangle$ (as we initialized it as such), but then as times goes on, between times of 0.0 and 1.5 units, we see a mixture of probability amplitudes. In fact, our state is in a superposition of product states $|00\rangle$ and $|11\rangle$. At time of 1.5 units, our state has a high probability to be in the $|11\rangle$ product state. We see the same relationship with the $|00\rangle$ and $|10\rangle$ product states.

In fact, at the vicinity of time of 1.5 units, the quantum state is in a superposition of $|11\rangle$ and $|10\rangle$ product states. This inherent oscillation that we see in our system is what is expected for this 2 level system, in fact Rabii oscillations is trivial for any 2 level systems.

If we focus on fig. 5.3 and look at the oscillations between the product states $|00\rangle$ and $|11\rangle$, and recall that the quantum state representation is $|CQRQ\rangle$. We see that the CQ, initially a state of $|0\rangle$ flips to a state of $|1\rangle$. If we were to stop our operation at that time of the flip where our quantum state has a high probability of being in the product state $|11\rangle$, then using the inherent property of our system, this oscillation, we have effectively induced a quantum X gate operation on the CQ. This begs the question, could different quantum gate operations be realised using this inherent property? Perhaps at later times, a Z gate could be realised. To look into this, we need to look at phase diagrams which will tell us about any phase changes that occurs as our system evolves in time, and from analysing these phase changes, see if they correspond to a specific quantum gate operation and if so then characterise our system in terms of this temporal behaviour, were we can stop our evolution of our system at the time of where this phase change occurs, effectively inducing the corresponding quantum gate operation.

5.3 Results: Further Analysis on 1+1 system

We can represent our 2 qubit system in terms of density matrices. Starting with an initial state of $|\psi\rangle = |00\rangle$, the density matrix is:

$$\rho_0 = |\psi\rangle\langle\psi| = |00\rangle\langle 00| \quad (5.5)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (5.6)$$

Realising that our evolution of density matrices can be found by applying the time evolution operator:

$$\rho(t) = \hat{U}(t)\rho_0\hat{U}^\dagger(t) \quad (5.7)$$

We can see how our matrix evolves in time. This matrix contains information of both our reservoir and computational layer. But we do not need information regarding our reservoir layer, in fact it is quite redundant, and so we can invoke the partial tracing method to trace out the reservoir layer, effectively yielding a reduced matrix, which contains information of just our computational layer (see section 1.2.1):

$$\rho_{CQ}(t) = Tr_{RQ}[\rho(t)] = \sum_j (\mathbb{1}_{CQ} \otimes \langle j|_{RQ})\rho(t)(\mathbb{1}_{CQ} \otimes |j\rangle_{RQ}) \quad (5.8)$$

Defining a quantity called Fidelity:

$$F(t) \equiv \langle \phi_{ideal} | \rho_{CQ}(t) | \phi_{ideal} \rangle \quad (5.9)$$

Which is a measure of how well does our reservoir layer act like a specific quantum gate, where:

$$F(t) = \begin{cases} 1 & \text{ideal operation} \\ < 1 & \text{non ideal operation} \end{cases}$$

For example, if our computational qubit is in a superposition of states, represented in vector form:

$$|\phi_{ideal}\rangle = \alpha|0\rangle + \beta|1\rangle \hat{=} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (5.10)$$

If we apply our X gate operation to such a qubit then:

$$X|\phi_{ideal}\rangle = \beta|0\rangle + \alpha|1\rangle \hat{=} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \quad (5.11)$$

So this Fidelity is measuring how close does our reservoir layer induce such quantum gate operation (for example the X gate) on our computational layer. The Purity of the quantum state (see section 1.2.1 on Purity):

$$P(t) = \text{Trace}(\rho_{CQ}(t)^2) \quad (5.12)$$

Where:

$$P(t) = \begin{cases} 1 & \text{pure state} \\ < 1 & \text{non pure state} \end{cases}$$

By using these parameters and looking at this simple 2 qubit system the Fidelity and Purity evolution is found as follows:

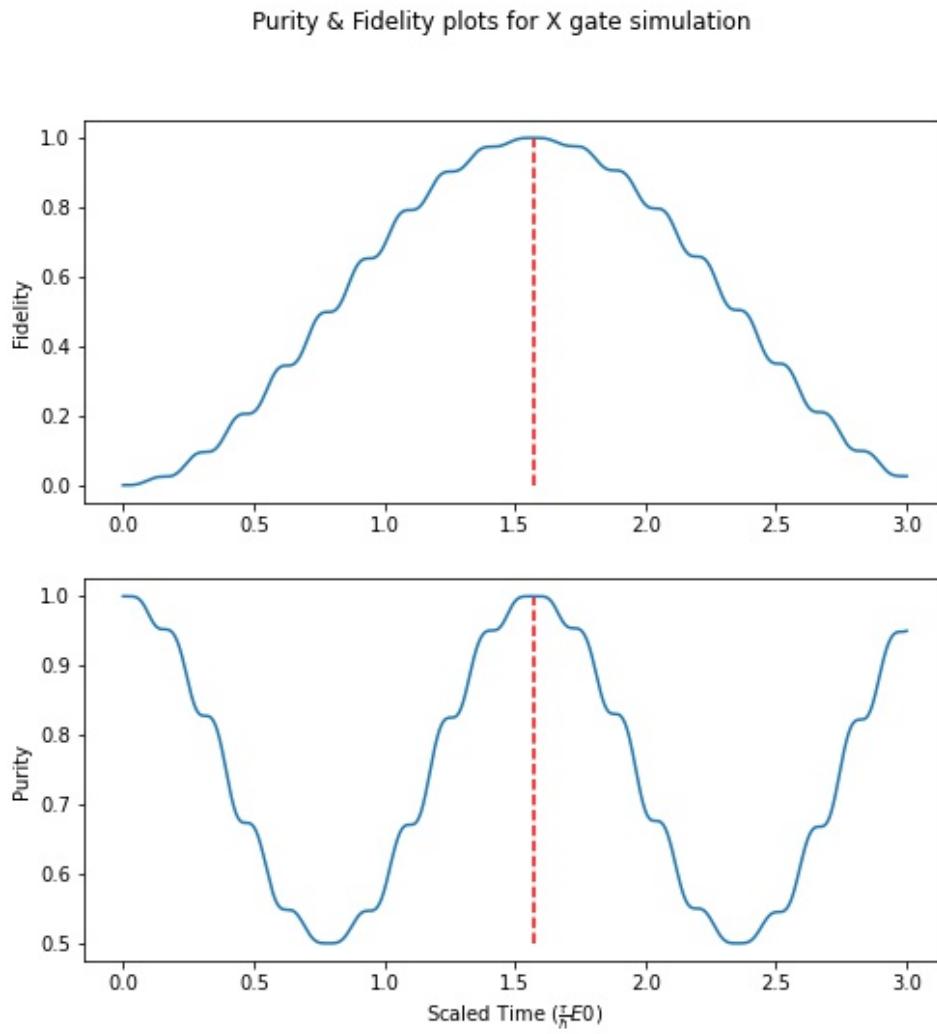


Figure 5.4: The Fidelity and corresponding Purity of our quantum state as it evolves in time with the same parameters seen in fig. 5.2. $P, J, E = (20/E0, 2/E0, 1/E0)$, where $E0 = 1$

Looking at fig. 5.4, we see that at the time (see fig. 5.3 of $|00\rangle$ and $|11\rangle$) of where our computational qubit flips from a state of $|0\rangle$ to a state $|1\rangle$, our Fidelity is maximum, which means that our reservoir layer at that moment in time does indeed act like the ideal X gate operation. Corresponding to that the Purity

is also maximum, which means that our computational qubit is in a pure state of $|1\rangle$, which is what we want when operating with qubits that we compute with (i.e. computational qubits). From analysing this Fidelity and Purity curves we can confirm that we can utilise this temporal behaviour that is inherent to our quantum system to do X gate operations (the reservoir layer induces such operations) on our computational layer. Now we need to look at phase diagrams of our oscillations and see if different quantum gate operations could be realised by analysing the corresponding phase changes in those diagrams, there after looking at the Fidelity and Purity curves to confirm if the observed quantum gate operation is viable (reservoir layer acts like the ideal operation) and if our computational qubits will be accessible (pure).

Chapter 6

Quantum Algorithm

6.1 Quiskit

This thesis is based on the paper by Sanjit Ghosh [22].

The laser excites the quantum nodes such that you suck¹. I can use² to make a placeholder and not define it until later².

6.1.1 Python

perhaps something about packages and reference it here?

6.2 Why is it Important?

6.3 Purpose of this paper?

¹a footnote with no socks lel!

²kek this is how this works

Chapter 7

things to do

- 1) Create chapter solely for explaining states and formalism? perhaps. Maybe too simple so perhaps not
- 2) Need to represent recent literature findings and such to QRC and the benefits/advantages
- 3) Can explain more on about qiskit perhaps, nice examples on how it works check out <https://qiskit.org/textbook/ch-quantum-hardware/density-matrix.html>
- 4) Need to talk about decoherence in density matrix section and the importance of mixed state representation when dealing with non ideal or noisy systems (i.e reality :D)
- 5) I need to rearrange the content in the paper, for now just putting all information in and rearrange later.
- 6) Look into 2 + 2 and get code working there, then try many body, machine learning etc, perhaps not enough time but attempt?
- 7) In supplementary materials perhaps show for different P,J values

Chapter 8

Conclusion

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8.1 Figures

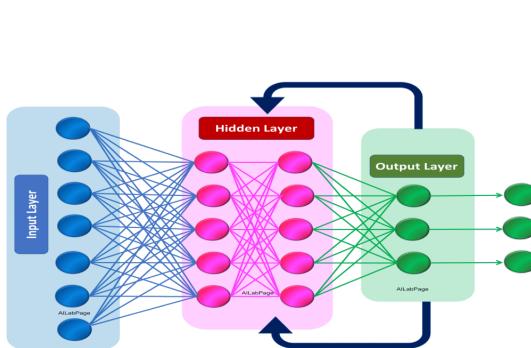


Figure 8.1: Caption

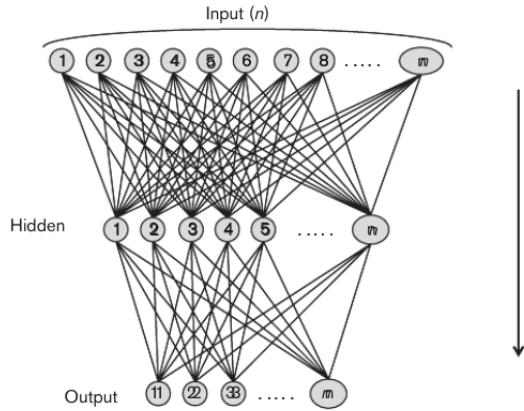


Figure 8.2: Caption

the graphics are gay fig. 8.1 and you are shit fig. 8.2 ok so i am understanding fig. 8.3a and not so much fig. 8.3b which are in fig. 8.3 the difference 8.3 is showing 8.3a and 8.3b Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Nunc ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam.



(a) Caption



(b) Caption

Figure 8.3: for subfigure representation

Appendix A

Extra Notes

this is not appendix

Appendix B

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