The Spiral of Silence and Its Application in Recommender System

Submitted for Blind Review

ABSTRACT

Are people in recommender systems less willing to speak out if they perceive they are in the minority? We testify the famous "spiral of silence" theory in actual recommender systems. We find that (1) People are less likely to express negative opinions and more likely to give positive responses, thus resulting in a spiral process where the average rating for each item increases given more ratings received. (2) People identify themselves as minority according to the perceived "opinion climate",which is highly affected by their communities and the opinion leaders. (3) A hardcore group of people, who are more active to reveal extreme opinions, exists in every recommender system. In general, hardcore is a consistent personality. (4)People are more willing to demonstrate dissents for popular items. The phenomenon of silent minority harms the performance of conventional recommender systems. We utilize our empirical discoveries to guide the recommender framework. We propose novel models which assume that the probability a user willing to express opinion is dependent on the rating and how the rating divergent to the perceived opinion climate. Furthermore, we model the formation of community and community specific opinion climate, opinion leaders, hardcore personality and item popularity to enhance recommendation performance. Experimental results demonstrate that our models outperform state-of-the-art recommendation models.

Categories and Subject Descriptors

H.4 [**Information Systems Applications**]: Miscellaneous

General Terms

Theory

Keywords

Spiral of Silence, Missing not at Random, Recommender System

1. INTRODUCTION

In 1974, German political scientist Elisabeth Noelle-Neumann proposed the Spiral of Silence Theory[1] [**?**]. The theory states that, due to the "fear of isolation", people are less willing to express their opinions if they perceive that they are not supported by the majority. It results in a spiral process in which the majority opinion receives growing popularity while other opinions are gradually pushed back. The process continues until the majority opinion ultimately becomes a social norm.

The theory has been acknowledged as "one of the most influential recent theories of public opinion formation" [**?**]. In the literature of mass communication and political science, many studies testified the theory. Typically, they conducted surveys and asked subjects to rate the willingness to speak out (i.e. enter a conversation, vote, donate, etc.) if their opinions are in the minority. However, this type of experiment protocols may be problematic, because the findings are based on hypothetical willingness instead of actual willingness [**?**]. The theory in its online form also triggers considerable critiques. Some researchers pointed out that, within the online context, factors such as anomity might decrease the fear of isolation and thus empower "people in the minority to speak up more" [**?**].

Our first contribution in this work is to empirically testify the theory in terms of actural willingness to speak out in Recommender Systems (RS). We use several real data sets to analyze response patterns given how the users' opinion diverge from the perceived "opinion climate". Our study suggests that, when people observe that their ratings are below the majority rating, they consider themselves as in the minority. Negative feedback tend to self-censor, which triggers an upward spiral of raising average rating for each item.

We then further examine some key components in the theory The key components include: (1) The perceptron of "opinion climate". The theory asserts that people use their "quasi-statistical" sense to assess current majority opinion. Our results reveal that, assessment of "opinion climate" might be related to the social group they identify themselves and the opinion leaders. (2) The existence of "hardcore groups". The theory presumes that some people are more active while the rest are more reluctant to respond. We empirically verify the existence of hardcore groups. We find out that "hardcore" is a consistent personality. (3) The strategies to remain silent. The theory presumes that users might choose different strategies to remain silent according to the nature of items. We observe that, for popular items, people are more willing to speak out extreme opinions.

The pehomenen of "silent minority" will harm the performance of RS. Fig. **??** gives an illustrative example. Suppose user u_1 is a sensitive user who only responds to items on which he agrees with the majority. Given his responses, a conventional recommender will estimate his preferences as the average user preference. For example, a collaborative fil-

[1]In the remaining of this paper, the Spiral of Silence theory will be referred as "the theory".

u/v	v_1	v_2	v_3	v_4	v_5
u_1	2	3		3	
u_2	5	3			2
u_3	5	3	5	3	
u_4			2	3	4
u_5	5		2		1

Figure 1: A toy example, ratings in gray are "hidden" opinions

tering recommender will determine u_1's nearest neighbor as u_2, u_3. It leads to a predicted rating $r_{1,1} = \frac{sim(u_2,u_1)r_{2,1}+sim(u_3,u_1)r_{3,1}}{sim(u_2,u_1)+sim(u_3,u_1)} = 5$ for item v_1. As we can see from the example, the prediction is highly biased.

To address the challenge of "spiral of silence", one need to model the missing ratings as missing not at random observations(a.k.a. MNAR). In the RS community, MNAR models explicitly generate user response by user ratings [?, ?, ?]. Instead of optimizing $p(R|\Theta)$, which is the likelihood of ratings, MNAR models optimize $p(R, X|\Theta)$, where X is the set of *response* variables indicating whether a rating is missing. Consequently, MNAR models achieve better performance in predicting both ratings and responses.

In the nutshell, our models fall into the MNAR framework. However, for the best of our knowledge, we are the first to apply "spiral of silence" theory in recommender systems and consider the perception of support as a factor in a MNAR model. we model the possibility of user "speaking out" as dependent on users' perception of opinion climate.

We also utilize the findings in empirical study to build the models. We incorporate hidden community, personality and item specific factors into the model. In the experiments on real data sets, our model achieves best results (NDCG 0.79) compared with other state-of-the-art MNAR models(best NDCG 0.7). The comparative performances of model variants support the discoveries about the impact of social identity, hardcore groups and silent strategy on self definition of minority. We believe our study sheds insights into bridging social science and computer engineering.

The paper is organized as follows. In Sec **??** we briefly introduce the related works. In Sec **??** we present our empirical study on several real data sets. In Sec **??** we propose several model variants, based on the findings in Sec. **??**. In Sec. **??** we demonstrate and analyze comparative performances of the models. Finnally, in Sec. **??** we conclude our contribution and look into the future work.

2. RELATED WORK

Recently, recommendation system has attracted a lot of research attentions. In the fruitful literature, matrix factorization [?] techniques have exhibited superior performances in rating predictions and become a standard approach. Probablistic matrix factorization [?] implement the idea of matrix factorization from a probabilistic generative perspective. It assumes that each rating is randomly sampled from a Gaussian distribution, whose mean is the product of user preference and item aspects. Opinion leaders and social communities can be included in this framework. For example, a SVD style model [?] factorizes the rating matrix to most representative users. The social recommender [?] models how a user's rating is affected by his/her trusted friends in a community.

When data is not missing at random, the probability of generating both observations and missing ratings (thus the "whole" data set) is not proportional to the likelihood of observed ratings. Marline et.al presented a piorneer work [?] of Missing Not At Random (MNAR) models. It assumes that response is a binary variable, which is generated by a Bernoulli distribution associated with the rating value. Along this line, several successive research works introduce new mechanism to generate responses from ratings. A continous rating is allowed in [?] with a step function for the probability of generating responses. Such a soft assignment model is further improved in [?] with a mini batch algorithm. [?] adopts an OR operation over per-item, per-user, per-rating-value parameters. The most complicated model is given by [?], which proposes seperate generative process for the complete ratings and the responses, and cover the rating matrix by the response matrix as a mask to form the observations.

Another line is to consider a semi-observed variable "exposure", which is related to response. Whether the item is exposed to the user, as well as the potential rating determine the response. A few recent works [?, ?] fall into this category. Instead of directly producing a response based on the hidden rating, an alternative approach is to probabilistically relate responses and ratings. For example, [?] presumes a parameter is involved in both processes of generating ratings and responses. Under the MNAR assumptions, conventional evaluation metrics, such as NDCG may fit better to response bias [?]. An approximate evaluation metric in top N observations is presented in [?].

The dynamics of public opinion is a long refreshing research topic. Many empirical studies are performed in various domains. Some researchers have observed the trend of increasing average ratings. They offered several explanations. The first one is selection bias hypothesis. It believes that users select and rate entities they are likely to like [?]. The second hypothesis is choice-supportive bias [?]. According to the choice-supportive bias hypothesis, since users take too much efforts in finging a product, they will refuse to admit their poor judgment. The third hypothesis is that more reviews bring in more self-promotion spams [?]. On the controry, some researchers find that the later ratings are on average lower than earlier ratings [?]. The possible explanation is that, the volumn of reviews restrict one's ability to diagnose previous reviews. Therefore when previous reviewers are very different, more reviews may lead to more purchase errors and lower ratings. Hu et.al observed a J-shaped distribution [?], presumably driven by purchasing bias (selecting higher product valuations) and under-reporting bias (report only when it is to "brag or moan").

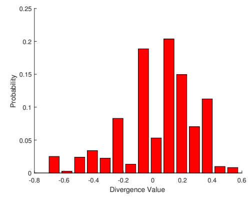
We should point out that, although the above biases make senses, we believe that our hypothesis in this paper is more reasonable. By introducing the "opinion climate", we provide a baseline for judging positive and negative opinions. Our experimental results prove that such a baseline adjustification improves models which consider the polarity of opinions simply by the rating values.

3. EMPIRICAL STUDY

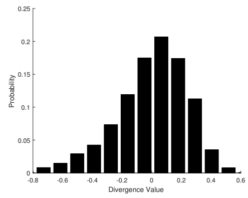
We use several real data sets. The first one is Movielens, which is a collection of ratings on movies in the movielens website. The ratings are made on a 5-star scale with timestamps. The second one is Eachmovie, which is a 18-month experiments on collaborative filtering to predict user ratings on films and videos. The timestamped ratings are in the

Table 1: Statistics of the data sets

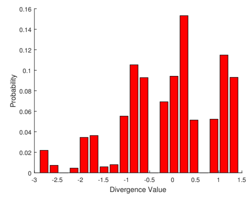
Dataset	#users	#Items	#Ratings
Movielens	6,040	3706	1,000,209
Eachmovie	61,131	1,622	2,559,107
Yahoo!user	15,400	1000	311,704
Yahoo!random	5400	1000	54,000



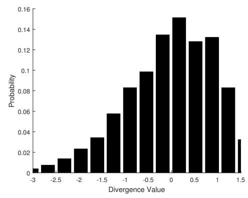
(a) Eachmovie timestamped



(b) Eachmovie background



(c) Movielens timestamped



(d) Movielens background

Figure 2: Distribution of rating divergence

range of $(0, 1)$. These are the most commonly used benchmarks in recommender systems.

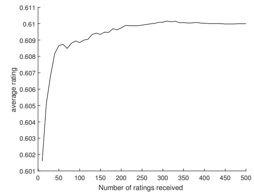
We also use the recent Yahoo! data set, which is a set of ratings on songs through the Yahoo! web-scope data sharing program. The data set contains two subsets of ratings. The first (Yahoo!user) set consists of ratings supplied by users during normal interaction with Yahoo! Music services. The second source consists of ratings collected during an online survey, when each of the first 5400 users in Yahoo!user set was asked to provide exactly ten ratings for randomly selected songs. The ratings in Yahoo! dataset do not come with timestamps.

The Yahoo! data sets provide unique opportunities to testify the spiral of silence theory. The random setting corresponds to a scenario where users are forced to response, against his actual willings. The user selected setting corresponds to a scenario where users are free to hide their responses. Therefore we can compare user's behaviors within different restrictions to study the key components of the theory.

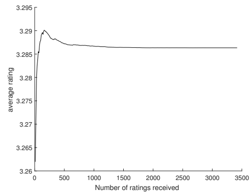
3.1 Silent Minority

It is worthwhile to investigate how users in recommender systems identify themselves as in the minority. We use the common RS data sets. For each rating, we compute its difference to previous average ratings on the item. We report the distribution of $d = r_{i,j,t} - \hat{r_{j,t}}$, where $r_{i,j,t}$ is the rating by user i on item j at timestamp t, and $\hat{r_{j,t}}$ is the average rating on item j by timestamp t (red bar). We also report a "background" distribution of $\bar{d} = r_{i,j,t} - \hat{r_j}$, where $\hat{r_j}$ is the average rating on item j at all times (black bar).

As shown in Fig. **??**, we have the following observations

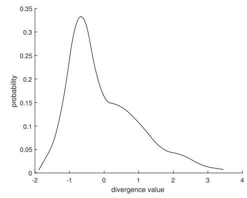


(a) Eachmovie

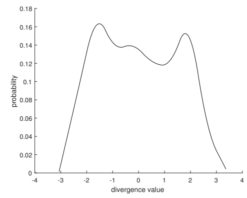


(b) Movielens

Figure 3: The spiral of silence



(a) Yahoo!random



(b) Yahoo!user

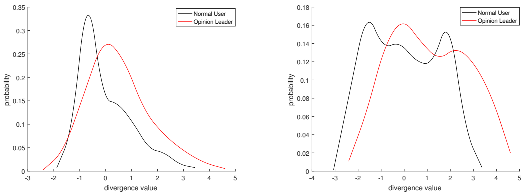
Figure 4: The comparative probability distribution of rating divergence in yahoo!

in all the data sets. (1) Compared with the "background" distribution of rating divergence, in user responses, the cumulative probability for relatively negative feedback (area left of zero) is much smaller. (2) In the "background" distribution, the probability of responding decreases as the rating divergence increases, for positive divergences. In the true distribution, user are equally likely to give relatively neutral and positive opinions. Taken together, people with negative opinions consider themselves as in the minority and fear more to be criticized by the majority. users are less willing to response if their ratings are lower than the perceived majority rating.

When users refuse to speak out relatively negative opinions, more positive feedback is observed while negative feedback is infrequently voiced. In theory this will lead to enlarging average rating. We plot the average rating for each item when they have $x = 10 \times k$ ratings, $k = 1, \cdots, 50$ for Eachmovie and $k = 1, \cdots, 350$ for Movielens. As shown in Fig. **??**, two discoveries are clear. (1) An upward spiral is indeed activated. With more ratings received, the average rating for each item increases. (2) In the tail (approximately more than 350 ratings in Eachmovie and 500 in Movielens), the average rating begins to decrease. This trend suggests that for popular items, the people are more incline to give negative feedback.

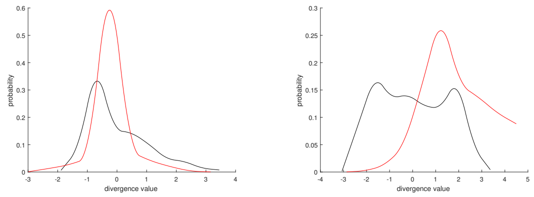
We have shown that the silent minority, defined as the group of users who hold relatively negative feedback, is universally observed in many recommender systems. For the yahoo! data set, there is not information about when the ratings are recorded. However, from the comparison between the random subset and user selected subset in Fig. **??**, we see that, when users are not forced to express their opinions, they produce significantly less negative feedback and much more positive feedback. Thus our discovery of spiral of silent negative feedback in the common recommender systems is verified.

3.2 Opinion Climate



(a) Yahoo!random (b) Yahoo!user

Figure 5: The distribution of rating divergence to opinion leaders



(a) Yahoo!random (b) Yahoo!user

Figure 6: The distribution of community specific rating divergence

Opinion climate is a coined term to describe the mainstream opinion. For a thorough understanding of how the opinion climate develops and influences response patterns, we conduct the following studies on the Yahoo! data set. In the theory, opinion climate is highly affected by the mass media. In recommender systems, opinion leaders play a similar role as mass media. We select 10 most active users O (with largest number of ratings) as opinion leaders, and compute the average rating within opinion leaders $\hat{r_j} = \frac{\Sigma_{i\in O} r_{i,j}}{|O|}$. From Fig. **??**, we see that, in the random setting, rating divergence to opinion leaders distribute as a normal distribution, while in the user selected setting, distribution of rating divergence to opinion leaders move to the right. The difference between two settings is more obvious than divergence to average people. It shows that opinion leaders magnify users' willingness to remain silence when they hold negative feedback and speak out positive opinions.

On the Internet, with the infinite flow of information, users tend to have a limited vision of how others think and behave. They might focus on the opinions alike theirs, and thus perceive a so called "looking-glass perceptron" of opinion climate. To study whether the opinion climate is dependent on user communities, we first represent each user as an $M-$dimensional rating vector on the item universe with M items, and proceed to cluster users by the KNN clustering algorithm. The clustering partition users into K communities. We then compute the average of community ratings $\hat{r_{j,c}} = \frac{\Sigma_{i\in c} r_{i,j}}{|c|}$ for community c. For each user u_i in community c, the rating divergence is $d = r_{i,j} - \hat{r_{j,c}}$.

As shown in Fig. **??**, we have the following observations. (1)In the random setting, rating divergence to community specific majority rating follows a shallow normal distribution, which centers at around 0. (2) In the user selected setting, distribution of rating divergence to community specific majority rating is asymmetric. The probability peak is at around 1.5. For negative opinions, the probability decreases as the divergence increases. For positive opinions, the probability increases as the divergence increases. For extreme positive opinions (at least 1.5 higher than community specific average rating), the probability decreases as the divergence increases. However, the probability is still larger than negative opinions with equal absolute divergence. These observations strongly indicate that users' perceived opinion climate is actually community specific opinion climate.

Table 2: Percentage of hardcore groups

Dataset	$h = 0.5$	$h = 0.6$	$h = 0.7$	$h = 0.8$
Movielens	0.006	0.0026	0.00099	0.0005
Eachmovie	0.0674	0.03174	0.01693	0.01219
Yahoo!random				
Yahoo!user				

Table 3: Percentage of hardcore group overlap

Threshold	$h = 0.5$	$h = 0.6$	$h = 0.7$	$h = 0.8$
Overlap	0.16377	0.11737	0.0908	0.07194

3.3 Hardcore Group

In the theory, hardcore group is a bunch of people who are brave and willing to express different opinions. We define a "hardcore" score for each user, which is $h = \frac{n_i^h}{n_i}$, where n_i is the number of ratings a user i gives to all items, n_i^h is the number of high divergent ratings of user i. In Eachmovie, $n_i^h = |N_i^h|$,where $N_i^h = \{|r_{i,j} - \hat{r_j}| > 0.3\}$. In Movielens, $n_i^h = |N_i^h|$,where $N_i^h = \{|r_{i,j} - \hat{r_j}| > 1.5\}$. We observe a considerable amount of hardcore users in both data sets. As shown in Tab. **??**, the size of group reasonably decreases as the threshold of hardcore score increases.

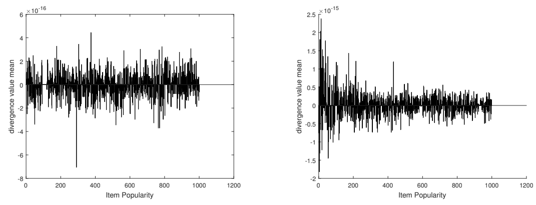
We detect hardcore users in the yahoo data sets, and compare the hardcore users in two subsets. We report the overlap percentage $p_h = \frac{|G_h^{random} \bigcap G_h^{user}|}{|G_h^{random} \bigcup G_h^{user}|}$ in Tab. **??**, where G_h^{random}, G_h^{user} are the set of hardcore users with hardcore score h in the random subset and the user selected subset respectfully. We find that hardcore is a personality of users. Despite of the threshold of hardcore score, hardcore users consistently express minority opinions in both settings.

We then plot the users in the Yahoo! data sets.

3.4 Silence Strategy

Finally we study whether the strategies of holding back are dependent on the items. We have already seen the changes in behavior patterns in Fig. **??**. Next we study whether the responding is directly related to the popularity of items. We order the items in the Yahoo! data sets, according to the number of ratings assigned to them. A box plot is shown in Fig. **??** to reveal the distribution of rating divergence versus the order position of popularity. We can see that, in the random subset, the scales of rating divergence are almost the same for popular and unpopular items. In the user selected subset, the scale of rating divergence narrows as the item becomes less popular. It shows that

Figure 7: Hardcore users



(a) Yahoo!random (b) Yahoo!user

Figure 8: The distribution of rating divergence upon item popularity

users adopt different strategies for popular items and unpopular items. When users are not restricted, they tend to speak out extreme opinions for popular items.

4. MODEL

As with most matrix factorization models, we assume there are K hidden aspects. The user preference is denoted as a vector $U_i \in R^K$, and each item's coefficient to the aspects is denoted as a vector $V_j \in R^K$. We assign a random variable $Bu_i \in R$ to each user and $Bv_j \in R$ to each item to represent the user bias and item bias. The rating given by user U_i to item V_j is denoted by $R_{i,j}$. The ratings are semi-observed. Whether the rating $R_{i,j}$ is observed is denoted by a response variable $X_{i,j}$, where $X_{i,j} = 1$ indicates the rating is observed and otherwise the rating is missing.

Intuitively, a user will give a high rating if the item matches user preference. If her/his opinion is more positive than the opinion climate, she/he is more likely to reveal this rating. Therefore our baseline model assumes the following three generative stages, as shown in Fig. **??**.

The preprocessing stage: For each user, generate the user preference from a Gaussian distribution with mean 0.

$$U_i \sim \mathcal{N}(0, \sigma_u^2) \tag{1}$$

The user bias, item bias and item factors are also generated from Gaussian distributions. $BU_i, BV_j \sim \mathcal{N}(0, \sigma_b^2)$, $V_j \sim \mathcal{N}(0, \sigma_v^2)$.

The rating generation stage: Generate $R_{i,j} \sim \mathcal{N}(U_i V_j + Bu_i + Bv_j, \sigma_r^2)$

The response generation stage: Generate the response value $X_{i,j}$ from Equ.**??**

$$P(X_{i,j} = 1 | R_{i,j}, E_j, \tau) = \frac{1}{1 + \exp(-\tau(R_{i,j} - E_j))} \tag{2}$$

where E_j is the opinon climate for the particular item. In the baseline model, we set the opinion climate to be the average observed ratings.

$$E_j = \frac{\Sigma_j R_{i,j} X_{i,j}}{\Sigma_j X_{i,j}} \tag{3}$$

4.1 Opinion Climate

To model the opinion climate formed by opinion leaders, we use a simple modification in the basic model in Fig. **??**. Instead of computing a global majority rating by Equ.**??**, the average is taken over all expert users e.

$$E_j = \frac{\Sigma_e R_{e,j} X_{e,j}}{\Sigma_e X_{e,j}} \tag{4}$$

To model the community specific ratings,we introduce a random variable γ_i to represent the community assignment by a 1-of-K coding,

$$\gamma_{i,c} = \begin{cases} 1 & \text{if user } i \text{ is in community } c \\ 0 & \text{else} \end{cases} \tag{5}$$

where $\Sigma_c \gamma_{i,c} = 1$. Intuitively, users in the same community share similar preferences. We assume that the "common" preference in community c, denoted by U_c is generated from a zero-mean Gaussian distribution $U_c \sim \mathcal{N}(0, \sigma_u^2)$. Thus we summarize the *preprocessing stage* as follows.

For each user, first generate the community indicator $\gamma_i \sim Discrete(\alpha), \Sigma_c \alpha_c = 1, \forall \alpha_c > 0$, then generate the user preference according to his/her community:

$$U_i \sim \Pi_c \mathcal{N}(U_c, \sigma_u^2)^{\gamma_{i,c}} \tag{6}$$

In the *response generation stage*, we replace the opinion climate in Equ.**??** by a community specific opinion climate.

$$E_{c,j} = \frac{\Sigma_j \gamma_{i,c} R_{i,j} X_{i,j}}{\Sigma_j \gamma_{i,c} X_{i,j}} \tag{7}$$

4.2 Hardcore Group

We further split the users as hardcore and normal people. The persona is denoted by π, which is a binary variable. When the persona is hardcore, the user is more likely to speak out regardless of the opinion climate. As shown in Fig. **??**, in the *preprocessing stage*, draw a persona from a Bernouli distribution $\pi_i \sim \mathcal{B}]\nabla\backslash(\beta), 0 < \beta < 1$. Draw a hardcore coefficient $\tau_z \sim Uniform(\tau_min, \tau_max), z \in \{0, 1\}$. In the *response generation process*, generate a response $X_{i,j}$, given the hidden or observed rating $R_{i,j}$, user persona π_i, and opinion climate by the following equation.

$$P(X_{i,j} = 1 | R_{i,j}, E_j, \pi_i) = \frac{1}{1 + \exp(-\tau_{\pi_i}(R_{i,j} - E_j))} \tag{8}$$

4.3 Silence Strategy

Finally we model the silence strategy as a factor of generating responses. we introduce a variable Vp_j to represent item popularity, we modified the response probability to

$$P(Vp_j | \sigma_{Vp}) = \mathcal{N}(Vp_j | 0, \sigma_{Vp}^2) \tag{9}$$

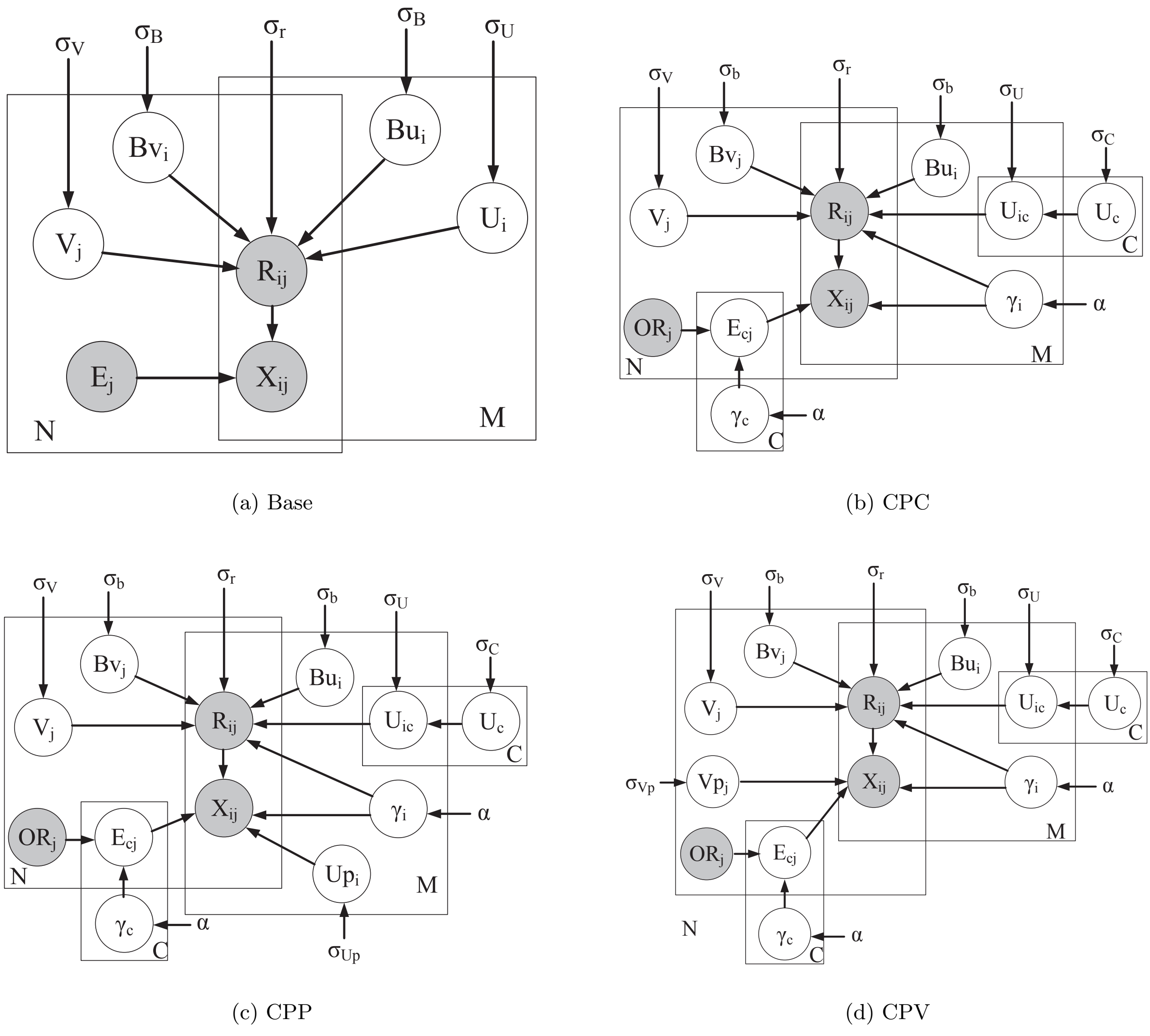
$$P(x_{i,j} | r_{i,j}, E_{c,j}, \gamma_{i,c}) = \frac{1}{1 + \exp(-\tau(r_{i,j} - E_{c,j} - Vp_j))} \tag{10}$$

As shown in Fig. **??** ...

4.4 Inference

$$\begin{aligned} \mathcal{L}_{CPC} = & \sum_{i=1}^{M} \sum_{c=1}^{C} \gamma_{i,c} (log(\alpha_c) + \sum_{j=1}^{N} log(\omega_{i,j,c})) \\ & + log(D(\alpha|\theta)) + log(\mathcal{N}(U_c | 0, \sigma_c^2)) \\ & + log(\mathcal{N}(U_{i,c} | U_c, \sigma_u^2)) + log(\mathcal{N}(V_j | 0, \sigma_v^2)) \\ & + log(\mathcal{N}(Bu_i | 0, \sigma_b^2)) + log(\mathcal{N}(Bv_j | 0, \sigma_b^2)) \end{aligned} \tag{11}$$

$$\omega_{i,j,c} = \begin{cases} P(X_{i,j} = 1 | R_{i,j}, E_{c,j}) P(R_{i,j}) & X_{i,j} = 1 \\ 1 - P(X_{i,j} = 1 | R_{i,j}, E_{c,j}) & X_{i,j} = 0 \end{cases}$$



(a) Base

(b) CPC

(c) CPP

(d) CPV

Figure 9: Graphic representation of models

As shown in Fig.**??**.

E-STEP:

$$\hat{\gamma_{i,c}} \leftarrow \frac{\alpha_c \prod_{j=1}^{N} \omega_{i,j,c}}{\sum_{c=1}^{C} \alpha_c \prod_{j=1}^{N} \omega_{i,j,c}}$$

M-STEP:

$$p_{i,j,c} = u_{i,c}^T v_j + bu_i + bv_j$$

$$E_{c,j} = \frac{(OR_j, \hat{\gamma_c})}{(sign(OR_j), \hat{\gamma_c})}$$

$$B_{i,j,c} = (r_{i,j} - p_{i,j,c})^{[x_{i,j}=1]}\left(-\frac{\tau}{1+e^{(-\tau(p_{i,j,c}-E_{c,j}))}}\right)^{[x_{i,j}=0]}$$

$$\alpha_c \leftarrow \frac{\sum_{i=1}^{M} \hat{\gamma_{i,c}} + \theta_c}{\sum_{c=1}^{C}(\sum_{i=1}^{M} \hat{\gamma_{i,c}} + \theta_c)}$$

$$u_c \leftarrow u_c + lr(-u_c + \sum_{i=1}^{M}(u_{i,c} - u_c))$$

$$u_{i,c} \leftarrow u_{i,c} + lr(\hat{\gamma_{i,c}} \sum_{j=1}^{N} B_{i,j,c} v_j - (u_{i,c} - u_c))$$

$$v_j \leftarrow v_j + lr(\sum_{c=1}^{C} \sum_{i=1}^{M} \hat{\gamma_{i,c}} B_{i,j,c} u_{i,c} - v_j)$$

$$bu_i \leftarrow bu_i + lr(\sum_{c=1}^{C} \hat{\gamma_{i,c}} \sum_{j=1}^{N} B_{i,j,c} - bu_i)$$

$$bv_j \leftarrow bv_j + lr(\sum_{c=1}^{C} \sum_{i=1}^{M} \hat{\gamma_{i,c}} B_{i,j,c} - bv_j)$$

Algorithm 1: EM Algorithm for the CPC model

we add the update formula in M-STEP of algorithm.**??**

$$BUP_{i,j,c} = \frac{\tau}{1+e^{(-\tau((r_{i,j}^{[x_{i,j}=1]} p_{i,j,c}^{[x_{i,j}=0]})-m_{j,c}-up_i))}} \quad (12)$$

$$up_i \leftarrow up_i + lr(\sum_{c=1}^{C} \hat{\gamma_{i,c}} \sum_{j=1}^{N} B_{i,j,c} - up_i) \quad (13)$$

we add the update formula in M-STEP of algorithm.**??**

$$BVP_{i,j,c} = \frac{\tau}{1+e^{(-\tau((r_{i,j}^{[x_{i,j}=1]} p_{i,j,c}^{[x_{i,j}=0]})-m_{j,c}-vp_j))}} \quad (14)$$

$$vp_j \leftarrow vp_j + lr(\sum_{c=1}^{C} \sum_{i=1}^{M} \hat{\gamma_{i,c}} B_{i,j,c} - vp_j) \quad (15)$$

5. EXPERIMENT

The data sets used in this section include Yahoo!, Mtweet ...

5.1 Comparative Study

The comparative models are (1)MF: the standard matrix factorization model [**?**]; (2)PMF: the probabilistic matrix factorization model [**?**]; (3)CPT-v and Logit-vd: the first MNAR models [**?**] (4) MF-MNAR [**?**]: the probabilistic MNAR model which masks the rating matrix by a response matrix. ...

The evaluation metric are (1) AUC: area under curve, which is a common measure to evaluate the performance of binary classifiers (2)NDCG@n: normalized discounted cumulative gain for top n results, which is a standard measure for ranking systems.

As shown in Tab. **??** ...

As shown in Fig. **??**

5.2 Parameter Tunning

6. CONCLUSION

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Table 4: Comparative AUC Performances

	CPT-v	MF-MNAR	Base	Community	Expert	User	Item
yahoo	0.825	0.863	0.803	0.880	0.882	0.865	0.345
Mtweet	0.828	0.856	0.823	0.864	0.863	0.847	0.823

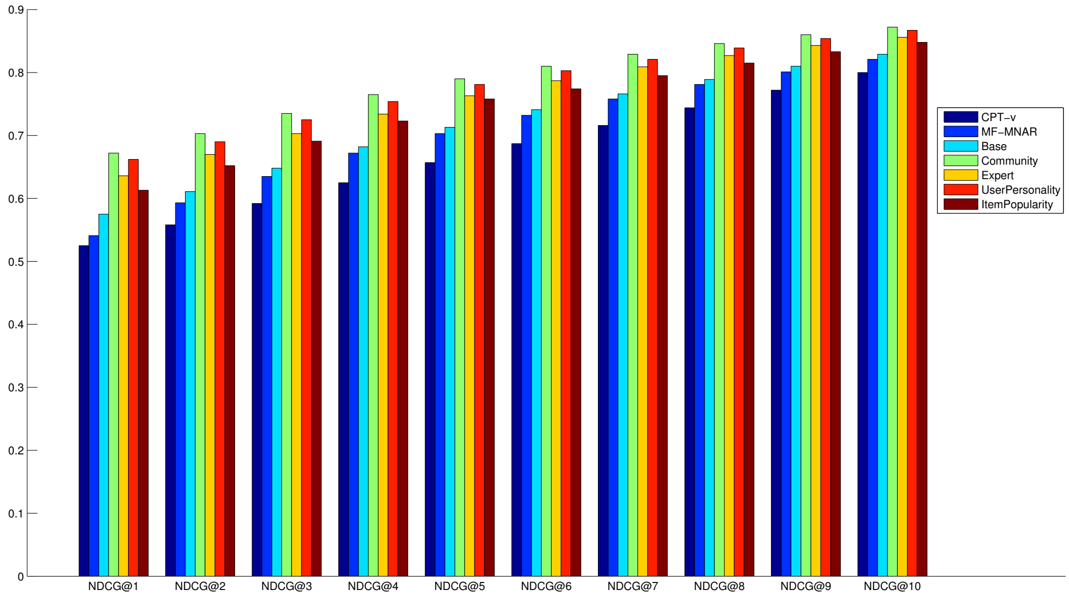
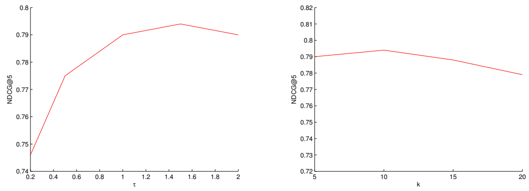


Figure 10: NDCG performance at top K items



(a) τ (b) K

Figure 11: NDCG@5 performance over different values of parameters

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