Demand Disagreement[*]

Christian Heyerdahl-Larsen[a], Philipp Illeditsch[b]

[a]*BI Norwegian Business School*
[b]*Mays Business School, Texas A&M University*

Abstract

Disagreement about macroeconomic fundamentals accounts for only part of the disagreement about future interest rates, creating a "disagreement correlation" puzzle. This puzzle arises because standard equilibrium models with belief differences predict a strong link between asset return disagreement and fundamental disagreement, a link not supported by the data. We address this puzzle by introducing a model where disagreement about future demand for savings—driven by disagreement over the prevalence of patient versus impatient investors in the economy—generates asset return disagreement. Our mechanism produces stochastic yield volatility, time-varying bond risk premia, and an upward-sloping yield curve. Empirically, we construct a proxy for demand disagreement by isolating the component of yield

disagreement unrelated to disagreement about macro-fundamentals. This proxy is positively related to yields and their volatilities, and predicts future bond risk premia, consistent with the predictions of our demand disagreement model.

I. Introduction

Investing involves balancing risk and return, which requires predicting future cash flows and prices. Beyond concerns about future economic conditions, investors may worry about who will be in the market when they sell an asset? Will demand be strong or weak? This demand uncertainty arises from unknown future investors' time or risk preferences, factors unrelated to macroeconomic conditions that are difficult to predict. Most asset pricing models overlook this uncertainty, assuming investor preferences and future demand are known, thus tightly linking asset prices to economic fundamentals.

Classical asset pricing models imply that variations in discount rates arise exclusively from fundamental shocks, leading to a high correlation between fundamentals and asset prices. Even when incorporating investor disagreement about future fundamentals, these models maintain a tight link between fundamental disagreements and disagreements about asset prices. Specifically, if investors agree on how fundamentals map into prices, despite disagreeing about the fundamentals themselves, no residual price disagreement remains, a pattern inconsistent with observed data (Section II and (Giacoletti, Laursen, and Singleton, 2021)). Kenneth Singleton highlighted this point in his 2021 Presidential Address to the American Finance Association, observing that variations in disagreement over inflation and output growth alone do not explain the full extent of disagreements about future interest rates. We refer to this challenge as the "disagreement

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correlation puzzle."

We address this puzzle by allowing random shifts in the population shares of investors with different time preferences, while individual preferences remain fixed. Because investors have incomplete information about these shifts, they form different views, resulting in persistent disagreement about the future aggregate demand for savings and consumption, even when fundamentals are known. This disagreement leads to speculative trade that moves asset prices independently of fundamentals, explaining key asset pricing anomalies. Our demand disagreement model thus accounts for the low correlation between asset returns and fundamentals and explains disagreements about future returns not driven by disagreement about macroeconomic fundamentals, resolving both the correlation puzzle and the disagreement correlation puzzle.

We show that demand disagreement generates stochastic yield volatility, time-varying bond risk premia, and an upward-sloping real yield curve. Increases in demand disagreement raise yield volatility across all maturities and predict higher future excess returns on nominal bonds. Using data from the Survey of Professional Forecasters (SPF), we construct a measure of demand disagreement by isolating the component of yield disagreement not explained by disagreement about macroeconomic fundamentals and empirically confirm its positive relationship with yield volatility and its predictive power for excess bond returns.

To formalize the effects of demand disagreement on asset prices, we develop a transparent heterogeneous agent model in an overlapping generation (OLG) framework (Blanchard (1985) and Gârleanu and Panageas (2015)). There are two types of investors, patient and impatient, whose population shares fluctuate randomly over time, generating an exogenous variation in aggregate asset demand that is independent of economic fundamentals. Investors observe current population shares but lack information about their future evolution. Each investor exhibits a "false consensus bias" as defined in Ross, Green, and House (1977), believing that their own type will dominate in the long run, leading to persistent disagreement about future

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aggregate demand for savings and, respectively, for consumption. All investors have log-utility, identical endowments, and trade in complete markets, enabling closed-form solutions for the stochastic discount factor and isolating the effects of demand disagreement from other sources of asset price variation. The OLG model ensures that neither type dominates in the long run and allows us to compute unconditional moments of equilibrium asset prices.

Unlike models such as in (Albuquerque, Eichenbaum, Luo, and Rebelo, 2016), where the representative agent's time-discount rate is exogenously time-varying and risk premia arise from preferences for early resolution of uncertainty, our model offers a more nuanced mechanism for pricing demand risk. In our framework, the market price of demand risk is determined by the consumption shares of patient and impatient investors. When both types have equal consumption shares, the market view (the consumption share weighted average belief) matches that of an outside observer (the econometrician), so the market price of demand risk is zero. In this case, patient investors, optimistic about future demand for savings, long the consol bond, while impatient investors, pessimistic, short the consol bond. As the consumption share of impatient investors increases, their influence on prices increases, pushing down the price of the consol. To the econometrician, the bond then appears underpriced, resulting in a positive market price of demand risk. In contrast, when patient investors have a larger consumption share, the bond appears overpriced, and the market price of demand risk turns negative.

While the conditional risk premium can be positive or negative depending on the dominance of patient or impatient investors, the unconditional risk premium is typically positive and increases with disagreement. This is because the demand risk exposure of an asset and its price of risk are both high during the same periods, which amplifies the average premium. This effect is more pronounced for long-duration bonds, as their sensitivity to demand shocks is greater, resulting in an upward-sloping yield curve. Thus, our model explains how demand disagreement generates both time-varying risk premia and excess volatility in

bond markets, with the magnitude and sign of the risk premium endogenously determined by the evolving distribution of investor types and their consumption shares.

Our paper relates to several strands of literature. First, it extends the heterogeneous beliefs literature (Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), Basak (2000)).[1] Unlike previous work, which primarily focuses on disagreement over macroeconomic fundamentals, we study disagreement about the future distribution of patient and impatient investors. This approach allows us to address both the correlation and disagreement correlation puzzles while matching key asset pricing facts. Our overlapping generations (OLG) framework ensures stationarity and enables analysis of unconditional asset pricing moments.

Second, while related to research on heterogeneous preferences such as Dumas (1989), Wang (1996), Chan and Kogan (2002), Gollier and Zeckhauser (2005), Gomes and Michaelides (2008), Weinbaum (2009), Zapatero and Xiouros (2010), Cvitanic, Jouini, Malamud, and Napp (2012), Longstaff and Wang (2013), Bhamra and Uppal (2014), Gârleanu and Panageas (2015), and Ehling and Heyerdahl-Larsen (2017), our focus is on how disagreement about the future distribution of investor preference types affects asset prices.

Third, we extend the literature on the role of preference shocks in asset pricing (Garber and King (1983), Campbell (1993)).[2] While Maurer (2012) and Albuquerque, Eichenbaum, Luo, and Rebelo (2016) use time discount factor shocks in representative agent models to address the correlation puzzle, we show that demand shocks are priced even without recursive preferences when investors differ in their time discount rates and beliefs. This highlights how our model captures the impact of heterogeneous preferences and

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[1]Models with disagreement include Scheinkman and Xiong (2003), Basak (2005), Berrada (2006), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic, Jouini, Malamud, and Napp (2012), Bhamra and Uppal (2014), Buraschi, Trojani, and Vedolin (2014), Cujean and Hasler (2017), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018), Bauer and Chernov (2024), and Xiouros and Zapatero (2024).

[2]Preference shocks have been frequently used in macroeconomics, international economics, and international finance (e.g. Pavlova and Rigobon (2007))

beliefs on asset prices, risk premia, and the disconnect between asset returns and fundamentals.

Fourth, our work also connects to the demand discovery literature (Grossman (1988), Gallmeyer, Hollifield, and Seppi (2017)), but differs by assuming that investors "agree to disagree" about future demand. This assumption increases tractability and enables us to analyze how demand disagreement affects a range of asset pricing outcomes.

Fifth, we contribute to the OLG asset pricing literature, including work on borrowing constraints (Constantinides, Donaldson, and Mehra (2002)), life-cycle effects (Gomes and Michaelides (2005)), innovation risk (Gârleanu, Kogan, and Panageas (2012)), heterogeneous agents (Gârleanu and Panageas (2015)), and experience-based learning (Ehling, Graniero, and Heyerdahl-Larsen (2018)). Although recent models (Gârleanu and Panageas (2021) and Gârleanu and Panageas (2023)) explain many asset pricing features without aggregate output risk, our approach is distinct in that asset prices are driven by disagreement about future asset demand.[3]

Finally, our paper contributes to the literature on investor sentiment, broadly defined as beliefs about future cash flows and discount rates that are not justified by fundamentals (see Baker and Wurgler (2007) and Zhou (2018)). Many theoretical models attribute sentiment to psychological biases, but these models typically imply a high correlation between price and fundamental disagreement. However, this contradicts empirical evidence. For example, disagreement about future output growth drives price disagreement in Dumas, Kurshev, and Uppal (2009) and economic activity in Angeletos and Lao (2013). In contrast, our model focuses on disagreement about the future distribution of investor preference types, enabling us to explain asset pricing anomalies while generating a lower correlation between disagreement over prices and fundamentals, consistent with observed data.

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[3]For a recent review of heterogeneity and inequality in asset pricing, see Panageas (2020).

II. Empirical Motivation - Disagreement Correlation Puzzle

We motivate our demand disagreement model by discussing the empirical limitations and challenges of existing asset pricing models. The fundamental pricing equation of an asset with future payoff x_{t+1} is $p_t = \mathrm{E}_t\left[m_{t+1}x_{t+1}\right]$ where m_{t+1} represents the stochastic discount factor or SDF (Cochrane (2005)). The SDF is a function of the fundamental variables of the economy, expressed as $m_{t+1} = f$ (fundamentals).

A. Data Availability

The data used in this paper are publicly available and can be downloaded from Mendeley Data (`https://data.mendeley.com/`) in three separate parts due to size constraints. The replication package, including code and data, is available at `https://github.com/cheyerdahllarsen/DemandDisagreement.git`.

Part A: Main Dataset (DOI: 10.17632/5vpb5rzv7s.1) - Complete data folder structure (2-3GB) containing SPF data, financial market data, MATLAB files, Excel files, and NumPy arrays. This is the core dataset required for replication.

Part B: HDF5 Simulation Data (DOI: 10.17632/md24pnhz67.1) - All HDF5 (.h5) files from model simulations (8-9GB). These files contain the computationally intensive simulation results and should be placed in the 'Data/Model Disagreement/' folder.

Part C: Consumption Share Distribution (DOI: 10.17632/rk343v5jk4.1) - Consumption share distribution files for the Online Appendix (100-200MB). These files are required for reproducing the consumption share analysis and should be placed in the 'Data/Model Disagreement/' folder.

To set up the complete replication environment, users should download all three parts from their respective Mendeley Data DOIs, extract the main dataset first, then add the HDF5 files and consumption share distribution files to the appropriate directories as specified in the GitHub repository.

B. Correlation Puzzle

In classical asset pricing models, all fundamental shocks are shocks to the supply side of the economy, such as output shocks in Lucas (1978) endowment economies or productivity shocks in Jermann (1998) production economies. In these models, stock market returns are highly correlated with measures of output growth, such as consumption and GDP growth, which is in stark contrast to the weak short- and long-term correlations between stock returns and output growth observed in the data. Cochrane and Hansen (1992), Campbell and Cochrane (1999), and Cochrane (2005) refer to this finding as the "correlation puzzle." For example, the first main column of Table I shows that the correlation between stock returns and consumption across one-, five-, and ten-year time horizons does not surpass 30%. Although the correlation with dividends is substantially greater, rising with the horizon, at 65% it remains significantly lower than estimates from classic asset pricing models.

| | Stock Returns | | Trading Volume | | One-Year Yield | |
	Dividends	Consumption	Dividends	Consumption	Dividends	Consumption
1 year	0.08	0.00	0.21	0.29	0.26	-0.16
5 year	0.46	0.28	0.35	0.38	0.20	0.04
10 year	0.65	0.11	0.31	0.40	0.24	0.06

Table I: Correlation Puzzle. This table presents the correlations between dividend and consumption growth with (i) stock returns, (ii) percentage changes in trading volume, and (iii) changes in the one-year real yield. The data are sourced from Shiller's and NYSE web pages and cover yearly observations from 1891 to 2009. The results indicate a low correlation between asset returns/trading volume and fundamental factors such as dividends and consumption growth.

While there is no trade in representative agent models like those by Campbell and Cochrane (1999) or Bansal and Yaron (2004), asset pricing models with heterogeneous agents predict a strong link between trading volume and fundamentals, which is at odds with the weak short-term and long-term correlations observed between stock market trading volume and both dividend and consumption growth, as displayed in the second main column of Table I.[4] The third main column of Table 1 shows low correlations between

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[4]We refer the reader to Panageas (2020) for a survey of the literature on heterogeneous agent models.

the one year real yield and consumption/dividend growth.

C. Disagreement Correlation Puzzle

There is also a tight link between disagreements on future asset prices and those on fundamentals in models with heterogeneous beliefs, which is at odds with the data, as we demonstrate below in the case of interest rates. Making matters even worse, this tight link in heterogeneous belief models is difficult to break, since all investors agree on the pricing functional that connects fundamentals to asset prices. This anomaly is similar to the correlation puzzle and has not been explored in the literature. We refer to it as the Disagreement Correlation Puzzle. Specifically, in a Markov equilibrium with disagreement about fundamentals, the price of an asset for every investor indexed by i is $p_t = \mathrm{E}_t^i \left[m_{t+1}^i x_{t+1} \right] = p \left(X_t^F \right)$, where $\mathrm{E}_t^i \left[\cdot \right]$ represents investor i's conditional expectation and m_{t+1}^i represents investor i's stochastic discount factor. All investors form beliefs based on the same information captured by the state vector X_t^F and agree on prices. Hence, they agree on the pricing functional $p(\cdot)$ and the asset price forecast of investor i is

$$\hat{p}_{t,t+\Delta t}^i = E_t^i \left[p_{t+\Delta t} \right] = E_t^i \left[\mathrm{E}_{t+\Delta t}^i \left[m_{t+1}^i x_{t+1} \right] \right] = E_t^i \left[p \left(X_{t+\Delta t}^F \right) \right], \qquad 0 < \Delta t \leq 1. \tag{1}$$

Investors who have different views on the distribution of fundamentals, represented by the state vector X_t^F, will disagree on the distribution of future asset prices. More importantly, any variation in disagreement about future asset prices is due to variation in disagreement about fundamentals. However, as pointed out by Kenneth Singleton in his 2021 AFA Presidential Address and further examined by Giacoletti, Laursen, and Singleton (2021), variations in disagreement over macroeconomic fundamentals alone do not fully explain the observed variation in disagreement over future interest rates.[5] We confirm this finding by analyzing a distinct set of survey data from forecasters, which encompasses a broader array of

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[5]Singleton's 2021 Presidential Address was titled "How Much 'Rationality' Is There In Bond Market Expectations."

macroeconomic variables and a longer sample. Specifically, we consider quarterly data from the Survey of Professional Forecasters (SPF) spanning from the third quarter of 1981 to the second quarter of 2024. In this data set, each forecaster provides predictions for both the three-month Treasury bill rate over each of the next four quarters and various macroeconomic fundamentals. The forecasts of macroeconomic fundamentals encompass the CPI, real GDP, aggregate consumption, federal government consumption and gross investments, corporate profits after tax, residential fixed investment, nonresidential fixed investments, unemployment rates, corporate bond yields, and housing starts.[6] For every quarter, disagreement for each forecast horizon and variable is defined as the cross-sectional standard deviation across forecasters in the panel. Subsequently, we regress the yield disagreement for forecast horizons of one quarter to a year on the corresponding disagreements about macroeconomic fundamentals.

Quarters	Level	Changes	Log
1	0.719	0.318	0.497
2	0.769	0.224	0.553
3	0.765	0.217	0.604
4	0.743	0.240	0.627

Table II: Disagreement Correlation Puzzle. This table presents the unadjusted R^2 values from regressions of the disagreement in the T-bill rate on fundamental disagreements for one to four quarters ahead. The fundamental variables include CPI, real GDP (RGDP), aggregate consumption (RCONSUM), federal government consumption and gross investments (RFEDGOV), corporate profits after tax (CPROF), residential fixed investment (RRESINV), nonresidential fixed investments (RNRESIN), unemployment (UNEMP), corporate bond yields (BOND), and housing starts (HOUSING). The regressions are conducted using the level, changes, and logarithmic transformations of these variables. The results indicate that fundamental disagreements explain only a modest portion of the variation in T-bill rate disagreements.

Table II shows the R^2 of these regressions using disagreement levels in the second column, changes in disagreement in the third column, and the logarithm of disagreement in the final column.[7] Despite the fact that we include significantly more explanatory variables in the regression than Giacoletti, Laursen, and

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[6]In the primary model, we incorporate corporate bond yields. Some might argue that this closely mirrors the T-bill disagreement. Thus, we present results excluding the corporate bond yield (BOND variable in SPF) in the Internet Appendix. Predictably, this omission decreases the regression R^2. For example, in the four-quarter regressions, the R^2 values are 0.592, 0.151, and 0.513 for the level, change, and log regressions, respectively.

[7]Details on the coefficients on the individual variables and their t-statistics can be found in the Internet Appendix.

Singleton (2021) to explain variations in yield disagreement, the results are consistent with their findings, that is, fundamental disagreement cannot fully explain disagreement about interest rates. Previous research has established that macroeconomic disagreement, and in particular inflation disagreement is an important driver of bond yields (see, for example, Xiong and Yan (2010) and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2018)). Although the R^2 values confirm the importance of macro-disagreement, a substantial residual component of yield disagreement remains unexplained. In the next section, we construct a measure for yield disagreement unrelated to macro-disagreement and interpret it as "demand disagreement", which our theoretical model in Section III is designed to capture as disagreement about future demand for savings and consumption. Although our novel economic mechanism provides a plausible explanation for this residual disagreement measure and accounts for stylized facts in the bond and equity markets, other mechanisms may also contribute to it.[8]

D.   Measuring Demand Disagreement

In the preceding section, we show that disagreement about macroeconomic fundamentals does not fully account for variations in interest rate disagreement. Our model attributes the remaining unexplained variation, termed demand disagreement, to differences in beliefs about future time preferences. In this section, we quantify demand disagreement using the residual component of yield disagreement, orthogonal to disagreement about fundamentals, as a proxy. We then calibrate our model to this residual variation and outline the specifics of our measurement approach in the following section.

In the SPF, each forecaster reports their forecast for the three-month Treasury bill for the next four quarters together with the forecast of macroeconomic fundamentals. Everybody observes and agrees on the yield and thus we know from Equation (1) that all forecasters should have the same mapping between

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[8]For instance, heterogeneity about the contribution of firms to aggregate cash flow may contribute to the residual, provided it does not lead to significant disagreement about aggregate cash flows. We thank an anonymous referee for this example.

macroeconomic fundamentals and the yield forecast if there is no demand disagreement. Hence, we run the pooled regression

$$\hat{y}^i_{t,t+\Delta} = \beta_0 + \beta'_X \hat{X}^{i,F}_{t,t+\Delta t} + \beta_y \hat{y}^i_{t,t} + \varepsilon^i_{t,t+\Delta t}, \qquad \Delta t \in \{1Q, 2Q, 3Q, 4Q\}, \tag{2}$$

where $\hat{X}^{i,F}_{t,t+\Delta t}$ denotes the vector of macroeconomic forecasts made by forecaster i at time t for Δt quarters ahead and $\hat{y}^i_{t,t+\Delta}$ denotes the corresponding Treasury yield forecast. We also include the nowcast of each forecaster for the yield, $\hat{y}^i_{t,t}$ to account for the persistence in yields.

For each forecaster i, the residual term $\varepsilon^i_{t,t+\Delta t}$ captures the yield forecast that is not related to their forecast of macroeconomic fundamentals. Hence, we use as a proxy for demand disagreement the cross-sectional standard deviation of this residual.[9] Specifically,

$$DD_{t,t+\Delta t} = \mathrm{SD}_t \left(\varepsilon^i_{t,t+\Delta t} \right), \tag{3}$$

where the standard deviation at time t, $\mathrm{SD}_t \, ()$, is calculated across all individual forecasters.

Figure 1 shows that both yield and demand disagreement increase with the forecast horizon. As the horizon increases, a larger fraction of yield disagreement is captured by demand disagreement. For instance, at the one-quarter horizon, the demand disagreement is 65% of the yield disagreement, but at the four-quarter horizon the demand disagreement is 76%.

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[9]In the Internet Appendix we consider an alternative based on the logarithmetic specification in Table II. In this case, we define the demand disagreement as $DD_{t,T} = e^{\epsilon_{t,T}}$ where $\epsilon_{t,T}$ is the error term of the spanning regression using logarithmic disagreements. This ensures that the disagreement is positive. The key finding is that qualitatively the results are the same. However, the amount of disagreement measured this way is much higher than the disagreement we measure in our main specification.
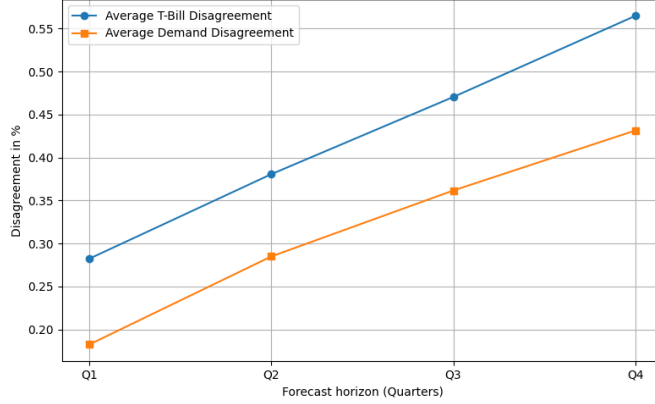
Figure 1: Measuring Demand Disagreement. This figure plots the average yield disagreement for the three-month Treasury bill and the demand disagreement across one- to four-quarter horizons. Yield disagreement is calculated as the cross-sectional standard deviation of yield forecasts, while demand disagreement is the cross-sectional standard deviation of the error terms from the regression in Equation (2). The results show that demand disagreement accounts for a significant portion of interest rate disagreement and increases with the forecast horizon.

III. Model

In this section, we present a parsimonious equilibrium overlapping generations (OLG) model with a continuum of investors who have different time preferences and beliefs. Asset price dynamics in this framework are driven entirely by demand disagreement arising from heterogeneous expectations about the future prevalence of patient and impatient investors in the economy rather than by differences in beliefs about macroeconomic fundamentals.

A. Consumers, Output and Asset Markets

We consider a continuous-time overlapping generations economy in the tradition of Blanchard (1985) and Gârleanu and Panageas (2015). Every agent lives until the stochastic time of death τ which is exponentially distributed with hazard rate, $\nu > 0$.[10] A continuum of agents of mass ν is born every period and therefore the total population size remains constant, that is, $\int_{-\infty}^{t} \nu e^{-\nu(t-s)} \, ds = 1$, where $\nu e^{-\nu(t-s)}$ denotes the population density.

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[10]If $\nu = 0$ agents live forever and we return to the classical infinite horizon economy.

We follow Gârleanu, Kogan, and Panageas (2012), Panageas (2020), and Gârleanu, Panageas, and Zheng (2021) by letting investors born at time t receive an equal share of a new "tree" that arrives at time t. The time t dividend of a tree arriving at time $s \leq t$ is $Y_{s,t} = \nu e^{-\nu(t-s)}Y_t$, where ν denotes the depreciation rate and, therefore, the aggregate dividend, which is equal to aggregate output, at time t is $\int_{-\infty}^{t} Y_{s,t}\, ds = \int_{-\infty}^{t} \nu e^{-\nu(t-s)}Y_t\, ds = Y_t$.[11] Aggregate output Y_t follows a Geometric Brownian Motion, that is, $dY_t = \mu_Y Y_t\, dt + \sigma_Y Y_t\, dZ_{Y,t}$. We refer to the Brownian motion $Z_{Y,t}$ as the supply shock and the constants μ_Y and σ_Y as the expected output growth rate and output growth volatility, respectively.

There are two types of agents with common risk-preferences but different time discount rates and beliefs. Patient type a agents have the low time discount rate ρ^a, satisfying $\nu + \rho^a > 0$ to ensure a finite wealth-consumption ratio in all states, and impatient type b agents have the high time discount rate $\rho^b > \rho^a$. The fraction of patient and impatient newborns is α_t and $1 - \alpha_t$, respectively. Let $\alpha_t = \alpha(l_t) = 1/(1 + \exp(-l_t)) \in (0,1)$, where l_t follows the mean reverting Ornstein-Uhlenbeck process

$$dl_t = \kappa\left(\bar{l} - l_t\right) dt + \sigma_l\, dZ_{\alpha,t}, \qquad \kappa, \sigma_l > 0. \tag{4}$$

The Brownian motion $Z_{\alpha,t}$ is independent of the supply shock, $Z_{Y,t}$. An increase in l_t increases the fraction of patient investors, and thus decreases the demand for current consumption, and thus we refer to $Z_{\alpha,t}$ as the demand shock.

Let $P_{s,t}$ denote the value of the vintage dividend tree of time s at time t. The instantaneous return of this stock is $dR_t^S = dP_{s,t}/P_{s,t} + Y_{s,t}/P_{s,t}dt$. We drop the s subscript because all stocks are perfectly correlated and thus the stock return, R_t^S, does not depend on the vintage s. We will therefore refer to this "representative" stock as the stock market. In addition to trading stocks in positive supply, agents can

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[11] We set the depreciation rate to be the same as the hazard rate.

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trade an instantaneously risk-free asset with real short rate, r_t, in zero-net-supply, and a consol in zero-net-supply with the time-dependent coupon rate $e^{(\mu_Y - \sigma_Y^2 - \nu)t}$ and price $B_t^C = \int_t^\infty e^{(\mu_Y - \sigma_Y^2 - \nu)(T-t)} B_t^T \, du$, where B_t^T denotes the price of a discount bond that matures at time T. We denote the instantaneous return of the consol bond by dR_t^C, derive its price in Corollary 2 in closed form, and show at the end of Section IV, that it is only exposed to demand shocks, $Z_{\alpha,t}$. Any other bond that is not instantaneously risk-free would also complete the market, but its price would not be given in closed form. The real short rate, the stock market diffusion coefficients, the expected stock market return, the consol diffusion coefficient, and the expected return on the consol are all determined in equilibrium. Finally, there is an annuity contract as in Blanchard (1985) and Gârleanu and Panageas (2015) that is offered by a competitive insurance industry that pays the actuarially fair rate ν per unit of wealth. Hence, the cash flows from this annuity contract from the perspective of investors for all $t \leq \tau$ are $d\mathcal{L}_t = \nu W_t^{\mathcal{L}} \, dt$ with $\mathcal{L}_\tau = -W_\tau^{\mathcal{L}}$.

Agent $i \in \{a, b\}$ born at time s chooses the consumption process $c_{s,t}^i$ to maximize the lifetime expected utility with respect to her belief $\mathbb{P}^i$ (determined in the next subsection)

$$E_s^i \left[\int_s^\tau e^{-\rho^i(t-s)} \log\left(c_{s,t}^i\right) dt \right], \tag{5}$$

subject to the dynamic budget constraint

$$dW_{s,t}^i = \left(r_t W_{s,t}^i - c_{s,t}^i\right) dt + \Phi_{s,t}^i \left(dR_t - r_t dt\right) + \Psi_{s,t}^i \left(dR_t^\alpha - r_t dt\right) + d\mathcal{L}_{s,t}^i, \tag{6}$$

where $W_{s,s}^i = P_{s,s}$, $W_{s,\tau}^i = W_{s,\tau^-}^i + \mathcal{L}_{s,\tau}^i = 0$, and $\Phi_{s,t}^i$ and $\Psi_{s,t}^i$ denote the dollar amounts held in the stock and the consol, respectively.

B. Information and Beliefs

There are two fundamental sources of heterogeneity in the model: investors differ in both their time preferences (patient vs. impatient) and their beliefs about the prevalence of their types (optimist vs. pessimist). In principle, this would allow for four types of agents in each cohort (high/low time discount rate ρ, optimistic/pessimistic about $\bar{l}$). To maintain tractability, we impose that each investor type believes their own preferences are more common than they actually are; a cognitive bias often referred to as the "false consensus bias" in the psychology literature (e.g. Ross, Green, and House (1977)).[12] Specifically, patient investors are optimistic about the long-run mean ($\bar{l}^a \geq \bar{l}$), while impatient investors are pessimistic ($\bar{l}^b \leq \bar{l}$). Importantly, we do not assume that either group has more accurate beliefs; rather, both patient and impatient investors are equally biased, so that $\bar{l}^a - \bar{l} = \bar{l} - \bar{l}^b$. This symmetry links time preference and belief heterogeneity, allowing us to summarize cross-sectional differences with a single endogenous state variable, the consumption share of patient investors f_t.

Formally, investors observe α_t and thus l_t, but disagree about the long-run mean of l_t. Investors of type $i \in \{a, b\}$ believe that l_t follows

$$dl_t = \kappa \left(\bar{l}^i - l_t \right) \ dt + \sigma_l \ dZ_{\alpha,t}^i, \tag{7}$$

where $Z_{\alpha,t}^i$ denotes the demand shock perceived by an investor with belief $\mathbb{P}^i$. The true and perceived demand shocks are linked by $Z_{\alpha,t}^i = Z_{\alpha,t} - \sigma_\Delta^i t$ with $\sigma_\Delta^i = \left(\bar{l}^i - \bar{l} \right) (\kappa/\sigma_l)$. Investors do not learn and only disagree about the long-run mean, so σ_Δ^i is constant.[13] Girsanov's theorem implies that the likelihood ratio between investor i's and the data-generating belief is $\eta_t^i \equiv (d\mathbb{P}^i/d\mathbb{P})_{\mathcal{F}_t} = \exp\left(-0.5 \left(\sigma_\Delta^i \right)^2 t + \sigma_\Delta^i Z_{\alpha,t} \right)$. We

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[12]In the Internet Appendix, we show that our results are qualitatively the same if we reverse the false consensus bias to $\bar{l}^a < \bar{l} < \bar{l}^b$ and discuss the small quantitative differences it implies for unconditional asset pricing moments.

[13]We discuss the effects of learning from experience which generates time-varying, cohort-dependent disagreement, and richer belief dynamics in Section VI and the Internet Appendix.

capture disagreement between patient and impatient investors with the likelihood ratio:

$$\eta_t \equiv \frac{d\mathbb{P}^b}{d\mathbb{P}^a} \mid_{\mathcal{F}_t} = \frac{\eta_t^b}{\eta_t^a} = e^{-\frac{1}{2}\sigma_\Delta^2 t - \sigma_\Delta Z_{\alpha,t}^a}, \qquad \text{and} \qquad \sigma_\Delta = \sigma_\Delta^a - \sigma_\Delta^b = 2\frac{\kappa}{\sigma_l}\left(\bar{l}^a - \bar{l}\right) = 2\frac{\kappa}{\sigma_l}\left(\bar{l} - \bar{l}^b\right). \qquad (8)$$

We use the parameter $\sigma_\Delta \geq 0$ to quantify the degree of demand disagreement between investor types.

C. Arrow-Debreu Price System

Before computing the equilibrium, it is convenient to summarize the price system in terms of the investor-specific state price density, ξ_t^i, which captures the investor-specific belief, but the common Arrow-Debreu price system. For example, the price of an asset with payoff x_{t+1} is $p_t = \mathrm{E}_t^a\left[\frac{\xi_{t+1}^a}{\xi_t^a}x_{t+1}\right] = \mathrm{E}_t^b\left[\frac{\xi_{t+1}^b}{\xi_t^b}x_{t+1}\right]$. The dynamics of the state price density of investor $i \in \{a,b\}$ are $d\xi_t^i = -r_t\xi_t^i dt - \theta_{Y,t}\xi_t^i dZ_{Y,t} - \theta_{\alpha,t}^i\xi_t^i dZ_{\alpha,t}^i$, where $\theta_{Y,t}$ denotes the market price of supply shock risk $Z_{Y,t}$ and $\theta_{\alpha,t}^i$ denotes the market price of demand shock risk $Z_{\alpha,t}^i$ perceived by investor i.[14] Individual state price densities are linked by the likelihood ratio η_t given in Equation (8) and, thus, the perceived market prices of demand risk are linked through the disagreement parameter σ_Δ, that is, $\xi_t^a = \eta_t\xi_t^b$ and $\theta_{\alpha,t}^a - \theta_{\alpha,t}^b = \sigma_\Delta$. Similarly, we present the Arrow-Debreu price system under the data generating belief by the state price density $\xi_t = \xi_t^a\eta_t^a = \xi_t^b\eta_t^b$ because the price of an asset with payoff x_{t+1} can be equivalently written as $p_t = \mathrm{E}_t\left[\frac{\xi_{t+1}}{\xi_t}x_{t+1}\right]$ with $d\xi_t = -r_t\xi_t - \theta_{Y,t}\xi_t dZ_{Y,t} - \theta_{\alpha,t}\xi_t dZ_{\alpha,t}$ and $\theta_{\alpha,t} = \theta_{\alpha,t}^i - \sigma_\Delta^i$.

IV. Equilibrium

In this section, we solve for the equilibrium that is defined in a standard way. That is, given preferences, endowments, and beliefs, an equilibrium is a collection of allocations and a price system such that all agents

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[14]Investors within a patient or impatient cohort agree on the dynamics of α_t and so their beliefs and, thus, state price densities do not depend on their date of birth s. In Section VI, we discuss a model that allows investors within a cohort to learn from their experience, and thus there is a difference in beliefs between agents with the same time preferences. Full details on this model can be found in the Internet Appendix.

maximize their expected lifetime utility and all markets clear. As soon as agents are born, they face a dynamically complete security market because the four securities described in the previous section allow agents to share supply shock, demand shock, and mortality risk. Hence, instead of solving the consumption-portfolio choice problem given in equations (5) and (6) for each agent, we can solve for the equilibrium by using the martingale methods developed in Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989). The static optimization problem is

$$\max_{\{c_{s,t}^i : s \leq t\}} E_s^i \left[\int_s^\infty e^{-(\rho^i + \nu)(t-s)} \log\left(c_{s,t}^i\right) dt \right] \qquad \text{s.t.} \qquad E_s^i \left[\int_s^\infty e^{-\nu(t-s)} \frac{\xi_t^i}{\xi_s^i} \left(c_{s,t}^i - Y_t\right) dt \right] = 0. \qquad (9)$$

The time of death τ is exponentially distributed, and thus we solve an infinite-horizon problem where the finite life effectively increases the time discount rate from ρ^i to $\rho^i + \nu$. Solving the FOC of the static optimization problem given in Equation (9) for individual consumption leads to

$$c_{s,t}^i = c_{s,s}^i e^{-\rho^i(t-s)} \frac{\xi_s^i}{\xi_t^i}, \quad \forall s \leq t \leq \tau^i \qquad \text{with} \qquad c_{s,s}^i = \frac{1}{\kappa_s^i}, \qquad i \in \{a,b\}. \qquad (10)$$

The individual consumption growth rate is inversely related to the investor's time discount rate and to the state price density. Moreover, the initial consumption share is inversely related to the Lagrange multiplier of the static budget constraint, κ_s^i, and needs to be solved in equilibrium because agents cannot hedge against endowment fluctuations prior to birth and thus initial consumption varies with the value of the endowment stream.

The total wealth at time t of a consumer of type i born at time s, $W_{s,t}^i$ is equal to the value of the consumption stream at time t. Specifically, $W_{s,t}^i = E_t^i \left[\int_t^\infty e^{-\nu(u-t)} \frac{\xi_u^i}{\xi_t^i} c_{s,u}^i \, du \right] = \frac{1}{\rho^i+\nu} c_{s,t}^i$. Hence, the individual consumption-wealth ratio is constant and equal to the effective subjective time discount rate, that is, $c_{s,t}^i / W_{s,t}^i = (\rho^i + \nu)$. Initial wealth is equal to the total wealth in the economy, i.e., $W_{t,t}^i = P_{t,t} =$

W_t.[15] The initial individual consumption share of agent i at time t is

$$\beta_t^i \equiv \frac{c_{t,t}^i}{Y_t} = \left(\rho^i + \nu\right)\phi_t, \qquad \forall\, i \in \{a,b\}, \tag{11}$$

where $\phi_t = \frac{W_t}{Y_t}$ is the aggregate wealth-consumption ratio. Hence, consumers born in times of high valuations have a higher welfare weight and consume more than consumers born in times of low valuations.

To derive the equilibrium, we proceed as follows. First, we solve for the stochastic discount factor as a function of the consumption share, f_t, the fraction of patient agents, α_t and the initial individual consumption shares β_t^a and β_t^b. Finally, we show that the wealth-consumption ratio is a consumption share weighted average of the wealth-consumption ratios in the two homogeneous agent economies, allowing us to determine the initial consumption shares β_t^a and β_t^b. To solve for the equilibrium stochastic discount factor, we insert the optimal consumption from Equation (10) into the aggregate resource constraint, $\int_{-\infty}^t \nu e^{-\nu(t-s)}(\alpha_s c_{s,t}^a + (1-\alpha_s)c_{s,t}^b)ds = Y_t$. The result is presented in the next proposition.

Proposition 1 (Stochastic discount factor)**.** *The equilibrium stochastic discount factor is $\xi_t = \frac{X_t}{Y_t}$ with*

$$X_t = \int_{-\infty}^t \nu e^{-\nu(t-s)} \left(\alpha_s \beta_s^a e^{-\rho^a(t-s)} \frac{\eta_t^a}{\eta_s^a} + (1-\alpha_s)\beta_s^b e^{-\rho^b(t-s)} \frac{\eta_t^b}{\eta_s^b} \right) X_s\, ds.$$

The stochastic discount factor in Proposition 1 is inversely related to aggregate output as in a standard infinitely lived representative agent economy with log utility and depends on the process X_t which captures variations in state prices due to differences in time discount rates and beliefs. To study the effects of this heterogeneity on asset prices, we define the consumption share of patient and impatient investors. Specifically, $f_t = \int_{-\infty}^t \nu e^{-\nu(t-s)}\alpha_s(c_{s,t}^a/Y_t)\,ds$ and $1 - f_t = \int_{-\infty}^t \nu e^{-\nu(t-s)}(1-\alpha_s)(c_{s,t}^b/Y_t)\,ds$, respectively. The endogenous consumption share f_t and the exogenous process α_t are the two state variables that

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[15]Define $\hat{Y}_{s,t}$ as the total endowment of all agents born up until time $s \leq t$. The total wealth at time s is the value of this endowment stream. We have $\hat{Y}_{s,t} = e^{-\nu(t-s)}Y_t$, which is equivalent to the endowment of an agent born at time s. Hence, their valuations are the same as well.

describe all the variation in asset prices. The consumption share is presented in the next proposition.

Proposition 2 (Consumption shares). *The consumption shares are $f_t = X_t^a/X_t$ and $1 - f_t = X_t^b/X_t$ with*

$$X_t^a = \int_{-\infty}^t \nu e^{-(\rho^a + \nu)(t-s)} \alpha_s \beta_s^a (\eta_t^a/\eta_s^a) X_s ds \ \text{ and } \ X_t^b = \int_{-\infty}^t \nu e^{-(\rho^b + \nu)(t-s)} \left(1 - \alpha_s\right) \beta_s^b (\eta_t^b/\eta_s^b) X_s ds.$$

Applying Ito's lemma to the stochastic discount factor $\xi_t = \frac{X_t}{Y_t}$ and matching the drift and the diffusion coefficients leads to the equilibrium real short rate and market prices of risk presented in Proposition 3.

Proposition 3 (Risk-free rate and market prices of risk). *In equilibrium, the real short rate is*

$$r_t = \mathcal{E}_f\left[\rho\right] + \mu_Y - \sigma_Y^2 + \nu\left(1 - \alpha_t \beta_t^a - \left(1 - \alpha_t\right)\beta_t^b\right), \tag{12}$$

with market time discount rate $\mathcal{E}_f\left[\rho\right] = f_t \rho^a + \left(1 - f_t\right)\rho^b$. The market price of output risk is $\theta_{Y,t} = \sigma_Y$ and the market price of demand shock risk is

$$\theta_{\alpha,t} = -\mathcal{E}_f\left[\sigma_\Delta\right] = \frac{\kappa}{\sigma_l}(\bar{l} - \mathcal{E}_f\left[\bar{l}\right]) = \sigma_\Delta\left(\frac{1}{2} - f_t\right). \tag{13}$$

with market views $\mathcal{E}_f\left[\bar{l}\right] = \bar{l}^a f_t + \bar{l}^b(1 - f_t)$ and $\mathcal{E}_f\left[\sigma_\Delta\right] = f_t \sigma_\Delta^a + \left(1 - f_t\right)\sigma_\Delta^b$.[16]

There are four economic effects that determine the risk-free interest rate: (i) a time-discounting effect, (ii) an intertemporal substitution effect, (iii) a precautionary savings effect, and (iv) an effect of the OLG structure. The market time discount rate, $\mathcal{E}_f\left[\rho\right]$ is the consumption share weighted average of the patient (ρ^a) and impatient (ρ^b) investors' time discount rates. The risk-free rate is inversely related to the consumption share of patient investors (f_t) because when f_t is high, the risk-free rate r_t is low to discourage excessive savings by patient investors, ensuring equilibrium of the bond market. The intertemporal smoothing and precautionary saving effects are captured by the term $\mu_Y - \sigma_Y^2$, and are the same as in a

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[16]For a detailed discussion of the market view, see Heyerdahl-Larsen and Illeditsch (2025).

standard infinitely lived representative agent economy with log utility. The effect of the OLG structure is captured by the term $\nu \left(1 - \alpha_t \beta_t^a - (1 - \alpha_t) \beta_t^b\right)$. If there is no time-preference heterogeneity, then this term is zero. This is no longer the case with preference heterogeneity as the expected consumption growth of agents currently alive no longer coincides with expected output growth. Hence, a displacement effect must be taken into account because the short-term interest rate reflects only expected consumption growth from agents currently alive. Specifically, if agents who are currently alive are expected to consume less on average than agents that are born next period, then the effective consumption growth is lower, and, therefore, the interest rate is also lower. Disagreement about the process α_t does not directly affect the real short rate. However, it significantly influences the distribution of the risk-free rate through its impact on the consumption share (f_t) and the relative consumption of newborns (β_t^i) as discussed in more detail in Section V.[17]

The market price of output shocks $(\theta_{Y,t} = \sigma_Y)$ is the same as in a standard log-utility economy. However, the price of demand shocks $(\theta_{\alpha,t})$ depends on the market view of $\bar{l}$, represented by the consumption-share-weighted average belief $\mathcal{E}_f\left[\bar{l}\right]$ rather than the statistical view $\bar{l}$. Demand shocks are not priced only when the market view aligns with the statistical view, which occurs if patient and impatient investors have equal consumption shares $(f_t = 0.5)$. When impatient investors consume a larger share $(f_t < 0.5)$, their beliefs have more impact, resulting in a positive market price of demand shock risk $(\theta_{\alpha,t} > 0)$. In contrast, when patient investors have a larger consumption share, the market price of demand risk turns negative. The intuition is straightforward: impatient investors, who overestimate future demand for consumption and, respectively, underestimate future demand for savings (lower α_t), short the consol, while patient investors, who underestimate it, go long. When impatient investors consume more, the price of the consol falls to

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[17] To isolate the effects of demand disagreement on asset prices, we assume that Y_t and α_t are independent; if they were correlated, the real short rate r_t would depend directly on disagreement.

clear the market, making it appear underpriced to an econometrician who is endowed with the statistical view, thus implying a positive covariance with the pricing kernel and a positive market price of demand risk. In contrast, if patient investors have a larger consumption share, they increase the price of the consol, making it appear overpriced to the econometrician, who then perceives a negative risk premium.[18]

To fully characterize the equilibrium stochastic discount factor, we need to derive the wealth-consumption ratio, ϕ_t, as $\beta_t^i = (\rho^i + \nu) \phi_t$. The following proposition shows that ϕ_t is a consumption share weighted average of the wealth-consumption ratios in homogeneous agent economies with only type a or b investors.

Proposition 4 (Wealth-consumption ratio). *Let $\phi^a \equiv \frac{1}{\rho^a + \nu}$ and $\phi^b \equiv \frac{1}{\rho^b + \nu}$ with $\phi^a > \phi^b$. Then the equilibrium wealth-consumption ratio is $\phi_t = \phi^a f_t + \phi^b(1 - f_t) = \phi^b + (\phi^a - \phi^b) f_t$.*

The risk-free rate r_t, the market price of demand shocks $\theta_{\alpha,t}$, and the wealth-consumption ratio are all affine functions of the economy's sole endogenous state variable: the consumption share f_t. The following corollary establishes the sign of their exposure to changes in f_t. This result will be particularly useful in Section V, where we analyze the unconditional distributions of the risk-free rate, the market price of demand shock risk, and the wealth-consumption ratio, as these inherit properties from the unconditional distribution of the consumption share.

Corollary 1. *The risk-free rate is strictly decreasing in the consumption share, that is, $\frac{\partial r(f)}{\partial f} = (\rho^a - \rho^b) - \nu \mathcal{E}_\alpha [\rho] (\phi^a - \phi^b) < 0$. The market price of demand shocks is strictly decreasing in the consumption share, that is, $\frac{\partial \theta_\alpha(f)}{\partial f} = -\sigma_\Delta$. The wealth-consumption ratio is strictly increasing in the consumption share, that is, $\frac{\partial \phi(f)}{\partial f} = \phi^a - \phi^b$.*

We conclude this section with Corollary 2 which shows that the price of the consol bond coincides with

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[18]This intuition is similar to Miller (1977) where short-sale constraints prevent pessimists from trading on their views, so the market view places greater weight on optimists' beliefs and the stock appears overpriced to an outside observer endowed with the statistical view.

the wealth-consumption ratio. Moreover, it decomposes the exposure of the stock return into cash flow and consol exposure since the wealth-consumption ratio and the price-dividend ratio coincide.

Corollary 2 (Consol price and stock return). *The price of the consol is the same as the wealth-consumption ratio, that is, $B_t^C = \phi_t$. The stock return is exposed to cash flow growth, and the consol return*

$$d \log P_{s,t} = \underbrace{d \log Y_{s,t}}_{\text{cash flow exposure}} + \underbrace{d \log B_t^C}_{\text{exposure to consol}} . \tag{14}$$

The first term shows that the price of the stock is exposed to the supply shock $Z_{Y,t}$ through the cash flows, and the second term shows that the price of the stock is exposed to the demand shock $Z_{\alpha,t}$ through the consol. Hence, the instantaneous risk-free asset, the stock, and the consol are sufficient to complete the asset market driven by the two Brownian shocks $Z_{Y,t}$ and $Z_{\alpha,t}$.

V. Analysis

In this section, we analyze the equilibrium derived in the previous section and show how the model is consistent with both the correlation puzzle and the disagreement correlation puzzle while at the same time reproducing many stylized asset pricing facts.

A. Parameters

The model has a total of nine parameters $\left(\mu_Y, \sigma_Y, \nu, \rho^a, \rho^b, \kappa, \bar{l}, \sigma_l, \sigma_\Delta\right)$. We set expected output growth to $\mu_Y = 2\%$ and output growth volatility to $\sigma_Y = 3.3\%$, which is similar to the estimates in Campbell and Cochrane (1999) that are based on a long data sample. The birth and death intensity is set to, $\nu = 2\%$ which implies an expected life of 50 years from the start of trading, as in Gârleanu and Panageas (2015). We assign a time preference parameter of $\rho^a = -0.015$ to the patient type a agent, ensuring a positive effective time discount rate of $\rho^a + \nu = 0.005$. The impatient type b agent has a time preference

parameter of $\rho^b = 0.025$. In the literature, time preferences parameters vary substantially. For example, the time preference parameters in Bansal and Yaron (2004), Chan and Kogan (2002), and Campbell and Cochrane (1999) are 2.4%, 5.2%, and 11.6%, respectively. For comparison, the 5% and 95% percentiles of the annualized time discount factor in Albuquerque, Eichenbaum, Luo, and Rebelo (2016) are -6.4% and 11.2% with a mean of 2.4%. In order to capture a slow moving change in the composition of types with a long run average for α_t of 50%, we set the parameters of the process l_t as follows: long run mean $\bar{l} = 0$, local volatility $\sigma_l = 0.1$, and speed of mean reversion $\kappa = 0.01$.[19] To examine the impact of varying levels of demand disagreement on asset prices, we explore different parameter values for σ_Δ ranging from zero $(\sigma_\Delta = 0)$, indicating no disagreement, to $\sigma_\Delta = 0.8$, our baseline parameter choice. Demand disagreement is not directly observable, so we estimate it using survey data as described earlier. Figure 2 shows the average implied demand disagreement from both the data and the model across forecast horizons of one to four quarters. The results show that the disagreement is greater for longer forecast horizons, both in the model and in the data. Furthermore, the model generates modest disagreement levels compared to the data, except when $\sigma_\Delta = 0.8$. At the one-year horizon, the model aligns closely with the data when $\sigma_\Delta = 0.8$, designating it as the base case.[20] Finally, in the base case with $\sigma_\Delta = 0.8$, the mean, standard deviation and autocorrelation of the pricing relevant time discount factor, $\mathcal{E}_f[\rho]$, is 1.3%, 1.2% and 0.92 at an annual horizon.[21]

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[19]The long-run average of α is not precisely 0.5 when $\bar{l} = 0$ due to a Jensen's correction term, but its impact is minimal.

[20]The Internet Appendix includes additional calibrations, considering different ρ^a and ρ^b values alongside various σ_Δ levels. Although such calibrations yield results similar to the base case, they show lower equity premium, bond risk premia, and volatilities.

[21]For comparison, in the representative agent model of Albuquerque, Eichenbaum, Luo, and Rebelo (2016) with stochastic time preferences, the time preference process has a mean, standard deviation and autocorrelation at the annual horizon of 2.4%, 1.5% and 0.9.
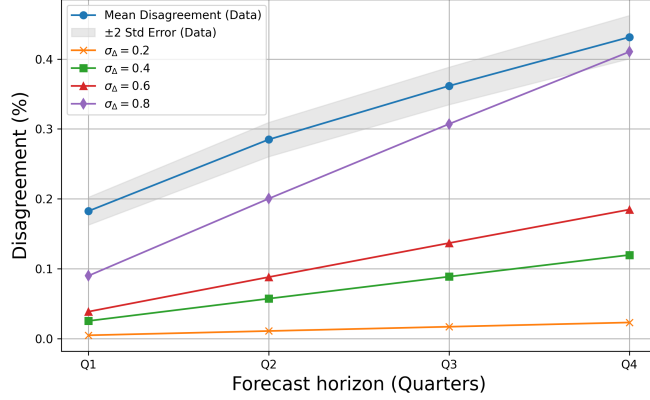
Figure 2: Yield Disagreement due to Demand Disagreement. This figure compares yield disagreement due to demand disagreement in both the model and data across forecast horizons of one to four quarters. Yield disagreement is measured as the cross-sectional standard deviation of residuals from individual yield forecasts based on macroeconomic fundamentals (Section II). In the model, yield disagreement is derived from the cross-sectional standard deviation of real short rate forecasts by patient and impatient agents at four demand disagreement levels ($\sigma_\Delta = 0.2, 0.4, 0.6, 0.8$), all below empirical observations. Consequently, $\sigma_\Delta = 0.8$ is selected as the baseline level for demand disagreement in subsequent analysis.

B. Asset Pricing Moments

Before detailing the economic mechanism of our demand disagreement model, we first present key unconditional asset pricing moments as a function of demand disagreement σ_Δ. We denote by $\tilde{\alpha} = 1/(1 + \exp(-\tilde{l}))$ the long-run fraction of newborn patient agents and by $\tilde{f}$ their consumption share, both drawn from their unconditional distributions. For each level of demand disagreement, σ_Δ, we simulate $500,000$ years of monthly data for l_t, which follows the dynamics given in Equation (4), and for f_t, which follows the dynamics specified in Proposition 5 below. Figure 3 shows the unconditional mean and volatility of asset price moments based on $\tilde{\alpha}$ and $\tilde{f}$, while Figure 4 presents a histogram of the consumption share $\tilde{f}$.

Figure 3 illustrates how increasing demand disagreement σ_Δ affects key asset pricing moments. As σ_Δ rises, the stock market's risk premium and volatility both increase (top-left and top-right panels), as do the risk premium and volatility of a 5-year discount bond (middle-left and middle-right panels). Similarly, yields across maturities of one to five years and their volatilities rise with disagreement, and rising σ_Δ also leads to a steeper yield curve (bottom-left and bottom-right panels).
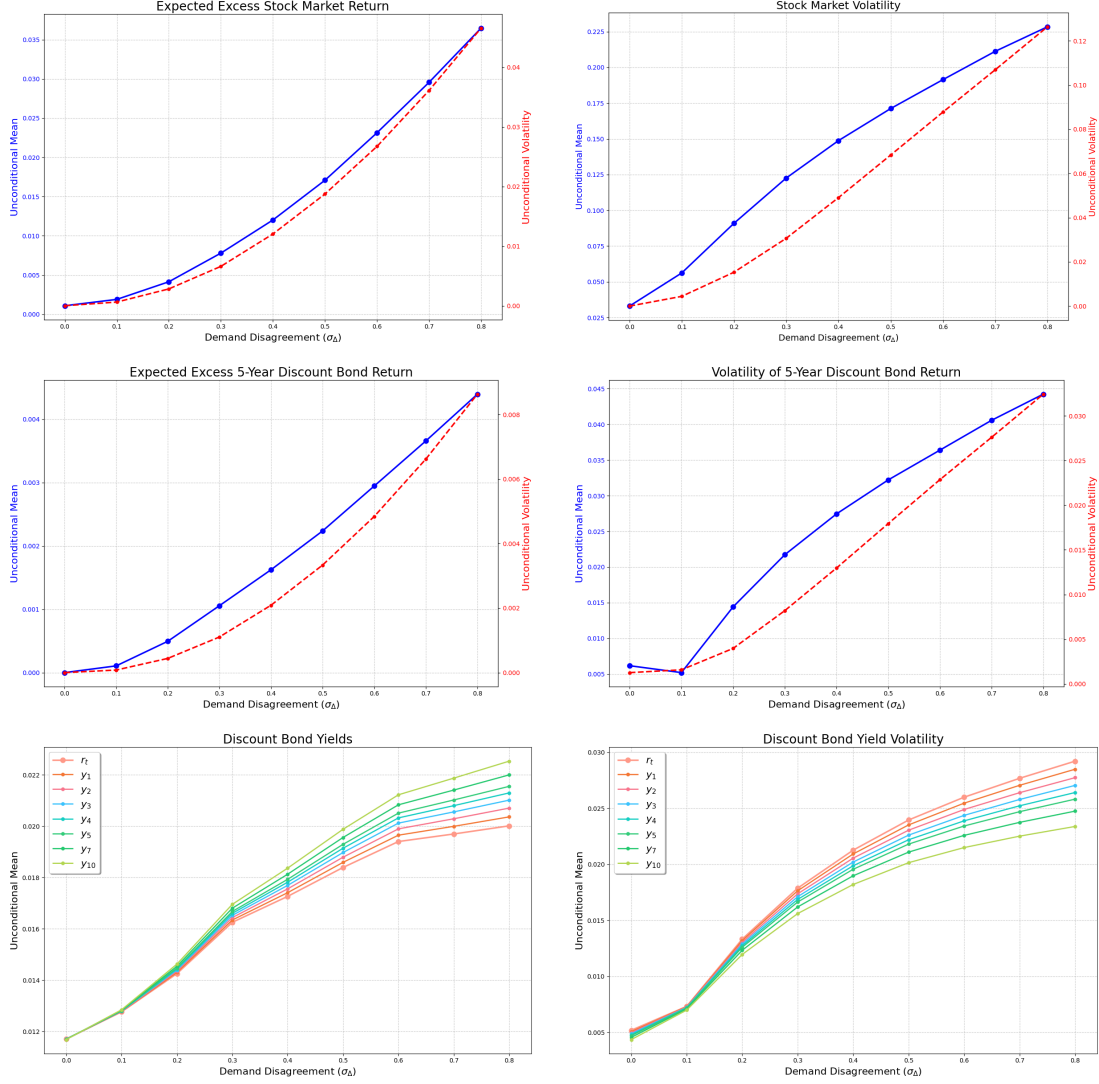
Figure 3: Unconditional Asset Pricing Moments. This figure presents six key asset pricing moments: expected excess stock market return (top-left), stock market volatility (top-right), excess return on five-year bonds (middle-left), five-year bond return volatility (middle-right), the yield curve (bottom-left), and yield volatility (bottom-right). These moments are derived from simulated unconditional distributions of $\tilde{\alpha}$ and $\tilde{f}$ across various demand disagreement levels (σ_Δ), based on $500,000$ years of monthly model simulations. Demand disagreement increases expected stock and bond returns, yields, the slope of the term structure, and yield volatilities.

For the baseline calibration with $\sigma_\Delta = 0.8$, the stock market risk premium is 3.6%, and the stock market volatility is 22.8%. While the risk premium is lower than the historical equity premium observed in postwar U.S. data, it is significantly higher than the no-disagreement value of 0.11%. Achieving an increasing risk premium with higher disagreement is typically challenging in models with disagreement, whereas increasing volatility is more common.[22] However, our model exhibits a risk premium that rises with increasing disagreement. The intuition behind this result is that when the market price of demand shock risk is high, the stock and bond loadings on demand shocks are also high; conversely, when the price is low or negative, their exposures are low. We discuss this novel mechanism in detail in Section V.E.

The middle panels show that both the risk premium and return volatility of the 5-year bond increase with disagreement. This similar behavior in the bond and stock markets follows from Equation (14). Specifically, stock returns are exposed to fundamental shocks through cash flow risk and to demand shocks through their exposure to the price risk of the consol bond. Since disagreement is independent of fundamental shocks, all effects from disagreement manifest through the consol bond.

The bottom left panel shows that as disagreement increases, the slope of the yield curve becomes steeper. Our model with demand disagreement thus generates an unconditionally upward-sloping yield curve. This happens because a higher disagreement decreases the demand for long-term bonds, increasing their yields relative to short-term bonds, and steepening the yield curve. We will provide more intuition for the increase of risk premia and the steeping of the yield curve with rising demand disagreement in Section V.E.

In addition, as disagreement increases, the volatility of risk premia in both the stock and bond markets increases. While this may not be surprising, it is significant because these premia are not constant in the

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[22]Disagreement does not always lead to excess volatility. For example, in a model with disagreement about dividends and CRRA utility with $\gamma > 1$, cash flow volatility can exceed stock volatility (see).

data. More importantly, as we show in Section V.G, demand disagreement predicts future bond returns and variations in bond return volatility in both our model and the data.[23]

C.   Economic Mechanism – The Consumption Share

In this section, we analyze the economic mechanism underlying the demand disagreement model, focusing on its two state variables: the exogenous fraction of the newborn patient agents, α_t and the endogenous consumption share of the patient agents, f_t. The following proposition presents the dynamics of the endogenous state variable, f_t, to further clarify its properties.

Proposition 5 (Consumption Share Dynamics). *The dynamics of the consumption share of patient agents, f_t, in equilibrium are $df_t = \mu_{f,t}\,dt + \sigma_{f,t}\,dZ_{\alpha,t}$ with*

$$
\begin{aligned}
\mu_{f,t} &= \nu\left(\alpha_t\beta_t^a\left(1-f_t\right)-\left(1-\alpha_t\right)\beta_t^b f_t\right)+\left(\rho^b-\rho^a\right)f_t\left(1-f_t\right)\\
&+\sigma_\Delta^2\left(\frac{1}{2}-f_t\right)f_t\left(1-f_t\right) \qquad and \qquad \sigma_{f,t}=f_t(1-f_t)\sigma_\Delta.
\end{aligned}
\tag{15}
$$

Output shocks do not affect the consumption share because investors have the same risk preferences and agree on the output shock, and thus have the same exposure to output shocks. The left graph of Figure 4 shows the volatility of the consumption share, $\sigma_{f,t}$, as a function of the consumption share, f_t, for different disagreement σ_Δ. There are no shocks to the consumption share if there is no disagreement, that is, $\sigma_{f,t}=0$ if $\sigma_\Delta=0$. The impact of shocks to the consumption share reaches its maximum when the consumption share is 0.5 and vanishes when it approaches 0 or 1because investors take large speculative positions of equal size in the former and have nobody to speculate with in the later. Moreover, speculative positions increase with disagreement σ_Δ and, thus, magnify shocks to consumption shares as wealth shifts

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[23]Earlier versions of this paper focused on stock markets, showing that demand disagreement predicts higher future stock returns when the price-dividend ratio is low and explains Black's leverage effect. These results are now reported in the Internet Appendix, as our main focus has shifted to bond markets.

from one speculator to the other.
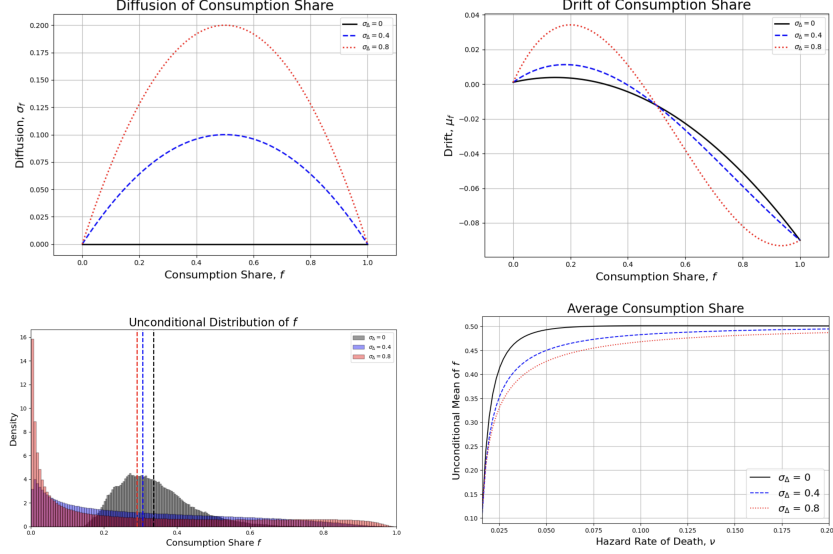
Figure 4: The consumption share. The top left graph shows the volatility $\sigma_{f,t}$ and the top right graph shows the drift $\mu_{f,t}$ of the consumption share dynamics as a function of the consumption share for three levels of disagreement σ_Δ. The drift of the consumption share $\mu_{f,t}$ in the top right graph depends on the fraction of patient newborns α_t which is fixed at 0.5. The left graph shows the unconditional distribution of the consumption share of patient investors $\tilde{f}$ and the right graph shows the unconditional mean of the consumption share of patient investors $\mathrm{E}[\tilde{f}]$ as a function of the birth/mortality rate ν. The histograms and unconditional means in the bottom row are based 500K years of monthly observations for each value of disagreement σ_Δ.

There are three economic effects that influence the consumption share drift, $\mu_{f,t}$, that is shown for different disagreement σ_Δ as a function of f_t in the right graph of Figure 4. Specifically, there is (i) an OLG effect, (ii) a preference heterogeneity effect, (iii) and a disagreement effect. The first term of the drift in equation (15), which is due to the OLG effect, leads to the time variation of the drift even without disagreement and it does not vanish when the consumption share approaches 0 or 1 because every instant new agents are born and the stochastic fraction, α_t, measures how many of them are patient. Hence, the OLG effect leads to reflecting boundaries at 0 or 1 and to a stationary distribution of the consumption share, f_t. Moreover, the push upward at the lower bound is on average smaller than the push downward at the upper bound, that is, $-0.09dt$ vs $0.0011dt$ in the top right graph of Figure 4. This holds more generally because the downward push when f_t approaches one $(-\mathrm{E}[\lim_{f_t \to 1} \mu_{f,t}] = \nu\,(1 - \mathrm{E}[\alpha_t])\,(\rho^b + \nu)\phi^a)$ is greater

or equal to the upward push when f_t approaches zero $(\mathrm{E}[\lim_{f_t \to 0} \mu_{f,t}] = \nu\mathrm{E}[\alpha_t](\rho^a + \nu)\phi^b)$. The average fraction of newborn patient investors is the same as for impatient investors, and so the economic reason for the different boundary behavior is twofold. First, impatient investors consume at a higher rate out of their initial wealth. Second, initial wealth is more valuable at the extinction boundary for impatient investors, where patient investors have all the price impact, and so valuation ratios are higher.

The second term of the drift in Equation (15) is due to investors having different time discount rates, and is always positive because $\rho^b > \rho^a$. This upward drift reflects the market selection force that favors the patient agent who saves more than the impatient agent, which in an infinite-horizon economy would lead to extinction of the impatient agent. The third term of the drift in Equation (15) is due to disagreement and disappears if the consumption share is 0 or 1 because neither patient nor impatient investors have someone to speculate with at the extinction boundaries. In stark contrast to the consumption share volatility, disagreement has no effect on the drift when the consumption share is 0.5 because from the econometricians' perspective speculating on the demand shock is a fair gamble in this case and, hence, neither patient nor impatient investors' financial wealth, and, thus, consumption share is expected to change.

We simulate $500,000$ years of monthly data for the consumption share f_t using its dynamics given in Proposition 5 for three levels of disagreement σ_Δ. Figure 4 shows the histogram of $\tilde{f}$. In an infinite-horizon economy, this distribution would not exist, with the share converging to zero or one. The black dashed line indicates an unconditional mean of $\tilde{f}$ at 33.4% without disagreement, below the 50% mean of newborn patient investors $\mathrm{E}[\alpha_t]$, as the OLG effect outweighs the market selection force.

Disagreement further reduces the average consumption share of patient investors because speculation leads to large and persistent wealth swings, and thus higher probabilities of very low consumption shares. This effect results in long-run means of $\tilde{f}$ even further below 50% as disagreement increases, shown in

Figure 4 (bottom left). The unconditional mean drops from 33.4% without disagreement (black line), to 31.7% with medium ($\sigma_\Delta = 0.4$, blue), and to 29.0% with high disagreement ($\sigma_\Delta = 0.8$, red). Disagreement also increases the volatility of consumption shares from 10% to 24.5% and 29.4%, significantly increasing the likelihood of extreme consumption share realizations. The bottom right graph of Figure 4 shows the unconditional mean of $\tilde{f}$ as a function of the birth/mortality rate ν, ranging from 0.016 to 0.2. As ν increases, the patient type's consumption share approaches the mean of newborn patient investors, since shorter agent lifespans reduce the impact of market selection.[24]

D. Valuation Ratio, Risk-free Rate, and Market Price of Demand Risk

In equilibrium, the wealth-consumption ratio, the risk-free rate, and the market price of demand shocks are all affine functions of the consumption share $\tilde{f}$. Therefore, these quantities inherit the properties of the unconditional distribution of $\tilde{f}$, which we now briefly discuss.

From Corollary 1, we know that the wealth-consumption ratio is strictly increasing in $\tilde{f}$. Consequently, as disagreement increases, the unconditional mean of the wealth-consumption ratio $\tilde{\phi}$ decreases, while its volatility increases. In other words, disagreement reduces the average valuation ratio and makes it more volatile. Moreover, the probability of observing a very low valuation ratio increases significantly with higher disagreement.

Similarly, Corollary 1 shows that the risk-free rate is strictly decreasing in $\tilde{f}$. Thus, increasing disagreement raises the average interest rate and makes it more volatile. However, unlike the substantial increase in stock market volatility with greater disagreement, the effect on interest rate volatility is moderate. However, the likelihood of experiencing very low or high risk-free rates increases notably with greater

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[24]In an alternative calibration (Internet Appendix) with $\rho^a = 0.001$ and $\rho^B = 0.05$, we consider the infinitely lived case ($\nu = 0$), where disagreement leads to a non-monotonic relation: the most patient initially dominates, then the mean dips below 0.5 before rising toward the unconditional mean of α.

disagreement.

The market price of demand risk $\tilde{\theta}_\alpha$ is a strictly decreasing linear function of the consumption share $\tilde{f}$. From Equation (13), its unconditional mean, $\mathrm{E}^{\sigma_\Delta}\left[\tilde{\theta}_\alpha\right] = \sigma_\Delta\left(0.5 - \mathrm{E}^{\sigma_\Delta}\left[\tilde{f}\right]\right)$, is a nonlinear function of σ_Δ. As σ_Δ increases, the unconditional mean of the market price of demand risk initially decreases, then becomes positive, and is positive at the baseline setting $\sigma_\Delta = 0.8$.

E. Bond and Stock Returns

In this subsection, we discuss the properties of bond and stock returns. We obtain closed-form solutions for the price of the stock market and consol bond and present their return dynamics in the next proposition.

Proposition 6 (Consol and Stock Market). *The price of the consol is $B_t^C = \phi_t$ and its return dynamics are $dR_t^C = \left(r_t + \lambda_{R^C,t}\right) dt + \sigma_{R^C,t}^\alpha dZ_{\alpha,t}$ with $\sigma_{R^C,t}^\alpha = \frac{\phi^a - \phi^b}{\phi_t} f_t (1 - f_t) \sigma_\Delta \geq 0$ and $\lambda_{R^C,t} = \sigma_{R^C,t}^\alpha \theta_{\alpha,t}$. The loading of the consol on the demand shock attains its maximum at $f_{max} = \left(1 + \sqrt{\frac{\nu + \rho^b}{\nu + \rho^a}}\right)^{-1} \leq 0.5$. The stock market price is $\phi_t Y_t$ and the return dynamics are $dR_t^S = \left(r_t + \lambda_{R^S,t}\right) dt + \sigma_{R^S,t}^Y dZ_{Y,t} + \sigma_{R^S,t}^\alpha dZ_{\alpha,t}$ with $\sigma_{R^S,t}^Y = \sigma_Y$, $\sigma_{R^S,t}^\alpha = \sigma_{R^S,t}^\alpha$, and $\lambda_{R^S,t} = \sigma_Y^2 + \lambda_{R,t}^\alpha$.*

We do not have closed-form solutions for the prices of discount bonds, and thus we use Monte Carlo simulations to determine the diffusion coefficient of the instantaneous discount bond return with maturity T defined as $\sigma_{R^T,t}$. Hence, the risk premium of the discount bond is $\lambda_{R^T,t} = \theta_{\alpha,t} \sigma_{R^T,t}$. Figure 5 shows the volatility (left plot) and the risk premia (right plot) of bonds with maturities ranging from one to ten years, the consol bond, and the stock market as functions of the consumption share, f_t.

If there is no disagreement, then the instantaneous return of the consol is not exposed to the demand shock and, thus, locally risk-free. Disagreement and the resulting speculative trade increases the exposure to the demand shock, and so leads to volatile bond returns. Reducing the maturity of the bond lowers the exposure to demand shock risk, and thus lowers the volatility of the bond. The bond return exposure
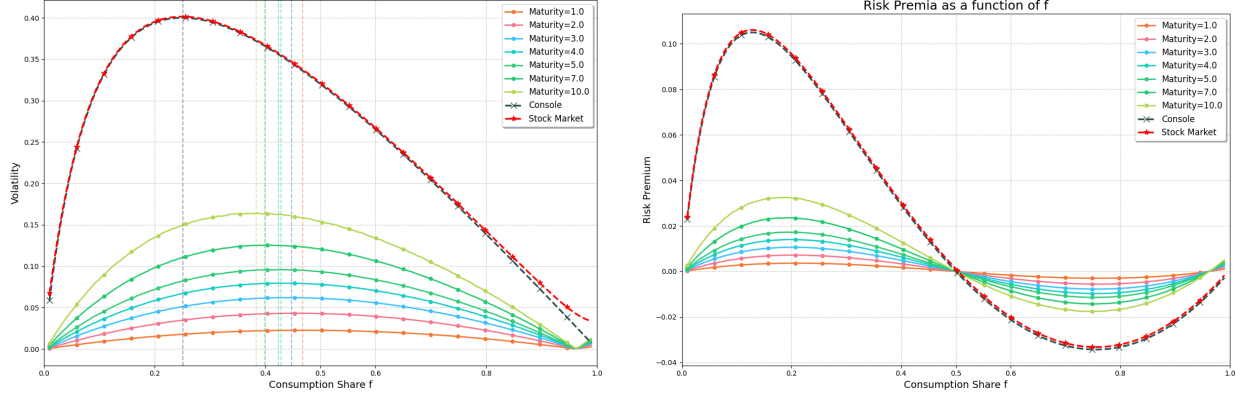
Figure 5: Conditional Return volatility and risk premium of bonds and stocks. The left graph shows bond return volatility and the right graphs shows the bond risk premium as a function of the consumption share f_t for different maturities. In all figures we keep $\alpha = 0.5$.

to demand shocks vanishes if the consumption share approaches either zero or one because in this case investors have no one to speculate with. Importantly, the volatility of the consol (and the stock market) is maximized at the value $f_{\max} = 0.25$ (vertical black line) rather than at the peak of the speculative potential of investors when $f = 0.5$. Shorter bond maturities shift this peak closer to 0.5.[25] Stock market volatility primarily reflects consol volatility, inheriting all its characteristics; as a result, the stock market exhibits excess volatility that mirrors the fluctuations observed in a consol bond.[26]

The right plot of Figure 5 shows risk premia conditional on the consumption share f. When the consumption shares of patient and impatient investors are equal ($f = 0.5$), the market price of demand shock risk vanishes. In this case, bond risk premia are zero because bonds are exposed only to demand shocks (and the equity premium is minimal due to the small market price of output risk under log utility). The conditional bond risk premium is positive when $f_t < 0.5$ and negative when $f_t > 0.5$. Importantly, bonds and stocks have a higher exposure to demand shock risk when $f_t < 0.5$, making the absolute value of

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[25]The short rate volatility, being linear in the consumption share, peaks when consumption volatility is at its maximum, which occurs at $f_t = 0.5$.

[26]Section VI.A discusses extensions to our model that reduce this co-movement, leading to distinct differences between stock market and bond volatilities as well as risk premia.

the risk premium much higher in this scenario. Therefore, the stock market risk premium is not monotonic in the consumption share. Moreover, it is zero at the extinction boundaries $f = 0$ and $f = 1$, where exposure to demand shock risk disappears.

Although the conditional risk premia are positive for $f_t < 0.5$ and negative for $f_t > 0.5$, we can explain why the unconditional bond and stock risk premia are positive and increase with demand disagreement, as shown in Figure 3, along with an unconditional upward sloping yield curve. This arises from the unconditional risk premium equation:

$$\mathrm{E}\left[\tilde{\lambda}\right] = \mathrm{E}\left[\tilde{\sigma}_\alpha\right]\mathrm{E}\left[\tilde{\theta}_\alpha\right] + \mathrm{Cov}\left(\tilde{\sigma}_\alpha, \tilde{\theta}_\alpha\right),$$

where $\tilde{\lambda}$ is the risk premium and $\tilde{\sigma}_\alpha$ is the demand risk exposure of a stock or bond. The positive unconditional risk premium results from two factors: (i) $\mathrm{E}\left[\tilde{\sigma}_\alpha\right]\mathrm{E}\left[\tilde{\theta}_\alpha\right] > 0$ because the expected consumption share is below 0.5 and demand risk exposure is always positive; and (ii) $\mathrm{Cov}\left(\tilde{\sigma}_\alpha, \tilde{\theta}_\alpha\right) > 0$ because maximum volatility occurs when $f_t < 0.5$, creating a positive correlation between risk exposure and the price of risk. Increased disagreement amplifies both effects, leading to higher unconditional asset risk premia.

The upward-sloping yield curve for maturities between one and ten years results from the peak in volatility shifting to lower consumption shares as bond maturity increases. This shift enhances the covariance with the price of demand shocks, increasing the risk premium for longer-maturity bonds.

F. Yield Volatility and Bond Return Predictability

In this section we show empirically and in our model, that higher demand disagreement is robustly associated with greater yield volatility and predicts higher excess bond returns.

We find a strong, positive relationship between demand disagreement and the volatility of discount bond yields across maturities from one to five years. In the data, we estimate nominal yield volatility

using an AR(1)-GARCH(1,1) model and regress it on our measure of demand disagreement, as described

in Section II.D. In the model, where disagreement is entirely unrelated to macroeconomic fundamentals,

demand disagreement reflects differences in one-year real short-rate forecasts between patient and impatient

investors. Because there are no closed-form solutions, we use Monte Carlo simulations to compute both

conditional yield volatility and demand disagreement. Table III shows that, both empirically and in the

model, higher demand disagreement is consistently associated with higher yield volatility at all maturities.

Quantitatively, a one-standard deviation increase in demand disagreement raises yield volatility by about

0.7 standard deviations in the data and nearly one-for-one in the model. The positive relationship in the

data is robust to alternative disagreement measures and remains significant after controlling for yields and

macroeconomic disagreement, as shown in Tables 8–11 of the Internet Appendix.

Maturity	1	2	3	4	5
Data	0.7334	0.7284	0.7204	0.6978	0.6618
	(14.108)	(13.902)	(13.584)	(12.738)	(11.544)
Model	0.9938	0.9937	0.9928	0.9913	0.9892

Table III: Yield Volatility Regression Coefficients. This table presents the standardized regression coefficients of nominal yield volatility for discount bonds with maturities from one to five years, regressed on yield disagreement due to demand disagreement as implied by both the model and the data. In the model, yield volatility is estimated using a simulation of 500,000 years of data, regressing actual conditional volatility on yield disagreement based on one-year real yield forecasts. In the empirical analysis, nominal yield volatility is estimated using an AR(1)-GARCH(1,1) model and regressed on the demand disagreement measure constructed in Section II.D. The model's standardized coefficients (population values) are consistent with those observed in the data. Newey-West corrected standard errors with four lags are used, and t-statistics for the data are reported in brackets.

We also find a strong, positive relationship between demand disagreement and future excess bond returns

across maturities from two to five years. In the data, we define the excess return of a T-year discount bond

as the one-year holding period return of the T-year bond in excess of the one-year yield, and regress these

on our measure of demand disagreement, as described in Section II.D. In the model, we use Monte Carlo

simulations to compute both the model-implied bond risk premium and demand disagreement. Table IV

shows that both empirically and in the model, a higher demand disagreement is consistently associated with higher future bond risk premia at all maturities. Quantitatively, the standardized coefficients in the data are substantial and statistically significant, while the model coefficients, though smaller, are consistent with the empirical findings. This predictive relationship in the data is robust to alternative disagreement measures and remains significant after controlling for yields and macroeconomic disagreement, as demonstrated in the Internet Appendix (see Tables 12-15 of the Internet Appendix). Notably, while disagreement spanned by macroeconomic fundamentals is associated with higher yield volatility, it does not predict bond excess returns, underscoring the unique predictive power of demand disagreement (see Table 10 for yield volatility and Table 14 for bond return predictability in the Internet Appendix).

| Maturity	2	3	4	5
Data	0.4364	0.4085	0.3932	0.3837
	(5.845)	(5.361)	(5.084)	(4.922)
Model	0.1510	0.1505	0.1494	0.1478

Table IV: Bond Risk Premium Regression Coefficients. This table presents the standardized regression coefficients of excess bond returns for discount bonds with maturities, T, ranging from two to five years, regressed on yield disagreement due to demand disagreement as implied by both the model and the data. In the model, we use a simulation of $500,000$ years of data to regress the instantaneous bond risk premium, $\lambda_{R^T,t}$, on yield disagreement based on one-year real yield forecasts, as all yield disagreement in the model arises from demand disagreement. In the empirical analysis, bond risk premia are calculated as $rx_{t,t+4}^{(T)} = Ty_t^{(T)} - (T-4)y_{t+4}^{(T-4)} - y_t^{(1)}$, where each period is a quarter, and are regressed on our demand disagreement measure constructed in Section II.D. The standardized coefficients from both the model (population values) and the data are reported. Newey-West corrected standard errors with four lags are used, and t-statistics for the empirical data are reported in brackets. The model's coefficients are consistent with those observed in the data.

G. *Estimating Latent States and Assessing Model Fit*

We show in this section that, despite its parsimony, our demand disagreement model - with just one exogenous and one endogenous state variable, both driven by a single demand shock - delivers remarkable explanatory power for bond market dynamics, particularly for bond risk premia and the yield curve slope.

Our demand disagreement model features two latent state variables: the exogenous fraction of patient

newborn investors (α_t) and the endogenous consumption share of patient investors (f_t), both driven by a single Brownian shock ($Z_{\alpha,t}$). To estimate these unobservable states, we use the Unscented Kalman Filter (UKF) with only two observable time series: our demand disagreement measure and the two-year Treasury Inflation-Protected Security (TIPS) yield. We intentionally exclude longer-maturity TIPS yields (3, 5, 7, and 10 years) and nominal bond yields in the estimation for out-of-sample validation. This filtering approach is necessary due to the nonlinear relationship between the observables and the latent state variables.[27]
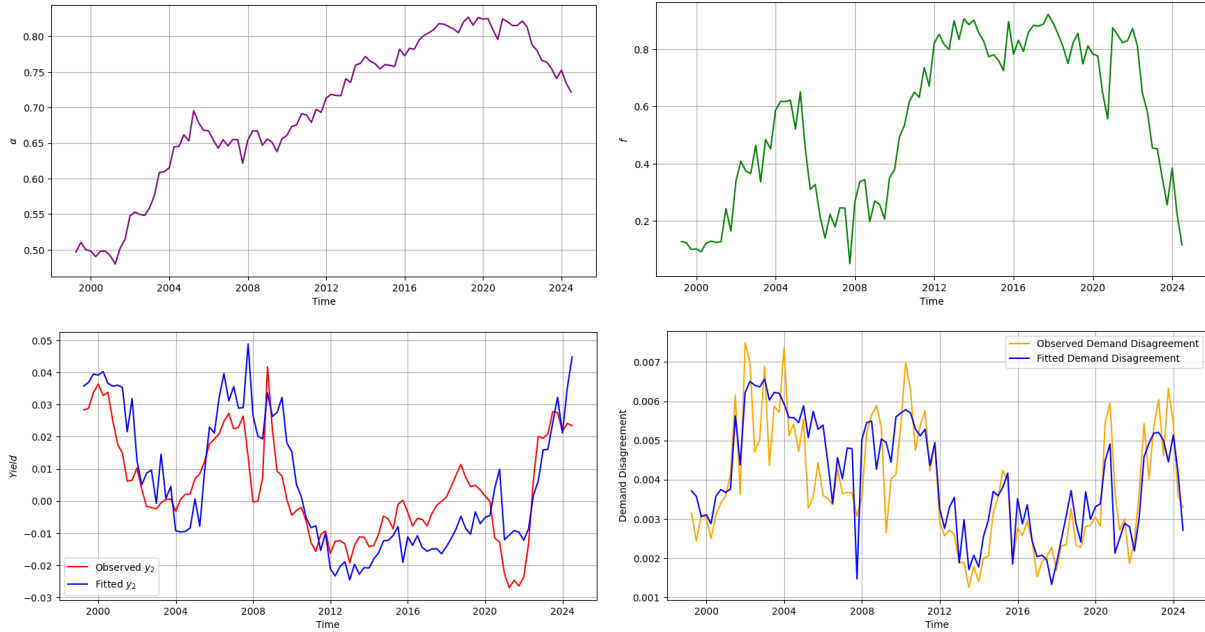
Figure 6: Estimated state variables and fit to demand disagreement and the two year yield. The figure shows the estimated α (top-left), the estimated f (top-right), the observed and implied two year real yield (bottom-left) and the observed and implied demand disagreement (bottom-left). We estimate the latent state variables using UKF. The observation equations are $y_t = h(l_t, f_t) + e_{y,t}$ and $DD_t = g(l_t, f_t) + e_{DD,t}$. We estimate the model using l_t rather than α_t. We use the Euler discretization of the latent state variables with quarterly frequency as in the data. We assume a diagonal variance-covariance matrix for the observation noise with $Var(e_{y,t}) = 0.001 * var(y_{2,t})$ and $Var(e_{DD,t}) = 0.0005 * var(DD_t)$, We also assume a diagonal variance-covariance matrix of the prior variance in the two state variables with a variance of both set to 0.01. We use data from 1999Q1-2024Q2 in the filtering.

The plot on the top left of Figure 6 shows an upward trend in implied α from 1999 to 2022, indicating an increasing fraction of better savers entering the economy. The consumption share exhibits similar but less

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[27]For a detailed explanation of the estimation using the Unscented Kalman Filter, please refer to the Internet Appendix.

pronounced trends. While the fraction of patient newborns has risen above the long-run mean, increasing the demand for savings, the consumption share is less persistent and more volatile than the newborn fraction. Notable fluctuations include a significant drop in the share of patient agents' consumption around 2005, coinciding with a temporary α decline and a sharp f_t decrease in recent years as real rates increased. The bottom panels demonstrate that, despite the parsimony of the model having only one shock to the exogenous and endogenous state variables, the filtered model provides a reasonable fit to both the observed demand disagreement and the real two-year yield. Notably, because both state variables are driven by a single shock in this nonlinear model, it is less flexible in fitting both observables.

Building on these filtered state variables, we generate model-implied time series for real yields (2, 3, 5, 7, and 10 years), their estimated AR(1)-GARCH(1,1) volatilities, and realized annual holding period returns of nominal bonds in excess of the one-year nominal yield. To evaluate the empirical performance of the model, we regress the observed data for these variables on their model-implied counterparts. Table V summarizes these results. In an ideal scenario, the regression slope would be one and R^2 high, indicating a great fit.[28] While omitted factors and the simplicity of the model naturally limit explanatory power, the results are surprisingly strong: all regression coefficients are positive and statistically significant, and for both nominal bond risk premia and the slope of the real yield curve, we cannot reject the null hypothesis that the coefficient equals one. The model also captures the cyclicality of the slope of the real yield curve, a challenging feature for general equilibrium models, and effectively predicts bond risk premia over 98 quarters. Although real yield volatilities show weaker explanatory power, especially at the two-year maturity, the overall findings underscore the model's ability to capture key features of the data with minimal structure and only a single source of uncertainty.

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[28]We do not expect a very high R^2 for risk premia since we regress realized returns in the data on the model implied risk premium.

Maturity	Real Yields			Real Yield Volatilities			Nominal Bond Risk Premia		
	Coeff.	Std. Dev.	R^2	Coeff.	Std. Dev.	R^2	Coeff.	Std. Dev.	R^2
2 years	0.621	0.062	0.667	0.141	0.093	0.035	1.354	0.543	0.178
3 years	0.593	0.057	0.694	0.156	0.073	0.086	1.540	0.672	0.143
5 years	0.558	0.054	0.687	0.136	0.045	0.155	1.281	0.708	0.084
7 years	0.535	0.053	0.663	0.111	0.038	0.136	–	–	–
10 years	0.517	0.052	0.629	0.079	0.032	0.088	–	–	–
Slope	1.147	0.275	0.236	–	–	–	–	–	–

Table V: Regression Results: Yields, Yield Volatilities, and Bond Risk Premia

The table displays regression results of observed data against model-implied values based on filtered α_t and f_t. Coefficients are estimated from the regression $y_t^{\text{Data}} = a + b y_t^{\text{Model}} + u_t$. Dashes indicate missing data for certain maturities. Although coefficients differ from one, their statistical significance and correct signs demonstrate that the model captures essential data features.

Finally, we note that the above filtering relies on our measure of demand disagreement (see Section II for details). In the Online Appendix, we present an alternative filtering approach that uses yield disagreement directly as an observable in the estimation. Because yield disagreement also reflects macroeconomic disagreement, we include an additional observation equation and latent state variable. Since our model does not feature macroeconomic disagreement, we assume it does not affect bond yields. The results from this alternative approach are very similar to those reported here.

VI. Discussion and Extensions

In this section, we explore potential extensions to our model that offer additional insight into asset pricing, as well as address some limitations of our current framework and potential improvements. Our primary goal has been to develop a tractable model with clear economic forces that reduces the strong correlation between fundamentals and asset returns, as well as the correlation between fundamental disagreement and returns that many models imply. Furthermore, we demonstrate that demand disagreement plays a significant role in bond pricing, both in the model and in the empirical data. A natural starting point to achieve these objectives is an exchange economy with log utility, and different time preferences.

Moreover, investors have dogmatic beliefs and a false consensus bias. While these simplifying assumptions improve model transparency and tractability, they involve certain trade-offs.

Here, we briefly discuss three possible extensions: (1) a more generalized form of demand disagreement, (2) time-varying disagreement, and (3) a production economy extension of our model. Through these extensions, we demonstrate that while our baseline model is intentionally simplistic, its core insights remain robust even within more generalized frameworks. For brevity, we present only an overview here, with detailed derivations and extended discussions available in the Internet Appendix.

A. Demand Disagreement from Heterogeneous Risk Aversion

In this paper, we focus on demand disagreement that arises from differing beliefs about the future distribution of patient and impatient investors. This limits our model's ability to capture certain asset price moments, such as the equity premium. In this framework, demand shocks only affect the yield curve, which implies that both the stock risk premium and its volatility are largely determined by the risk premium and volatility of the consol. This aligns with recent evidence from Van Binsbergen (2021), showing that duration-matched bond portfolios are as volatile as stocks and yield similar excess returns. Consequently, our analysis emphasizes the effects of demand disagreement on bond pricing.

There are several ways to extend our model to drive a wedge between stock and bond returns. First, allowing for preferences with constant relative risk aversion (CRRA) preferences and letting the two investor types differ in risk aversion would break this link. In this scenario, demand disagreement would also affect the pricing of fundamental (cash flow) shocks, as the effective risk aversion in the economy would become the consumption-weighted harmonic average of the risk aversions of the two types. Specifically, the market prices of output and demand risk would take the form $\theta_{Y,t} = \mathcal{R}_t \sigma_Y$ and $\theta_{\alpha,t} = -\bar{\sigma}_{\Delta,t}^{\mathcal{R}}$, with consumption-

and risk-tolerance-weighted risk aversion and estimation error defined as:[29]

$$\mathcal{R}_t = \frac{1}{f_t \frac{1}{\gamma^a} + (1 - f_t) \frac{1}{\gamma^b}} \qquad \text{and} \qquad \bar{\sigma}_{\Delta,t}^{\mathcal{R}} = \frac{f_t \frac{1}{\gamma^a} \sigma_{\Delta}^a + (1 - f_t) \frac{1}{\gamma^b} \sigma_{\Delta}^b}{f_t \frac{1}{\gamma^a} + (1 - f_t) \frac{1}{\gamma^b}}. \tag{16}$$

The consumption share, f_t, would now depend on both fundamental output shocks from differences in risk aversion and demand shocks from distribution shifts of preference types. Additionally, the price of risk for output shocks, $\theta_{Y,t}$, would indirectly reflect demand shocks through its dependence on consumption shares, leading to disagreement about the future risk premium for output shocks. This divergence would introduce a wedge between the stock and bond markets.

Another possible extension is to incorporate recursive preferences or to introduce output disagreement alongside demand disagreement. With recursive preferences, demand shocks would typically be priced even without demand disagreement. Moreover, allowing heterogeneity in time preferences, risk aversion, and the intertemporal elasticity of substitution (IES) would further increase the disconnect between stock and bond markets.

B. Time-Varying Disagreement—Learning from Experience

Until now, we have assumed that agents have dogmatic beliefs about the long-run mean of l_t, and thus the fraction of patient newborns, α_t. This leads to constant estimation errors and non-stochastic disagreement, a parsimonious approach given the exogenous nature of the process. Alternatively, one could assume exogenously that agents have differing beliefs about persistence, resulting in stochastic disagreement. Hence, we treat the estimation error for type $i = a, b$ agents, $\sigma_{\Delta,t}^i$, as stochastic, so that $\sigma_{\Delta,t} = \sigma_{\Delta,t}^a - \sigma_{\Delta,t}^b$ captures the stochastic disagreement between types. Where closed-form solutions apply,

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[29]See Heyerdahl-Larsen and Illeditsch (2023) for derivations of a general OLG model with differences in beliefs and preferences.

substituting the constant σ_Δ with $\sigma_{\Delta,t}$ provides comparable results.

In the Internet Appendix, we endogenize disagreement through learning. Here, agents observe lifetime data to infer the long-run mean of l_t, leading to time-varying, cohort-dependent disagreement. Using Bayes' rule and experience-based learning as in Ehling, Graniero, and Heyerdahl-Larsen (2018), agents initially share identical priors for the unknown long run mean of l_t but there is disagreement over time across birth cohorts. This results in a stochastic $\sigma_{\Delta,t}$ without the assumption of a false consensus bias. Furthermore, consumption shares become cohort-dependent: older agents typically have higher consumption shares due to lower estimation errors. This approach reproduces the qualitative findings of the baseline model, extending it to accommodate richer dynamics in beliefs and consumption patterns across cohorts.

C. Production Economies

Our prior analysis assumed an exchange economy with zero correlation between aggregate consumption and demand shocks, which naturally lacks a correlation puzzle. In the Internet Appendix, we extend this model to include production, following Brunnermeier and Sannikov (2014) for capital dynamics and investment adjustment costs. Capital evolves as $dK_t = (\psi(\iota_t) + \mu_K - \sigma_\Delta) K_t \, dt + \sigma_K K_t \, dZ_{K,t}$, where $\psi(\iota) = \frac{\log(\kappa\iota+1)}{\kappa}$ defines how the investment rate ι_t influences capital growth. With $\kappa \to 0$, the adjustment costs disappear and the output, as a fraction of capital, is given by $Y_t = aK_t$.

The production model shows that, with endogenous capital choice, the correlation puzzle remains. For a range of κ values, dividend-stock return correlations are low or negative, yet demand shock risk premia stay positive. Contrary to expectations, shocks negatively correlated with consumption growth do not inherently imply negative risk premia. The demand disagreement model highlights that speculative dynamics can reverse the price of risk, potentially intensifying the correlation puzzle in a production setting.

VII. Conclusion

We develop an overlapping generations (OLG) model in which the cross-sectional distribution of investor preferences and beliefs evolves over time. In our model, investors hold incomplete information and disagree on these changes, creating persistent disagreement about future asset demand—even when economic fundamentals are known. This demand disagreement gives rise to speculative trade that is disconnected from fundamentals, introducing priced demand shocks into the asset markets. Demand disagreement accounts for the low correlation between asset returns and economic fundamentals and explains yield disagreements that are not driven by macroeconomic factors. Moreover, it increases risk premia in both stock and bond markets, generates an upward-sloping real yield curve, establishes a positive relationship between demand disagreement and yield volatilities, and produces bond risk premia that are predictable by yield disagreement.

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Appendix A.   Appendix

Proofs

Proof of Proposition 1. To solve for the equilibrium stochastic discount factor, we insert optimal consumption from Equation (10) into the aggregate resource constraint, $\int_{-\infty}^{t} \nu e^{-\nu(t-s)}(\alpha_s c_{s,t}^a + (1-\alpha_s)c_{s,t}^b)ds = Y_t$ and rearrange terms. We get $\xi_t = \frac{X_t}{Y_t}$ where X_t solves the integral equation

$$X_t = \int_{-\infty}^{t} \nu e^{-\nu(t-s)} \left(\alpha_s \beta_s^a e^{-\rho^a(t-s)} \frac{\eta_t^a}{\eta_s^a} + (1-\alpha_s)\beta_s^b e^{-\rho^b(t-s)} \frac{\eta_t^b}{\eta_s^b} \right) X_s ds. \qquad (A1)$$

$\square$

Proof of Proposition 2. We use type a's optimal consumption demand, $c_{s,t}^a = c_{s,s}e^{-\rho^a t}\frac{\eta_t^a}{\eta_s^a}\frac{\xi_s}{\xi_t}$ to get

$$\xi_t X_t^a = \int_{-\infty}^{t} \nu e^{-(\rho^a+\nu)(t-s)}\alpha_s \beta_s^a \frac{\eta_t^a}{\eta_s^a} X_s ds = \xi_t \int_{-\infty}^{t} \nu e^{-(\rho^a+\nu)(t-s)}\alpha_s c_{s,t}^a ds. \qquad (A2)$$

Similarly, X_t^b follows from optimal consumption of type b agents. Hence, we get

$$f_t = \frac{\xi_t X_t^a}{\xi_t X_t^a + \xi_t X_t^b} = \frac{X_t^a}{X_t} \qquad (A3)$$

$\square$

Proof of Proposition 3. Applying Ito's lemma to X_t given in Equation (A1) leads to

$$dX_t = \left(-\left(f_t \rho^a + (1 - f_t) \rho^b \right) - \nu \left(1 - \alpha_t \beta_t^a - (1 - \alpha_t) \beta_t^b \right) \right) X_t dt + \mathcal{E}_f \left[\sigma_\Delta \right] X_t dZ_{\alpha,t}, \quad \text{(A4)}$$

with $\mathcal{E}_f \left[\sigma_\Delta \right] = f_t \sigma_\Delta^a + (1 - f_t) \sigma_\Delta^b$. Applying Ito's lemma Ito's o $\xi_t = \frac{X_t}{Y_t}$ leads to

$$
\begin{aligned}
d\xi_t &= \frac{dX_t}{Y_t} + X_t d\left(\frac{1}{Y_t} \right) + \frac{dX_t}{Y_t} d\left(\frac{1}{Y_t} \right) \\
&= \frac{X_t}{Y_t} \left(-\left(f_t \rho^a + (1 - f_t) \rho^b \right) - \nu \left(1 - \alpha_t \beta_t^a - (1 - \alpha_t) \beta_t^b \right) \right) dt \\
&\quad - \frac{X_t}{Y_t} \left(\mu_Y - \sigma_Y^2 \right) dt - \frac{X_t}{Y_t} \left(\sigma_Y dZ_{Y,t} - \bar{\sigma}_{\Delta,t} X_t dZ_{\alpha,t} \right) \\
&= \xi_t \left(-r_t dt - \theta_{Y,t} dZ_{Y,t} - \theta_{\alpha,t} dZ_{\alpha,t} \right), \quad \text{(A5)}
\end{aligned}
$$

where r_t, $\theta_{\alpha,t}$ and $\theta_{Y,t}$ are as in Proposition 3. $\square$

Proof of Proposition 4. We have $c_t^i = \left(\rho^i + \nu \right) W_t^i$ for $i \in \{a, b\}$ and define $\phi^i = \frac{W_{s,t}^i}{c_{s,t}^i} = \frac{1}{\rho^i + \nu}$. To derive the wealth-consumption ratio, we multiply the consumption share f_t and $(1 - f_t)$ with ϕ^a and ϕ^b, respectively. Then we add up both terms and impose market clearing to get

$$\phi^a f_t + \phi^b (1 - f_t) = \frac{W_t^a}{c_t^a} f_t + \frac{W_t^b}{c_t^b} (1 - f_t) = \frac{W_t^a + W_t^b}{Y_t} = \frac{W_t}{Y_t}. \quad \text{(A6)}$$

Aggregate consumption and wealth of all agents of type i are c_t^i and W_t^i for $i \in \{a, b\}$, respectively. $\square$

Proof of Corollary 1. Taking the first derivative of (i) the risk-free rate $r(f)$ given in Equation 12, the market price of demand shock risk $\theta_\alpha(f)$ given in Equation (13), and (iii) the wealth-consumption ratio $\phi(f)$ given in Proposition (4) w.r.t. the consumption share f proves the result. $\square$

Proof of Corollary 2. First, note that we can write, ϕ_t, as

$$
\begin{aligned}
\phi_t &= E_t \left[\int_t^\infty \frac{\xi_u}{\xi_t} e^{-\nu(u-t)} \frac{Y_u}{Y_t} du \right] = \int_t^\infty E_t \left[\frac{\xi_u}{\xi_t} e^{-\nu(u-t)} \frac{Y_u}{Y_t} \right] du \\
&= \int_t^\infty E_t \left[\frac{\hat{\xi}_u}{\hat{\xi}_t} \right] e^{-\nu(u-t)} E_t \left[e^{-\frac{1}{2} \sigma_Y^2 (u-t) - \sigma_Y \left(Z_{Y,u} - Z_{Y,t} \right)} \frac{Y_u}{Y_t} \right] du = \int_t^\infty e^{(\mu_Y - \sigma_Y^2 - \nu)(u-t)} B_t^u du
\end{aligned}
\quad \text{(A7)}
$$

where we have defined $\xi_t = \hat{\xi}_t e^{-\frac{1}{2} \sigma_Y^2 t - \sigma_Y \left(Z_{Y,u} - Z_{Y,t} \right)}$. $\square$

Proof of Proposition 5. Applying Ito's lemma to the consumption share $f_t = \frac{X_t^a}{X_t}$ leads to its dynamics. $\square$

Proof of Proposition 6. The wealth-consumption ratio is $\phi_t = \phi^a f_t + \phi^b (1 - f_t)$ and thus applying Itô's lemma to the price of the market portfolio $S_t = Y_t \phi_t$ and using the dynamics of the consumption share f_t given in Proposition 5 leads to the exposures of stock market returns to the supply $(\sigma_{R,t}^Y)$ and demand $(\sigma_{R,t}^\alpha)$ shock as well as the risk premium $\lambda_{R,t}$. $\sigma_{R,t}^\alpha$ is a concave function of the consumption share f_t and,

thus, taking the first derivative w.r.t. f leads to the global maximum of the demand shock exposure and thus the the conditional stock market volatility. □