Generalization of the main model

Magnus Dahlquist* Christian Heyerdahl-Larsen‡
Anna Pavlova† Julien Pénasse§

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*Stockholm School of Economics and CEPR, magnus.dahlquist@hhs.se.
†London Business School and CEPR, apavlova@london.edu.
‡BI Norwegian Business School, christian.heyerdahl-larsen@bi.no.
§University of Luxembourg, julien.penasse@uni.lu.

This document presents a generalization of the main model in Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse (2025).

1 Generalization of the main model

This Appendix generalizes the main model in Dahlquist et al. (2025).

$$U = \mathbb{E}\left[\int_0^\infty e^{-\rho t}\frac{\left(\hat{C}_t\right)^{1-\gamma}}{1-\gamma}\,dt\right], \tag{E1}$$

$$U^* = \mathbb{E}\left[\int_0^\infty e^{-\rho t}\frac{\left(\hat{C}^*_t\right)^{1-\gamma}}{1-\gamma}\,dt\right], \tag{E2}$$

where $\hat{C}_t$ and $\hat{C}^*_t$ are CES aggregators of the habit-adjusted consumption, i.e.,

$$\hat{C}_t = \left(\Theta_t^{1-\beta}\left(C_{H,t} - \phi H_t\right)^\beta + \left(1-\Theta_t\right)^{1-\beta}\left(C_{F,t} - (1-\phi)\,H^*_t\right)^\beta\right)^{\frac{1}{\beta}}, \tag{E3}$$

$$\hat{C}^*_t = \left(\left(1-\Theta_t\right)^{1-\beta}\left(C^*_{H,t} - (1-\phi)\,H_t\right)^\beta + \Theta_t^{1-\beta}\left(C^*_{F,t} - \phi H^*_t\right)^\beta\right)^{\frac{1}{\beta}}. \tag{E4}$$

This nests our main model as a special case when γ approaches one (log utility) and the β approaches zero (Cobb-Douglas aggregator).

We solve the model numerically using Monte-Carlo simulations. While the basic Monte-Carlo approach is simple, it is very computer-time intensive. This is further exacerbated by the fact that the model is highly nonlinear. Even when applying various variance reduction techniques such as control variates and antithetic variates, the model is still very time-consuming to solve. Therefore, we have focused on the key quantities that we believe are most important for our main results.

Table 1 shows the sensitivity of the model to the parameters of the utility function, β and γ. We have included the baseline model ($\gamma = 1$, $\beta = 0$). We look at the initial response to a one standard deviation negative Home-specific shock (shock to x_t) starting the state variables at their medians. We report the response to six key quantities: the log real exchange rate, net exports over GDP, net foreign assets over GDP, the log wealth ratio, the log consumption ratio, and the log GDP ratio.

The results are qualitatively similar to the results in the main text. First, the wealth, consumption,

and GDP ratios all increase after a negative Home-specific shock (shock to x_t). Second, the log real exchange rate is always decreasing, i.e., the Home currency appreciates following a negative Home-specific shock. Note that as we increase the substitutability between the home and the foreign good, the initial response is lower. This is expected, and in the limit when the goods are perfect substitutes, there would be no effect at all. The net export over GDP and NFA over GDP have the same sign as in the main model for elasticities that are not too high, where both NFA and NX changes sign.

Table 1: Sensitivity to parameters of the utility function, β and γ.

β	γ	$\log Q$	NX / GDP	NFA / GDP	log W_R	log C_R	log GDP_R
-0.2500	0.7500	-0.0058	0.0006	-0.0107	0.0043	0.0047	0.0058
-0.2500	1.0000	-0.0058	0.0006	-0.0071	0.0019	0.0048	0.0058
-0.2500	1.2500	-0.0058	0.0005	-0.0071	0.0022	0.0049	0.0058
0.0000	0.7500	-0.0046	0.0002	-0.0058	0.0026	0.0036	0.0039
0.0000	1.0000	-0.0046	0.0001	-0.0035	0.0016	0.0037	0.0039
0.0000	1.2500	-0.0046	0.0001	-0.0032	0.0010	0.0038	0.0039
0.2500	0.7500	-0.0035	-0.0002	-0.0019	0.0020	0.0025	0.0020
0.2500	1.0000	-0.0035	-0.0003	0.0003	0.0005	0.0026	0.0020
0.2500	1.2500	-0.0035	-0.0003	0.0006	0.0004	0.0027	0.0020

$\log Q$ denotes the log real exchange rate. NX / GDP is net exports over GDP, NFA / GDP is net foreign assets over GDP. W_R, C_R, and GDP_R refer to wealth, consumption, and GDP ratios in logs, respectively. The highlighted row corresponds to our main model in Section 4.

References

Dahlquist, Magnus, Christian Heyerdahl-Larsen, Anna Pavlova, and Julien Pénasse, 2025, The dollar and the US wealth share, Working paper.