1. Find $\int \sec^6 x \, \mathrm{d}x$

Answer. Use the identities $\frac{\mathrm{d}}{\mathrm{d}x} \tan x = \sec^2 x$ and $\tan^2 x + 1 = \sec^2 x$. Rewrite the integral as

$$\int \sec^4 x \sec^2 x \, \mathrm{d}x = \int (\tan^2 x + 1)^2 \sec^2 x \, \mathrm{d}x.$$

Then substitute $u = \tan x, \ \mathrm{d}u = \sec^2 x \, \mathrm{d}x$. This gives:

$$\begin{aligned} \int (u^2+1)^2 \, \mathrm{d}u &= \int (u^4 + 2u^2 + 1) \, \mathrm{d}u \\ &= u^5/5 + 2u^3/3 + u + C \\ &= \frac{\tan x(3\tan^4 x + 10\tan^2 x + 15)}{15} + C. \end{aligned}$$

$\square$

Answer: $\frac{\tan x(3\tan^4 x+10\tan^2 x+15)}{15} + C$

2. True or False?

a) If $\sec\theta = \frac{5}{3}$, then $\cot\theta = \frac{3}{4}$.

Answer: True

b) $\int e^{x^2} \, \mathrm{d}x = \dfrac{e^{x^2}}{2x}$

Answer: False

3. Find $\displaystyle\int_0^{\pi/2} e^x \cos x \,\mathrm{d}x$

Answer. First find the indefinite integral, and then plug in the values at the end. Integrate by parts, setting

$$\begin{aligned} u &= \cos x & \mathrm{d}v &= e^x \,\mathrm{d}x \\ \mathrm{d}u &= -\sin x & v &= e^x \end{aligned}$$

Then we get that

$$\int e^x \cos x \,\mathrm{d}x = e^x \cos x + \int e^x \sin x \,\mathrm{d}x.$$

Now, do the same thing with the integral at the end, and get

$$\int e^x \sin x \,\mathrm{d}x = e^x \sin x - \int e^x \cos x \,\mathrm{d}x.$$

Now just put it all together:

$$\begin{aligned} \int e^x \cos x \,\mathrm{d}x &= e^x \cos x + \left(e^x \sin x - \int e^x \cos x \,\mathrm{d}x\right) \\ \int e^x \cos x \,\mathrm{d}x &= e^x \cos x + e^x \sin x - \underbrace{\int e^x \cos x \,\mathrm{d}x}_{\text{add this to both sides}} \\ \int e^x \cos x \,\mathrm{d}x + \int e^x \cos x \,\mathrm{d}x &= e^x \cos x + e^x \sin x \\ 2\int e^x \sin x \,\mathrm{d}x &= e^x \sin x + e^x \cos x \\ \int e^x \sin x \,\mathrm{d}x &= \frac{e^x \sin x + e^x \cos x}{2}. \end{aligned}$$

Finally, plug in $\pi/2$ and 0 to get

$$\frac{e^{\pi/2}}{2} - \left(\frac{1}{2}\right).$$

$\square$

Answer: $\frac{e^{\pi/2}-1}{2}$