1. Solve $\displaystyle\int \frac{6}{(x-1)(x+5)}\,\mathrm{d}x$

Answer. Use partial fractions to break up the integrand.

$$\begin{aligned}
\frac{6}{(x-1)(x+5)} &= \frac{A}{x-1} + \frac{B}{x+5} \\
(x-1)(x+5)\left(\frac{6}{(x-1)(x+5)}\right) &= (x-1)(x+5)\left(\frac{A}{x-1} + \frac{B}{x+5}\right) \\
6 &= (x+5)A + (x-1)B.
\end{aligned}$$

Now just plug in -5 and 1 to find that $B = -1$ and $A = 1$. So we've reduced the original problem to

$$\int \frac{1}{x-1} - \frac{1}{x+5}\,\mathrm{d}x = \ln(x-1) - \ln(x+5) = \ln\left(\frac{x-1}{x+5}\right).$$

$\square$

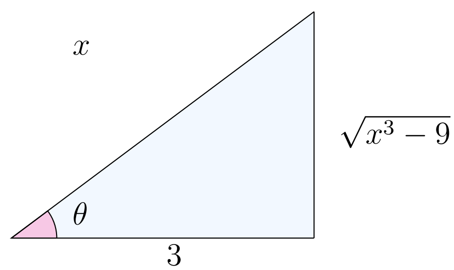
Answer: $\ln\left(\frac{x-1}{x+5}\right)$

2. Solve $\int \frac{1}{\sqrt{x^2-9}}\,\mathrm{d}x$

Answer. Use the substitution $x = 3\sec\theta,\ \mathrm{d}x = 3\tan\theta\sec\theta\,\mathrm{d}\theta$. This gives

$$\int \frac{3\tan\theta\sec\theta}{\sqrt{(9)(\sec^2\theta-1)}}\,\mathrm{d}\theta = \int \sec\theta\,\mathrm{d}\theta = \ln|\sec\theta+\tan\theta|.$$

To substitute back for x, draw a triangle, and use the identity $\sec\theta = \frac{x}{3}$.



Then $\ln|\sec\theta+\tan\theta| = \ln|\frac{x+\sqrt{x^2-9}}{3}|$. □

Answer: $\ln|\sec\theta+\tan\theta| = \ln|\frac{x+\sqrt{x^2-9}}{3}|$