1. Evaluate $\displaystyle\int \frac{e^x \, \mathrm{d}x}{e^{2x} - 1}$

Answer. Set $u = e^x$, and so $\mathrm{d}u = e^x \, \mathrm{d}x$. Then the integral becomes

$$\int \frac{\mathrm{d}u}{(e^x)^2 - 1} = \int \frac{\mathrm{d}u}{u^2 - 1} = \int \frac{\mathrm{d}u}{(u+1)(u-1)}$$

Now we use partial fractions to break up this fraction:

$$\frac{1}{(u+1)(u-1)} = \frac{A}{u+1} + \frac{B}{u-1}$$

$$1 = (u-1)A + (u+1)B$$

Plugging in $u = 1, u = -1$ gives $A = -1/2, B = 1/2$.
So now, the problem is

$$-1/2 \int \frac{\mathrm{d}u}{u+1} + 1/2 \int \frac{\mathrm{d}u}{u-1}$$

which equals

$$-1/2 \ln|u+1| + 1/2 \ln|u-1| = 1/2 \ln\left|\frac{u-1}{u+1}\right|.$$

Substituting back in for u gives

$$1/2 \ln\left|\frac{e^x - 1}{e^x + 1}\right| = \ln\sqrt{\frac{e^x - 1}{e^x + 1}}.$$

$\square$

Answer: $\ln\sqrt{\frac{e^x-1}{e^x+1}}$

2. Evaluate $\int x \sec x \tan x \, \mathrm{d}x$

Answer. Integrate by parts, with

$$\begin{array}{ll} u = x & \mathrm{d}v = \sec x \tan x \, \mathrm{d}x \\ \mathrm{d}u = 1 & v = \sec x \end{array}$$

Then we get

$$\begin{aligned} \int x \sec x \tan x \, \mathrm{d}x &= x \sec x - \int \sec x \, \mathrm{d}x \\ &= x \sec x - \ln|\sec x + \tan x| \end{aligned}$$

□

Answer: $x \sec x - \ln|\sec x + \tan x$

3. True or False?

a) The polynomial $2x^4 - 3x^3 - 19x^2 - 6x + 8$ is divisible by $(x + 1)$ (being divisible means there is 0 remainder after division).

Answer: True

b) The following is a correct first step for breaking up a fraction:

$$\frac{1}{(x-1)^2(x^2+1)(x^2-2x+12)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x^2+1} + \frac{Dx+E}{x^2-2x+12}.$$

Answer: False