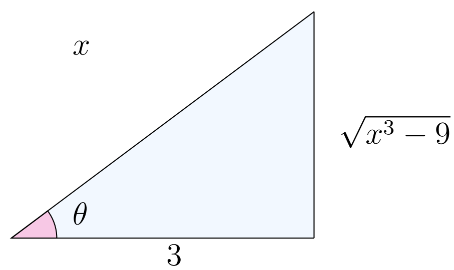
1. Solve $\displaystyle\int \frac{1}{\sqrt{x^2-9}}\,\mathrm{d}x$

Answer. Use the substitution $x = 3\sec\theta,\ \mathrm{d}x = 3\tan\theta\sec\theta\ \mathrm{d}\theta$. This gives

$$\int \frac{3\tan\theta\sec\theta}{\sqrt{(9)(\sec^2\theta-1)}}\,\mathrm{d}\theta = \int \sec\theta\ \mathrm{d}\theta = \ln|\sec\theta+\tan\theta|.$$

To substitute back for x, draw a triangle, and use the identity $\sec\theta = \frac{x}{3}$.



Then $\ln|\sec\theta+\tan\theta| = \ln|\frac{x+\sqrt{x^2-9}}{3}|$. □

Answer: $\ln|\sec\theta+\tan\theta| = \ln|\frac{x+\sqrt{x^2-9}}{3}|$

2. Solve $\displaystyle\int \frac{5}{(x+3)(x-2)}\,\mathrm{d}x$

Answer. Use partial fractions to break up the integrand.

$$\begin{aligned}\frac{5}{(x+3)(x-2)} &= \frac{A}{x+3}+\frac{B}{x-2}\\ (x+3)(x-2)\left(\frac{5}{(x+3)(x-2)}\right) &= (x+3)(x-2)\left(\frac{A}{x+3}+\frac{B}{x-2}\right)\\ 5 &= (x-2)A+(x+3)B.\end{aligned}$$

Plugging in 2 and -3 for x tells us that $B=1$ and $A=-1$.
So we have reduced the original problem to

$$\int \frac{-1}{x+3}+\frac{1}{x-2}\,\mathrm{d}x = -\ln(x+3)+\ln(x-2)=\ln\left(\frac{x-2}{x+3}\right).$$

□

Answer: $\ln\left(\frac{x-2}{x+3}\right)+C$