

1 CCD Gain Lab: The Theory.

The CCD consists of a rectangular array of $N_{row} \times N_{col} = M$ picture elements, or *pixels*. (In the case of the Photometrics CCD used on our 20 inch telescope, there are $576 \times 384 = 221,184$ pixels.) After the CCD chip has been exposed to the light of the optical image focused upon it, the charge accumulated by each of the pixels is read out. Because these small amounts of charge must be greatly amplified before they are turned into an array of numbers, and because a constant “bias” voltage is added in the process, the numbers which are output by the CCD electronics is not simply the number of charges accumulated by the pixels. Rather, the output numbers are in what are called “analog-digital units” (or ADU’s) and consist of numbers which are *proportional to* the electron charge created by the photons, plus the constant bias offset (so that negative numbers will never arise).

We first define some quantities. Most of the following symbols (except G and \overline{B}) represent *arrays* of numbers. The arrays will be of size M (the number of pixels in the CCD, or in some section of it).

n = the actual number electron charges accumulated by each pixel

S = the output signal of the CCD electronics in ADU’s (“analog-digital units”)

B = the output signal for the case of a “bias” frame

\overline{B} = the bias level in ADU (an average over the array)

G = the “gain” of the amplifier in ADU/electron

Δ = the (random) read-out noise

$\sigma_B = \sqrt{\overline{\Delta^2}}$ = standard deviation of the read-out noise.

One indispensable step in observing with a CCD is to take a *bias* image, that is, we simply read out the CCD without letting any light fall on the pixels. Generally, this is an exposure of 0 seconds. (Astronomers also obtain so-called “dark” frames, where no light is admitted but the CCD is integrated for a length of time equal to the exposures being taken. The dark frame will thus also include the thermal electrons which accumulate during the exposure. Because our exposures for this lab are short, we do not need to take dark frames.)

Because of the constant bias voltage, the bias frame will not produce an array of zeros; instead we get array of numbers that represent the bias. Further, the numbers will not be exactly equal for each pixel due to random noise in the electronics, called “read-out noise”. Let us call the output of the electronics for a bias image B . Further, let \overline{B} represent the average value of B over the CCD chip:

$$\overline{B} = \frac{1}{M} \sum_{i=1}^{N_{row}} \sum_{j=1}^{N_{col}} B_{ij} \quad (1)$$

Then we can write the elements of B as

$$B = \overline{B} + \Delta_B \quad (2)$$

where the array Δ_B represents fluctuations about the mean value from pixel to pixel. This “read-out noise” for a particular pixel will differ from one bias image to the next, but once the CCD has cooled down and is stable, \overline{B} should be constant, and the *probability distribution* of Δ_B should also be stable. Clearly, the mean of Δ_B must vanish:

$$\overline{\Delta_B} = \frac{1}{M} \sum_{i=1}^{N_{row}} \sum_{j=1}^{N_{col}} [\Delta_B]_{ij} = 0 \quad (3)$$

Now suppose we expose the CCD to uniform illumination at some intensity. We will now have the signal due to the electrons liberated by the photons and collected in the potential well of each pixel. But we will still have read-out noise (call it Δ_S). Thus we can write the basic equation relating the observed output, S , to the photon events, n , as

$$S = G \times n + \overline{B} + \Delta_S \quad (4)$$

Here, G is the **gain**. It is a number which relates the number of electron charges on a pixel to the number that comes out of the CCD electronics box; it depends upon the amount of amplification, etc. This is a number we want to determine.

Let us subtract the bias frame from the frame exposed to light; this will remove the constant bias level \overline{B} . Subtracting eq(2) from eq(4):

$$S' = S - B = G n + \Delta_S - \Delta_B \quad (5)$$

Now let us average over the pixels (remember that S' is really an array S'_{ij}):

$$\overline{S'} = G \overline{n} + \overline{\Delta_S} - \overline{\Delta_B} = G \overline{n} \quad (6)$$

since the Δ 's are fluctuations about the mean and average to zero.

Now that we have the mean bias-subtracted signal $\overline{S'}$, let us consider the *variance* in S' :

$$V_{S'} = \overline{(S' - \overline{S'})^2} = \overline{(Gn + \Delta_S - \Delta_B - G\overline{n})^2} = \overline{(G\Delta_n + \Delta_S - \Delta_B)^2} \quad (7)$$

where $\Delta_n = n - \overline{n}$. We expand the square and have

$$V_{S'} = G^2 \overline{\Delta_n^2} + 2G \overline{\Delta_n(\Delta_S - \Delta_B)} + \overline{(\Delta_S - \Delta_B)^2} \quad (8)$$

Now consider what happens if we multiply one of the Δ terms with any quantity *that is not correlated with it*, and take the average of this product. There will be as many positive Δ 's as negative ones, and since M is large, the average will tend to zero. Thus the $\overline{\Delta_n \Delta_B}$ and $\overline{\Delta_n \Delta_S}$ terms will vanish. So we have

$$V_{S'} = G^2 \overline{\Delta_n^2} + \overline{\Delta_S^2} - 2\overline{\Delta_S \Delta_B} + \overline{\Delta_B^2} \quad (9)$$

But Δ_S is not correlated with Δ_B since they are fluctuations that came from different exposures. Thus $\overline{\Delta_S \Delta_B}$ also vanishes and we are left with

$$V_{S'} = G^2 \overline{\Delta_n^2} + 2 \overline{\Delta_B^2} \quad (10)$$

Here, we use the fact that the *variance* of the Δ_S is the same as that of Δ_B , even if the individual elements of the arrays differ. This variance we will just call σ_B^2 . That is, σ_B is the *root mean square of the bias frame*:

$$\sqrt{\frac{1}{M} \sum_{ij} (B_{ij} - \bar{B})^2} = \sqrt{\frac{1}{M} \sum_{ij} [\Delta_B]_{ij}^2} = \sqrt{\overline{\Delta_B^2}} = \sigma_B \quad (11)$$

We now come to the key point of this project. We introduce the idea that the electrons ejected by the photons striking the CCD will have some mean value corresponding to the intensity of the light, but that the individual numbers of electrons counted by the pixels will obey a *Poisson distribution*. But for a Poisson distribution, $\sigma_n = \sqrt{N} = \sqrt{\bar{n}}$. Thus we can say that in equation (10), $\overline{\Delta_n^2} = \bar{n}$. Hence,

$$V_{S'} = G^2 \bar{n} + 2 \sigma_B^2 \quad (12)$$

Furthermore, we can use equation (6) to replace $G\bar{n}$ with $\overline{S'}$ to obtain

$$V_{S'} = G \overline{S'} + 2 \sigma_B^2 \quad (13)$$

The importance of this result is that we can *measure* $\overline{S'}$ and $V_{S'}$. We would then like to use this equation to determine the gain G of our system and the read-out noise σ_B . It would seem that we have only one equation but two unknowns. But suppose we change the intensity of the light. Then the measured $\overline{S'}$ and $V_{S'}$ will change, but G and σ_B would remain the same, so we now have two equations for our two unknowns.

1.1 A More Realistic Analysis

The problem with the preceding analysis is that we have assumed that all the pixels are alike, so that if we illuminate the CCD with a uniform intensity of light, all the pixels would eject electrons at the same average rate \bar{n} . But the sensitivity of the pixels is not truly constant, but can vary by $\sim 10\%$ over the chip. This means that the average n will tend to be larger than \bar{n} for some pixels and smaller for others. Thus $\Delta_n = n - \bar{n}$ will be systematically correlated with the position of the pixel. As a result, our measured $\overline{\Delta_n^2}$ will be too large.

We can eliminate these systematic variations by taking *two* images with identical conditions. Then we will have

$$S_1 = G n_1 + \bar{B} + \Delta_{S_1} \quad n_1 = \bar{n} + \Delta_{n_1} \quad (14)$$

$$S_2 = G n_2 + \bar{B} + \Delta_{S_2} \quad n_2 = \bar{n} + \Delta_{n_2} \quad (15)$$

$$B = \bar{B} + \Delta_B \quad (16)$$

Here we can allow \bar{n} to be an array of numbers, i.e., each pixel will have its own average number of electrons. But this average will not change from one exposure to the next. So, if we subtract the first image from the second, we obtain

$$S_1 - S_2 = G (\Delta_{n_1} - \Delta_{n_2}) + (\Delta_{S_1} - \Delta_{S_2}) \quad (17)$$

and the Δ_n are just Poisson noise, without any of the systematic \bar{n} variation. We then may take the variance of this difference in the images:

$$V_s = \overline{(S_1 - S_2)^2} = G^2 \overline{(\Delta_{n_1} - \Delta_{n_2})^2} + 2G \overline{(\Delta_{n_1} - \Delta_{n_2})(\Delta_{S_1} - \Delta_{S_2})} + \overline{(\Delta_{S_1} - \Delta_{S_2})^2} \quad (18)$$

Just as in the simple case, all the cross terms disappear, and we are left with

$$V_s = 2G^2 \overline{(\Delta_n)^2} + 2\sigma_B^2 \quad (19)$$

As before, our knowledge that we are dealing with a Poisson distribution allows us to replace $\overline{(\Delta_n)^2}$ with the mean \bar{n} . Here we cheat a little, because each pixel has its own \bar{n} , but since the pixel to pixel variations are not large, that will not be very important. Again, we replace $G\bar{n}$ with \bar{S}' and obtain

$$\boxed{V_S = 2G \bar{S}' + 2\sigma_B^2} \quad (20)$$

To get a better value of \bar{S}' , we should average both exposures:

$$\bar{S}' = \frac{1}{2} \{ \overline{(S_1 - B)} + \overline{(S_2 - B)} \} \quad (21)$$

Written out explicitly, this would be

$$\bar{S}' = \frac{1}{2M} \sum_{i=1}^{N_{row}} \sum_{j=1}^{N_{col}} \{ ([S_1]_{ij} - B_{ij}) + ([S_2]_{ij} - B_{ij}) \} \quad (22)$$

Likewise, the expression for the computation of V_S is

$$V_S = \frac{1}{M} \sum_{i=1}^{N_{row}} \sum_{j=1}^{N_{col}} ([S_1]_{ij} - [S_2]_{ij})^2 \quad (23)$$

In the lab exercise, instead of the whole CCD chip, the $M = N_{row} \times N_{col}$ array will be a smaller rectangular area on the chip. We will illuminate different areas of the chip with different intensities. For each such region we can obtain a value of \bar{S}' and of V_S . We may then plot these pairs of values on a graph with axes \bar{S}' and V_S . Because of equation (20), these points should lie on a straight line: the slope of the line is $2G$ and the intercept with the V_S axis ($\bar{S}' = 0$) gives us $2\sigma_B^2$. Another way of obtaining σ_B is directly from the bias frame using equation (11).

The final result is usually expressed as the reciprocal of G , called K :

$$K = \frac{1}{G} \quad \text{electrons/ADU}$$

and the quantity R

$$R = K \sigma_B = \frac{\sigma_B}{G} \quad \text{read-out noise, in electrons}$$

The read-out noise in electrons, R , is the number generally quoted which characterizes how noisy the CCD is. At faint light levels, it limits what can be detected. In particular, when the number of electrons collected is of order R^2 or less, the read noise limits detection. The read noise is particularly important for spectroscopic (narrow wavelength band), high time resolution, or high angular resolution observations: situations where only a few photons may strike an individual pixel. It is less important for broad-band, long exposures, or low angular resolution observations.