Astrophysics II: Laboratory 2

Black Body Spectrum and The HR Diagram

January 27, 2012

1 Objectives:

- Learn functionality of MATLAB.
  - Write and run functions (.m files).
  - Learn how Planck Function changes with Temperature.
  - Motivate optical astronomical observations using filters.
  - Explore parameter space via visualization.
  - Create theoretical HR diagram using Planck function.
- Prepare to create HR Diagram from SDSS stellar cluster data in future lab.

2 Theory

A physical body is said to emit thermal radiation as a Black Body (BB) if it is radiating with maximal efficiency for a particular temperature. Correspondingly, such a body is also a perfect absorber, absorbing all incident radiation (hence the name black body). There is no mechanism for cooling of the body via thermal radiation which can release more heat energy per unit time than BB radiation, which has a specific spectrum dependent only on temperature. This spectrum is given by Planck's law

$$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1},$$

where T is the temperature of the body, k_B is Boltzmann's constant, ν is the frequency of radiation, c is the speed of light, h is Planck's constant, and I is the energy per unit time radiated per unit area of emitting surface in the normal direction per unit solid angle per unit frequency (i.e., the intensity).

Associated with the Planck black body spectrum is a formula relating the frequency of the peak of the spectrum to the temperature, Wien's law,

$$\nu_{max} = T \times 59\text{GHz K}^{-1}$$

and the Stefan-Boltzmann law giving the power emitted per unit area of the surface of the black body,

$$\text{Power per unit area} = \sigma T^4$$

where σ is the Stefan-Boltzmann constant.

3 Procedure

3.1 MATLAB: Further down the path...

1. Open Matlab. Recall sections of the window: editor, command line, history, data window, etc.

2. Write a new function definition file that defines the Planck Function. This function should accept two arguments – Temperature and frequency – and return the specific intensity, as described in the theory intro.

3. At the command line create a log value array using logspace() as you did last time using linspace(). logspace([lower],[upper],[num]) is a built-in Matlab function with returns a list of values logarithmically spaced starting with *lower*, ending with *upper*, and the list is of length *num*.

4. Now try using the command semilog() rather than plot() to plot your list against exp(<your list>).

5. Now try using the plot command loglog(). Note that all three of these plot commands use roughly the same syntax. (i.e. you don't have to change the way you call them)

6. Now, use your new plotting skills to plot the Planck Function with a range of frequency values = [10e4, 10e24]. Use 200 data points in a list made with logspace().

   (a) You will need to set the vertical range using ylim([ymin ymax]).

   (b) Type hold on

   (c) Plot the Planck Function for several black body temperatures in the range 100 K to 10 billion K.

7. Estimate g-r values:

   (a) Choose a frequency which would be green to the eye. Do the same for a frequency in red.

   (b) In MATLAB, set up a vector including 20 values of temperature in the range typical of main sequence stars, 3000 to 18000 K.

   (c) The HR diagram requires luminosities and effective temperatures. Typically, the magnitude in R band (red) roughly scales as the luminosity, and the effective temperature can be estimated from $G - R$ (green minus red) as the color index. Recall that the stellar magnitudes scale as $-\log(\text{intensity})$.

(d) Plot these data points: each point should roughly represent (units may be wrong but behavior close to correct) luminosity ($\log R$) on the y-axis and the color index $((-\log G) - (-\log R))$ on the x-axis.

(e) The x-axis is now reverse of what we want. So, to avoid this use the following order of commands:

```
>> clf
>> set(axes, 'XDir', 'reverse')
>> hold on
>> plot(g-r, r, '+')
```

(f) Take note of the pattern this data illustrates in your linear-linear plot. This IS effectively a theoretical HR diagram. However, we should note that this is only roughly accurate for the main sequence for actual stars. On the main sequence, to 0th order, the surface area of the star doesn't change enough to significantly effect our simplified HR diagram in this lab. However, for more massive, less massive, and stars near the end of their lives (on Red Giant branch) in order to make a true theoretical HR diagram, we would have to take into the account the total flux of the star as a function of both surface temperature AND surface area. This is why, despite relatively low surface temperatures, red giants have highest luminosity on true HR diagrams.

4 Questions

Now let's think about the usefulness of what we've learned.

Why did we plot r and g-r on their respective axis? What physical properties does each value represent? What do r and T, from the plot in the previous section of this laboratory, have in common?