# Lab #2:
# Parallax & Introduction to Error Analysis

February 6, 2012

Due February 10, 2012 by 11am

Objectives:

To understand and apply the techniques of parallax to measure the distances to terrestrial objects.

To understand the importance of accurate measuring tools and practices.

To understand and apply methods of data analysis.

Equipment:

One pentant and tape measure for each group of 5 to 6 students.

One meter stick per group.

Chalk, string, binder clips

# Introduction

**Part 1: Parallax**

Parallax is the apparent (observed) shift of an object as compared to a more distant object as the position of the observer changes. In other words, it is the shift of an object compared to the horizon as you move about. Parallax allows you to measure the distance to the object which appears to move. Our eyes use parallax everyday while driving, walking, or bicycling to judge distances to objects so that we can avoid crashing into them.

You can try this on your own right now. Choose a far wall or chalkboard as your background. Hold up both of your thumbs, each at different distances from your eyes. Hold your right eye open and close your left eye and note where on the wall each thumb appears to rest. Now close your right eye and open the left. Your thumbs will have shifted with respect to the background wall. Note that the thumb closest to your eyes has shifted the most. This is true of any of these types of measurements. The closer the object is, the more it appears to shift.

The parallax is found using the simple trigonometry of a right triangle. From the tangent of the angle $p$, we get:

$$\tan p = \frac{b}{d}$$

where $d$ is the distance to the object of interest, and $b$ is the baseline.

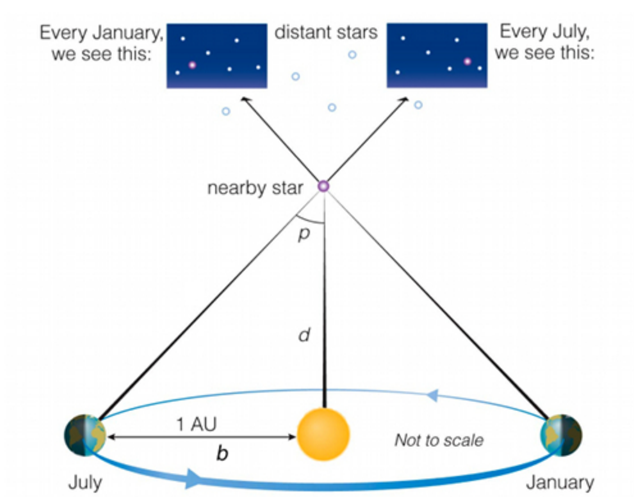


Figure 1: Parallax using Earth's orbit as the baseline. Adapted from *The Cosmic Perspective, Sixth Edition*

In astronomical cases, the angles are typically small enough that the small angle approximation

($\tan p \sim p$) can be used. Then the equation can be written as:

$$p = \frac{b}{d}$$

where $p$ must be in RADIANS, and the units of the baseline $b$ and distance $d$ must match, so that they cancel out. For this lab, we will be restricting ourselves to small angles as well.

In the special case where one uses the units of AU for the $b$, arcsec for $p$, and parsec for $d$, the equation becomes:

$$p_{\text{arcsec}} = \frac{b_{\text{AU}}}{d_{\text{pc}}}$$

resulting, for Earth-based observations of the stars, in the familiar expression from lecture:

$$p_{\text{arcsec}} = \frac{1}{d_{\text{pc}}}$$

**Part 2: Error Analysis**

The easiest way to calculate the error of any measurement is to find the difference between the true values and your measured value. This difference is then the error associated with the measurement. However, we don't always know what the correct answer should be before we conduct an experiment. We might have a general idea of the order of magnitude, but not the exact answer. In that case, we need a way to estimate the error from the data itself.

In order to calculate the error, the measurement must be made several times. This can be done by one person who is careful not to let previous measurements bias the current measurement, or by several different people, such as each person in this class.

The best answer is then calculated by adding all values and dividing that sum by the number of measurements:

$$\langle x \rangle = \frac{(x_1 + x_2 + x_3 + \ldots + x_N)}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

where $x_i$ is each individual measurement and $N$ is the total number of measurements. Your best answer is then $\langle x \rangle$, which is the same as the average value of $x$.

But how well does your data actually fit this answer? The standard deviation of your measurements

$$\sigma_x = \sqrt{\frac{(x_1 - \langle x \rangle)^2 + (x_2 - \langle x \rangle)^2 + (x_3 - \langle x \rangle)^2 + \ldots + (x_N - \langle x \rangle)^2}{N-1}} = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \langle x \rangle)^2}{N-1}}$$

basically tells you how well your data agree with each other and the mean. The smaller the value, the less error in your individual measurements.

The next thing we must measure is the standard error of the mean. While the standard deviation is the uncertainty associated with your individual data points, the standard error of the mean is the uncertainty associated with your mean. We find this by dividing your standard deviation by the square root of the total number of measurements:

$$SE = \frac{\sigma_x}{\sqrt{N}} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \langle x \rangle)^2}{N(N-1)}}$$

Again, the smaller your value, the less error in your answer.

Note that the standard error of the mean has $N$ in the denominator. This means that as you increase the number of measurements ($N$), your standard error of the mean will become smaller, even though the standard deviation of your data points might not.

The last step to the error analysis is to determine how the error in your measurement propagates to your final calculated result. In general, the formula for error propagation is

$$\sigma_x = \sqrt{\sigma_u^2(\frac{\partial x}{\partial u})^2 + \sigma_v^2(\frac{\partial x}{\partial v})^2 + \dots}$$

where $u$ and $v$ are the measured quantities and $x$ is the quantity calculated from the measurements. In today's lab, we need 2 more specific solutions to this error propagation equation.

If we calculate the value $p = ka$, where $p$ is your parallax angle in radians, $a$ is your measurement in inches and $k$ is a constant (i.e. 0.0349 radians/inch), then the error propagation equation simplifies to:

$$\sigma_p = \sigma_a(\frac{\partial p}{\partial a})$$

$$\sigma_p = \sigma_a k$$

where $\sigma_a$ is the standard error of the mean (SE) of your measurements.

If we calculate the value $d = n/p$, where $d$ is the distance, $n$ is a constant (your baseline), then the error propagation equation becomes:

$$\sigma_d = \sigma_p(\frac{\partial d}{\partial p})$$

$$\sigma_d = \sigma_p \frac{n}{p^2}$$

where $\sigma_p$ is the same $\sigma_p$ you found from the previous equation.

## Laboratory Procedures

In this lab you will be using trigonometric parallax to determine the distances to two objects. You will do this using two different baselines. This can be related to taking the measurements from opposite sides of the Earth's orbit and the opposite sides of, say, Jupiter's orbit. Be sure to watch for the differences that baselines introduce.

In order to measure the angular shifts of the objects in question, you will need a wooden device called a pentant. This device looks a bit like a crossbow with a bent yardstick at one end. To use the pentant, place the handle end on your cheek just under one eye (being careful, of course, not to poke an eye out!). Line the beginning of the yardstick (zero inches) with one of the objects in question and record the number of inches to the other object. The pentant has been calibrated to subtend 1/5 of a circle on your horizon or $\frac{2\pi}{5}$ radians. So 36 inches spanning $\frac{2\pi}{5}$ radians gives 0.0349 radians per inch. Be sure to remember to convert your inches to radians!

**Procedure Steps:**

(1) Examine your pentant. Try measuring angular separations between various objects to get an idea of how the pentant works. When you have a good feel for how to make a measurement, record the smallest angle (in inches) you believe you can accurately measure using the pentant. We will use this value in the Questions section.

Smallest angle you can measure:________________________

(2) In a group of 5 or 6 students, stand outside the building and examine your horizon. Choose some distinct object in the far horizon to be your reference point. The farther, the better.

(3) Now, choose another object (a tree, a lamppost, anything) that is considerably closer to you, but make sure that it is at least, say, 3 or 4 car lengths away. You will be calculating the distance to this object. Keep in mind that you will also have to measure the distance to the object with string, tape measures, etc. Record which object you chose.

Object chosen:________________________

(4) Now, visually line up the object with your reference point on the horizon. The point at which you are now standing will be somewhat like the position of the Sun in the parallax diagram from your textbook. Mark this point on the ground with chalk (or some small, heavy object that will not carry in the wind, if you are not on pavement/sidewalk).

(5) Now, use 1 meter as your baseline (to simulate the 1 AU baseline of Earth's orbit). Standing at the end of the baseline, measure and record (in the table on the last page, for Object 1: 1 m baseline) the angle of separation (in inches) between your object and your reference object using the pentant. MAKE SURE YOUR ANGLE IS LESS THAN 10.5 INCHES! If it is larger than that, your object is too close to you. You'll have to back away from it, or find a new object if you don't have room to back up. Each student in the group should make a measurement, but do not share your measurement with the other members of your group until everyone has done the measurement. We want to minimize the chances of bias in the measurements. To test who's reading these directions closely, please include the name of your favorite science fiction or fantasy television show below the table on the last page. Do not tell anyone else that you read this. Once you have all made the measurement, complete the table for Object 1: 1 m baseline by filling in the measurements from the rest of your group. You will need the measurements of everyone in the group to do the data analysis.

(6) Repeat Step 5 now using a baseline of 5 meters from your original reference point (to simulate the 5 AU baseline of Jupiter's orbit). Make a note of the differences in your measurements due to a larger baseline. Record your measurement in the table on the last page under Object 1: 5 m baseline, and, once everyone has finished doing the measurement, record the rest of your group's measurements in the table as well.

(7) Devise another means of measuring the distance to this object (say with a meter stick or tape measure). These are not to be included in your final mean for the distance to the object, but will be needed to answer question (3). Record these distances in the appropriate space in the table on the last page.

(8) Choose another object approximately double the distance of the first from your position. Repeat Steps (3) through (7) to measure the distance using both baselines. These measurements will be recorded in the table on the last page under Object 2 for the two baselines.

Object chosen:________________________

## Questions

Answer the following questions on a separate sheet of paper.

(1) Analyze your data; for Object 1: 1 m baseline, calculate the mean, standard deviation, and standard error of the mean for your original measurements, *in inches*. **Write these values in the appropriate spaces in the table on the last page, and be sure to hand in the table.** Then convert the mean to radians, and calculate the distance to Object 1. Use the error propagation equations, with the standard error of the mean, to estimate the uncertainty in your distance for Object 1. Repeat these steps for Object 1: 5 m baseline, Object 2: 1 m baseline, and Object 2: 5 m baseline. **Write your values for distance and uncertainty in distance in the table on the last page, in meters.**

You may do these calculations in MATLAB. You may have to work with a partner from your group, due to the limited number of computers. Make an m-file which contains all of the necessary commands to reproduce your results. That is, if you gave your m-file to me, I should be able to run it and get the same results that you do. To really test who's reading these instructions, also include the name of your favorite sci-fi or fantasy character, in any medium. Do not tell anyone else that you read this. The m-file should contain the arrays with your original data, the calculations of the mean, standard deviation, and the standard error of the mean, the conversions to radians, the distance calculations, and the error propagations for each object and for each baseline. PRINT A COPY OF THIS M-FILE FOR EACH PARTNER AND TURN IT IN, ALONG WITH THE TABLE FROM THE LAST PAGE, AND YOUR ANSWERS TO THE QUESTIONS BELOW.

The relevant commands in MATLAB are mean(x) and std(x), where x is the array of your measurements (in inches). To find the standard error of the mean, just divide std(x) by sqrt(N), where N is the number of measurements. See the lab website (`http://www.astro.umd.edu/~cychen/MATLAB/ASTR121/labError/`) for more details on the MATLAB commands.

(2) The smallest angle you believe you can accurately measure corresponds to the accuracy of your instrument (in the first case, your eyes and the pentant). This value places limits on the distances to which you can measure.
a) The standard deviation of your measurements (not the SE) should also be an indication of how accurately you can measure with your instrument. How does it compare to your estimate of the smallest angle you can measure?
b) Calculate the distance (**using the Earth's orbit as a baseline**) to a fictional object which has a parallax of the smallest angle (converted to arcseconds, so you can use the simplified equation) you can accurately measure with your pentant. **Show your work, either in your answer to this question, or in the MATLAB code that you will turn in. If you do the calculation in MATLAB, make sure you either publish the m-file to display the results, or write the result in by hand. Please suppress any unnecessary output if you publish.**
c) Is this the largest or smallest distance you can measure?
d) What kinds of modifications would you make to the instrument to improve your accuracy?

(3) Consider the direct distance measurements you made in part (7).
a) How well do your direct measurements agree with the distances you found via parallax?
b) How did your measurements change as a result of a longer baseline?
c) Which baseline better matched the direct measurement? Which baseline would you expect to be a better match? (Hint: Consider how the uncertainty in your measurements compare to the measurements for each baseline themselves.)

(4) The best telescopes on Earth can measure shifts to 0.01 arcsec. What is the furthest distance to which we can measure on the best telescopes using parallax? **Show your work, either in your answer to this question, or in the MATLAB code that you will turn in. If you do the calculation in MATLAB, make sure you either publish the m-file to display the results, or write the result in by hand. Please suppress any unnecessary output if you publish.**

(5) If stars are distributed uniformly about the Sun, which baseline (Earth's or Jupiter's) would yield distances for more stars? Explain your answer.

| Your Data (IN INCHES) | | |
|---|---|---|
| Object | Baseline 1 m | Baseline 5 m |
| 1 | | |
| 1 | | |
| 1 | | |
| 1 | | |
| 1 | | |
| 1 | | |
| mean | | |
| st dev | | |
| st err | | |
| 2 | | |
| 2 | | |
| 2 | | |
| 2 | | |
| 2 | | |
| 2 | | |
| mean | | |
| st dev | | |
| st err | | |

| Directly Measured Distance (IN METERS) | |
|---|---|
| 1 | |
| 2 | |

| Your results (IN METERS) | | |
|---|---|---|
| Distance | | |
| Object | Baseline 1 m | Baseline 5 m |
| 1 | | |
| 2 | | |

| Uncertainty in Distance | | |
|---|---|---|
| Object | Baseline 1 m | Baseline 5 m |
| 1 | | |
| 2 | | |