# A121 Lab #10:
# Superluminal Motion of Quasar Jets

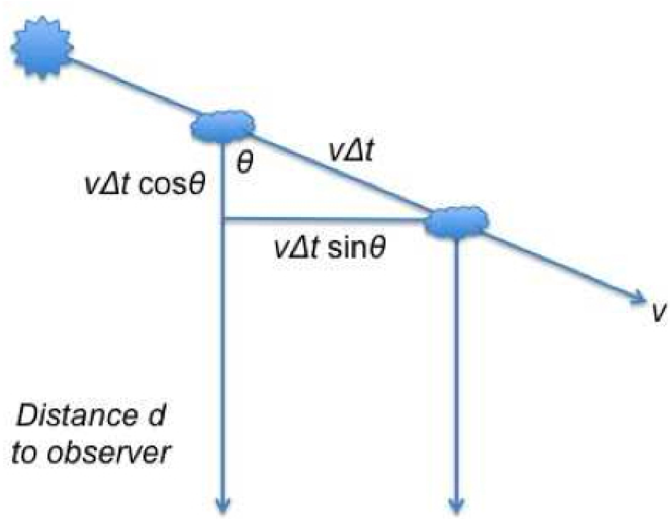
April 23, 2012
Due April 27, 2012

## 1 Objectives:

- Plot familiar and unfamiliar equations.
- Write and run functions (.m files).
- Explore parameter space via visualization.
- Learn simple 'for' loops.
- Measure superluminal motion and estimate the lower bound speed for bulk motion in the jet of 3C 279.

## 2 Theory

Consider the motion of the blob in the figure below.



At the first observation, the blob is at a distance $d$ from the observer and moving at an angle $\theta$ to the line of sight (where $\theta = 0$ degrees is motion completely along the line of sight, and $\theta = 90$ degrees is motion completely perpendicular to the line of sight). A second observation is taken after a length of time, $\Delta t$, has passed. In this amount of time, the blob has moved $v\Delta t$ along its path. From geometry, this means that the blob is now $v\Delta t \cos\theta$ closer to the observer than at the first observation. Thus it will take a time $t_1 = d/c$ for the light from the blob to reach the observer for the first observation, and a time $t_2 = (d - v\Delta t\cos\theta)/c + \Delta t$ for the light from the blob to reach the observer for the second observation. Meanwhile, in the time $t_2 - t_1$, the blob has moved across the sky a distance of $\ell = v\Delta t \sin\theta$, from the point of view of the observer. Therefore the observer sees an *apparent* motion across the sky of:

$$v_{app} = \frac{\ell}{t_2 - t_1}$$

Plugging in for $\ell$, $t_2$, and $t_1$ gives:

$$v_{app} = \frac{v\Delta t \sin\theta}{\frac{d - v\Delta t\cos\theta}{c} + \Delta t - \frac{d}{c}}$$

which simplifies to:

$$v_{app} = \frac{cv\Delta t \sin\theta}{\Delta t(c - v\cos\theta)}$$

resulting in the expression:

$$v_{app} = \frac{v\sin\theta}{1 - \beta\cos\theta}$$

where $\beta$ is defined as $v/c$. If we divide each side by $c$, we get our final expression in terms of the fraction of the speed of light, $\beta$, rather than $v$:

$$\beta_{app} = \frac{\beta\sin\theta}{1 - \beta\cos\theta}$$

One last thing we can do is determine what value of $\theta$ maximizes $\beta_{app}$. By taking the derivative of the right-hand-side of our $\beta_{app}$ equation and setting it equal to zero, we find:

$$\beta_{app,max} = \gamma\beta$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor. You can then solve this for $\beta$, which tells you the minimum true velocity ($v_{true} = \beta$*c) of the jet needed to produce the measured $\beta_{app}$. It is the minimum velocity because, if the actual angle $\theta$ of the jet is not the one that maximized $\beta_{app}$, then you need to have a higher true velocity to get the same $\beta_{app}$.

# 3 Procedure

## 3.1 Exploring the Parameter Space

1. Write a function m-file that will take two arguments, $\beta$ and $\theta$, and will calculate $\beta_{app}$. Write your function so that you can give it $\theta$ in degrees. Remember that MATLAB's trig functions expect their arguments to be in radians.

2. Test that your function works by checking for appropriate behavior at $\theta = 0$ degrees and $\theta = 90$ degrees, using a value of $\beta = .99$. If the jet is coming directly at us ($\theta = 0$ degrees), then there should be no apparent motion across the sky. If the jet is moving entirely perpendicular to our line of sight ($\theta = 90$ degrees), the apparent motion should be the same as the actual motion.

3. We don't necessarily know the angle $\theta$ that the jet makes to our line of sight, so we want to explore how $\beta_{app}$ depends on $\theta$ for a range of $\beta$. To do this:

   a) Start a blank m-file, and create an array for $\theta$ using linspace, with 200 values linearly spaced between 0 and 180 degrees. (From 90 to 180 degrees, we will be exploring what happens to the jet when it's moving away from us, instead of towards us.) **It is important to do all of these steps in an m-file, so that step d) will work.**

   b) Also create an array of $\beta$ values, using the values 0.5, 0.7, 0.9, 0.995, 0.999, and 0.9999 (that's 6 values).

   c) Also, for reasons which become clear in step d), create the following array:
   clrs = ['r','y','g','b','k','m'];
   You might recognize these values as the color options for **plot**...

   d) Now we want to plot $\beta_{app}$ (y-axis) vs $\theta$ (x-axis) for our 6 values of $\beta$. First, clear the figure space (**clf**) and turn **hold** on. We can then save a lot of typing by using a 'for' loop to do the plotting commands:

   ```
   for i= 1:6
       bA = betaA(beta(i),theta);
       plot(theta,bA,clrs(i))
   end
   ```

   assuming you named your function file betaA.m, your array of $\beta$ values beta, and your array of $\theta$ values theta.

   e) Turn off **hold**, and give your plot a title and axes labels. Also give it a legend, which labels the lines in the plot by their value of $\beta$.

   f) Create a second plot which shows the details of the plots for the smaller $\beta$ values by limiting the range of the y-axis to 0 to 20.

4. You should notice that there is an angle at which $\beta_{app}$ very strongly peaks, and this angle depends on $\beta$. Also notice that, for a given value of $\beta_{app}$, the further you are from the angle that maximizes $\beta_{app}$, the higher the $\beta$ needs to be in order to produce that $\beta_{app}$ value. For example, look at $\beta_{app}$ of about 10. This is roughly the $\beta_{app,max}$ for $\beta = 0.995$. The only other plotted curves that meet this value of $\beta_{app}$ are $\beta = 0.999$ and $\beta = 0.9999$, and they meet this value at an angle other than the angle at which their $\beta_{app,max}$ occurs. This is why the true velocity you calculate from $\beta_{app,max}$ is a minimum velocity for the jet.

## 3.2 Measuring Superluminal Motion

1. From Newtonian (and as it turns out, General Relativity gives the same answer) Cosmology we arrive at the equation

   $$v = H_0 D$$

   where $H_0 = 74.2 \pm 3.6$ km/s/Mpc.

   There is also a relation in cosmological physics between the redshift $z$ of light from distant objects and the velocity at which that object is receeding, the same velocity as in equation 2.

   $$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

Use these relations to find the distance in Mpc to 3C 279 if $z = 0.5362$.

2. Now, use this distance to find the scale between angle on the sky measured in arcseconds and distance at 3C 279 (i.e. find the physical diameter that corresponds to an angular diameter of 1 arcsecond at the distance of 3C 279). Your answer will have units pc/arcsec.

   Check your answer against that listed in the Nasa/IPAC Extragalactic Database (NED) if you have time by searching for the NED main page and searching for 3C 279 on that page. These values are listed under the "Quantities Derived From Redshift" section.

3. Now, study the handout image of the jet of 3C 279 showing its change over time. Use a ruler or other method to approximate the change in position of the center of the brightest knot between the top image and the second from the bottom image (i.e. ignore the bottom image), **measured in milliarcsec per second**.

   Note the scale bar at the bottom that tells you the conversion factor from a distance measured on the image to an angular measurement on the sky.

   Also note that there is a scale indicating time. To measure time, measure with your ruler how long 1 year is on the scale. Then measure with your ruler the distance on the page between the center of the quasar in the top image and the center of the quasar in the second from the bottom image, and convert this distance to years using your scale. Then convert to seconds. Combine this with your measurement of the movement of the knot in milliarcsec to get your change in position of the knot in milliarcsec per second.

4. Use the conversion from 3.2.2 and your result from 3.2.3 to find the apparent velocity of the knot in pc per second.

5. Divide your result from 3.2.4 by $c = 3 \times 10^8$ m per second to get $\beta_{app}$.

6. Now use the equation for $\beta_{app,max}$ and solve for $\beta$ to find the minimum true velocity ($v_{true}$) of the knot, where $v_{true} = \beta * c$. Leave your answer as a decimal times $c$, rather than actually multiplying by $c$ to get $v_{true}$ in m/s.

# 4 What you should turn in

You should turn in:

a) your code
b) your function file for $\beta_{app}$
c) your plot of $\beta_{app}$ vs $\theta$ for the 6 values of $\beta$, and your zoomed-in plot showing the detail of the lower $\beta$ values (it's OK this time that your lines will be indistinguishable when printed in black and white)
d) the answers to the following questions (show any work that is not included in your code):

1. What is the distance you found to 3C 279 in 3.2.1, **in Mpc and in pc**?

2. What is the conversion factor you found for 3.2.2, **in pc per arcsec**?

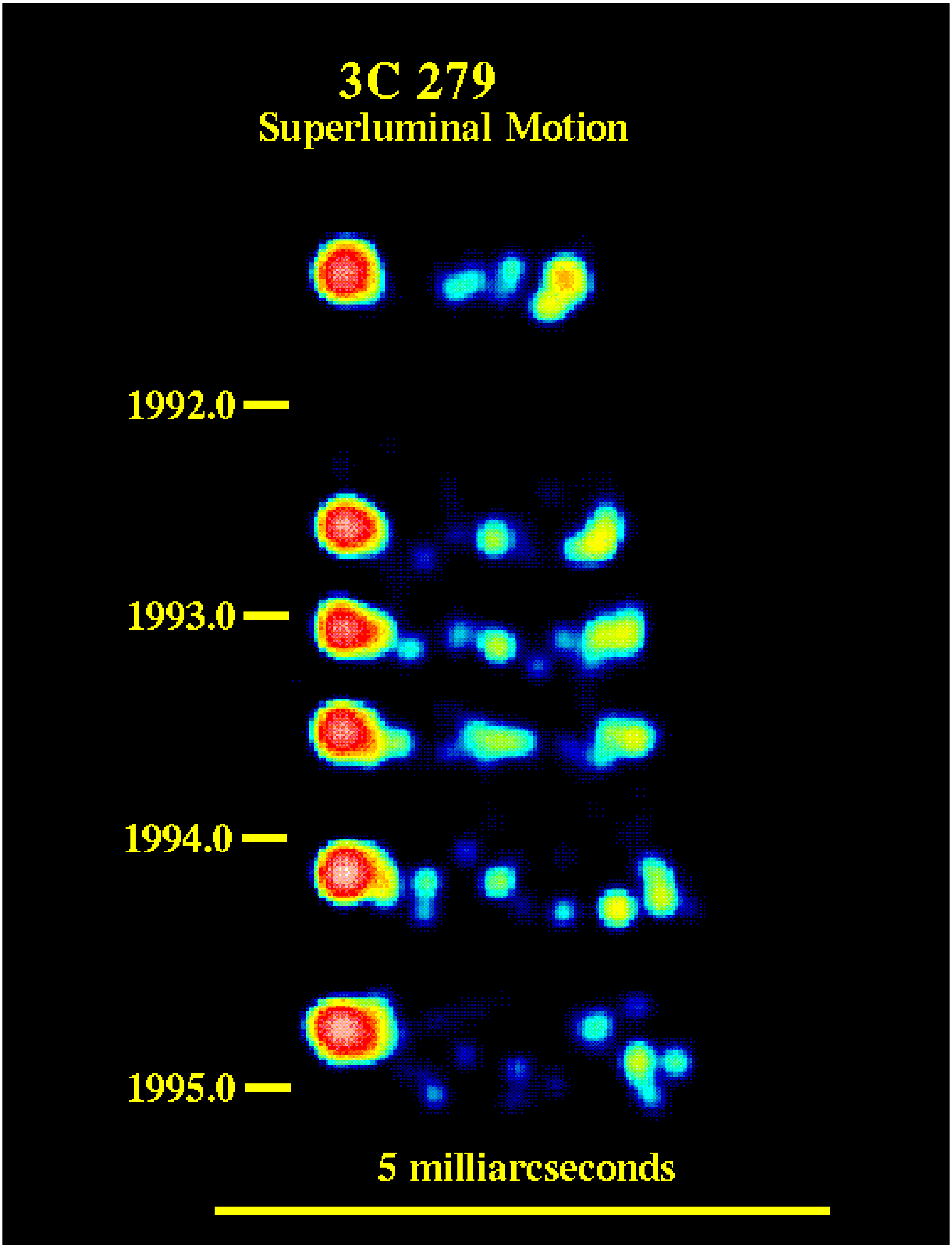
3. What is the motion of the knot from 3.2.3, **in milliarcsec per second and in arcsec per second**?

4. What is the motion of the knot from 3.2.4, **in pc per second and in m per second**?

5. What is the $\beta_{app}$ for 3C 279, from 3.2.5?

6. What is the lower limit on the true velocity $v_{true} = \beta * c$, from 3.2.6 (leave your answer as a decimal times $c$, rather than actually multiplying by $c$ to get $v_{true}$ in m/s)?

7. How can we increase our precision of the measurement of the transverse velocity of the jet knot?


3C 279
Superluminal Motion
1992.0
1993.0
1994.0
1995.0
5 milliarcseconds