A121: Lab 3
Binary Stars and Stellar Masses

February 13, 2012
Due February 17, 2012

1 Objectives:

- Continue learning functionality of MATLAB.
  - Read simple data files.
  - Visualize data.
  - Pull values from plots.
  - Fit a function to data.
- Use Spectroscopic measurements of a stellar absorption line to determine binary masses.

2 Introduction

Our sun appears to be a rarity in space. Approximately two-thirds of all solar-type field stars are members of binary systems, and recent studies suggest that virtually all stars begin life as members of multiple systems. Consequently, many of the stars you see at night are actually binaries, comprised of two stars gravitationally bound in orbit with one another. These binary systems are important astrophysical laboratories because they allow us to deduce the properties of the constituent stars more accurately than we can with single stars. The physics that governs how stars orbit one another was developed by Newton and Kepler over three hundred years ago, and can be summarized by the equation

$$P^2 = \frac{4\pi^2}{G(M_P + M_S)} a^3 \tag{1}$$

where P is the period of orbit, G is the gravitational constant, M_P and M_S are the masses of the primary and secondary stars respectively, and a is the sum of the semi-major axes of the two orbits about the center of mass, $a = a_P + a_S$ (i.e. the separation of the two stars). In mks units, $G = 6.67 \times 10^{-11}$ but these units are not the units of choice. If masses are measured in solar masses, distances in astronomical units, and periods in years, then the application of Newton's law to the Earth-Sun system gives $4\pi^2/G = 1$. The equation then simplifies to:

$$P^2_{\text{years}} = \frac{a^3_{\text{AU}}}{M_P + M_S} \tag{2}$$

where M_P and M_S are in solar masses ($M_\odot$).

Binary stars fall into several categories, depending on their observed properties: optical doubles, visual binaries, astrometric binaries, composite spectrum binaries, spectroscopic binaries, and eclipsing binaries (photometric binaries). Optical doubles merely appear to be next to each other on the sky, but they are not physically associated with each other. Visual binaries, on the other hand, appear next to each other on the sky, but they ARE gravitationally bound to one another. In an astrometric binary, only one of the two stars is bright enough to detect, but we can see the motion over time of that star around a fainter, unseen companion. In a composite spectrum binary, we cannot resolve the two stars in an image, but we can see hints in the spectrum of stars of two different spectral types. In a spectroscopic binary, we again cannot resolve the two stars in an image, but we can detect the motion of the stars in their mutual orbit by the Doppler shift of the spectral lines of one, if not both, of the stars. In an eclipsing binary, the orbit of the stars lines up with our line of sight, such that one star will pass between the other star and us, causing a dip in the overall light we detect from the system for the duration of the transit.

Here we focus on the case of spectroscopic binaries, in which the spectrum from the binary system exhibits a doublet of the same Hα absorption line. We assume here that the components of the binary in question follow circular orbits. This is not true for all binaries, but for the present system it is valid. This means that the orbital eccentricity is zero and that the orbital velocities are constant at all times. We also assume that the orbital plane coincides with our line of sight (i.e. it is not inclined to our line of sight).

3 Procedure

1. Create a folder in the A121 folder on the desktop using your (and your partner's) initials and the number 3, since this is lab 3.
2. Open MATLAB and **cd** into the folder you just created.
3. Download the seven data files at `http://www.astro.umd.edu/~cychen/MATLAB/ASTR121/labBinary/` and save them into your folder. Keep this site open, so that you can use the links for more information on some of the commands you will be using.
4. Type the following commands at the MATLAB command line to load the data in each of these files into MATLAB:
   >> load('binary1.dat')
   >> load('binary2.dat')
   etc.....
5. Note that these data files each contain an array of two columns: the first, binary1(:,1), signifying wavelength in angstroms and the second, binary(:,2), signifying a normalized flux.
6. Plot a few of these spectra to get a feel for what they're showing.
7. The major absorption feature in these plots is a doubling of the Hα line. Notice that there are two strong absorption features in each spectrum and they are of unequal depths.

The strength of the line depends on how much hydrogen is in the right state (neutral, atomic hydrogen with its electron in the n = 2 state) to absorb the Hα photon, which then depends on how hot the star is. Thus the strength of the line tells you which star you're looking at. Make sure you keep track of which measurement belongs to which star! For simplicity, rather than deciding which star is the primary and which is the secondary, let's just refer to them as the "shallow" star, and the "deep" star.

8. To measure the wavelength of each component of the Hα line, for each spectrum, first plot the normalized flux (y-axis) vs. the wavelength (x-axis). Recall that the **plot** command expects the x-axis values first, and then the y-axis values. Then use the zoom tool in the plot window to zoom in around Hα, so that you can see the two components in it. Now use the **ginput** command to find the wavelengths of the shallow and the deep absorption lines. Refer to the website above for more information on saving the wavelengths to a new array and on keeping the measurements for each star separate. For each spectrum, always click on the same absorption line first! It is ABSOLUTELY CRITICAL that you keep all of the shallow measurements in one array, and the deep measurements in a different array. It is ALSO CRITICAL that you proceed through the spectrum files in order from 1 to 7, so that the measurements will match up with the correct times that you will enter as an array later.

9. Use the equation

$$\frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}} = \frac{v}{c} \tag{3}$$

to determine the corresponding radial velocities of both components of the binary system for each spectrum. For Hα, the rest wavelength is $\lambda_{emitted} = 6562.79$ angstroms. The speed of light is $c = 2.99792 \times 10^5$ km/s.

10. The spectra, in order, were taken on the Julian Dates given in Table 1.

Table 1: Dates of Observations

Filename	Julian Date
binary1.dat	2441578.831
binary2.dat	2441579.581
binary3.dat	2441580.742
binary4.dat	2441581.943
binary5.dat	2441582.670
binary6.dat	2441582.982
binary7.dat	2441583.960

11. Plot the velocities for both stars vs the date, with both stars on the same plot. Use different symbols for each star. You'll need to create a new array to store the times. To do this, you would type
>> t = [78.831, 79.581, 80.742, 81.943, 82.670, 82.982, 83.960]
where I have subtracted the leading 2441500 from the Julian date. We're ultimately only interested in the difference between dates (the t-t_0 in the sine function), so the 2441500 isn't necessary to drag around.

12. Now you are going to fit a sine curve to each set of velocities, using the **fit** command. Refer to the website, under "Fitting model to data", to learn how to use this command. Note that the function you will using to fit assumes that the sine curves oscillate around zero, but your data points do not. You should have already noticed that spectrum 4 conveniently has the same measured velocity for both stars. This velocity is the velocity at which the sine curves of each star cross (i.e. both stars have zero *orbital* velocity along our line of sight). Subtract this velocity value from all of your velocities, to make your sine curves now oscillate about zero. Then the **fit** command with the sine function should work.

13. In the sine function $a1*sin(b1*x+c1)$, the amplitude $a1$ corresponds to the orbital velocity of the star. Since the fits likely did not give exactly the same period for both stars, take the mean of the results for $b1$ from both fits. The period is then related to the mean of $b1$, using the equation $P_{\text{days}} = (2\pi)/b1_{\text{mean}}$. Use our assumptions about this particular stellar binary system and the period and orbital velocities to calculate the semi-major axis of each orbit around the center of mass. Remember that the distance covered by each star in its orbit around the center of the mass is 2πr, where r is either a_{shallow} or a_{deep}. The definition of velocity, $v = d/t$ will also be useful. Then calculate the total separation between the stars, which is the semi-major axis $a = (a_{\text{shallow}} + a_{\text{deep}})$ for Newton's form of Kepler's third law from the introduction.

14. Convert your semi-major axis a from km to AU and your period P from days to years, and then use the simplified (units of solar mass, years, AU) version of Equation 1 to calculate the combined mass.

15. Calculate the individual masses of the two components of the binary, in solar masses ($M_{\odot}$). Remember, from the definition of center of mass, $a_S M_S = a_P M_P$.

4 Questions

Each partner turn in a plot with the data sets for both stars as well as your best fit for a sine curve for each star all plotted together. Be sure to label your axes and give the plot a title. **Make sure the plot uses different symbols for each star and different line styles for the best fit curves.** You should each also turn in a hardcopy of an m-file that shows all of your calculations, as well as the commands to do the sine curve fits and to create the plot. Since you will be getting the wavelengths from **ginput**, you will have to create the 2 arrays in your m-file that have the wavelength values that **ginput** gives you. Be sure to record the wavelengths to a tenth of an angstrom. (The idea here is that anyone should be able to run your m-file and reproduce your sine curve fits and plots.) You should also **clearly note** somewhere, whether in the title of the plot, in the m-file, or on the paper with your questions, the parameters you used for your sine curves. Then answer the following questions:

(1) If the orbital plane of the two stars was inclined to our line of sight, how would this affect the masses we measure? Explain. **Hint:** Remember that **radial** velocity is the motion directly towards or away from us. How would the inclination affect what velocity we measure? How would that then affect the calculations to find the masses?

(2) For what type of binary system (from the list in the introduction) can we be guaranteed that the orbits are not inclined to our line of sight, and why?

(3) What would the spectrum of a triple system look like? Sketch or explain in words.

(4) At what velocity (i.e. what is the value) do the sine curves cross? What does this velocity represent?

* Note that this lab has borrowed heavily from Dr. Christopher Palma's Lab 4 of Astro 293, spring 2003. http://www.astro.psu.edu/ cpalma/astro293/