Delaunay Triangulation

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March 13, 2017

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Why do we choose Delaunay Triangulation?

– In the paper named "***Action Selection for Hammer Shots in Curling***"

– "This paper makes two main contributions. The first is to adapt Surovik and Scheeres [2015]'s non-convex optimization method to our problem. They use Delaunay triangulation on a set of sampled points to discretize the continuous action space and focus subsequent sampling in regions that appear promising. Our contribution is to add a final step, in which a shot is selected by treating the most promising regions as "arms" in a multi-armed bandit problem. We call our method Delaunay Sampling (DS)."

Why do we choose Delaunay Triangulation?

– As iterations increases, refining the high-valued regions, and we will choose only the high-value regions to be used in the selection stage of the algorithm.

– The advantages of the algorithm:

- Maximize the minimum angle of all the angles of the triangles in the triangulation. So it is the most closest to the regularized triangulation.
- Except any four points on a circle, Delaunay Triangulation is only one for every determinate set of points.

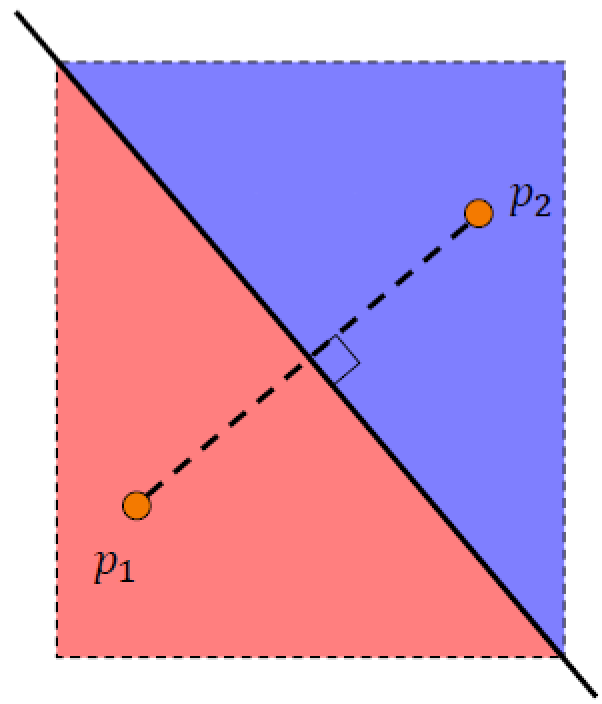
The principle of the Delaunay Triangulation.

- We consider the problem of triangulating a set of points in the plane.
- Using Voronoi Diagrams to get the Delaunay Triangulation.
- Let's see the Voronoi Diagrams first:
- Given points $P = \{p_1, \ldots, p_n\}$, the Voronoi region of point p_i , $V(p_i)$ is the set of points at least as close to p_i as to any other point in P :

$$V(p_i) = \left\{x \middle| |p_i - x| \leq |p_j - x| \; \forall 1 \leq j \leq n\right\}$$

The principle of the Delaunay Triangulation.

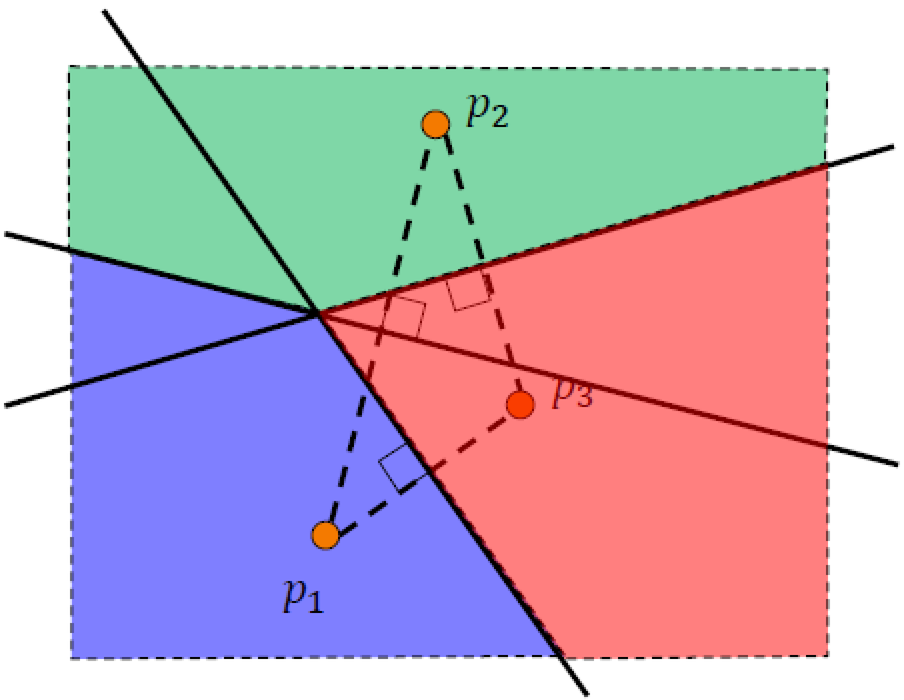
- The set of points with more than one nearest neighbor in P is the Voronoi Diagram of P. And the point P is called the ***sites*** of the Voronoi Diagram.
- For example:
- **<u>2 points:</u>**
- When $P = \{p_1, p_2\}$, the regions are defined by the perpendicular bisector.



The principle of the Delaunay Triangulation.

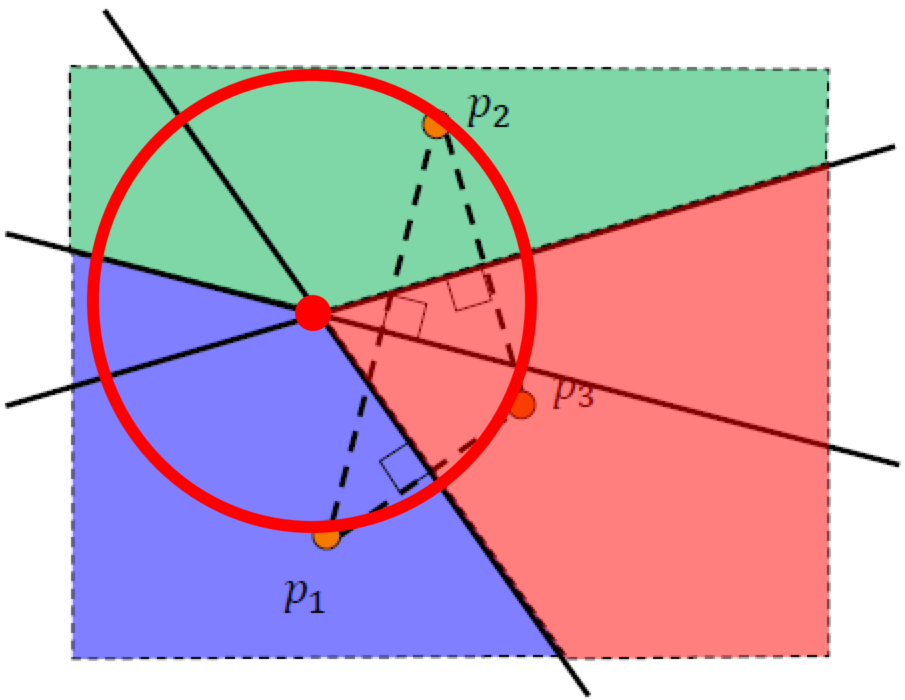
– **<u>3 points:</u>**

– When $P = \{p_1, p_2, p_3\}$, the regions are defined by the three perpendicular bisectors:



The principle of the Delaunay Triangulation.

- **<u>3 points:</u>**
- The three bisectors intersect at a point.
- The intersection can be outside the triangle.
- The intersection is the center of the circle passing through the three points.

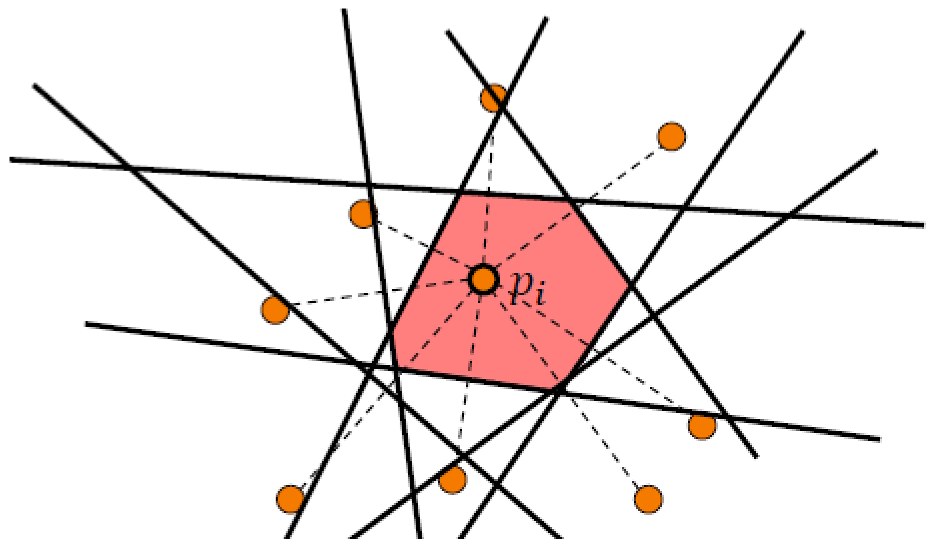


The principle of the Delaunay Triangulation.

– **More generally:**

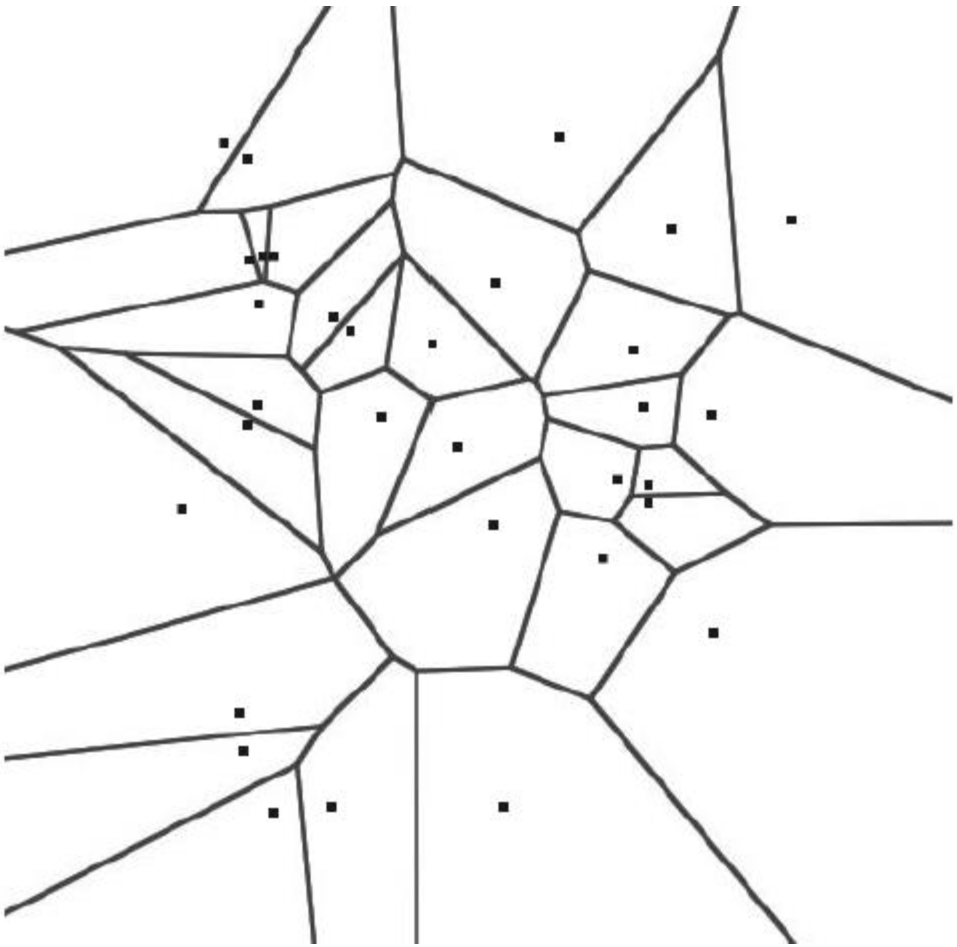
– The Voronoi region associated to point p_i is the intersection of the half-spaces defined by the perpendicular bisectors:

– Voronoi regions are convex polygons.



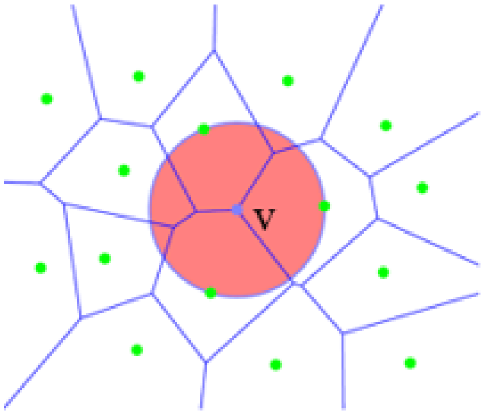
The principle of the Delaunay Triangulation.

- Voronoi regions are in 1-to-1 correspondence with points.
- Most Voronoi vertices have valence 3.
- Voronoi faces can be unbounded.



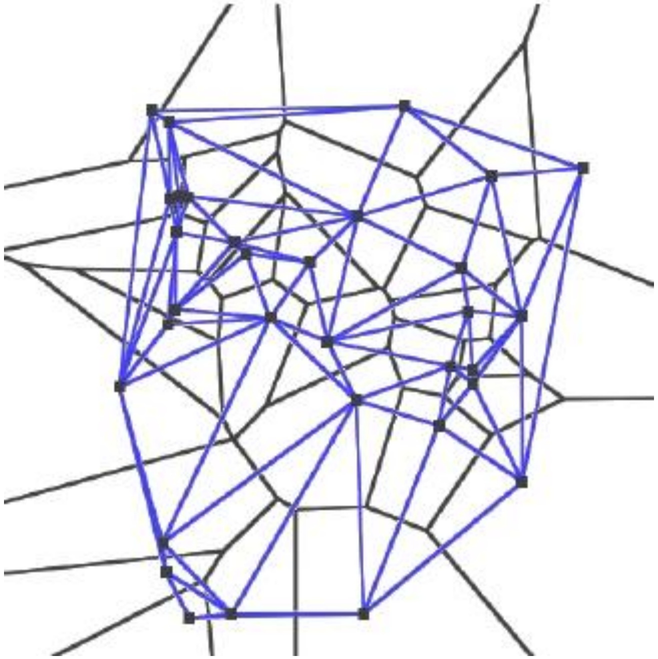
The principle of the Delaunay Triangulation.

- Now we get the Voronoi Diagrams, then let's see the definition of **Delaunay Triangulation**.
- If v is a point at the junction of $V(p_1)$,..., $V(p_k)$, with $k \geq 3$, then v is the center of a circle, $C(v)$, with p_1,...,p_k on the boundary.
- The most important property of the Delaunay Triangulation is the interior of $C(v)$ contains no points.



The principle of the Delaunay Triangulation.

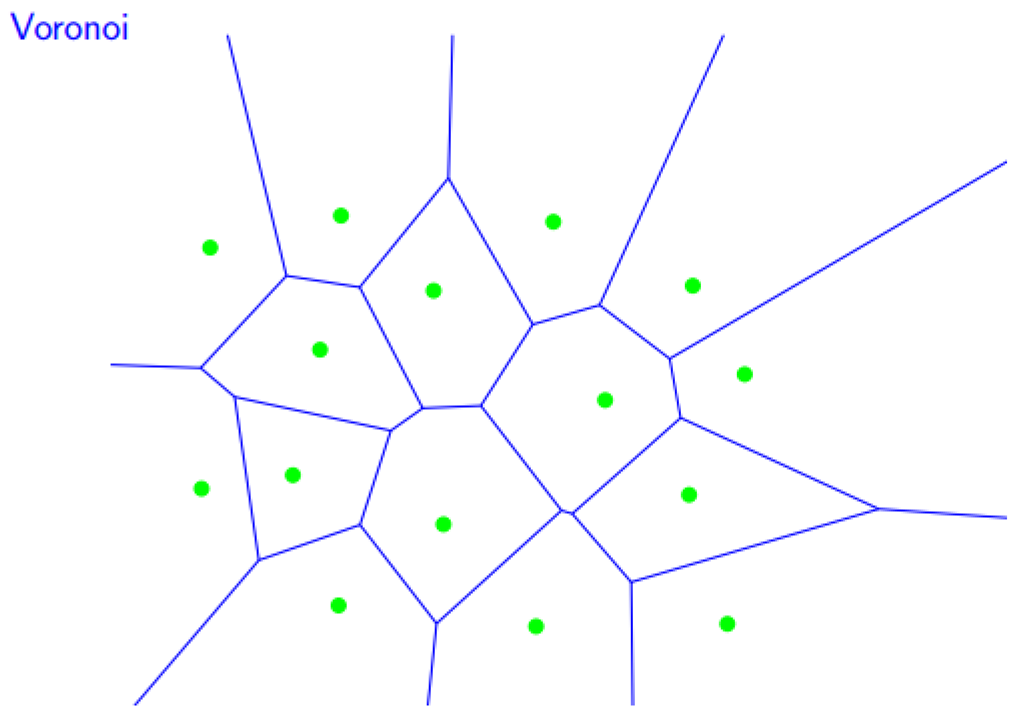
- And for every Voronoi Diagrams and Delaunay Triangulation are one-to-one corresponding. In other words, for every Voronoi Diagrams, there are only one Delaunay Triangulation.
- Maximize the minimum angle of all the angles of the triangles in the triangulation. So it is the most closest to the regularized triangulation.



The principle of the Delaunay Triangulation.

– After make sure the definition of Delaunay Triangulation, let's see how to generate it by Voronoi Diagrams.

– Here is a Voronoi Diagram.



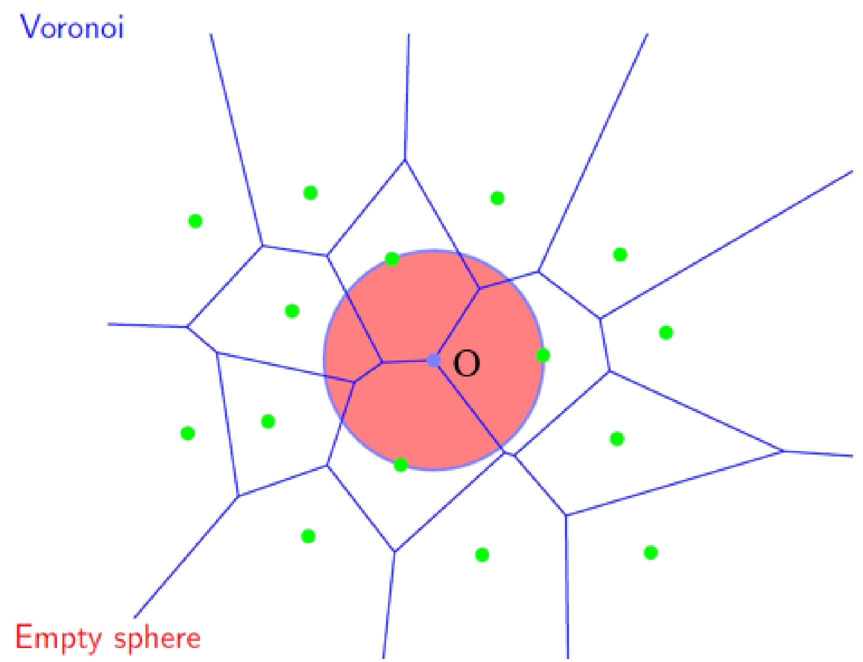
The principle of the Delaunay Triangulation.

- Step 1:
- For the node O(Voronoi vertices),

  there are 3 valence as usual.
- Selecting the sites of the 3

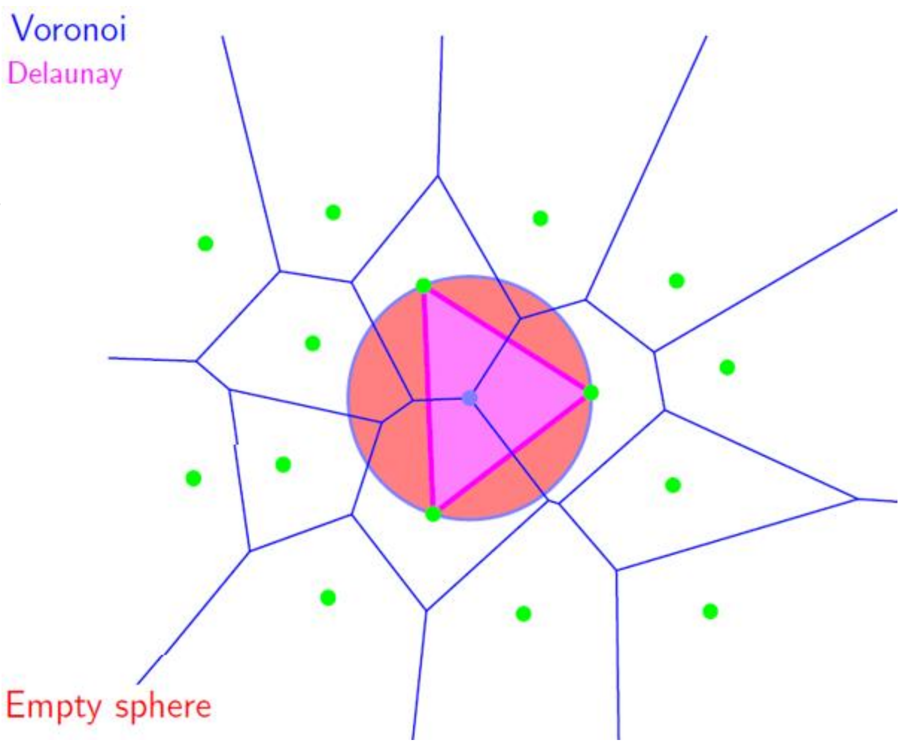
  valence's area to draw a circle.

  and O is not the center as usual



The principle of the Delaunay Triangulation.

– Connecting these three points, this triangle is the Delaunay Triangulation. There are no node in the circle.



The principle of the Delaunay Triangulation.

For example,

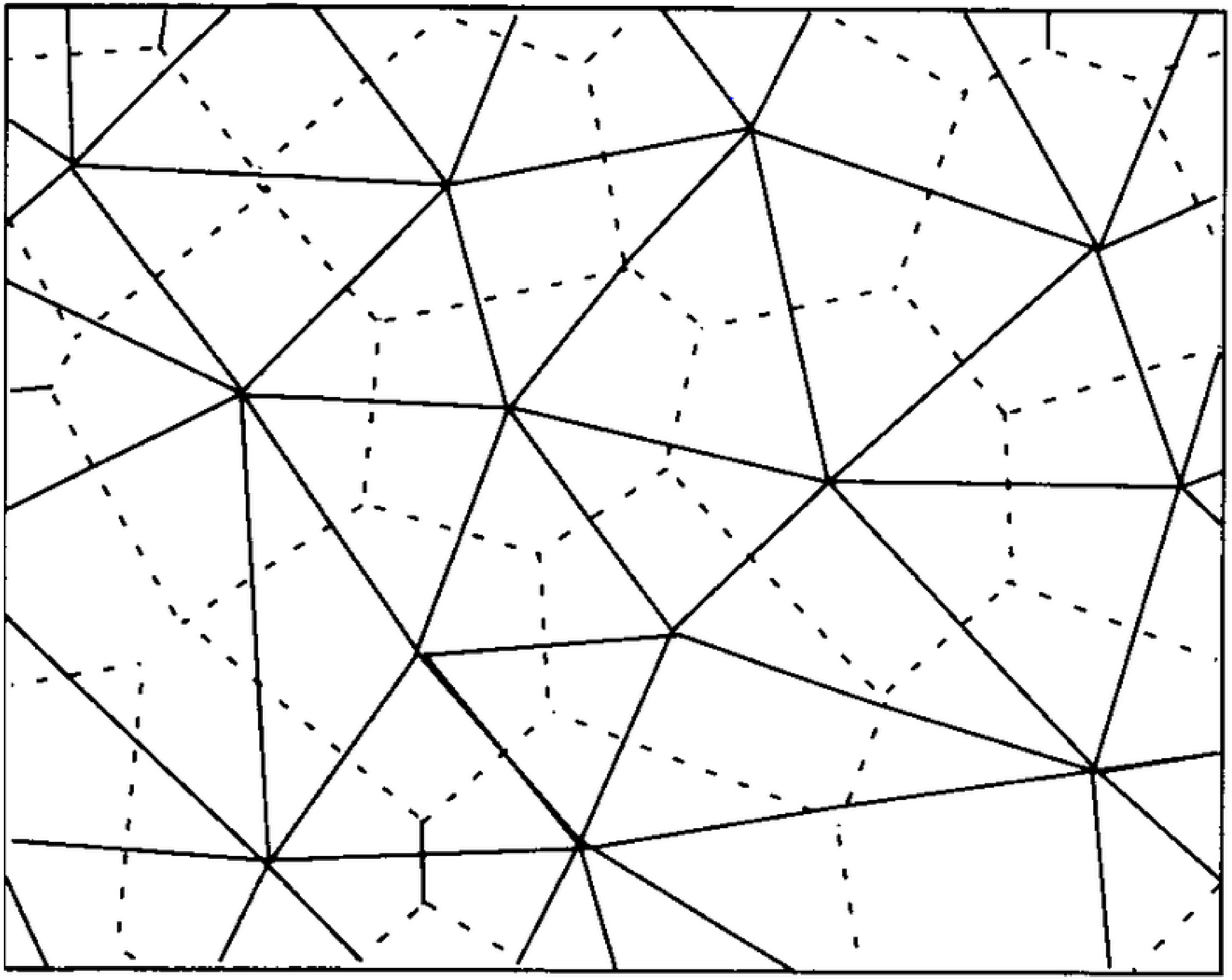
the full line shows the

Delaunay Triangulation,

and the imaginary line

shows the Vorinoi

Diagram.



Thank you!